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Large Scale Inversion using a Discontinuous Galerkin Method for Geophysics

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Sandia National Laboratories

Albuquerque, New Mexico

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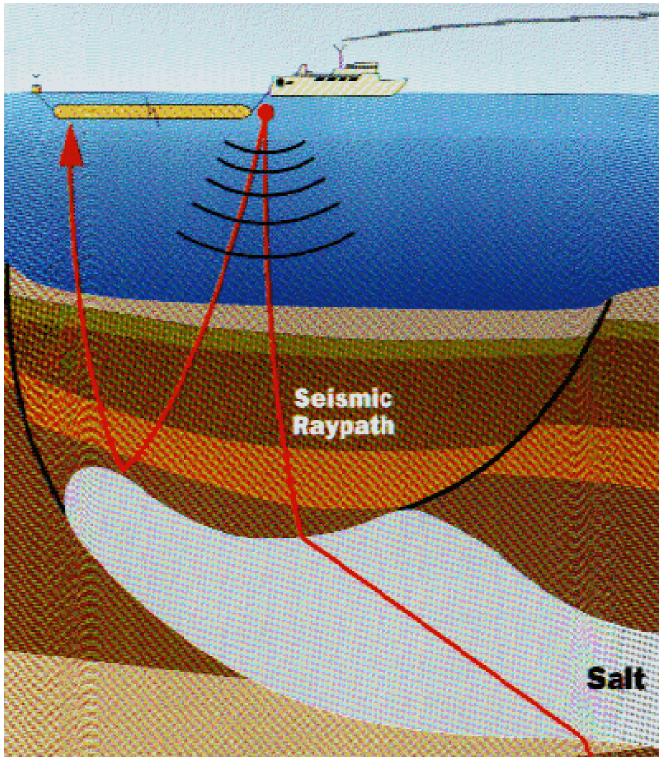
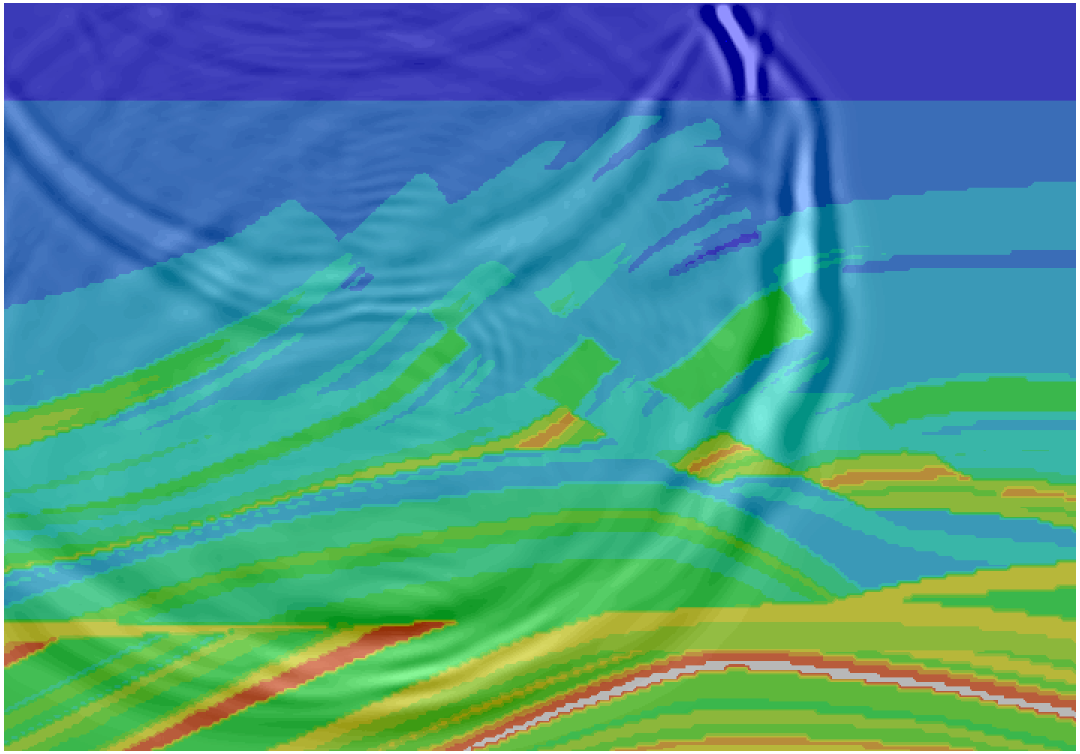
Geophysical Inversion

Seismic Experiment



Marmousi2 - Acoustic



Goal: determine material properties of the subsurface

Challenges and Strategies

Challenges

- complex media
- large inversion space
- multi-experiments
- Ill-posedness
- multiple minima
- non-linear
- computationally expensive
- modeling approximations
- measurement noise

Strategies

- discontinuous Galerkin
- variable media representation
- phase encoding
- trust region and line search
- second order algorithm
- different parameterizations
- parallel
- Griewank restart

Outline

- **Discontinuous Galerkin discretization**
  - formulation
  - fluxes
  - numerical examples

- **Inversion**
  - optimization formulation
  - phase encoding
  - parameterization
  - numerical examples

- **Conclusions and Future Work**

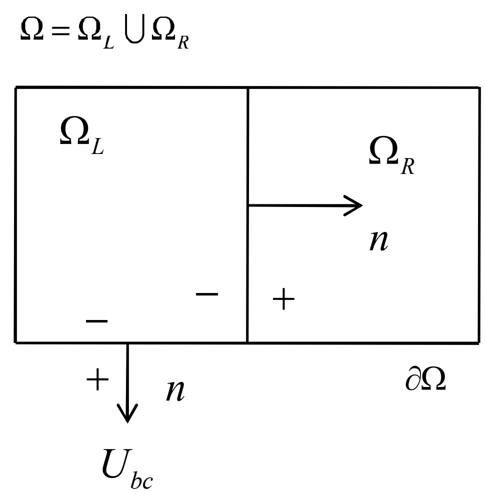
Discontinuous Galerkin Method

Start with strong form of acoustic equations:

$$\mathbf{U}_{,t} + \mathbf{A}_i \mathbf{U}_{,i} = \mathbf{S}$$

$$\mathbf{U}(\mathbf{x},0) = \mathbf{U}_0(\mathbf{x}), \quad \text{at t=0}$$

defined on Ω with appropriate boundary conditions on $\partial\Omega$.



Define the following in 3D:

$$U = \begin{bmatrix} p \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad A_i = \begin{bmatrix} 0 & \kappa\delta_{1i} & \kappa\delta_{2i} & \kappa\delta_{3i} \\ \nu\delta_{1i} & 0 & 0 & 0 \\ \nu\delta_{2i} & 0 & 0 & 0 \\ \nu\delta_{3i} & 0 & 0 & 0 \end{bmatrix} \qquad S = \begin{pmatrix} m^s_{,t} \\ f + m^a_{,j} \end{pmatrix}$$

p - pressure
u_i - velocity component
κ - ρc^2
ρ - density
c - wave speed
ν - $1/\rho$
S - source term
m - moment tensor
f - force vector

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Discontinuous Galerkin Method

$$\sum_{e=1}^{e=N_{el}} \int_{\Omega_e} \mathbf{W}^T \left(\mathbf{U}_{,t} + \mathbf{A}_i \mathbf{U}_{,i}\right) d\Omega + \sum_{e=1}^{e=N_{el}} \int_{\partial\Omega_e} \mathbf{W}^T \left(\hat{\mathbf{F}}_n\left(\mathbf{U}^-, \mathbf{U}^+\right) - \mathbf{F}_n\left(\mathbf{U}^-\right)\right) d\partial\Omega = \sum_{e=1}^{e=N_{el}} \int_{\partial\Omega_e} \mathbf{W}^T \mathbf{S}\, d\Omega \quad \forall \mathbf{W} \in V$$

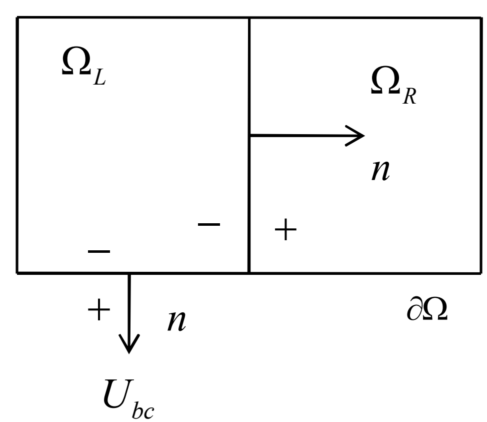
where $\mathbf{F}_n\left(\mathbf{U}^-\right)$ is the real flux and $\hat{\mathbf{F}}_n\left(\mathbf{U}^-, \mathbf{U}^+\right)$ is the numerical flux function which is designed to add stability for under-resolved wave fields.

Lax Friedrich flux:

$$\tilde{\mathbf{F}}_i = \frac{1}{2}(\mathbf{A}^L \mathbf{U}^L \cdot n^l + \mathbf{A}^L \mathbf{U}^R \cdot n^l + \lambda(U^l + U^R))$$

where λ represents the maximum eigenvalue of A

$\Omega = \Omega_L \cup \Omega_R$



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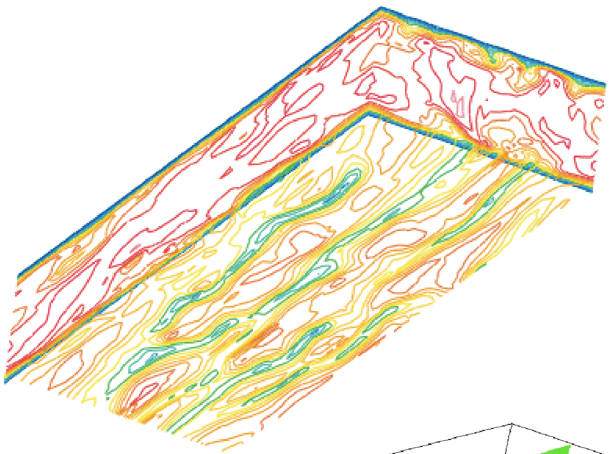
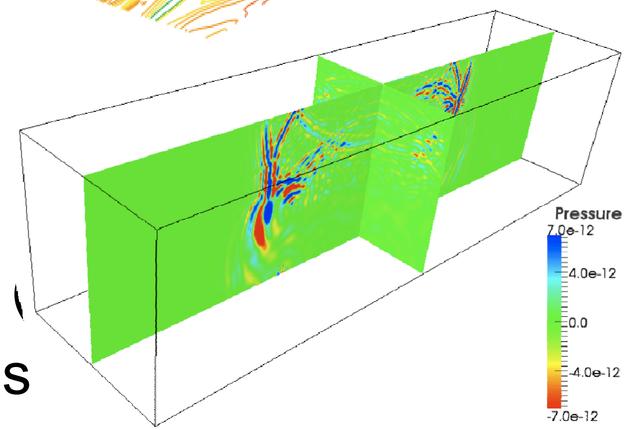
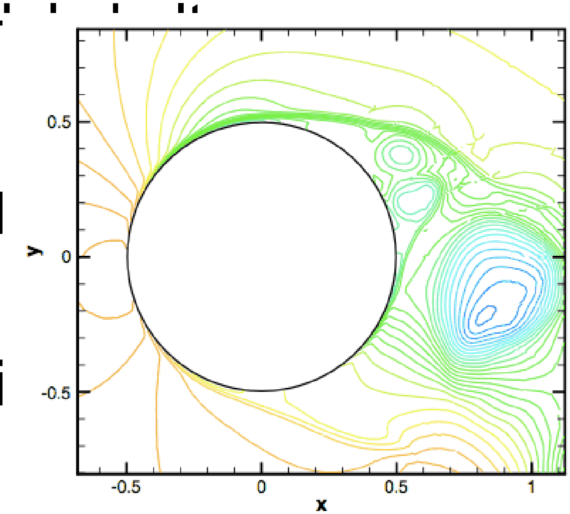
DGM Toolkit

- High-order on unstructured meshes
  - Line, Quad, Tri, Hex elements
- Supports local, *p*-refinement
- Object-oriented software design
- Physics independent: examples for
  - Compressible Euler & Navier-Stokes
  - Incompressible Euler & Navier-Stokes
  - Advection-diffusion, Burgers, Darcy, H
- Designed for adjoint-based optimizati
  - Steady-state and transient with check
- MPI with MPI-IO
- Version 0.0 released open-source (Ri
- Version 1.0 on the way…

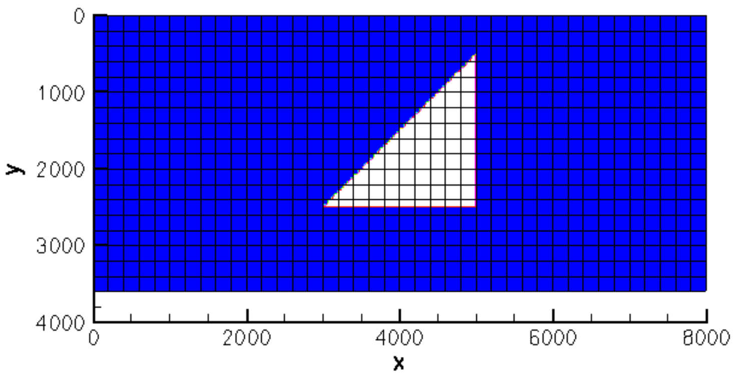


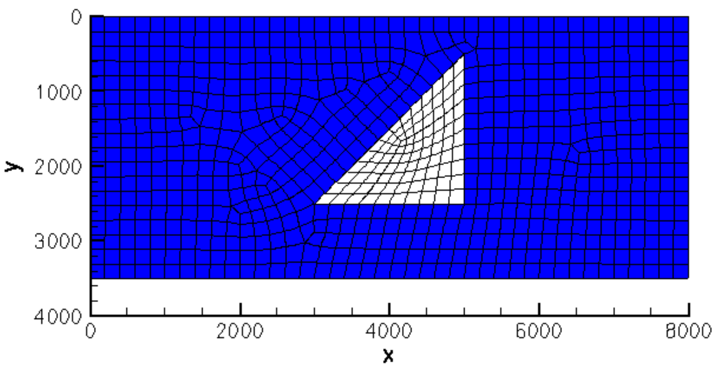


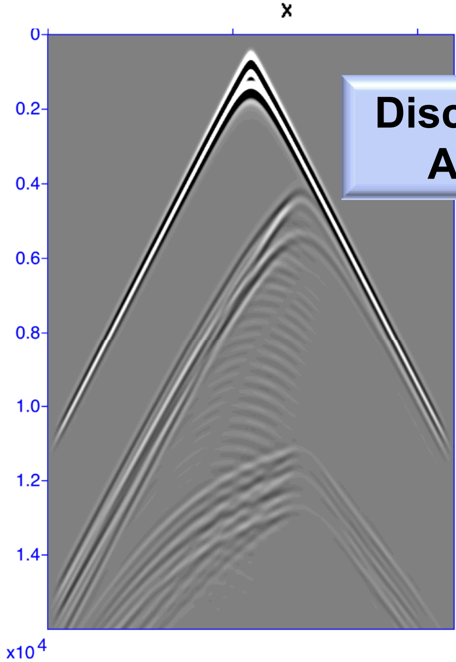


Discretization Challenges

Traditional structured meshes have difficulty capturing geological features accurately

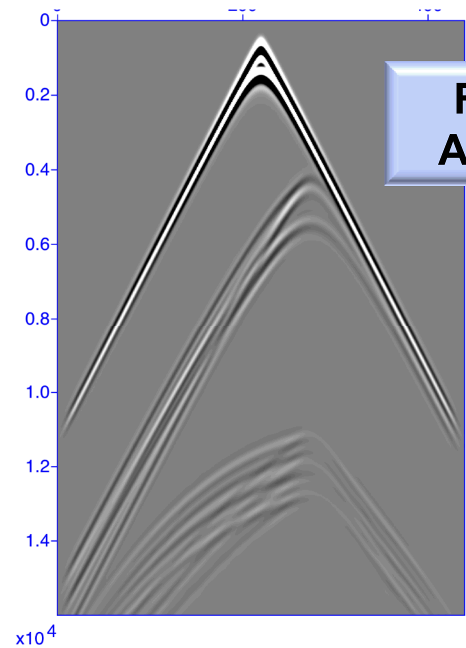






Discretization Artifacts

Triangle: p=10 uniform mesh



Free of Artifacts

Triangle: p=10 unstructured

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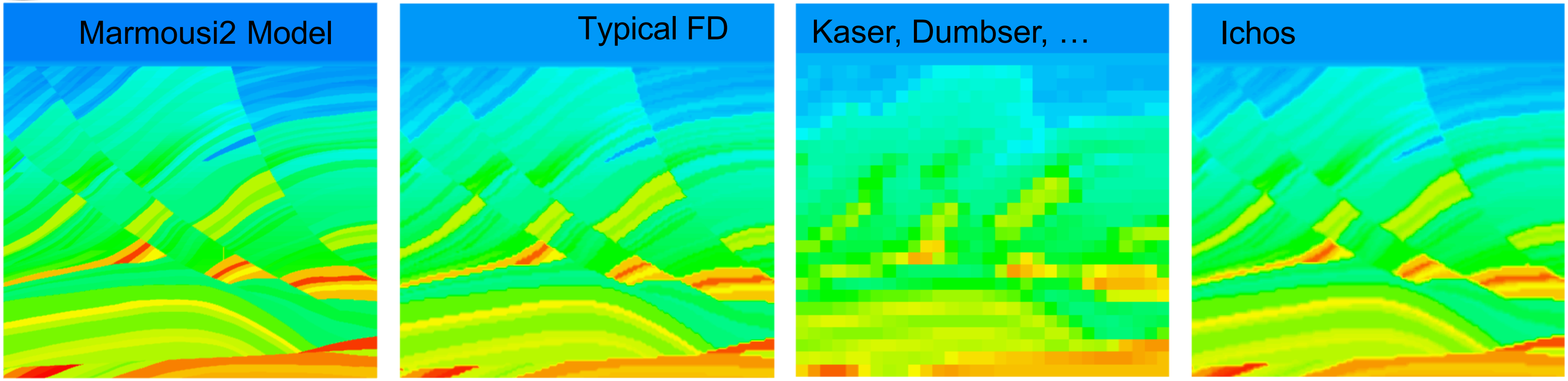
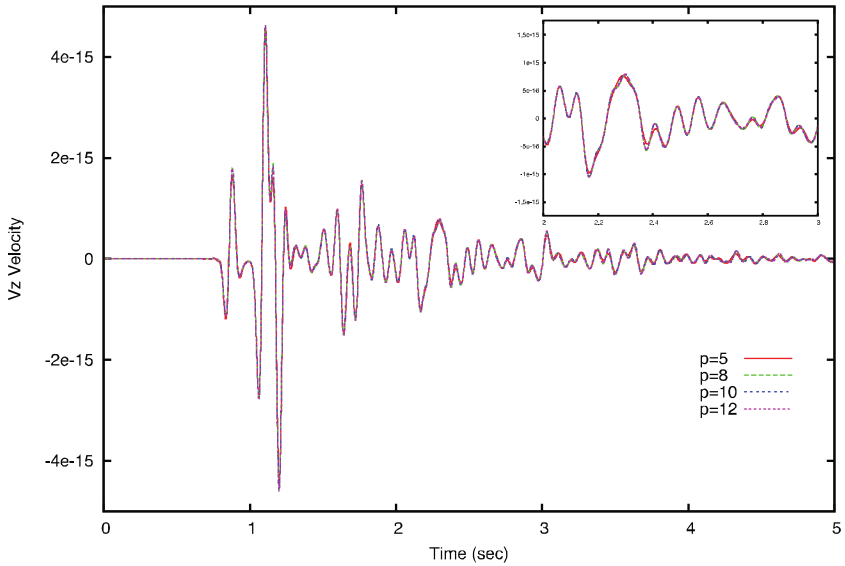
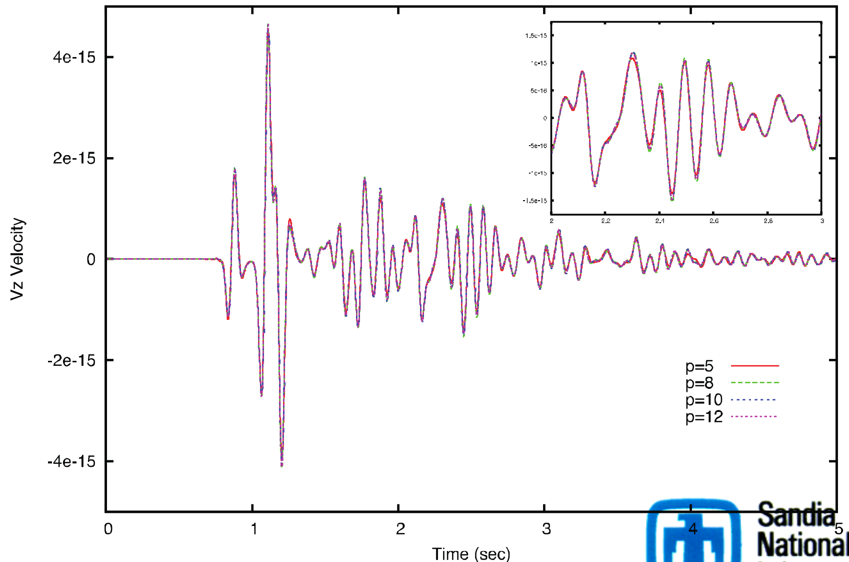
Better Media Representation



h = 1.25m | h = 20m | h = 100m, Mp = 0 | h = 100m, Mp = 8



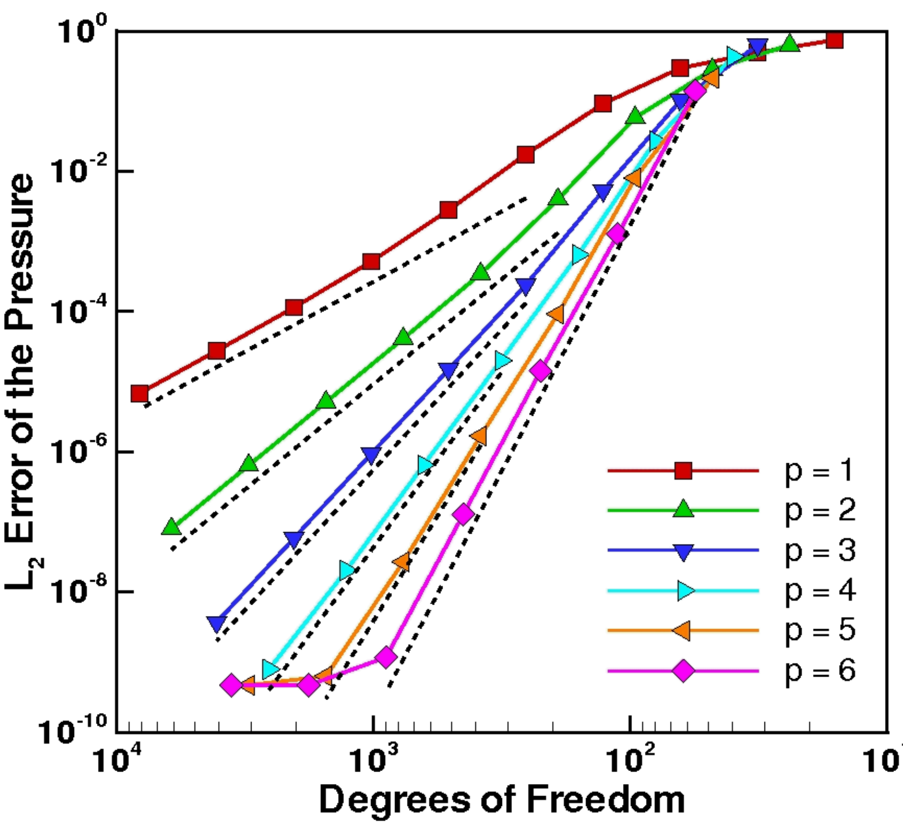
h = 100m, Mp = 0



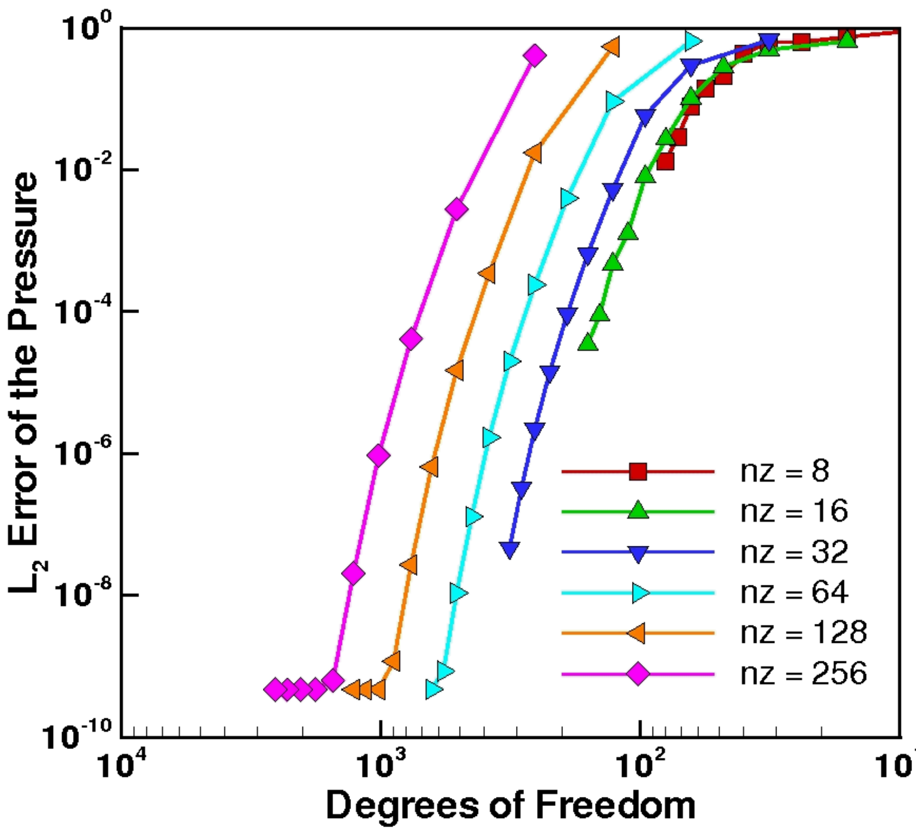
h = 100m, Mp = 8

Code Verification

Mesh Refinement



Polynomial Refinement



See: Ober, Collis, van Bloemen Waanders, Marcinkovich, SEG 2009.

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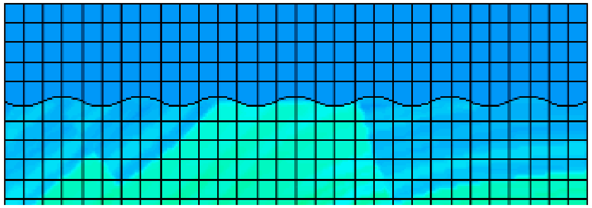
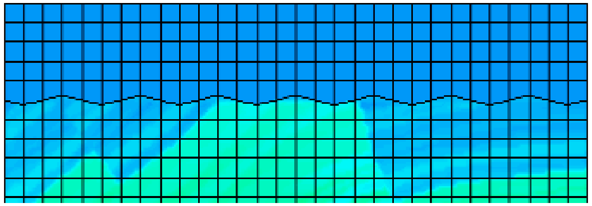
Topology Capturing

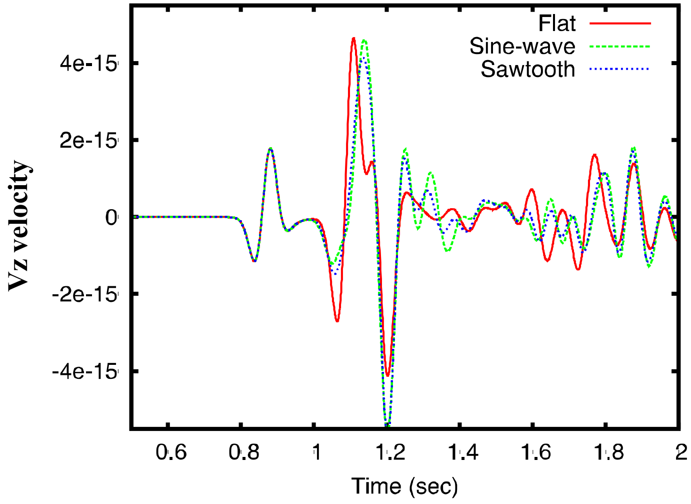
- For example:
  - Ocean floor
  - Faults
  - Salt structures
  - Even ocean waves…
- Elastic and Acoustic

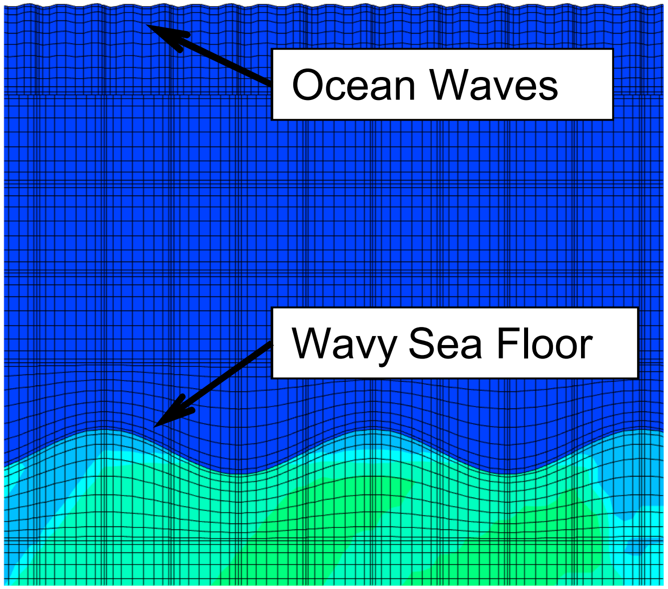
Sinusoidal Ocean Bottom



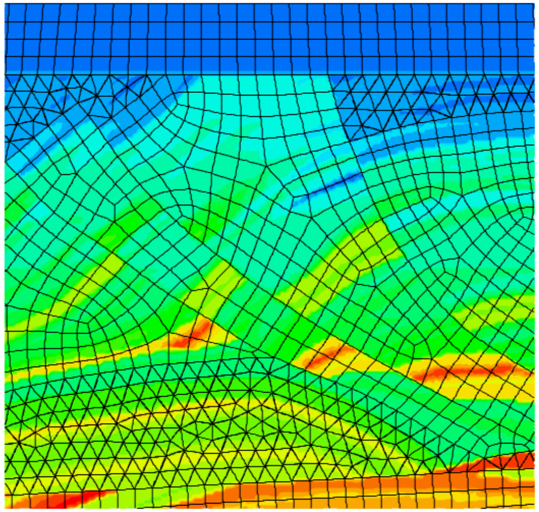
Sawtooth Ocean Bottom



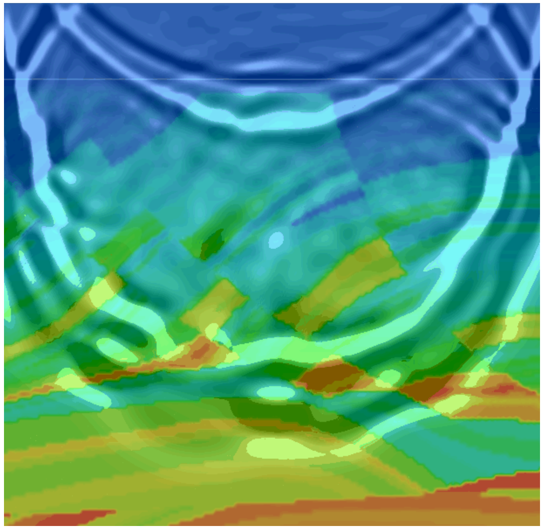




Hybrid Mesh, h = 100 m

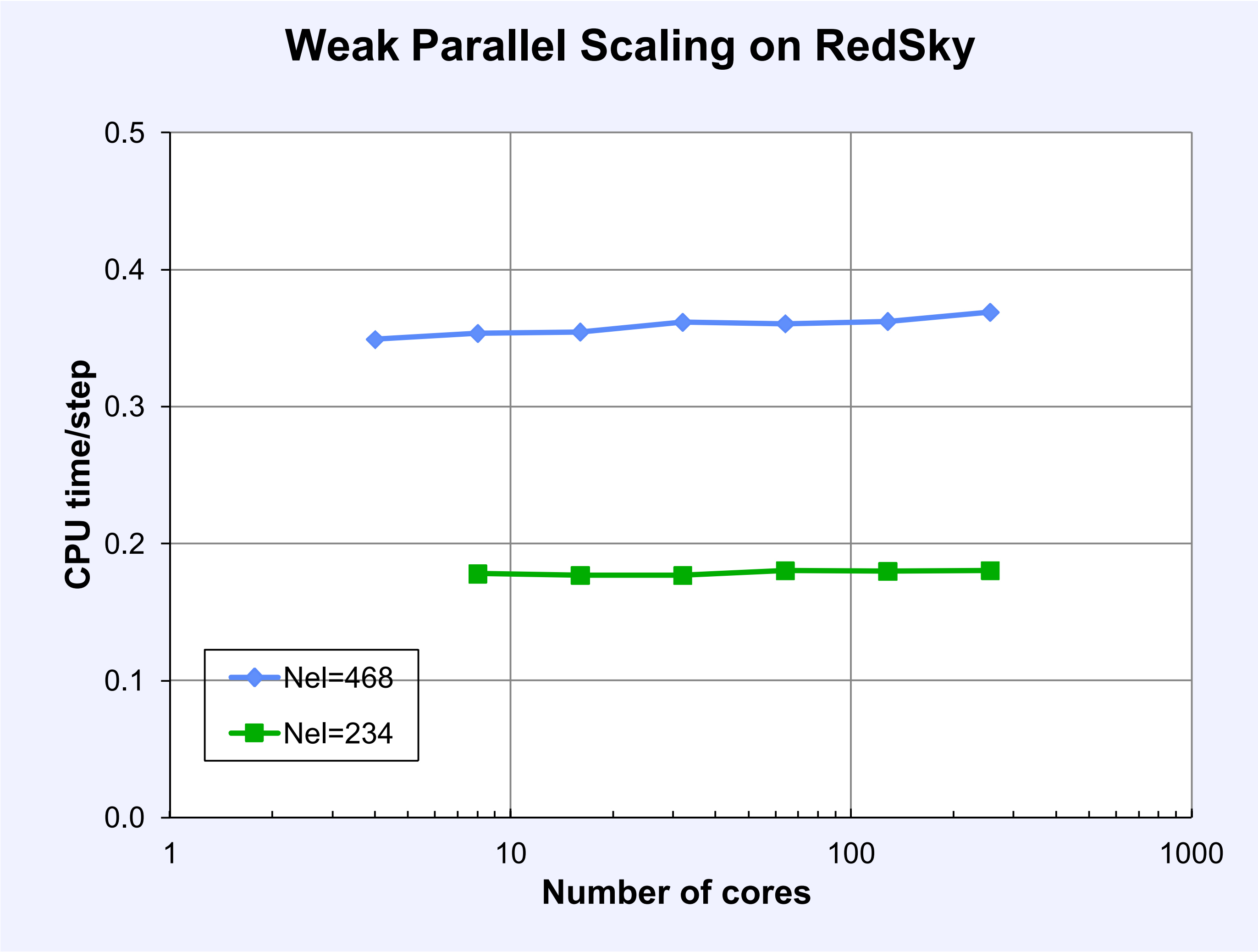


Pressure Wavefield, t=1.25 s



See: Smith, Collis, Ober, Overfelt, Schwaiger, SEG 2010.

Ichos Performance



Parallel efficiency = 84%

Inversion Formulation
Sequential

$$\min_{\kappa} \; \frac{1}{2}\int_0^T\int_\Omega (U(x,t)-\tilde{U}(x,t))^2\,\delta(x-x^*)\,dx\,dt+\frac{1}{2}\int_\Omega R^2\,d\Omega$$

$$s.t. \quad U_{,t} + A_i U_{,i} = S \quad \text{in } \Omega \times (0,\mathrm{T}]$$

$$U(x,0) = 0 \quad \text{for } x \in \Omega$$

where

$$U = \begin{bmatrix} p \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad A_i = \begin{bmatrix} 0 & \kappa\delta_{1i} & \kappa\delta_{2i} & \kappa\delta_{3i} \\ \nu\delta_{1i} & 0 & 0 & 0 \\ \nu\delta_{2i} & 0 & 0 & 0 \\ \nu\delta_{3i} & 0 & 0 & 0 \end{bmatrix} \qquad S = \begin{pmatrix} m^s_{,t} \\ f + m^a_{,j} \end{pmatrix}$$

p - simulated pressure

$\tilde{p}$ - measured pressure

δ - Direc Delta function

R - regularization

Solution Strategy

- **Forward**

$$U_{,t} + A_i U_{,i} = S \quad \text{in } \Omega \times (0,\text{T}]$$

- **Adjoint**

$$-\lambda_{,t} - A_i \lambda_{,i} + (U - U^*)\delta(x - \tilde{x}) = 0 \quad \text{in } \Omega \times (\text{T},0]$$

- **Gradient**

$$R + U_{,i}\lambda = 0 \;\; \text{in } \Omega \text{ x } (0, T]$$

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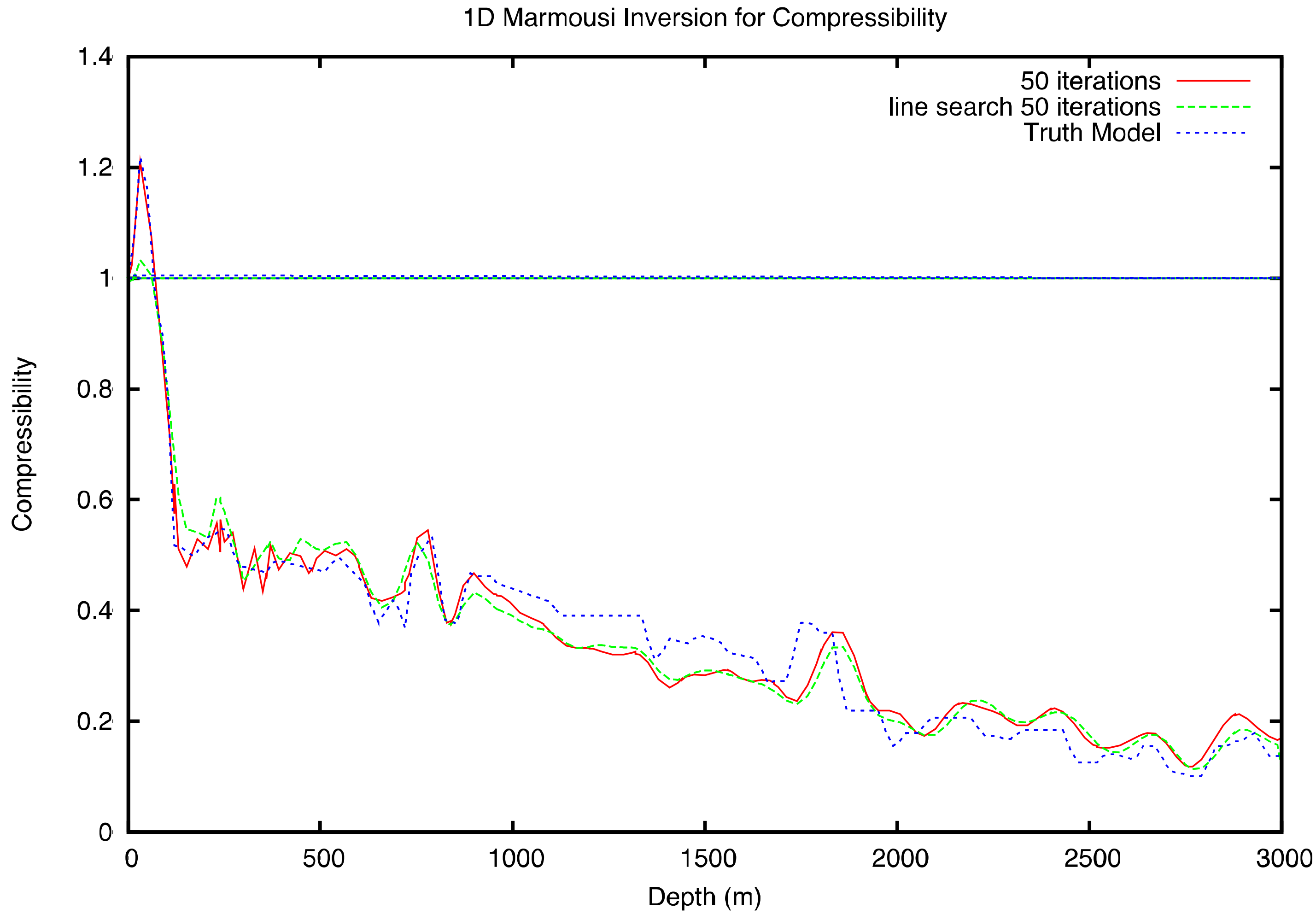
Implementation Details

- Adjoint time integration - RK4
- NonLinear Conjugate Gradient
- Brent Line search
- Trust Region with SR1, BFGS
- Phase encoding
- Griewank restart
- Different parameterizations
- Bounding
- Physics: acoustic, elastic

Sequential Numerical Result 1D
Trust region versus line search



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Inversion Formulation
Multi-Experiment

$$\min_{\kappa} \frac{1}{2}\sum_{r=1}^{N_r} \int_0^T \int_\Omega \xi_r(x) \left(p(x,t) - \sum_{s=1}^{N_s} \omega_s \tilde{p}(x,t) \right)^2 dxdt + \int_\Omega R d\Omega$$

$$s.t. \quad U_{,t} + A_{,i} U_{,i} = S_s \quad \text{in } \Omega \times (0,\mathrm{T}]$$

$$U(x,0) = 0 \quad \text{for } x \in \Omega$$

where

N_r - number of receivers

p - simulated pressure

$\tilde{p}$ - measured pressure

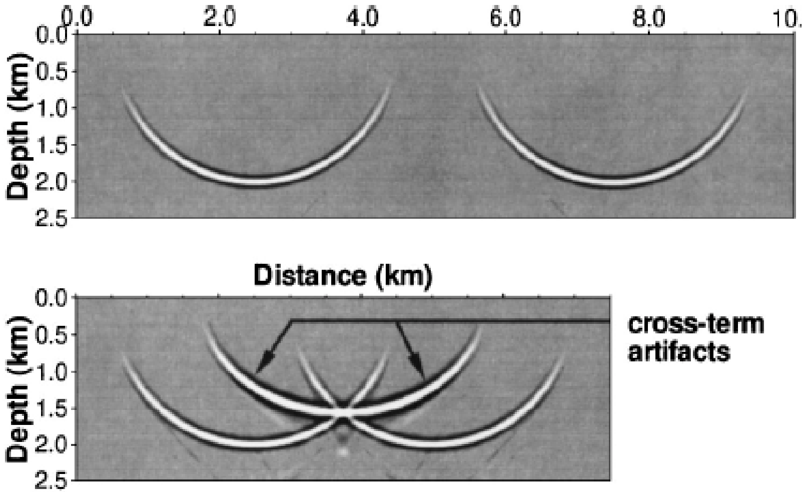
ω_s - encoding operator

R - regularization

ξ_r, ξ_s - spatial kernel

Phase Encoding

- Romero et al. "Phase encoding of shot records in prestack migration", Geophysics 2000
- Krebs et al. "Fast full-wavefield seismic inversion using encoded sources", Geophysics, 2009
- Haber et al. "An effective method for parameter estimation with PDE constraints with multiple right hand sides", tech report 2010.



Phase Encoding as a Stochastic Formulation (Haber et al.)

$$\min_u J(u) = \| PA(u)^{-1}Q - D \|_F^2 + R(u)$$

where $Q = (q_1, \ldots, q_N), D = (d_1, \ldots, d_N), P$ is a projection rewrite deterministic problem as a stochastic one using:

$$\mathrm{E}_w(w^T S(u)^T S(u) w) = trace(S(u)^T S(u)) = \| S(u) \|_F^2$$

then obtain:

$$\min_u E_w J(u,w) = \frac{1}{2} \| (PA(u)^{-1}Q - S)w \|^2$$

w is a random vector with 0 mean and cov matric = I. This problem is solved using SAA:

$$E_w J(u,w) \approx \frac{1}{K} \sum_{j=1}^{K} J(u, w_j)$$

and by choosing w from a distribtion of ± 1, the variance is minimized

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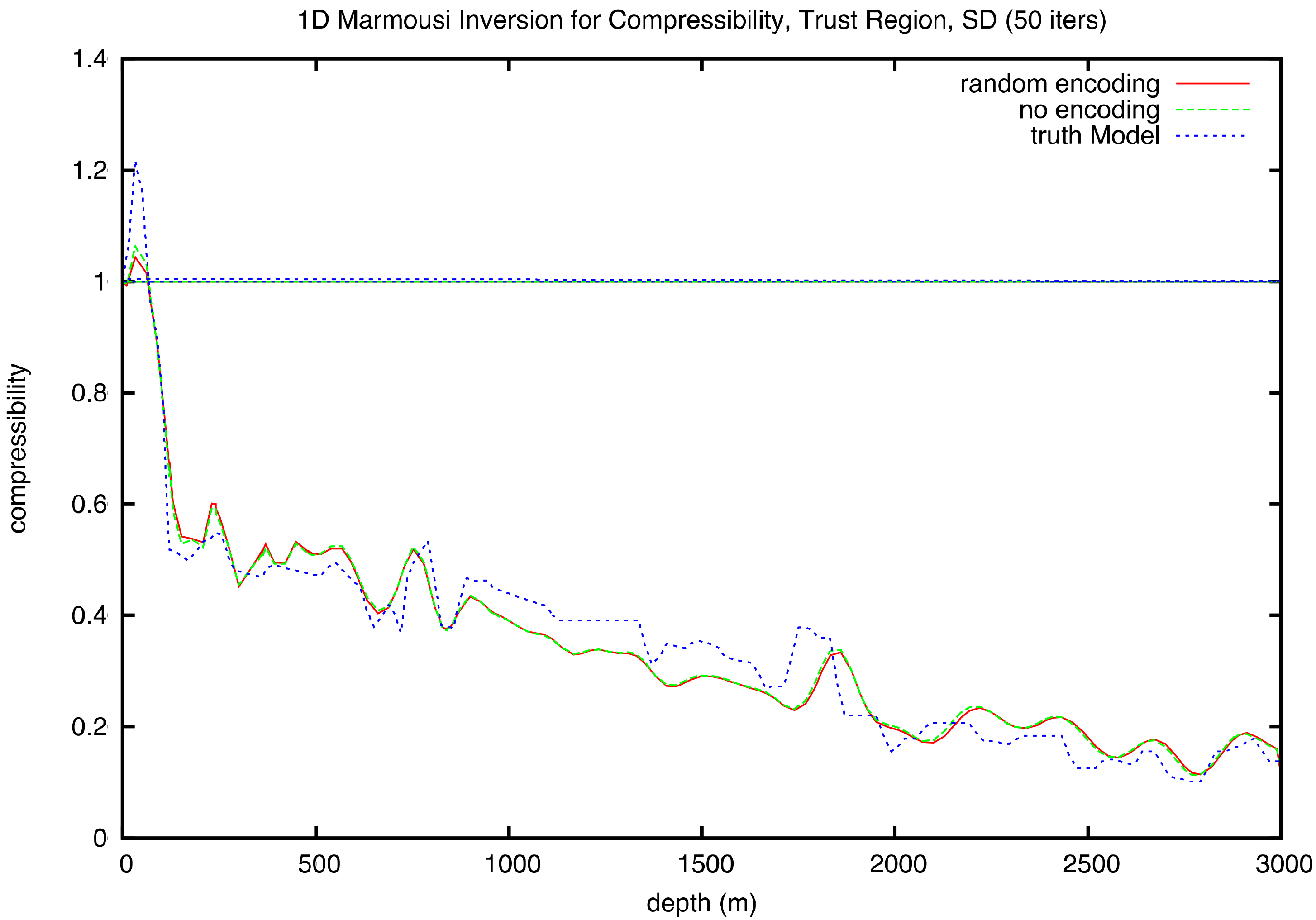
Multi-Experiment Numerical Results 1D comparison to sequential



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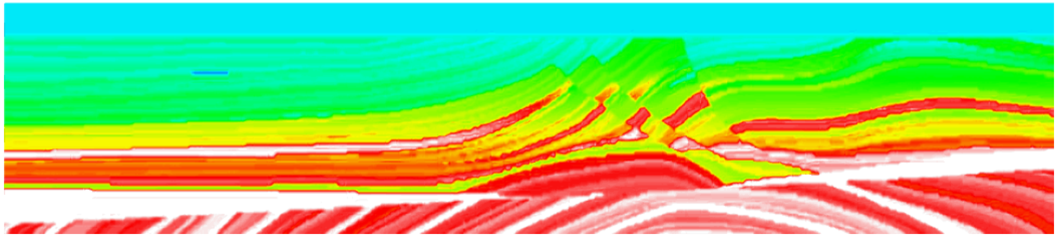
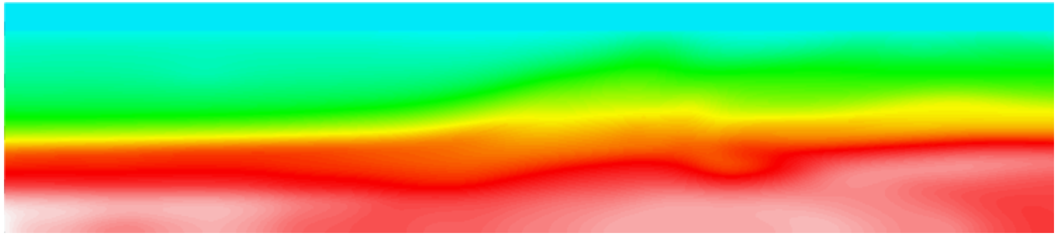
Numerical Results Comparison
1D, 50 iterations

Method	global	Wall clock	Obj fcn	encode
SD	line	0.77	0.269	no
SD	trust	0.43	0.159	no
SR1	trust	0.46	0.015	no
BFGS	trust	0.42	0.0085	no
SD	line	0.25	0.62	yes
SD	trust	0.17	0.59	yes

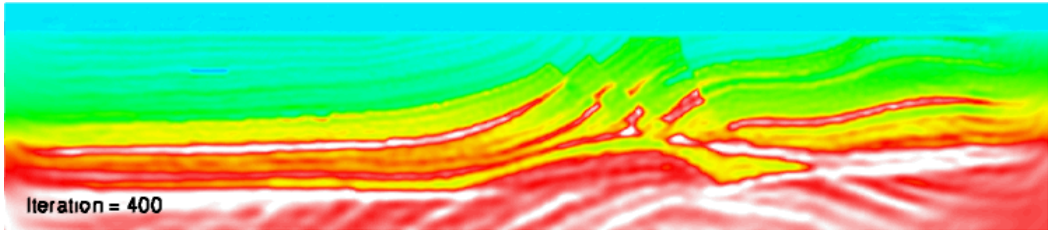
2D Numerical Results – Marmousi



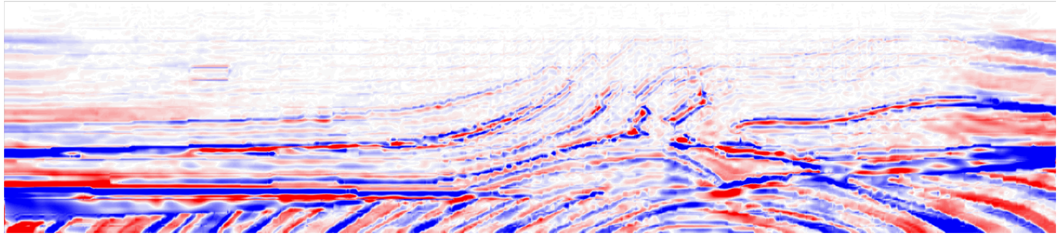
Truth Model



Smoothed Initial Model



Inverted Model

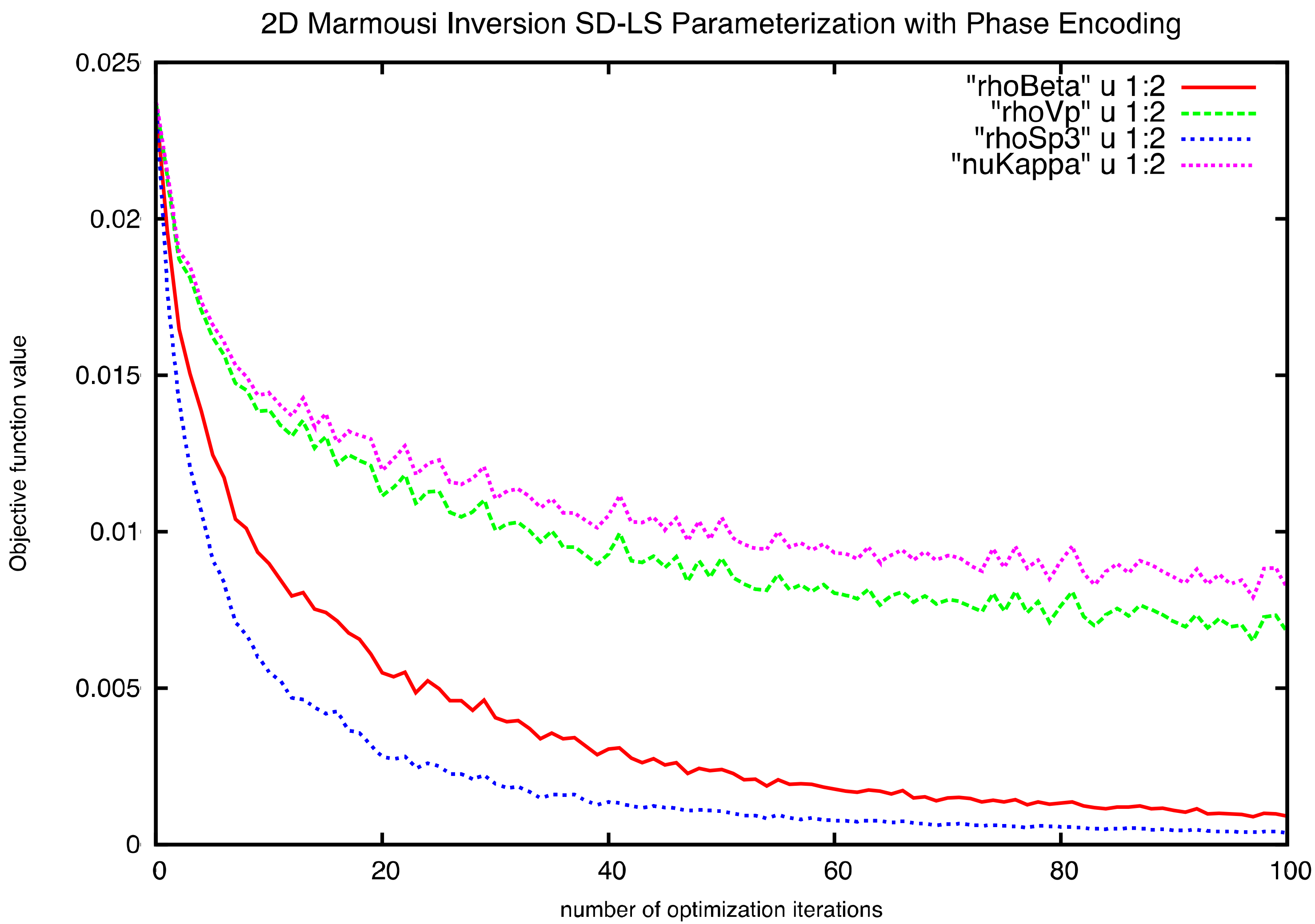


Inverted - True

Details:

- 500 m sponge
- 76 receivers
- 15 sources
- 1600 elmts
- p = 5
- 200 m cells
- $RhoSp^3$

Parameterization Study
(Redsky 96 cores)



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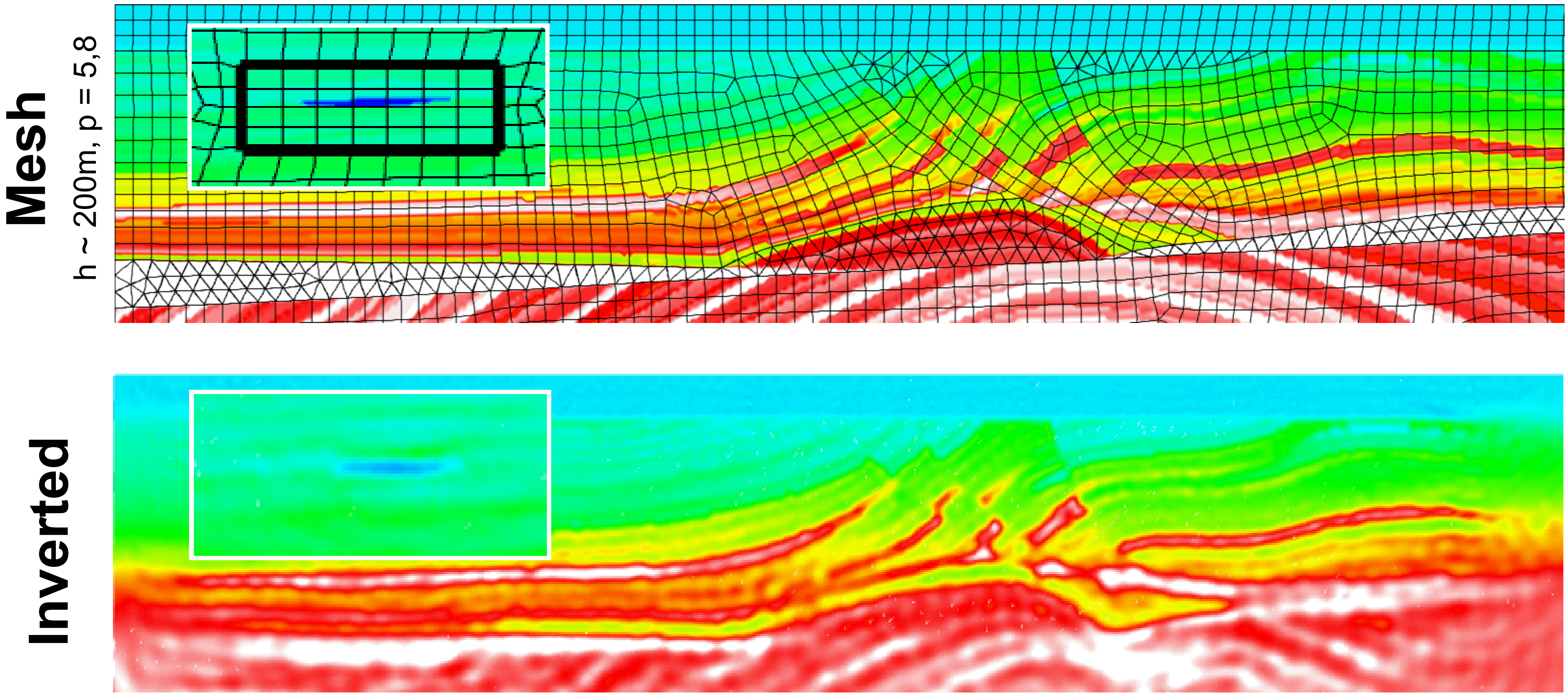
Acoustic Inversion

Unstructured Hybrid Mesh



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Conclusions

- Discontinuous Galerkin discretization has been applied to solve the wave equations, providing high resolution and unstructured capabilities.
- Full wave form inversion using DG has been demonstrated on complex 2D datasets.
- Computational accelerations are realized through phase encoding, parallelism, parameterizations, trust region, and second order algorithms.