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# Diffusion Monte Carlo calculations of xenon melting under pressure

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# Xenon: Rare gas with rare behavior under pressure

- **Closed shell insulator at ambient conditions**
- **Under static compression**
  - FCC  $\rightarrow$  HCP Phase transition
  - Isostructural insulator to metal transition
- **Can form compounds with  $H_2$  under pressure**
  - Somayazulu et al. Nature Chemistry 2, 50 (2010)
- **Liquid phase may exhibit anomalous behavior**
  - Very narrow temperature range at ambient pressure
  - Potentially flat melt curve at moderate pressures

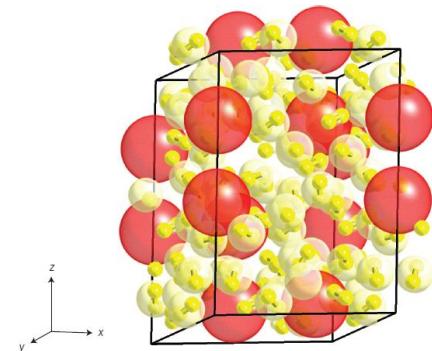
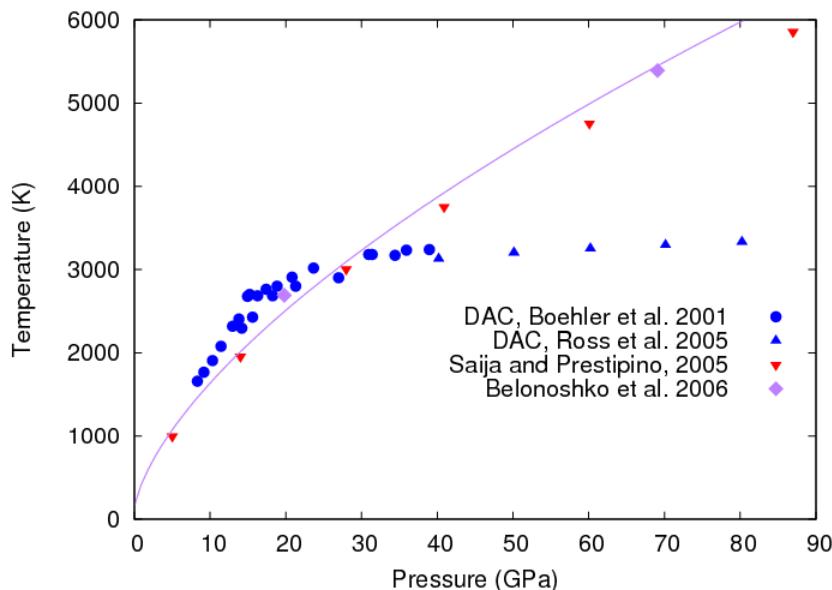
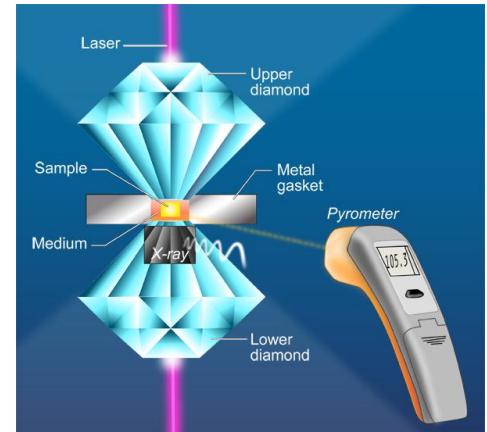


Figure 3 | Model structure of  $Xe(H_2)_7$ . The xenon atoms are surrounded by dumbbell-shaped, freely rotating hydrogen molecules represented by the spherical shells.



# Xe melting: disagreement between DAC and DFT

- Disagreement between melting under pressure between DAC, ab initio calculations and shock measurements is common
  - See for example Ta, Fe, MgO and Xe
- Many sources of uncertainty
  - DAC
    - Anisotropic Stress, Reactivity, Ambiguous Phase Assignment
  - Shock
    - Temperature measurements
  - Ab Initio
    - DFT Approximations, convergence
- Xe : Demanding for ab initio
  - Van der Waals effects in DFT
  - Low number density requires large simulation cells
  - But the Hugoniot was calculated and measured with great accuracy and agreement
    - Root *et al. PRL 105, 085501 (2010)*
    - Constrained EOS at high temperatures and pressures

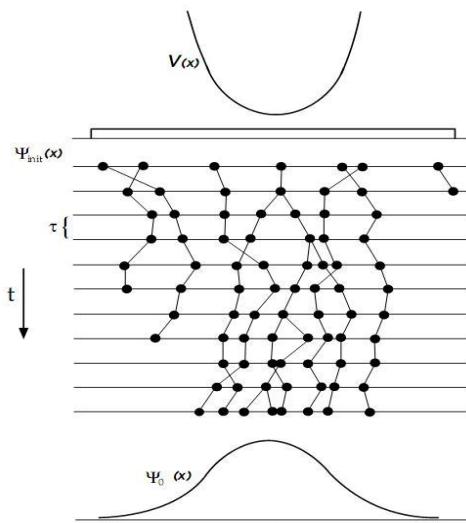


Klug, Physics. 3, 52 (2010)

# Assessing Quantum MD melting with Quantum Monte Carlo

- **Quantum Monte Carlo is an appealing alternative**

- Use stochastic projection to solve many body Schrodinger equation exactly
- Only uncontrolled approximation, fixed node approximation, does not involve Hamiltonian (interactions)
- Limitations
  - *Significantly more expensive than DFT*
  - *Forces not currently available*
  - *Only norm-conserving pseudopotentials*

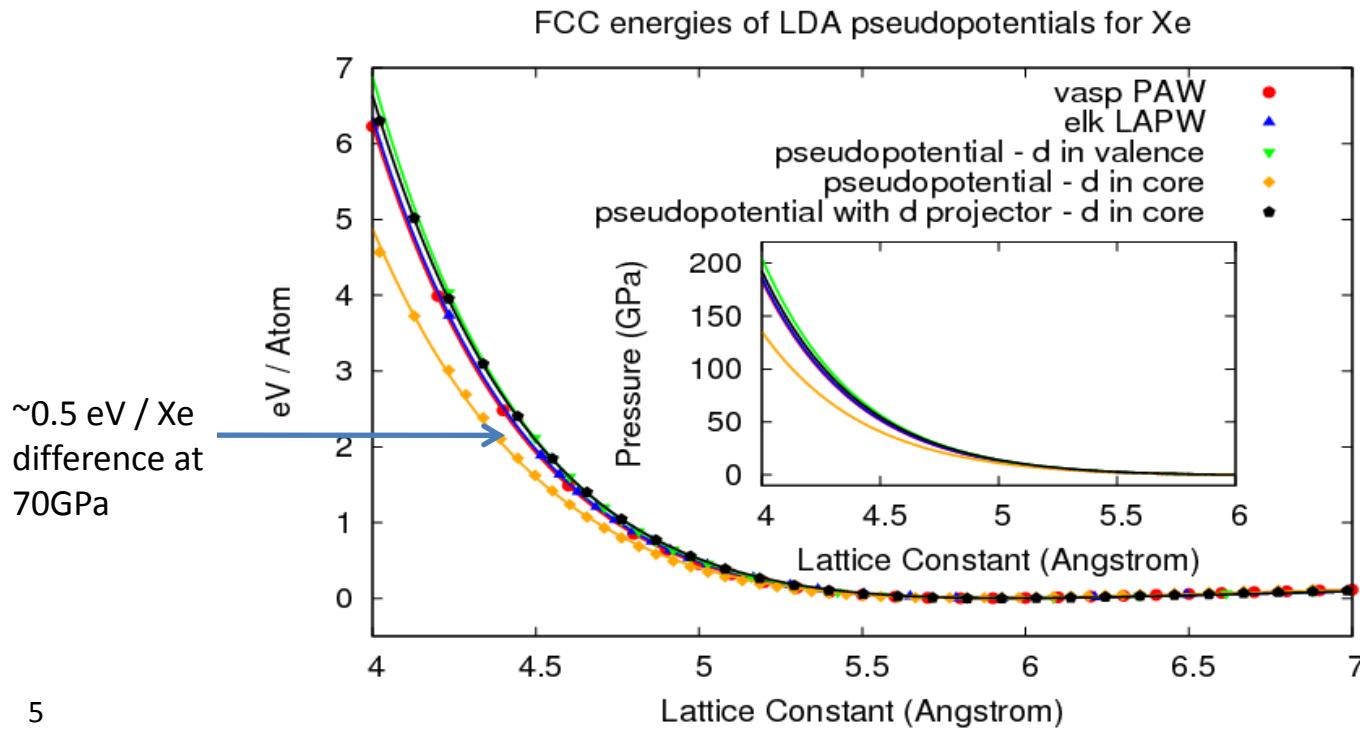


J. Needs, M. D. Towler, N. D. Drummond,  
and P. Lopez-Rios, Casino Version 2.2  
User Manual, University of Cambridge,  
Cambridge (2008)

- **Use snapshots from quantum MD calculations**
  - Assess the ability of DFT to determine proper energy landscape
  - Use thermodynamic integration approach of Sola et al to estimate change in melting temperature
    - Sola and Alfe, PRL. 130, 078501 (2009)
  - Assume electronic excitations are treated appropriately within DFT

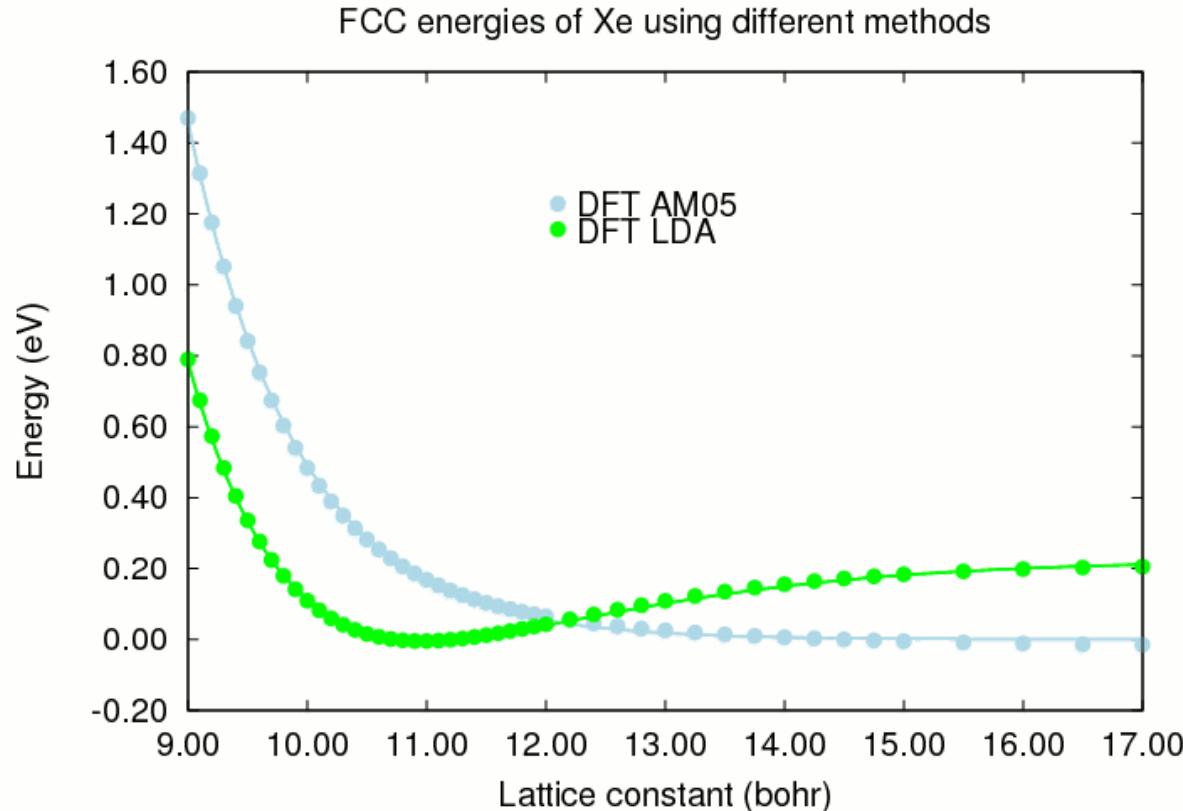
# Importance of pseudopotential in QMC

- Must strip out core electrons to make problem computationally tractable
  - Core does not contribute to chemistry at these pressures
- Validated norm conserving Xe pseudopotentials not widely available
- D-states well removed from valence, but d-projector is crucial
  - Increasing d-hybridization suggested as cause of flat melt line
    - Ross *et al.* PRL 95. 257801 (2005)



# Fixed node approximation and DFT Functional

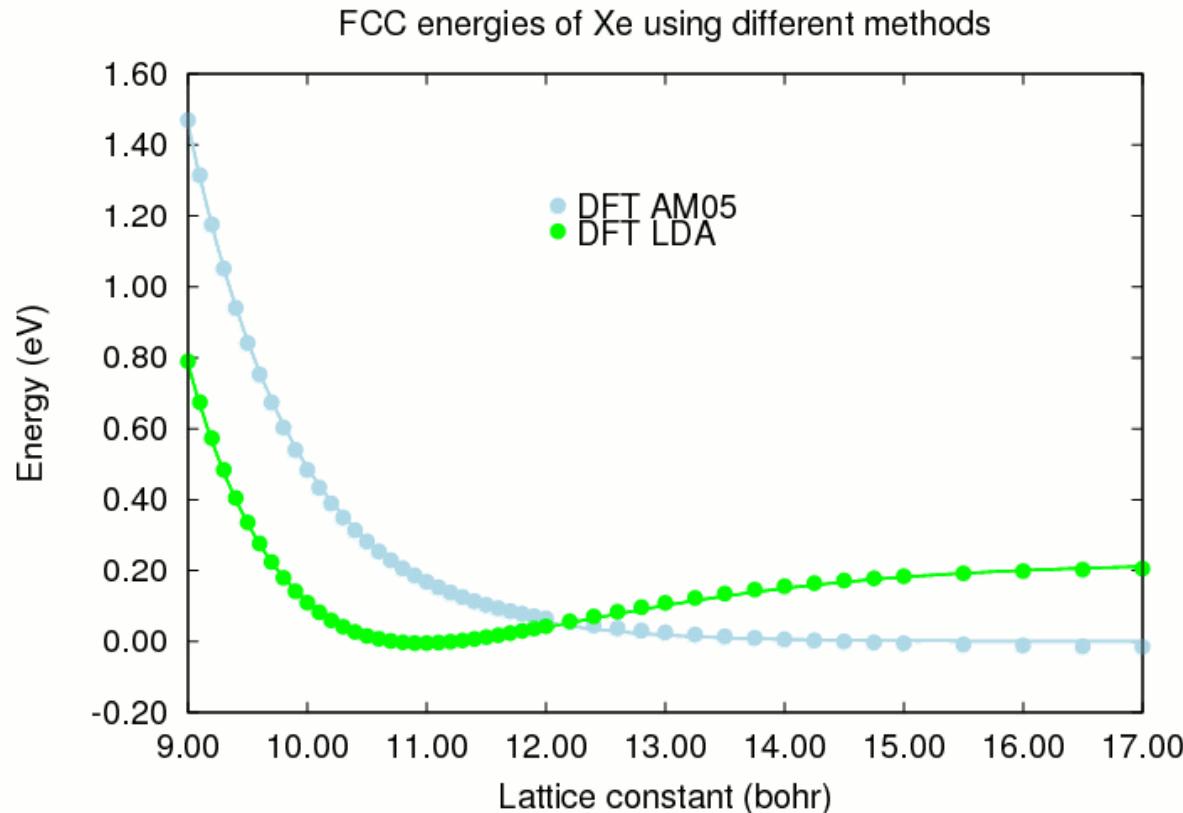
- FCC equation of state
  - LDA → no long range correlation, but self interaction in low density regions
  - AM05 → subsystem based functional, van der Waals is completely absent



# Fixed node approximation and DFT Functional

- **FCC equation of state**

- LDA → no long range correlation, but self interaction in low density regions
- AM05 → subsystem based functional, van der Waals is completely absent
- DMC with nodes and pseudopotentials taken from above calculations
  - *Very small dependence on DFT trial wavefunction*



# Thermodynamic Integration approach to melting

- Use thermodynamic integration to calculate relative change in Helmholtz free energy going from DFT to QMC

$$\Delta F = \int_0^1 d\lambda \langle \Delta U \rangle_\lambda \approx \langle \Delta U \rangle_{\lambda=0} - \frac{1}{2k_B T} \langle (\Delta U - \langle \Delta U \rangle_{\lambda=0})^2 \rangle_{\lambda=0}$$

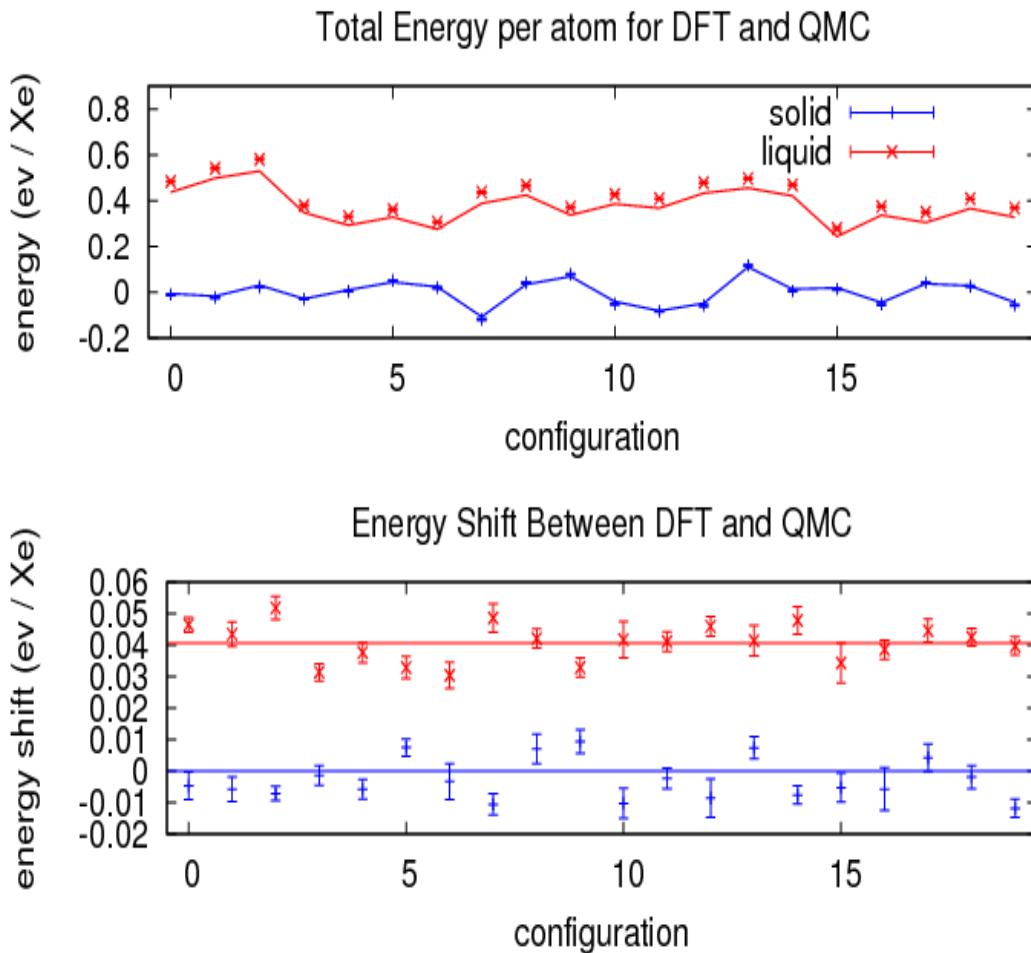
- The change in melting temperature between DFT and DMC is

$$\Delta T_m \approx \frac{\Delta G^{ls}}{S_{DFT}^{ls}} \quad \Delta G \approx \Delta F - V\Delta p^2 / 2K_T$$

- Assume that difference in dynamics between DFT and DMC is small (fluctuation terms above are small)
- Take two long molecular dynamics calculations at coexistence with electronic temperature equal to ionic temperature
- Extract snapshots from liquid and solid phase and calculate change in melting temperature

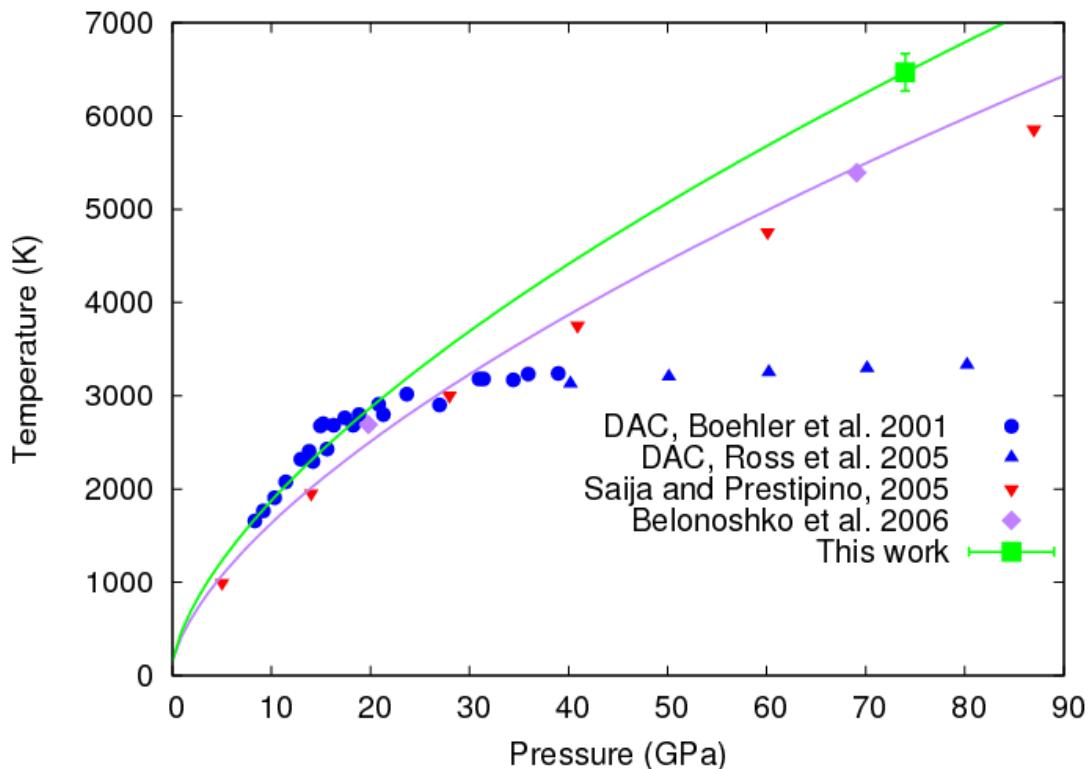
# DMC calculations of sold and liquid snapshots

- 20 snapshots from 108 atom solid and liquid LDA runs at 6000 K
- Fluctuations of QMC energy about LDA energies are small
- Energy difference between liquid and solid  $0.0406 \pm 0.0027$  eV / Xe greater in DMC
- Assuming a rigid shift of the enthalpy curves  $\rightarrow$  Increase in melting temperature by  $470 \pm 30$  K at 73 Gpa
- Magnitude of correction similar to Sola and Alfe. PRL 130, 078501 (2009)



# Conclusions

- Diffusion Monte Carlo can accurately treat Xe under pressure
  - Pseudopotential Approximation is small
  - Fixed node approximation is likely a small error
- Relative energies from DFT/LDA are accurate near 1 Mbar
- Errors in total energies from DFT/LDA will increase melting temperature



# QMC Calculation Details

- Trial wavefunctions for qmcpack using hybrid LMTO like and b-spline representation - 22 GB of memory per node for the wavefunction
- Slater-Jastrow form used with independent one body jastrow factors for each Xe and an overall two body jastrow
- Each DMC calculation required 15,000 CPU hours
- Timestep (0.01 Ha) converged to within 0.0001 Ha / Xe
- Finite size correction using MPC and Chiesa corrections