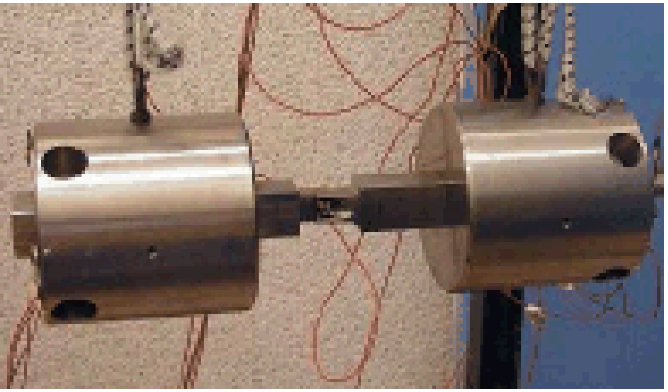
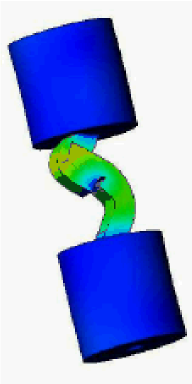
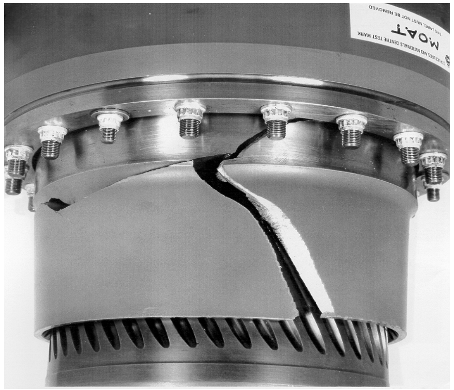
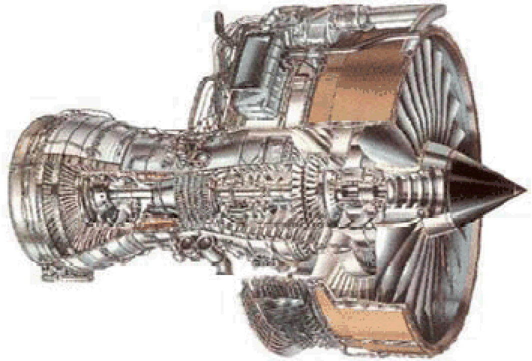

Reduced Order Modeling of Nonlinear Structures with Frictional Interfaces

M. R. Brake and D. J. Segalman, Sandia National Laboratories

P. Reuss and L. Gaul, the University of Stuttgart

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Overview

- How can we efficiently model systems with strong nonlinearities?
  - Frequency Based Substructuring
  - Discontinuous Basis Functions

- How can we compare two different nonlinear models?
  - Time histories
  - FRFs
  - Nonlinear normal modes

Frequency Based Substructuring

- Craig-Bampton reduction for the ***linear*** substructures

$$\begin{bmatrix} \boldsymbol{M}_{\mathrm{ff}} & \boldsymbol{M}_{\mathrm{fi}} \\ \boldsymbol{M}_{\mathrm{if}} & \boldsymbol{M}_{\mathrm{ii}} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{x}}_{\mathrm{f}} \\ \ddot{\boldsymbol{x}}_{\mathrm{i}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{\mathrm{ff}} & \boldsymbol{K}_{\mathrm{fi}} \\ \boldsymbol{K}_{\mathrm{if}} & \boldsymbol{K}_{\mathrm{ii}} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{\mathrm{f}} \\ \boldsymbol{x}_{\mathrm{i}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{f}_{\mathrm{i}} \end{bmatrix}$$

- Frequency Based Substructuring based on a Harmonic Balance Method approach

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{D}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} + \boldsymbol{f}_{\mathrm{T}}(\dot{\boldsymbol{x}}, \boldsymbol{x}) = \boldsymbol{f}_{\mathrm{exc}}$$

$$\boldsymbol{f}_{\mathrm{T}}(\dot{\boldsymbol{x}}, \boldsymbol{x}) \approx \boldsymbol{D}_{\mathrm{hbm}}\dot{\boldsymbol{x}} + \boldsymbol{K}_{\mathrm{hbm}}\boldsymbol{x}$$

- This yields a frequency domain equation that is solved linearly

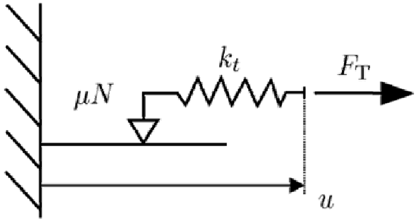
$$\left(\boldsymbol{L}_{\mathrm{p}}^{\mathrm{T}}(\boldsymbol{K}+\boldsymbol{B}_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{K}_{\mathrm{hbm}}\boldsymbol{B}_{\mathrm{f}})\boldsymbol{L}_{\mathrm{p}}+i\omega\boldsymbol{L}_{\mathrm{p}}^{\mathrm{T}}(\boldsymbol{D}+\boldsymbol{B}_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{D}_{\mathrm{hbm}}\boldsymbol{B}_{\mathrm{f}})\boldsymbol{L}_{\mathrm{p}}-\omega^2\boldsymbol{L}_{\mathrm{p}}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{L}_{\mathrm{p}}\right)\hat{\boldsymbol{x}} = \boldsymbol{L}_{\mathrm{p}}^{\mathrm{T}}\hat{\boldsymbol{f}}$$

Interface Flexibility and Damping

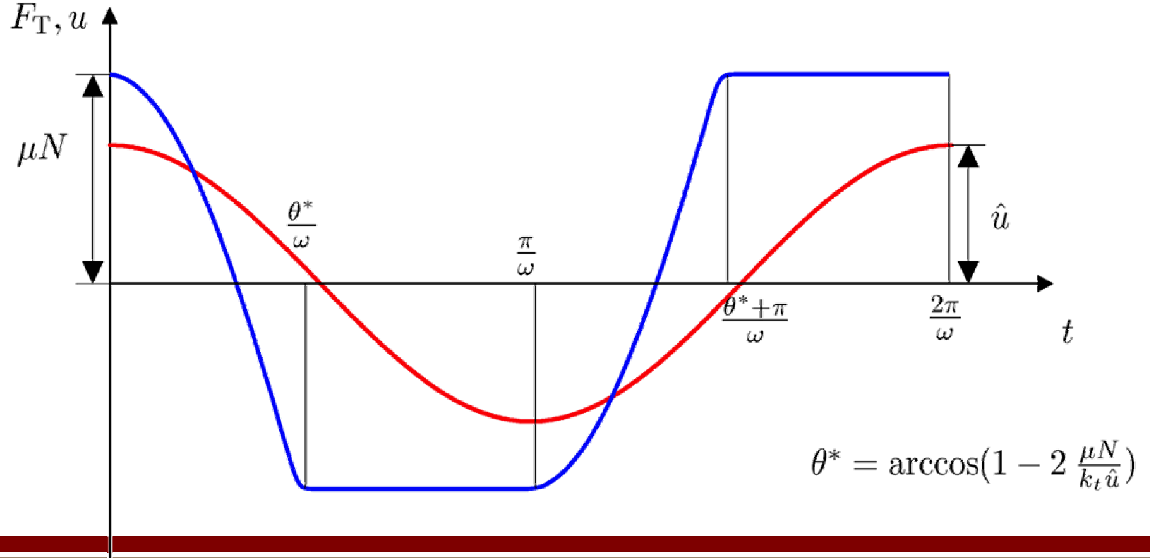
- Linearization of friction forces by Harmonic Balance Method

$$F_{\mathrm{T}}(u) \approx k_{\mathrm{hbm}} u + d_{\mathrm{hbm}} \dot{u}$$

- Jenkins friction element



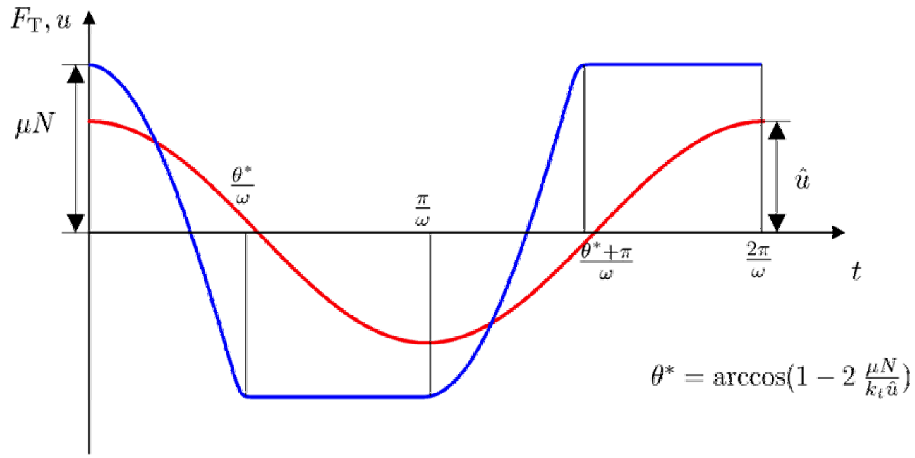
- Friction force for slip-stick



Harmonic Linearized Coefficients

- Normalized critical amplitude

$$u^* = \frac{\mu N}{k_{\mathrm{T}} \hat{u}}$$



- Sticking $u^* \geq 1$

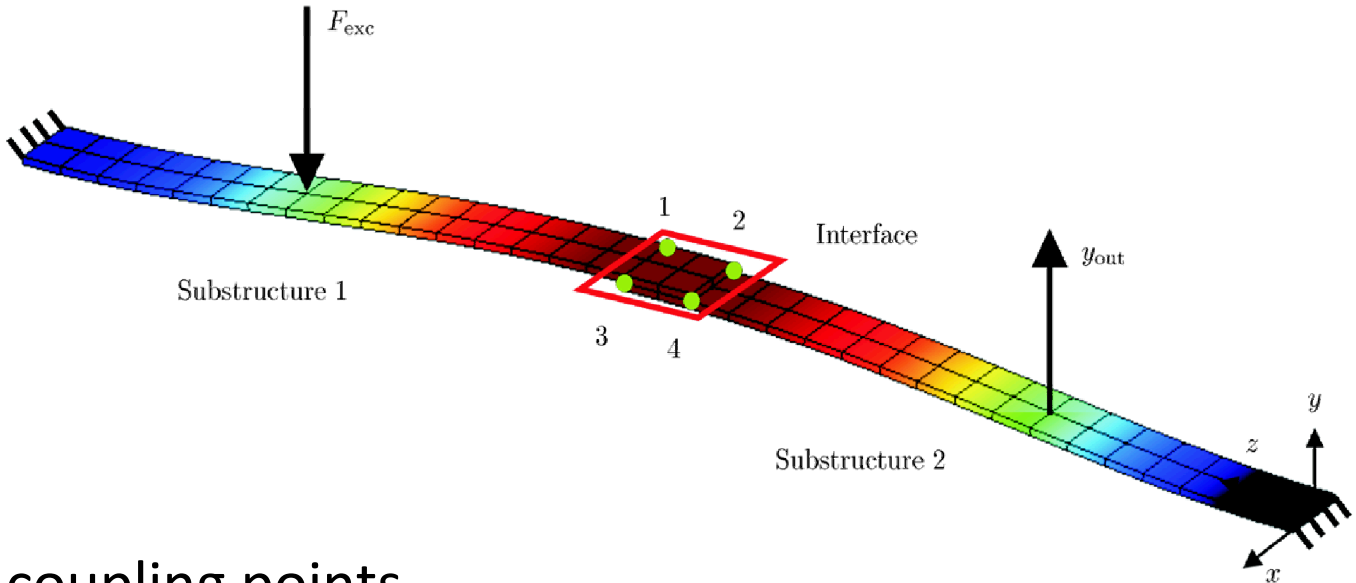
$$k_{\mathrm{hbm}} = k_{\mathrm{T}}$$
$$d_{\mathrm{hbm}} = 0$$

- Stick-slip $u^* < 1$

$$k_{\mathrm{hbm}} = \frac{k_{\mathrm{T}}}{\pi}\left(\arccos(1 - \frac{2}{u^*}) - \frac{2}{u^*}(1 - \frac{2}{u^*})\sqrt{u^* - 1}\right)$$
$$d_{\mathrm{hbm}} = \frac{4k_{\mathrm{T}}}{\pi\omega u^*}(1 - \frac{1}{u^*})$$

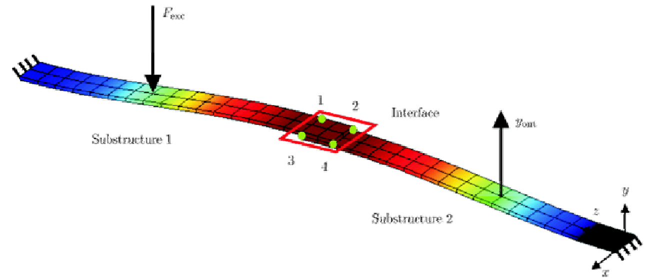
Numerical Test Cases

- Finite Element Model

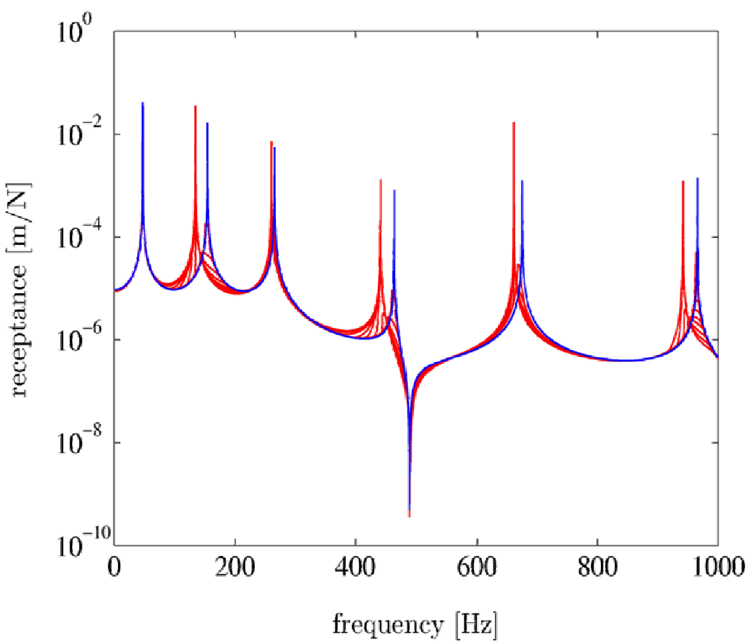


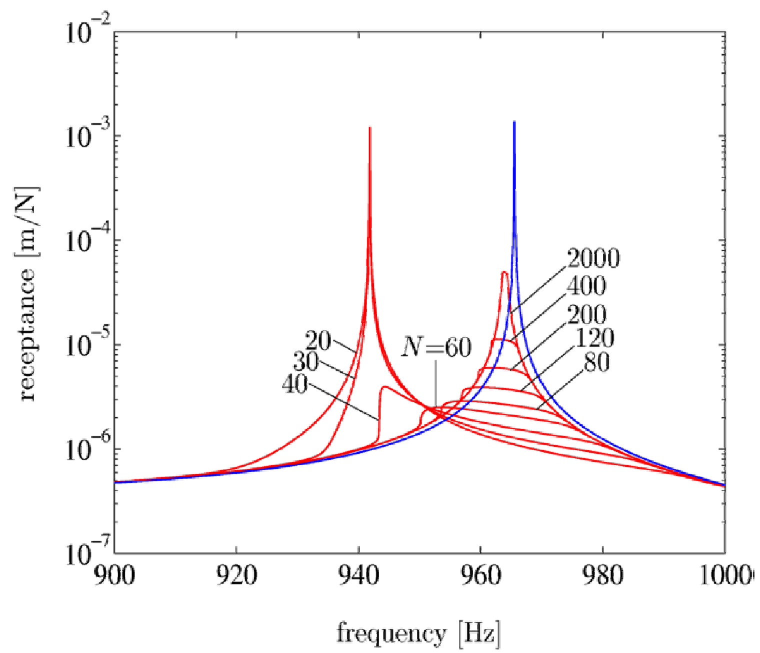
- 4 coupling points
- Excitation of bending modes

Numerical Test Cases



- FRF with excitation at substructre 1



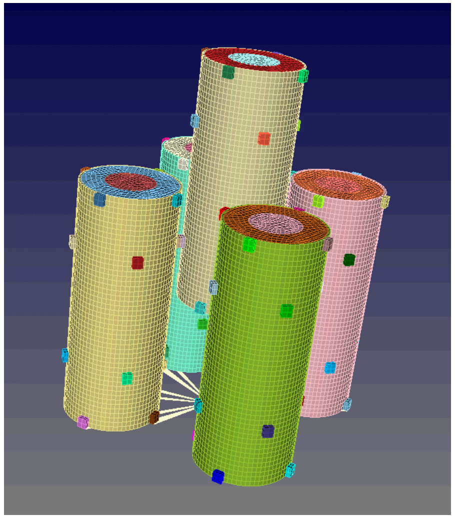


Discontinuous Basis Functions

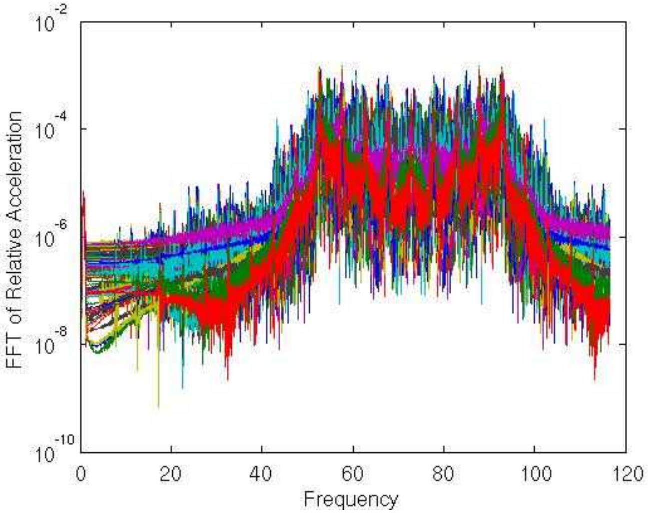
- Based on component mode synthesis
- CMS model augmented by a set of Milman Chu modes
- Only a few discontinuous basis functions needed for convergence,
- Can easily handle arbitrary nonlinearities

Assessment of Efficiency

- 5 Body Example: 5 linear substructures connected via Iwan elements
- Effective computational savings of 24,000x.

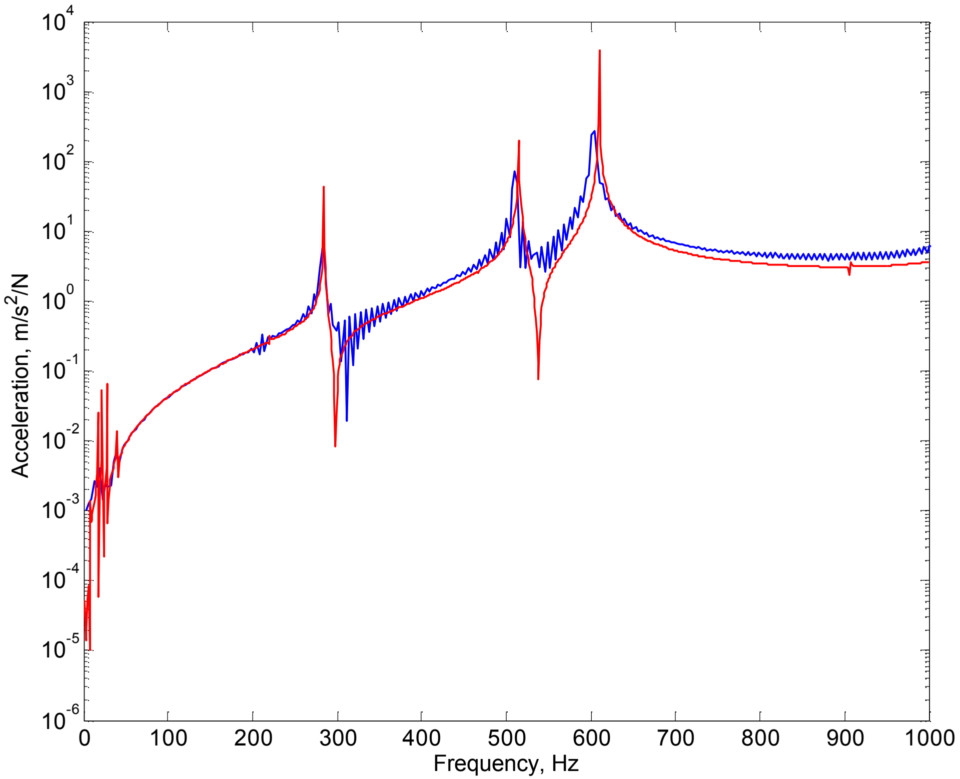


Time Step	Salinas	Reduced Order Model
1e-4	Still running	312 minutes
2e-4	Still running	156 minutes
4e-4	7347 minutes	79 minutes
8e-4	Unstable	39 minutes
2e-3	Unstable	19.5 minutes
4e-3	Unstable	Unstable



Comparison of Models

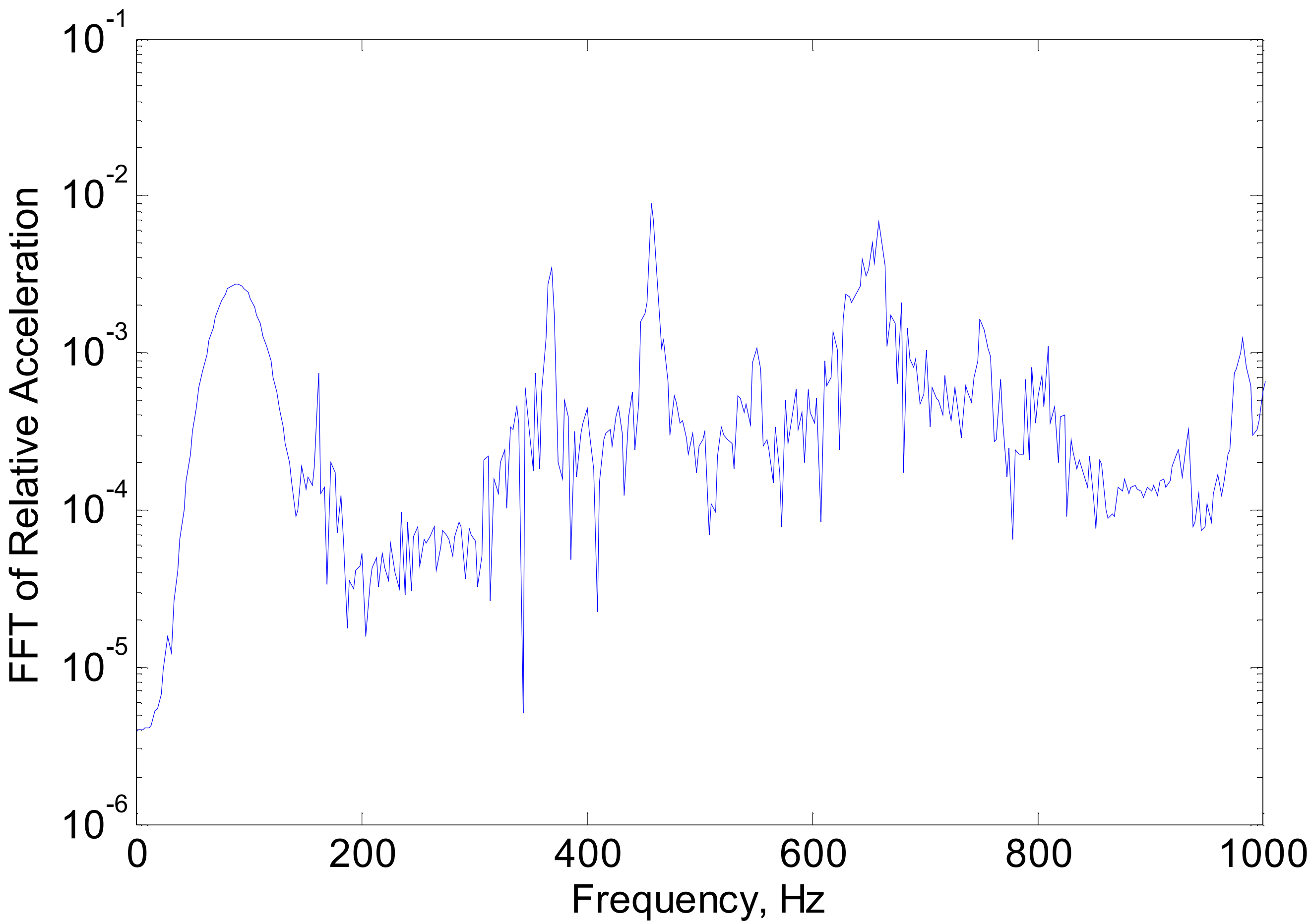
- Good agreement seen for linear springs connecting the interfaces



- Red: HBM, Blue: DBF.

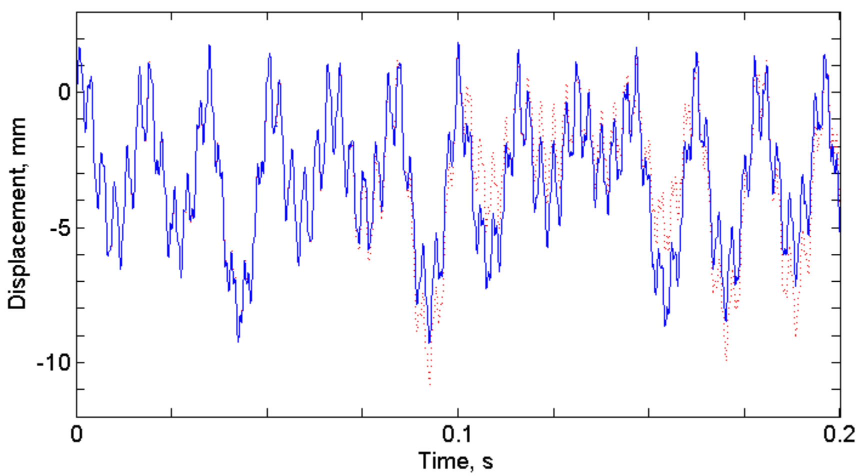
Comparison of Nonlinear Models in Progress

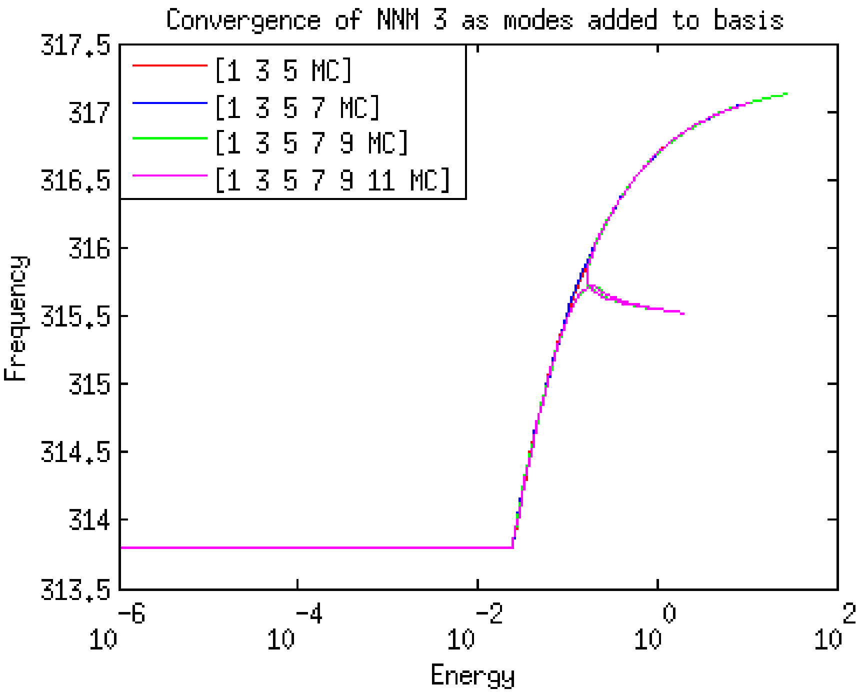
- Sample result for a shock response with Iwan elements using DBF:



Comparison of Nonlinear Systems

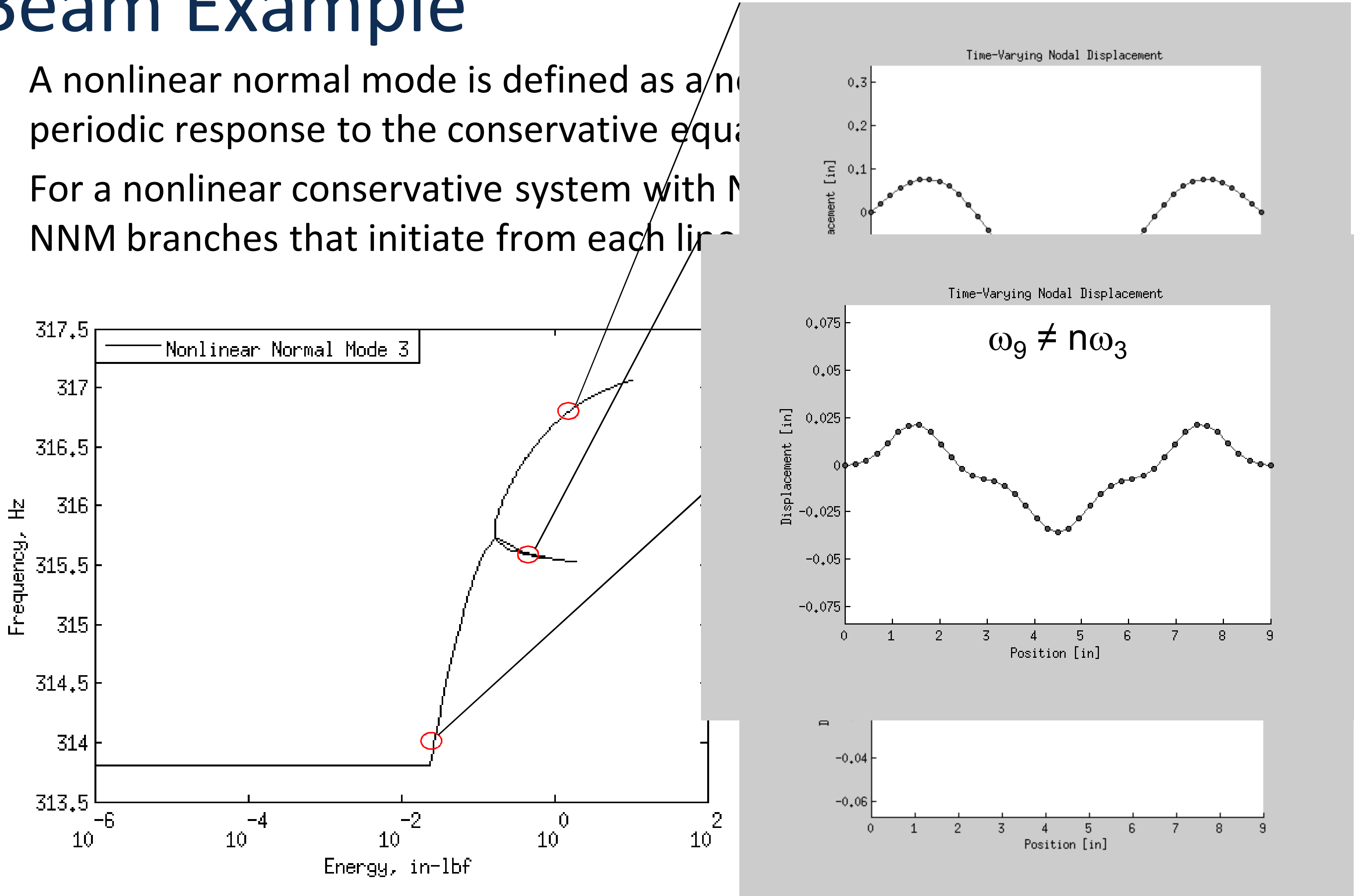
- How do you compare two different models of the same nonlinear system?
  - Time histories, dissipation, strain energy, L_2 norm, etc.
  - Use of nonlinear normal modes to measure convergence





Nonlinear Normal Modes Applied to a Beam Example

- A nonlinear normal mode is defined as a n
periodic response to the conservative equa
- For a nonlinear conservative system with N
NNM branches that initiate from each lin



Summary

- Two methods compared for modeling nonlinear systems
- Frequency based substructuring very efficient when excitation and nonlinearities are able to be expressed as harmonic terms
- Method of discontinuous basis functions better suited for transient simulations and arbitrary nonlinearities
- Both methods approximately 24,000x faster than full FEA model