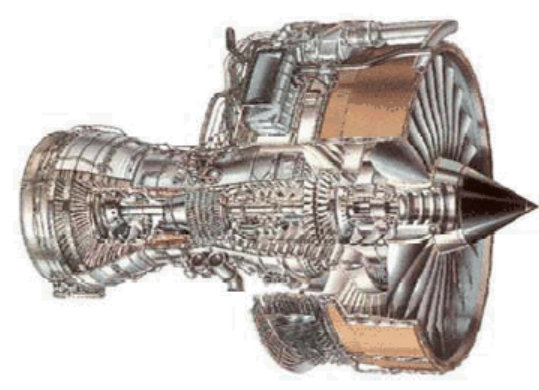
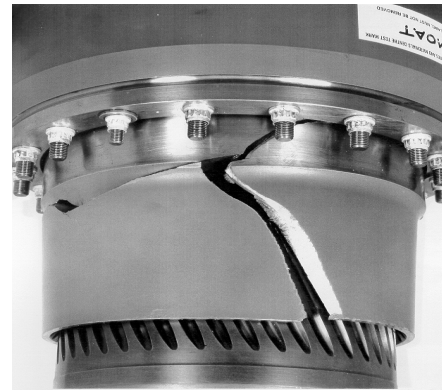
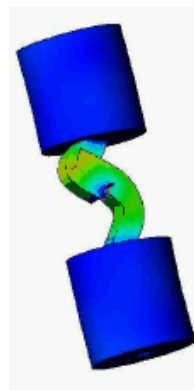
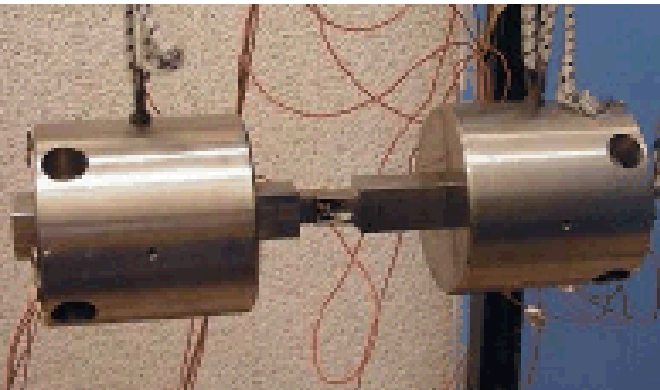


Exceptional service in the national interest



Development of a Coupling Metric to Assess the Shakedown Limits for a Contact Interface

M.R. Brake, Sandia National Laboratories

R.C. Flicek and D.A. Hills, University of Oxford



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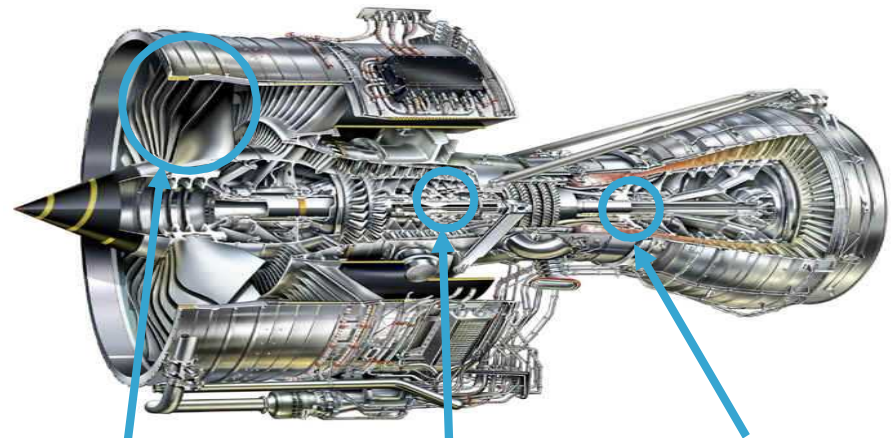
Motivation & Aim

Motivation:

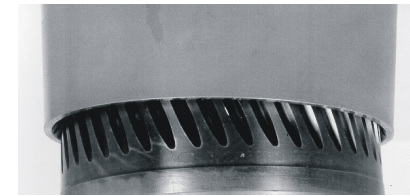
- To predict wear damage from high cycle fatigue in jet engines or other cyclically loaded friction interfaces
- Transient simulations of frictional contact are computationally intensive

Aim:

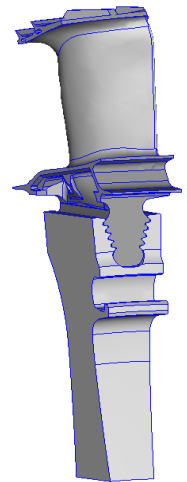
- Develop efficient methods for predicting steady state behavior



Fan Blade to Disc Joint



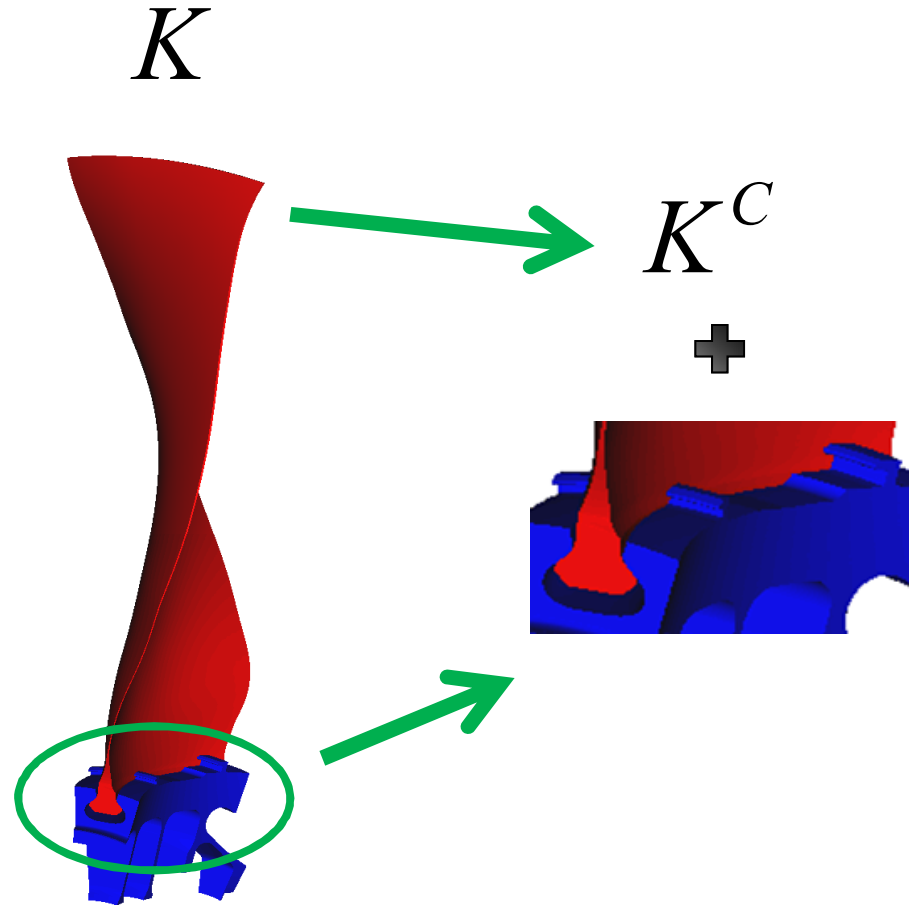
Shaft Drive
Splines



Turbine
Blade
to Disc Joint

Methodology

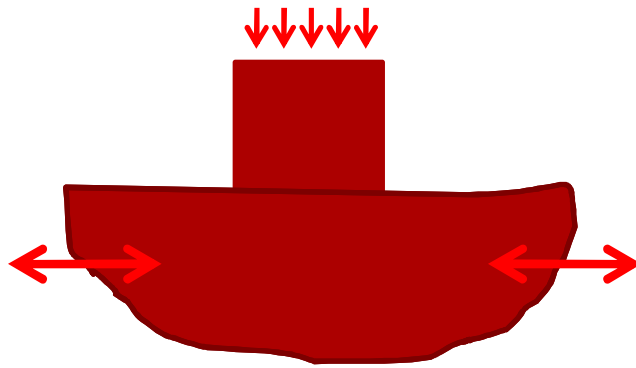
- Many components are loaded quasi-statically
 - Neglect inertial effects
 - Use to develop a model reduction technique
- **Quasi-Static Reduction**
 - Eliminates all “internal” degrees of freedom
 - Reduces dimensionality by orders of magnitude



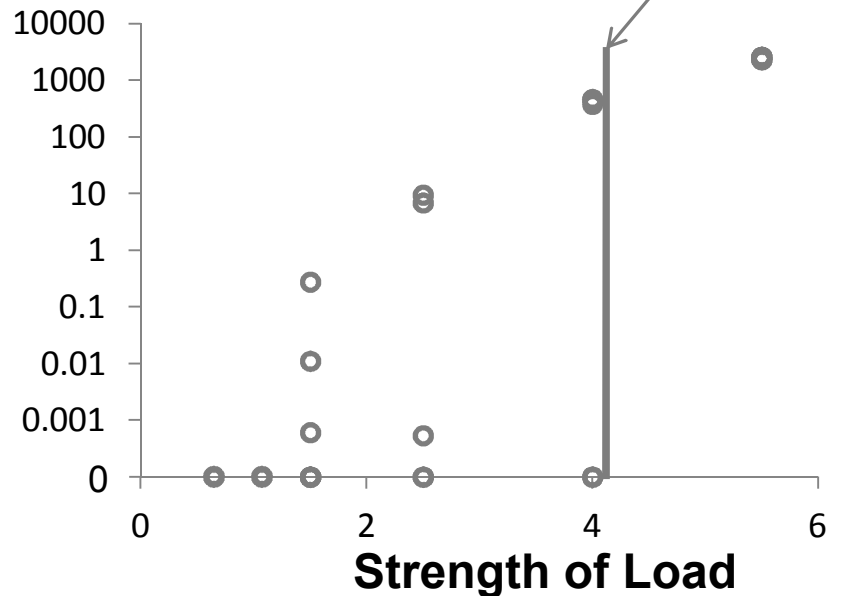
The Shakedown Limit

Shakedown limit:

- Load above which only dissipative solutions exist
- Can calculate from reduced stiffness matrix
 - Frame as an optimization
 - Computes \approx as fast as one transient simulation



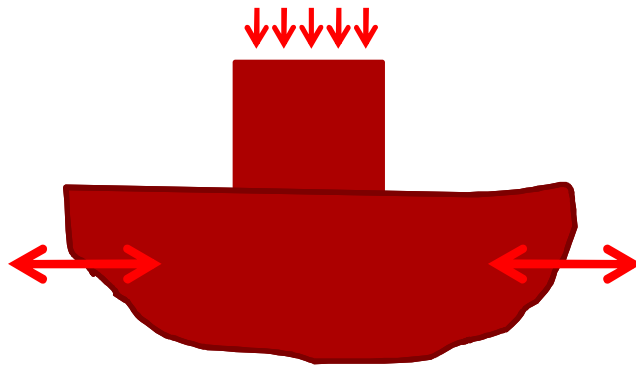
Steady State Dissipation



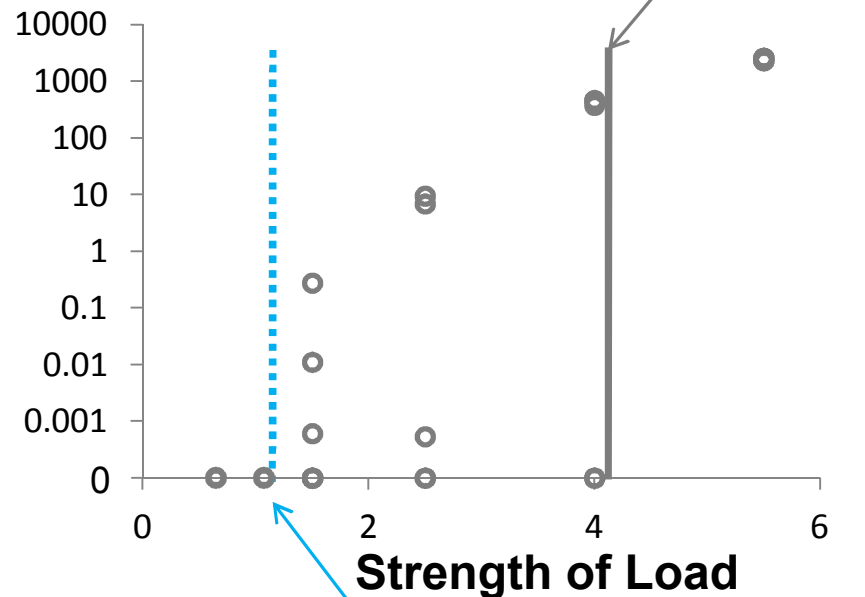
Unconditional Shakedown Limit

Unconditional shakedown limit:

- Load below which only non-dissipative solutions exist
- Analytical calculation too computationally expensive ($N \cdot 2^N$ equations for N interface nodes)



Steady State Dissipation



Hypothesized unconditional shakedown limit

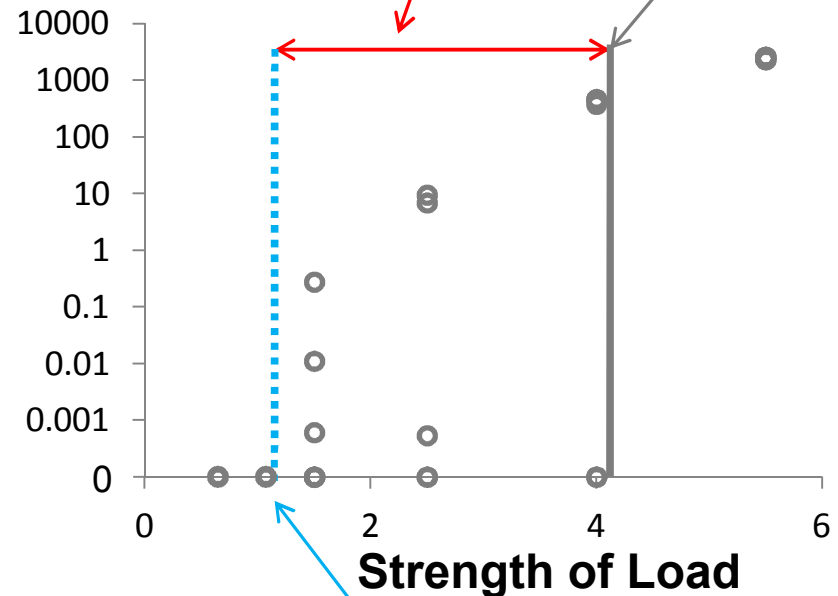
Coupling Metric

- Hypothesized to predict the relative difference between the Shakedown and Unconditional limits
- An *a priori* calculation
- Result converges quickly

$$f^C = K^C u^C, \quad K^C = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$$

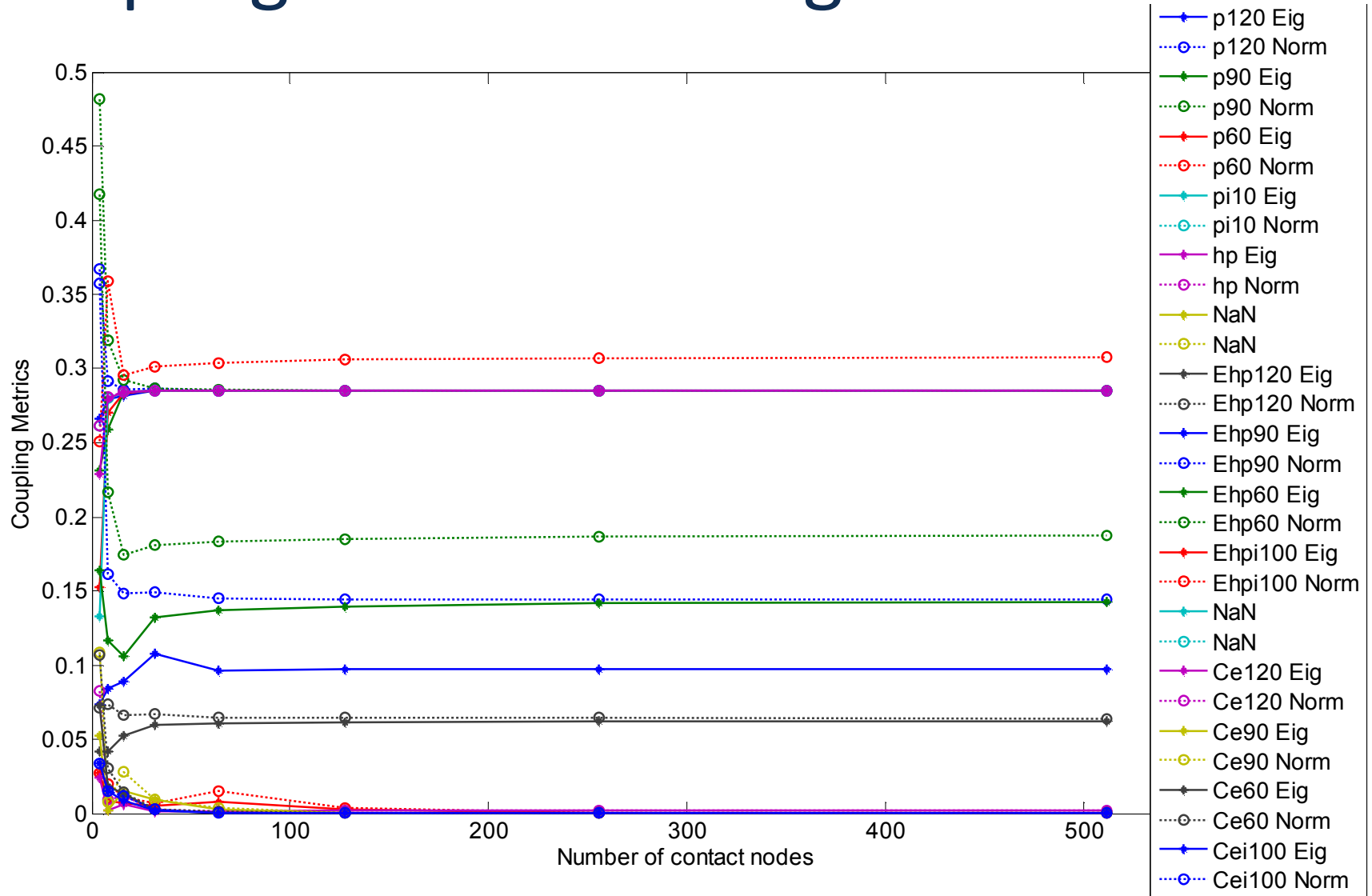
$$\zeta_{eig} = \frac{eig(B)}{eig(A)}, \quad \zeta_{norm} = \frac{norm(B)}{norm(A)}$$

**Steady State
Dissipation**



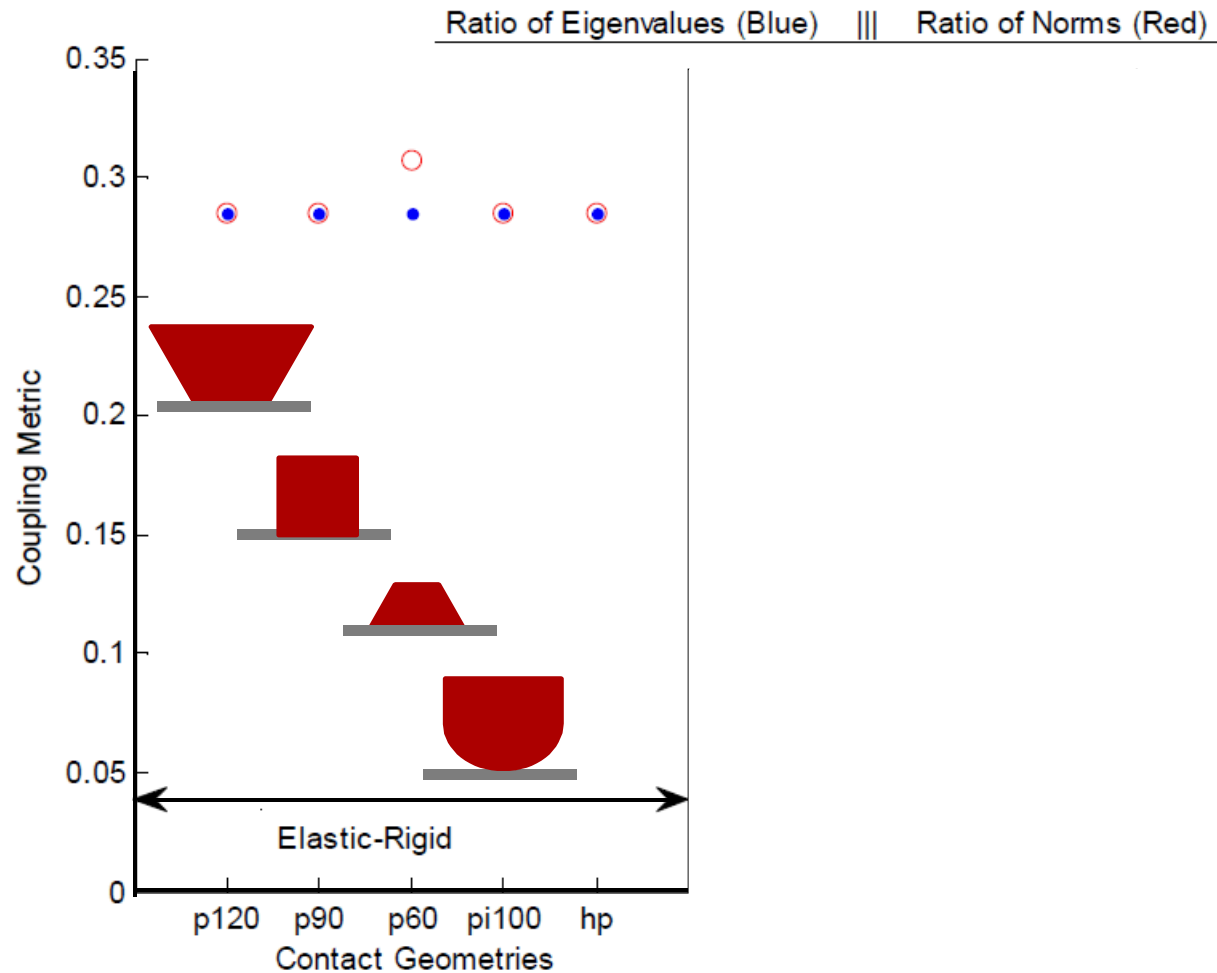
Hypothesized unconditional
shakedown limit

Coupling Metric Convergence



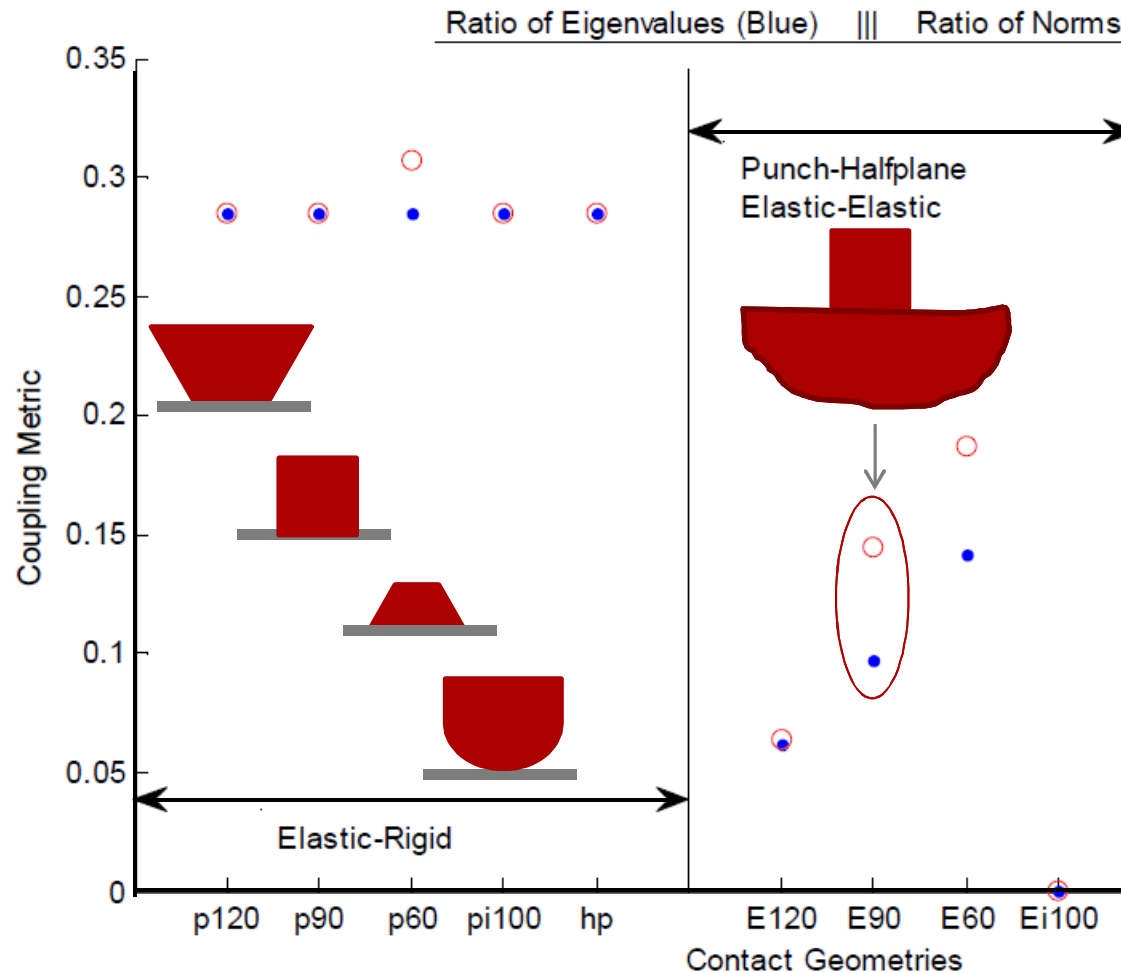
Coupling Metric

Comparison of two Metrics of Frictional Coupling



Coupling Metric

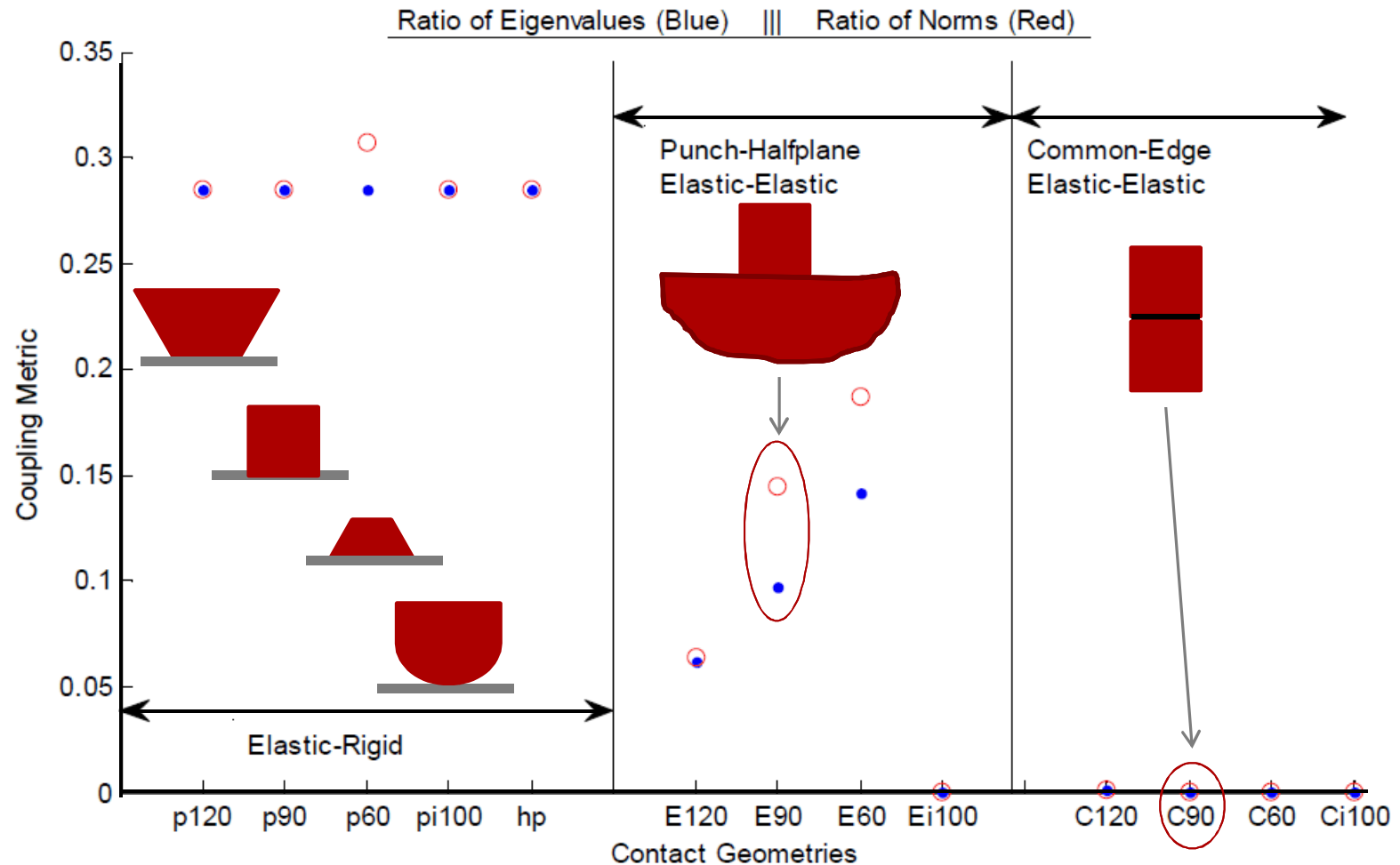
Comparison of two Metrics of Frictional Coupling



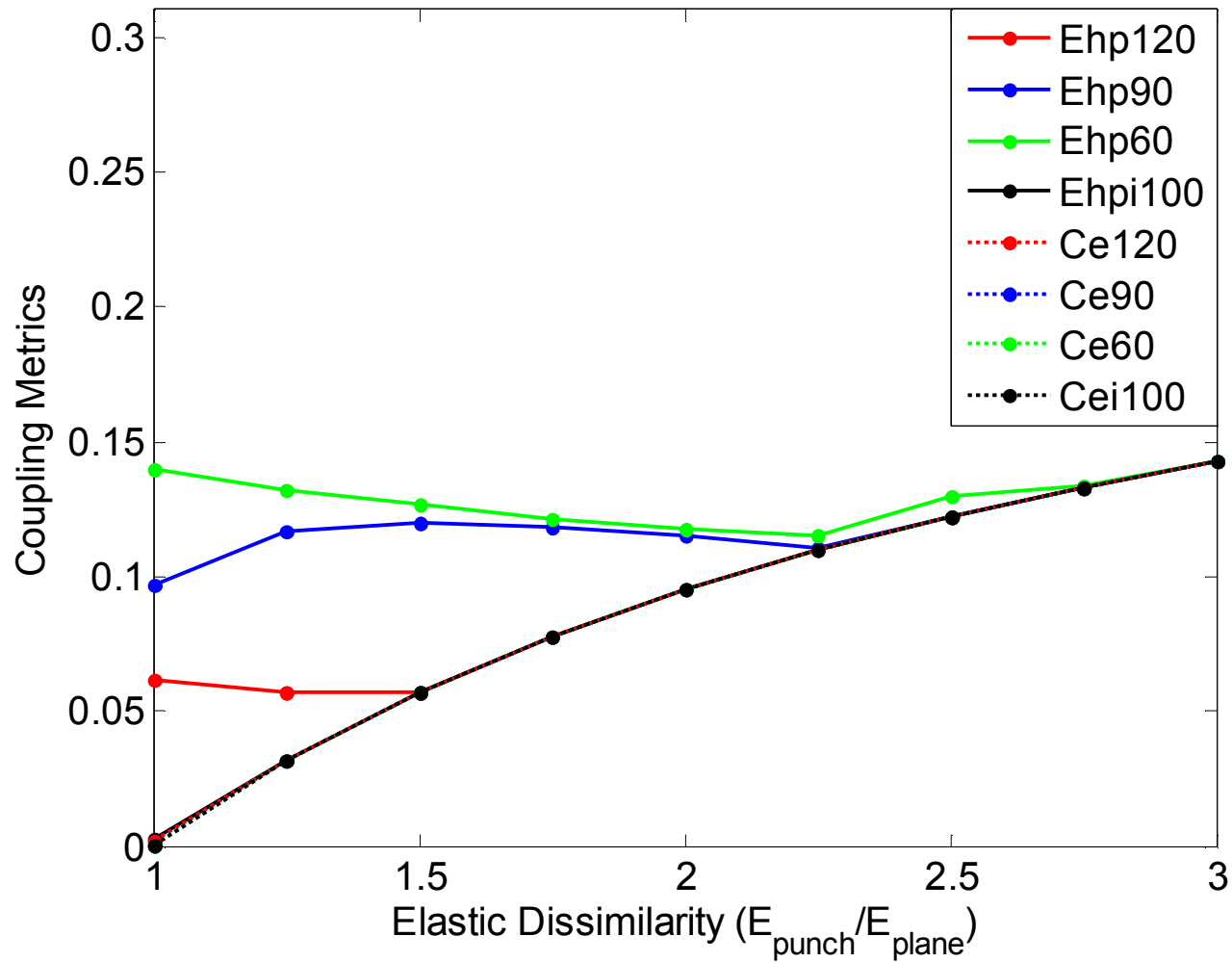
- Elastically identical materials
- Geometrically mismatched at interface

Coupling Metric

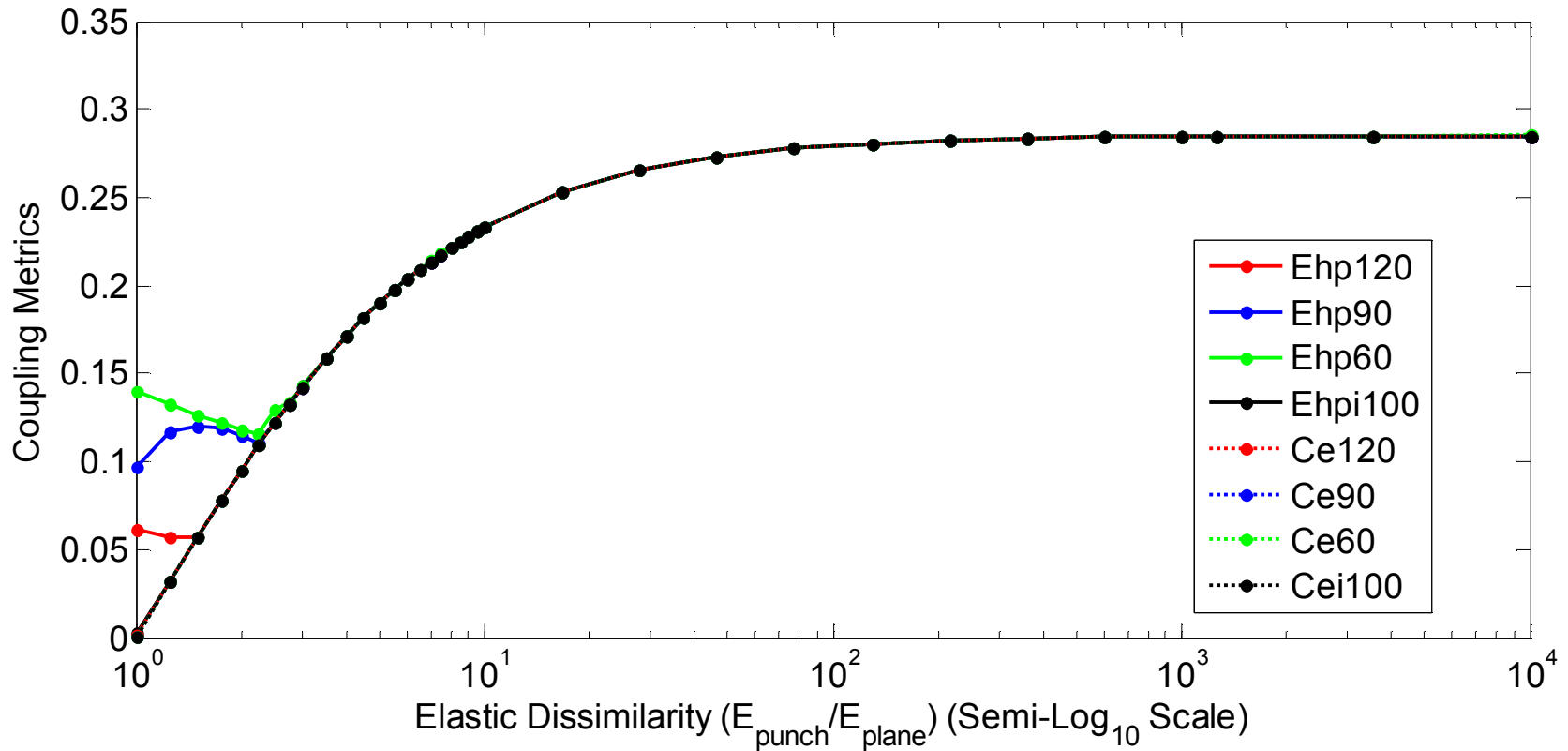
Comparison of two Metrics of Frictional Coupling



Effect of Material Dissimilarity

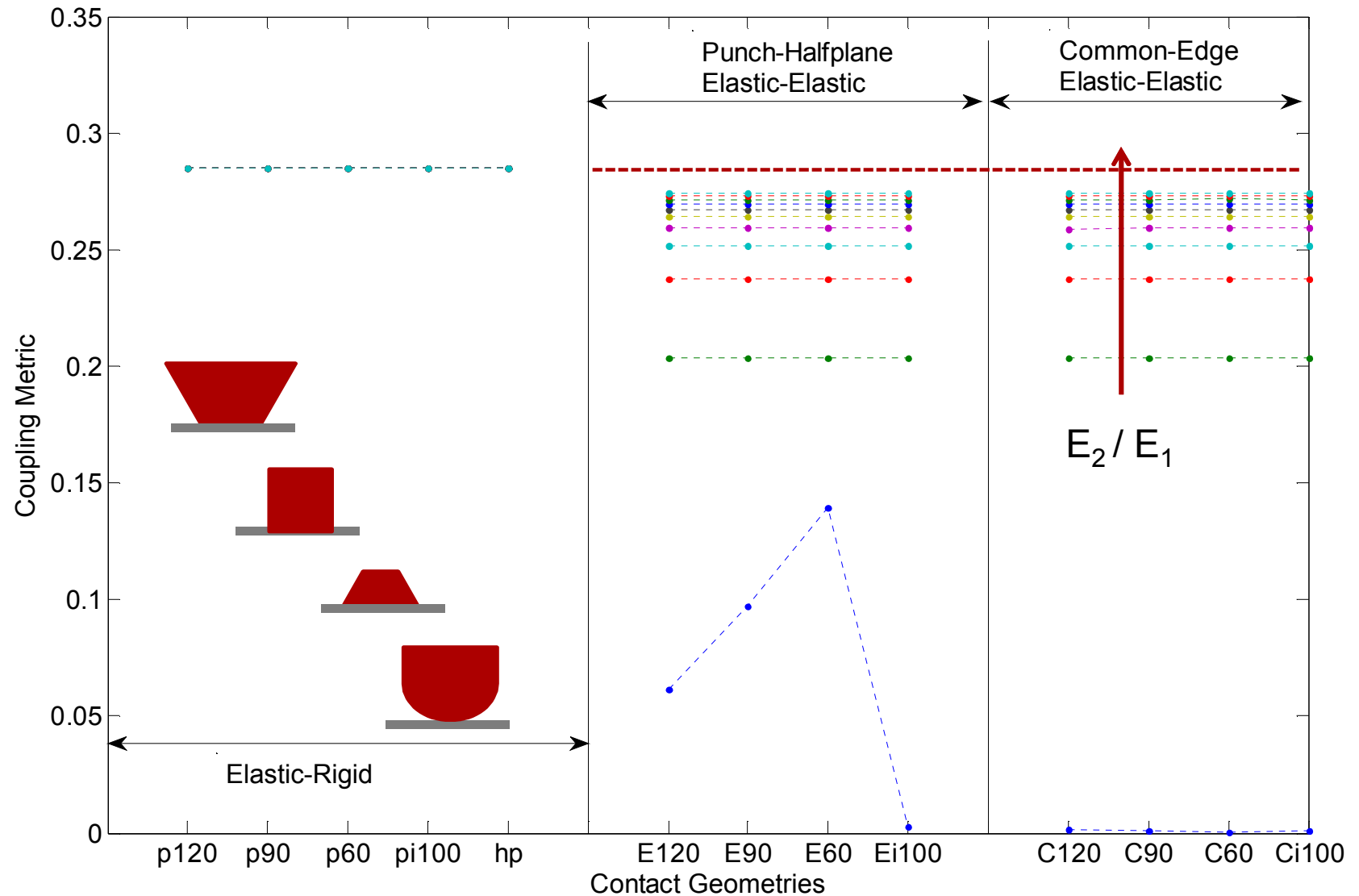


Effect of Material Dissimilarity

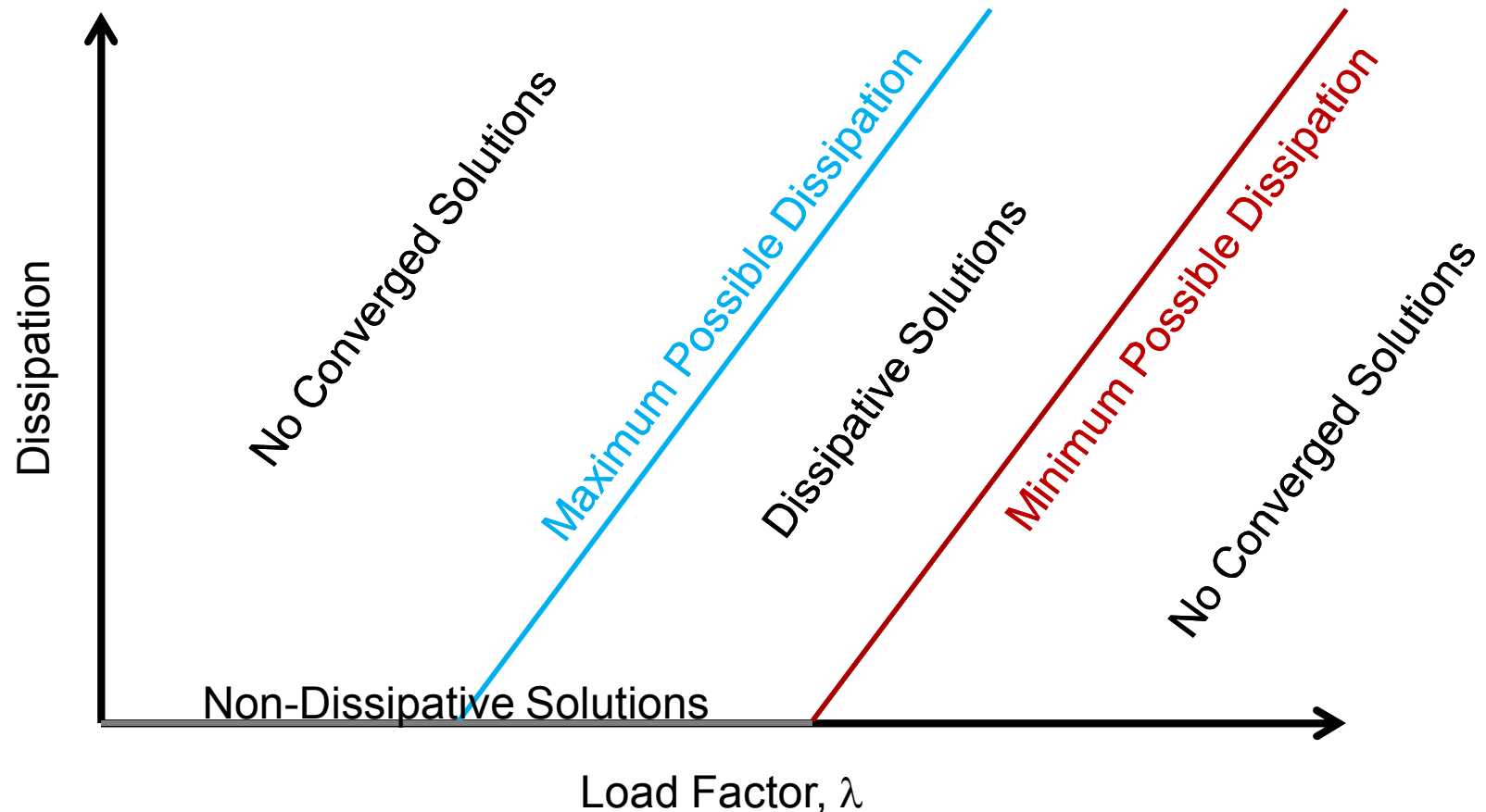


- All models converge to the same coupling metric value for sufficiently large dissimilarity
- “Sufficiently large dissimilarity” is non-representative of engineering materials

Coupling Metric

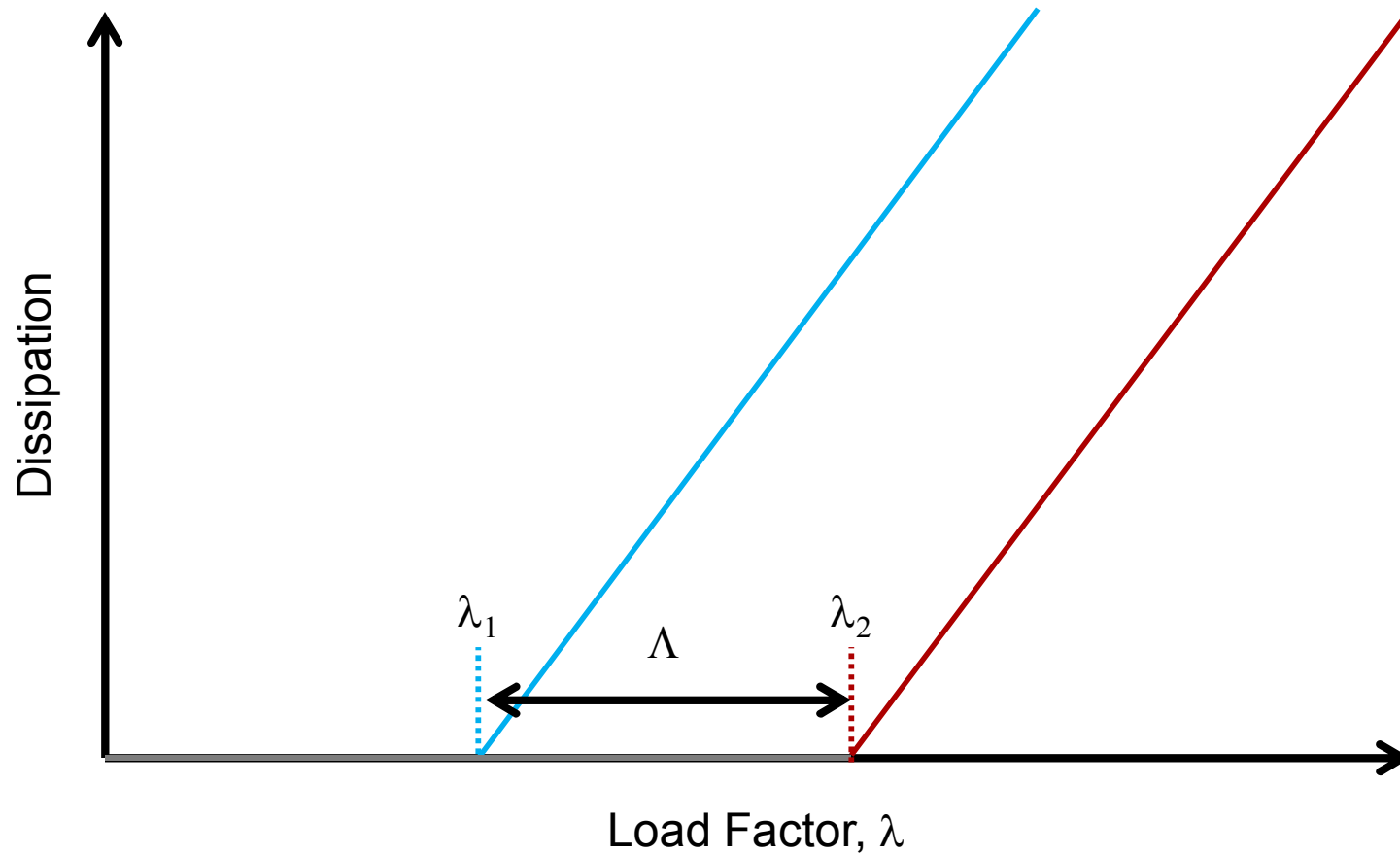


Determination of Shakedown Limits



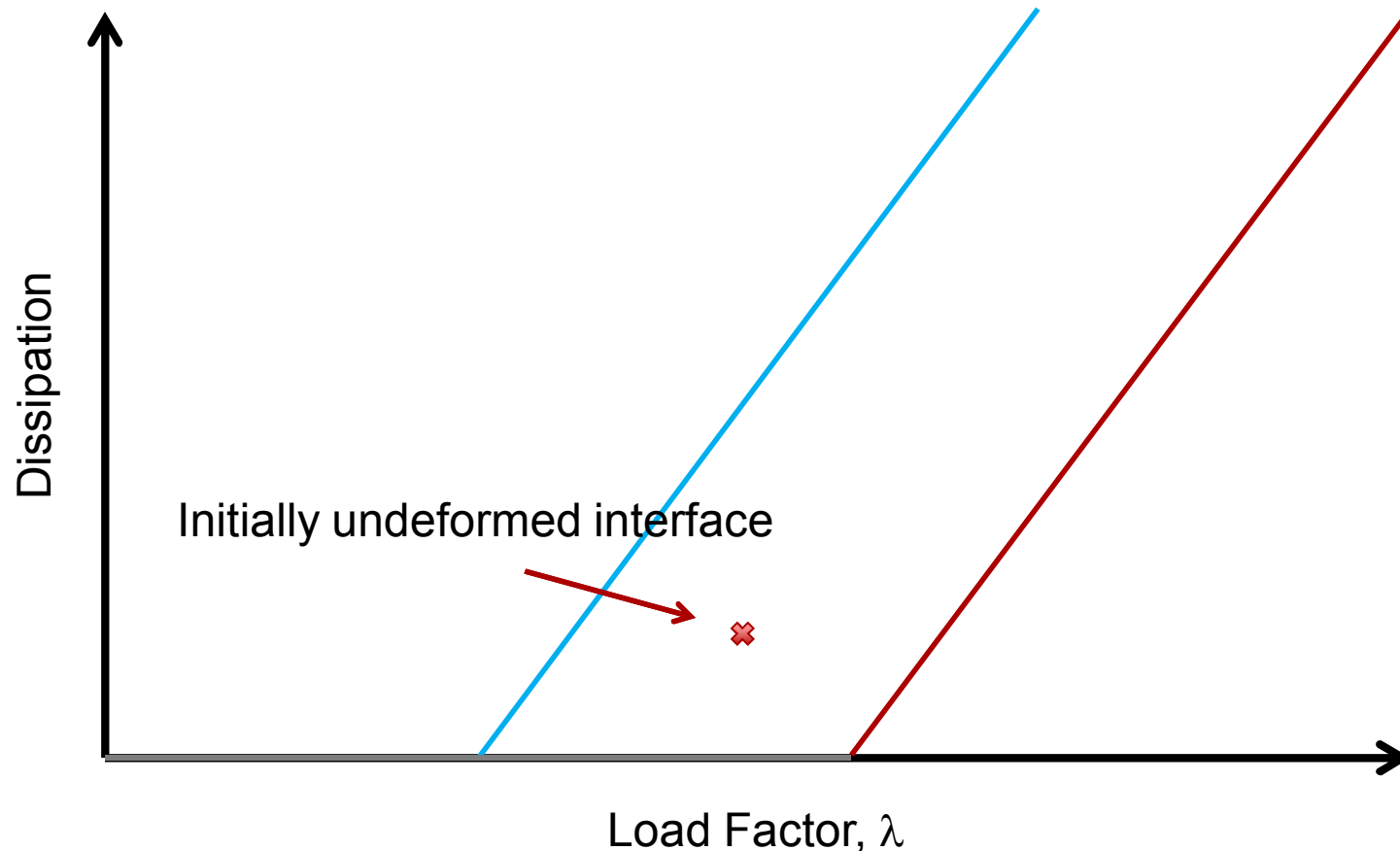
- For a specific geometry and coefficient of friction

Determination of Shakedown Limits



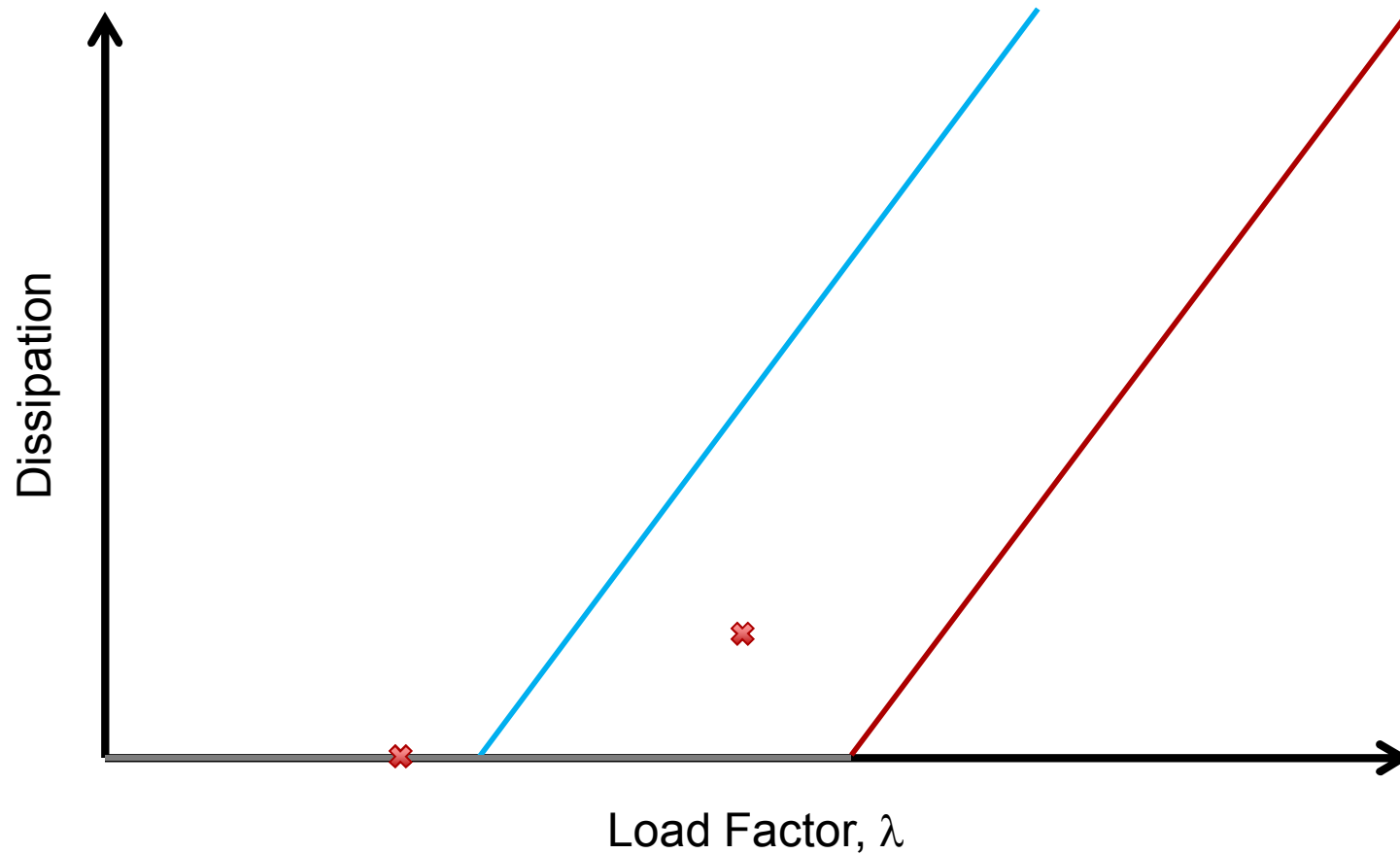
- λ_1 : Unconditional shakedown limit
- λ_2 : Shakedown limit

Determination of Unconditional Shakedown Limit



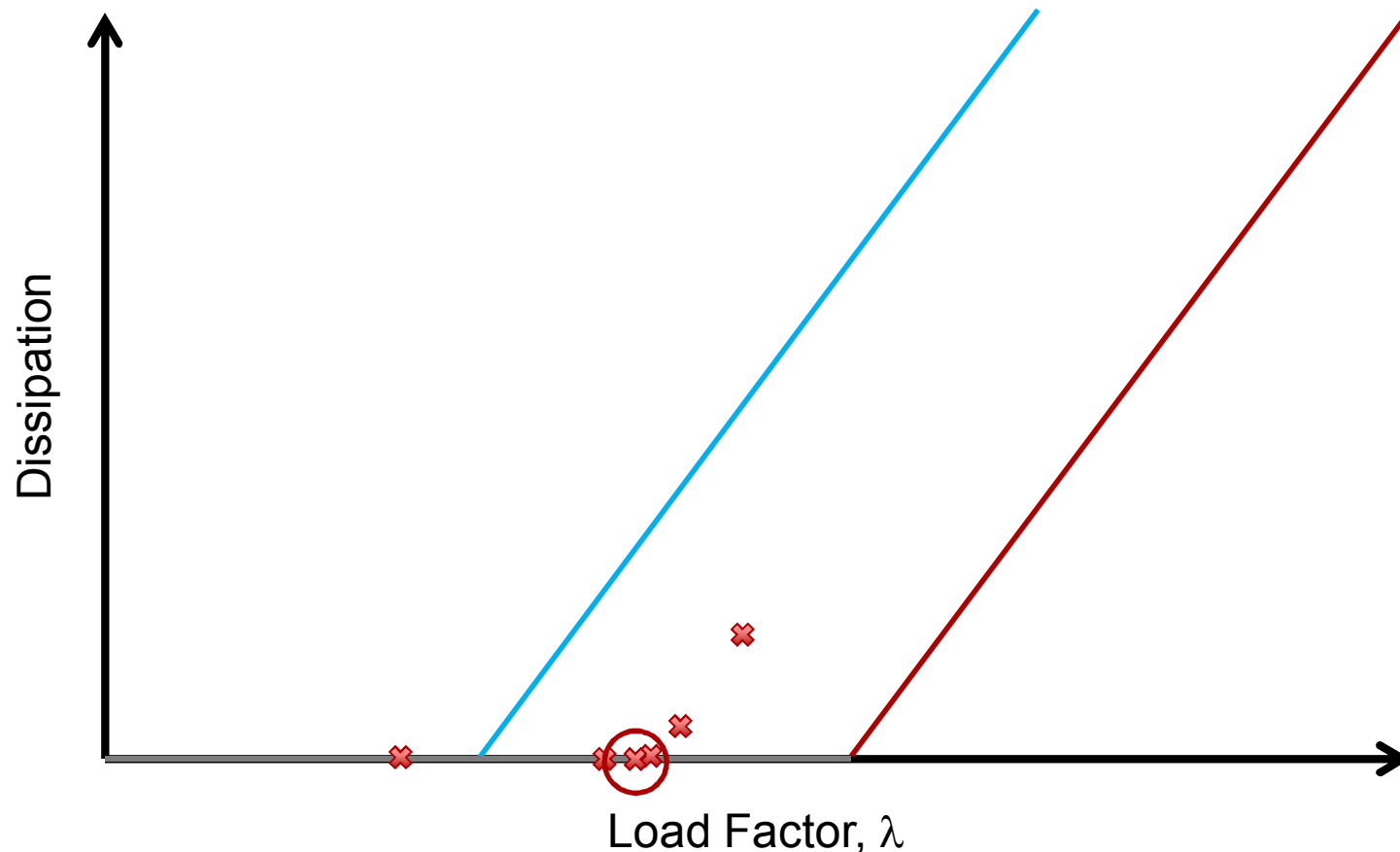
- Guided search method using efficient transient solutions

Determination of Unconditional Shakedown Limit



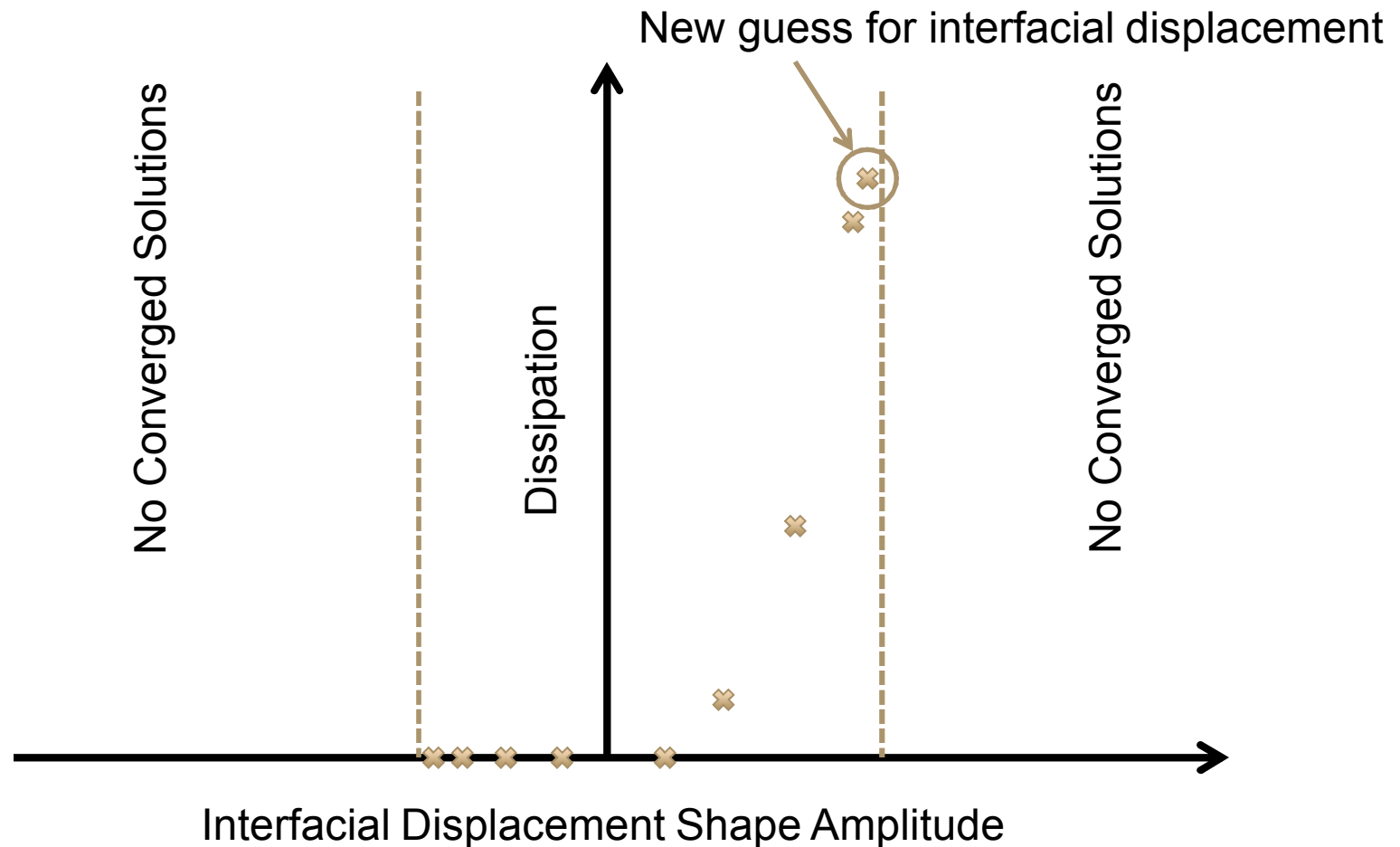
- First step is a bisection method

Determination of Unconditional Shakedown Limit



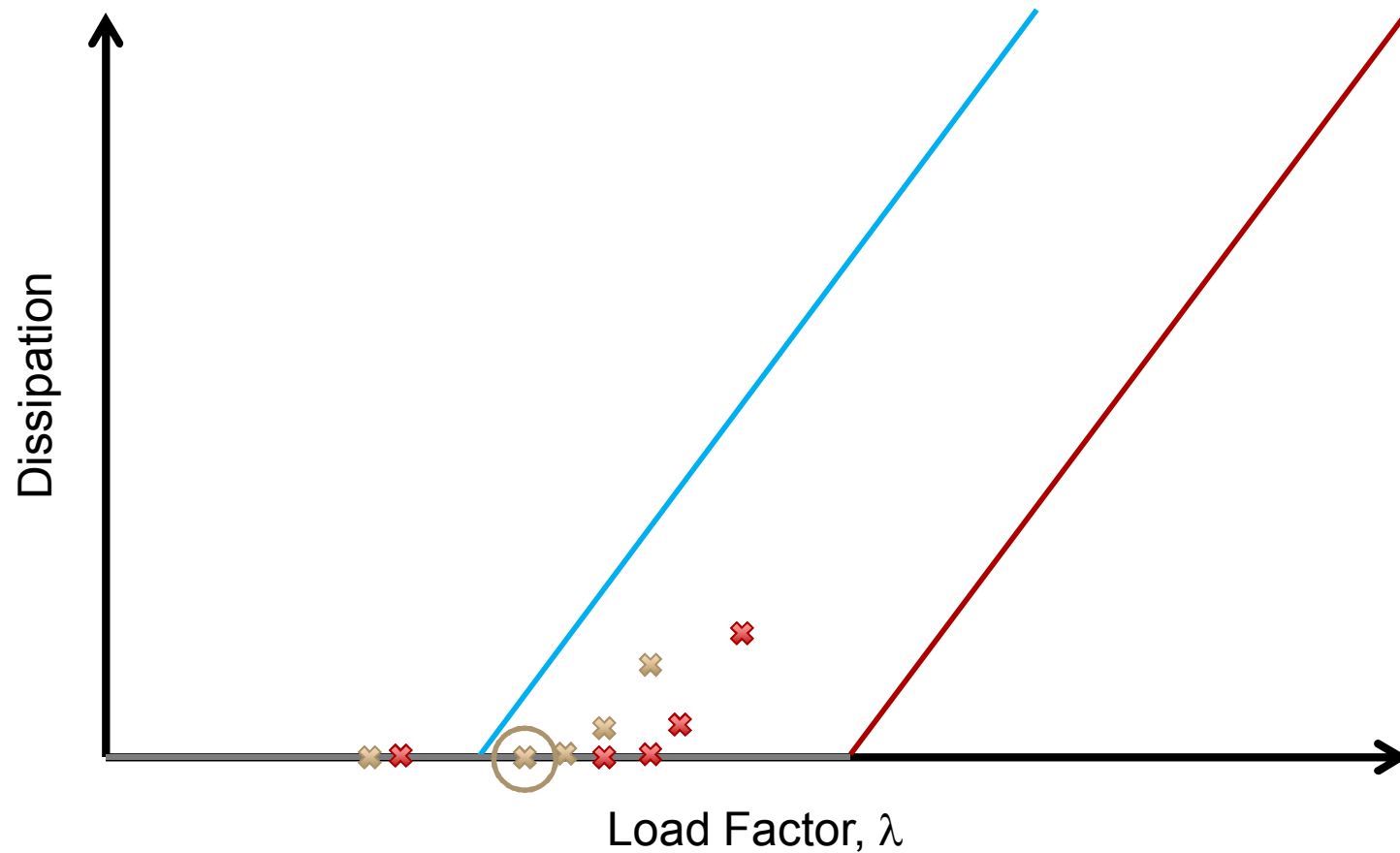
- The unconditional shakedown limit for a given interfacial displacement shape is found (using both a bisection and Newton iteration)

Determination of Unconditional Shakedown Limit



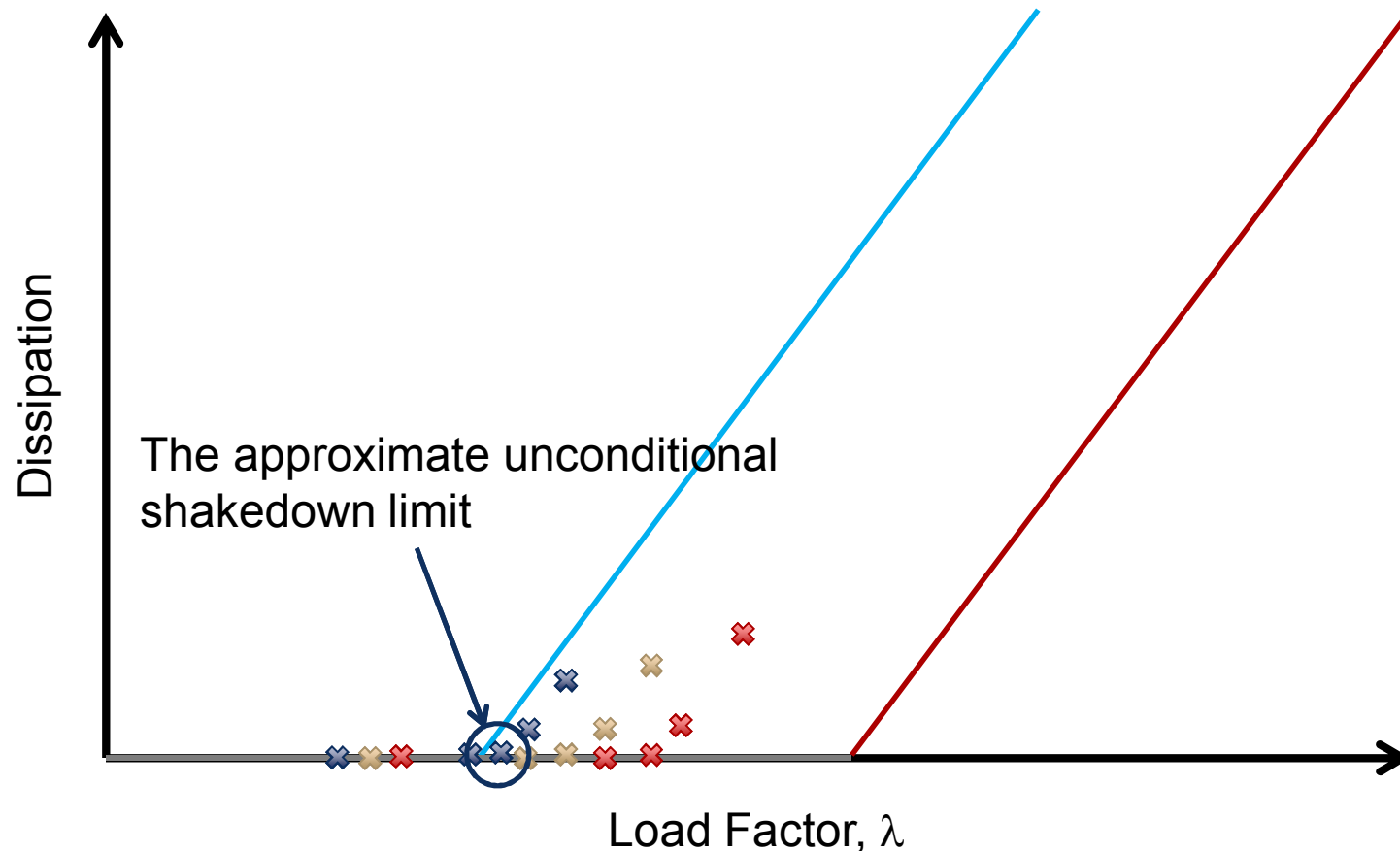
- The unconditional shakedown limit for a given interfacial displacement shape is found (using both a bisection and Newton iteration)

Determination of Unconditional Shakedown Limit



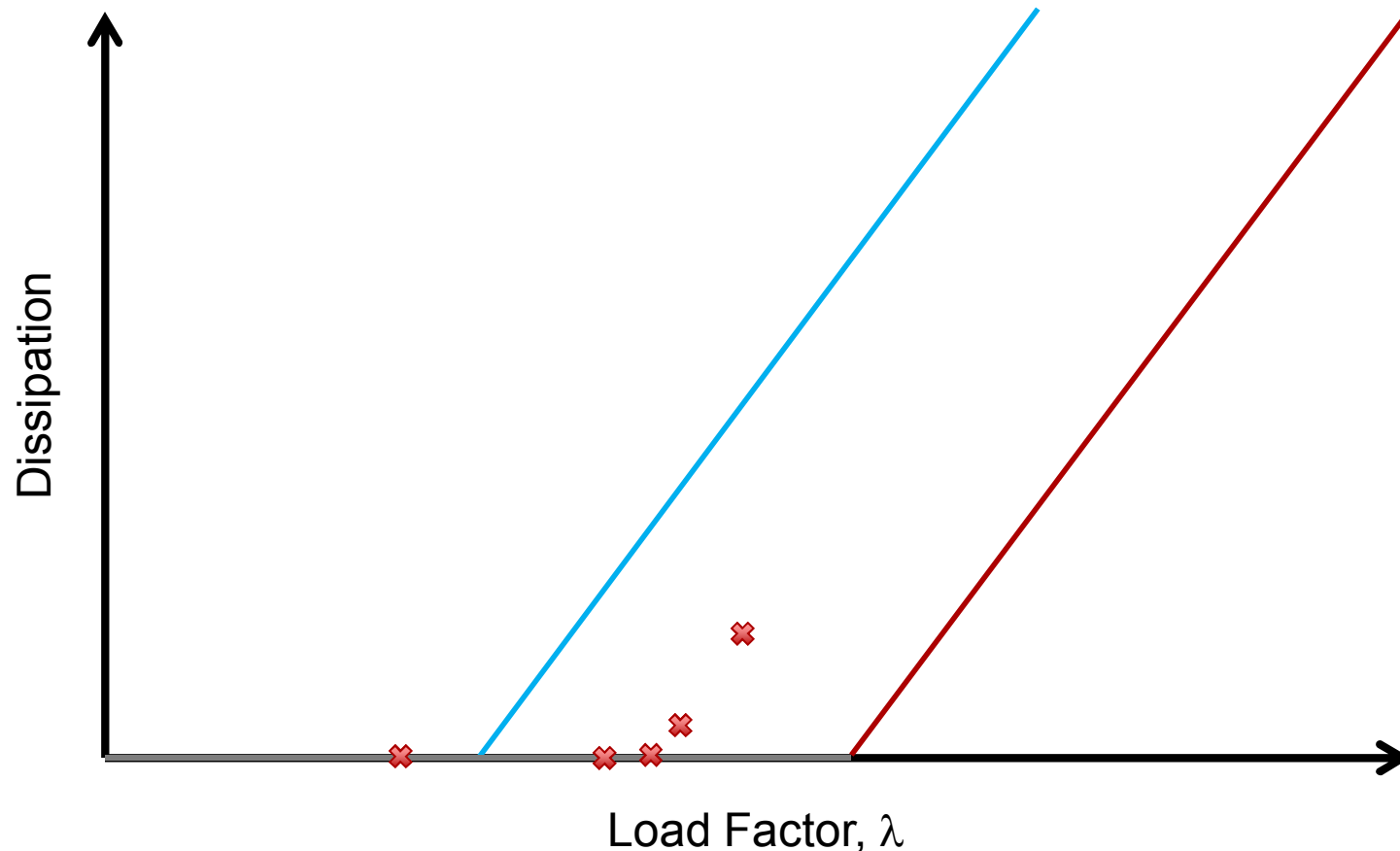
- The interfacial displacement shape is modified and the process is repeated

Determination of Unconditional Shakedown Limit



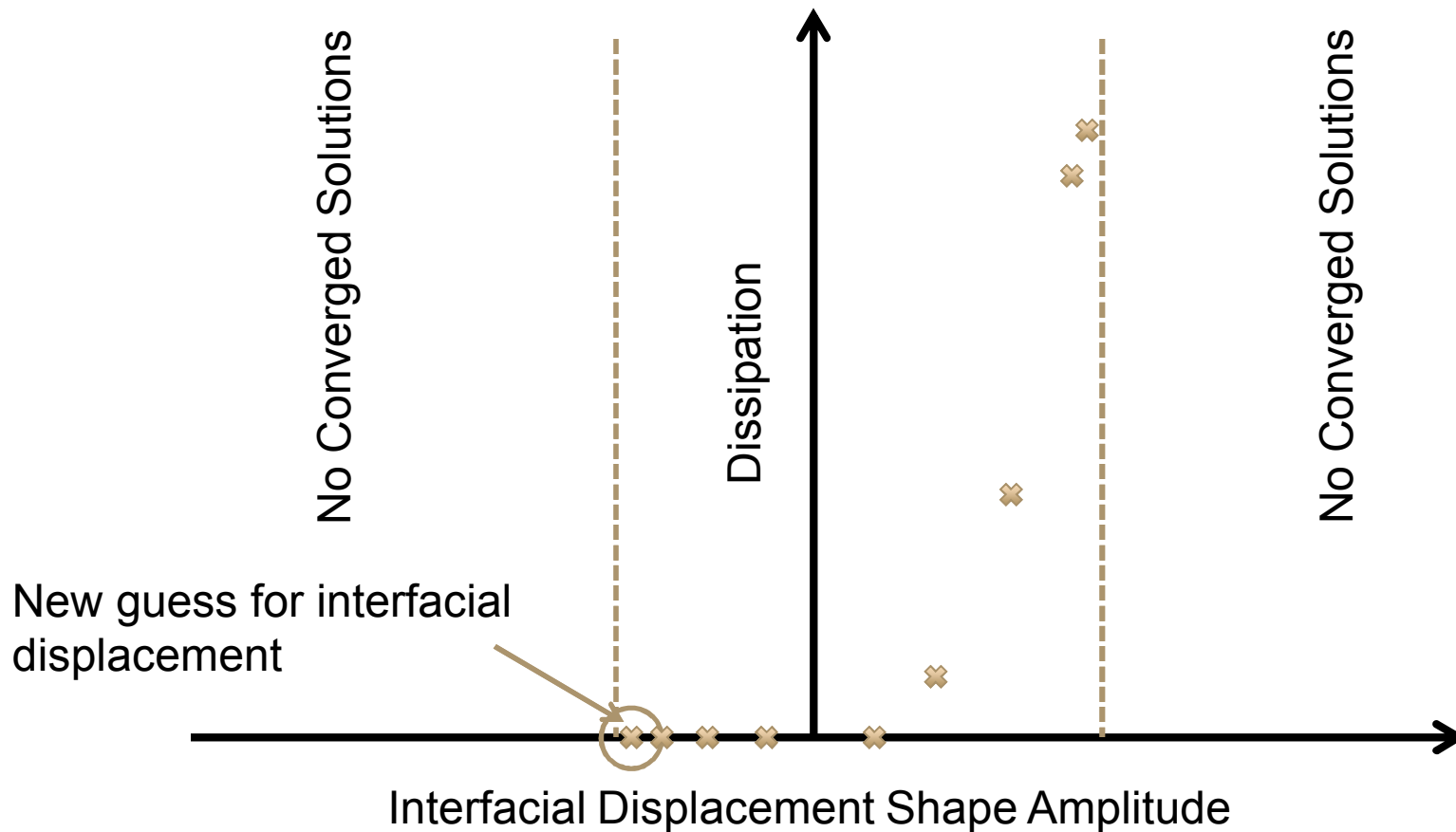
- The interfacial displacement is expressed as a modified Fourier series
- The analytical solution for verification is too computationally expensive to solve

Determination of Shakedown Limit



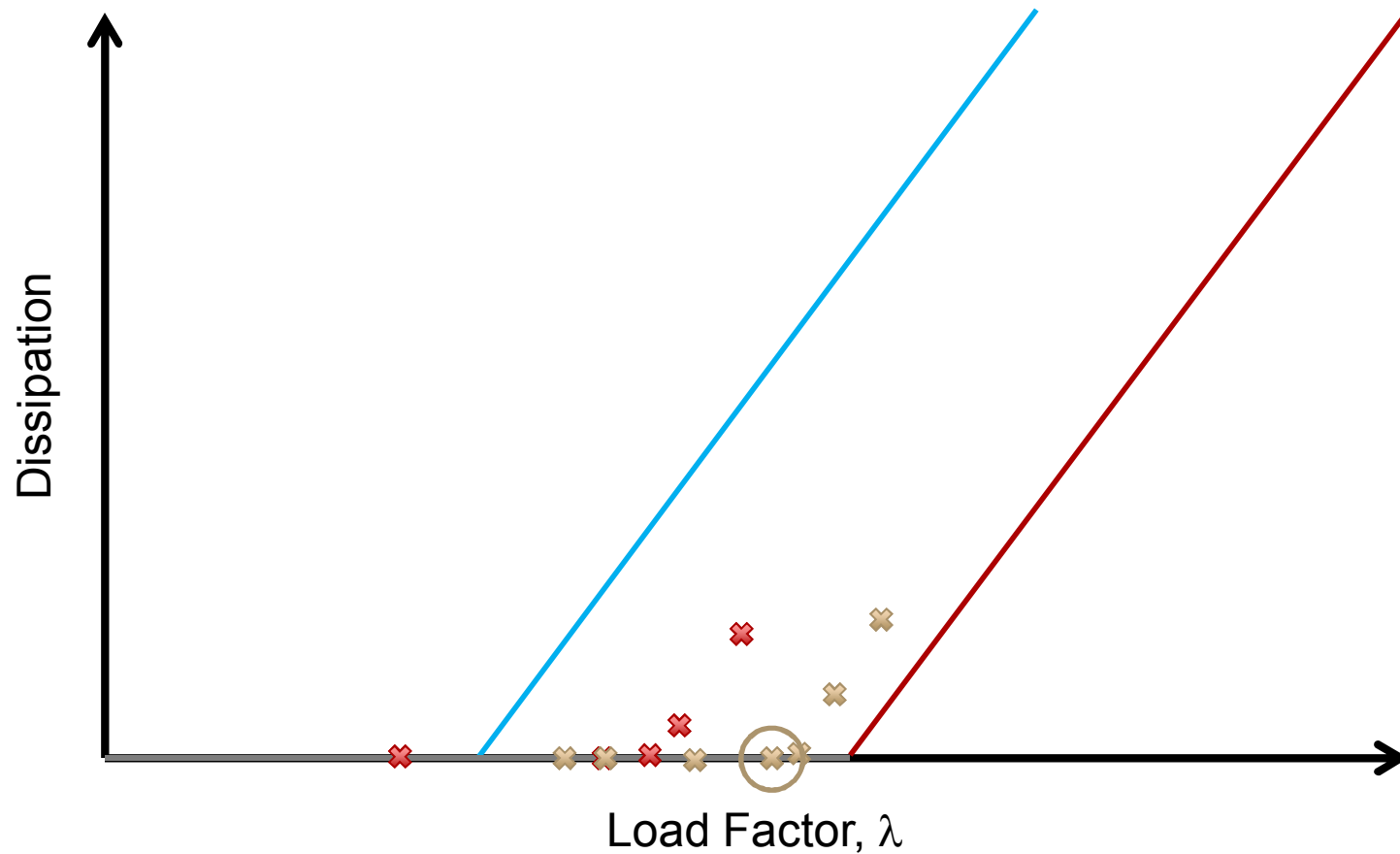
- Similar process as for the unconditional shakedown limit
- Starts with results from first step of unconditional shakedown limit search

Determination of Shakedown Limit



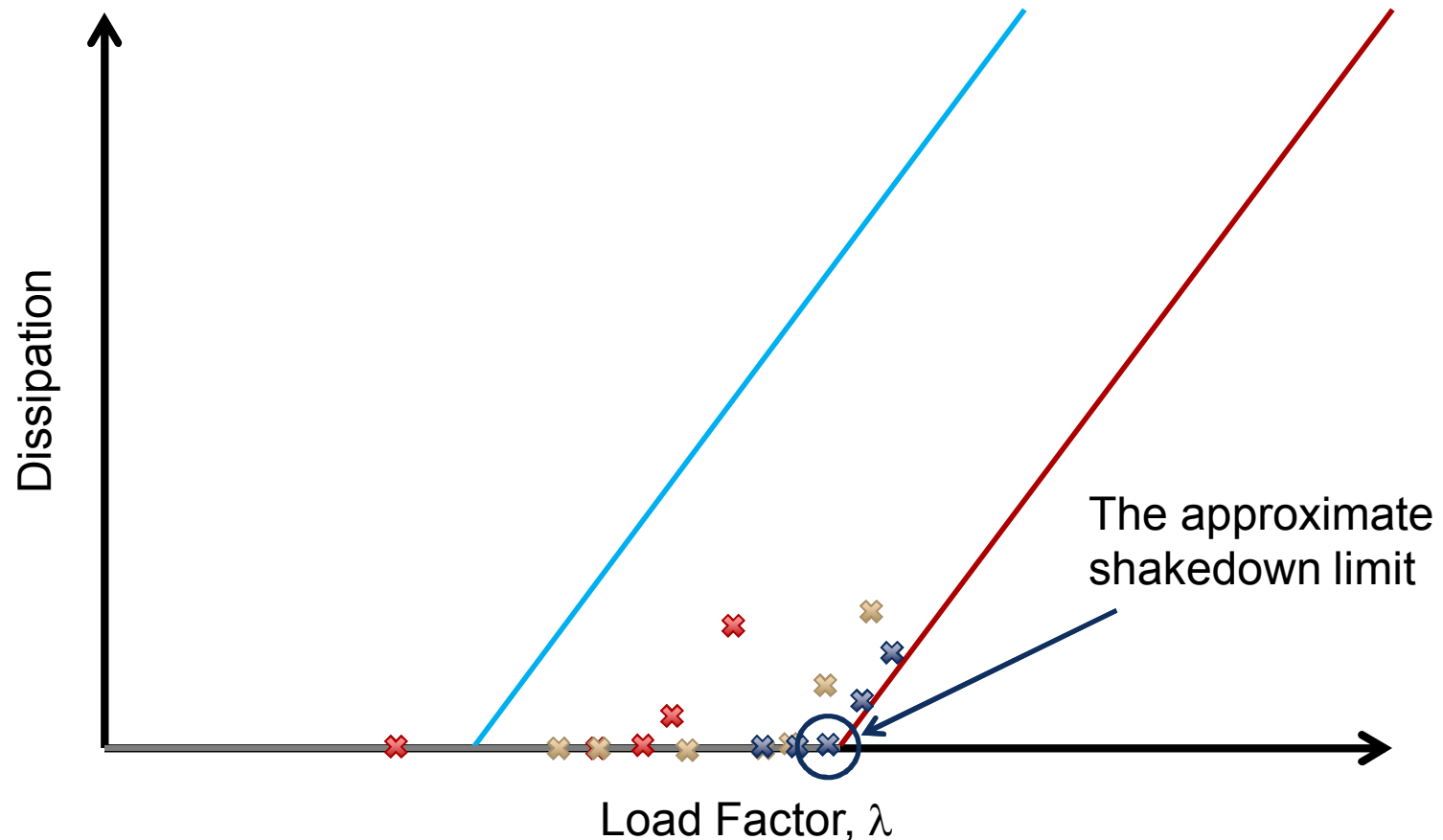
- The method of determining the new guess is changed though

Determination of Shakedown Limit



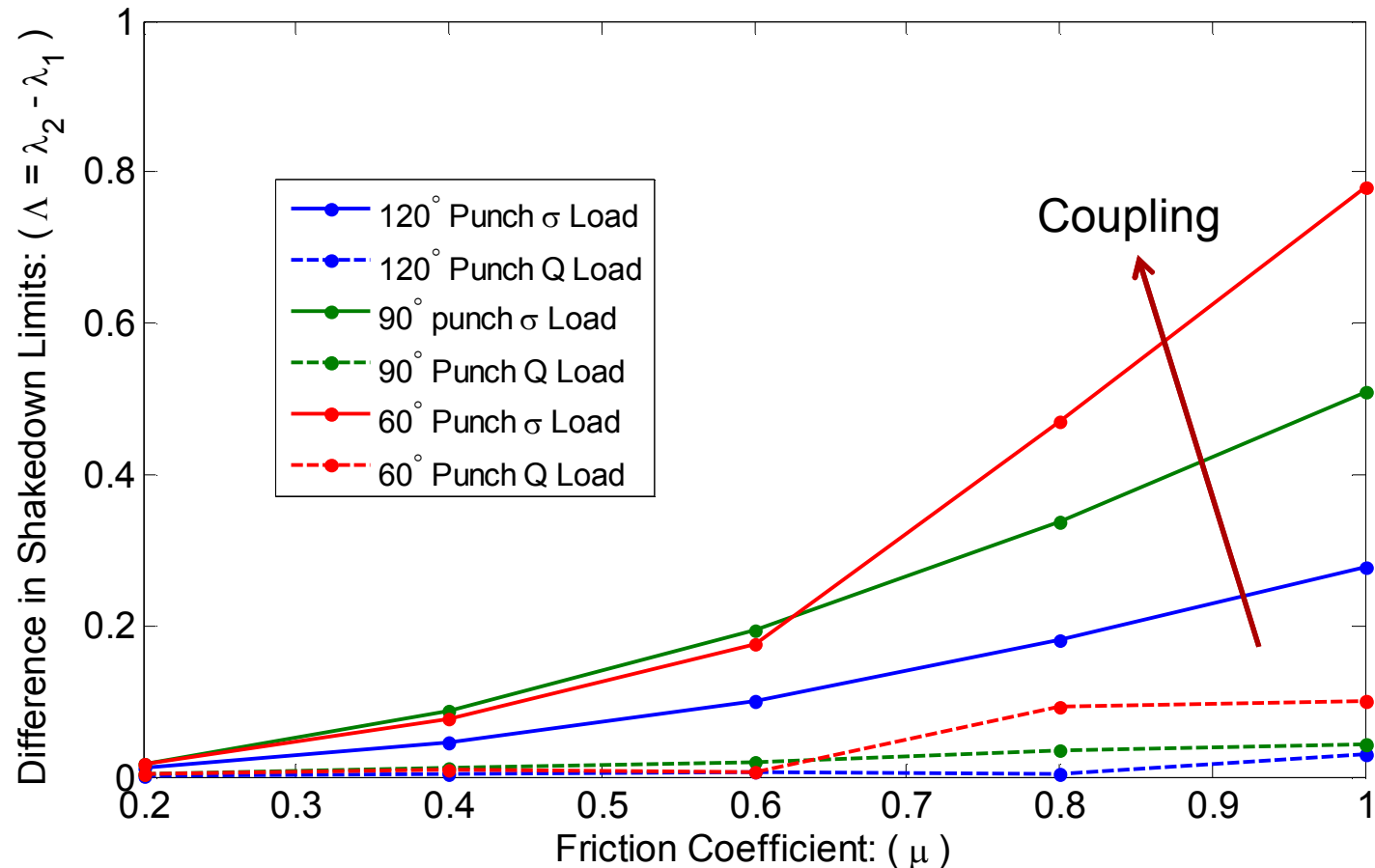
- The interfacial displacement shape is modified and the process is repeated

Determination of Shakedown Limit



- An exact solution is available for verification of the shakedown limit

Relation Between Shakedown Limits and Coupling



- Preliminary result for an 8 node interface
- Results for more nodes show similar trends

Summary and Conclusions

- A reduced order model is formulated using quasi-static reduction -> Very efficient transient simulations
- A scalar coupling metric is proposed to determine properties of shakedown *a priori* for the first time
- An approximate method for determining shakedown limits is demonstrated
- As material or domain dissimilarity is increased, coupling approaches an asymptote equal to the elastic-rigid case
- The coupling metric indicates the relative magnitude of the difference between the shakedown limits
- The coupling metric is hypothesized to indicate adequate mesh convergence of the full order system *a priori*