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Diffusion Monte Carlo calculations of xenon melting under pressure

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Xenon: Rare gas with rare behavior under pressure

- Closed shell insulator at ambient conditions
- Under static compression
 - FCC -> HCP Phase transition
 - Isostructural insulator to metal transition
- Can form compounds with H_2 under pressure
 - Somayazulu et al. Nature Chemistry 2, 50 (2010)
- Liquid phase may exhibit anomalous behavior
 - Very narrow temperature range at ambient pressure
 - Potentially flat melt curve at moderate pressures

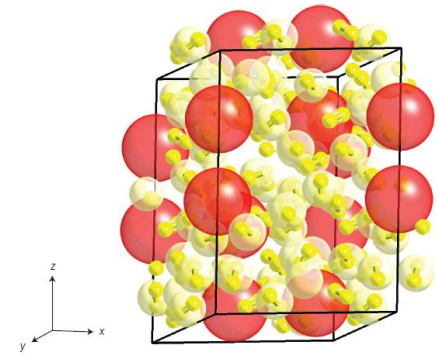
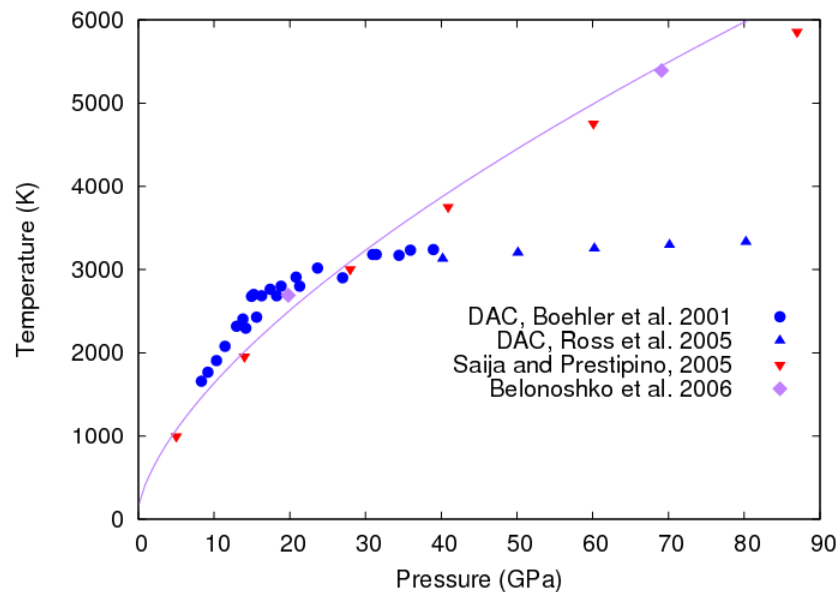
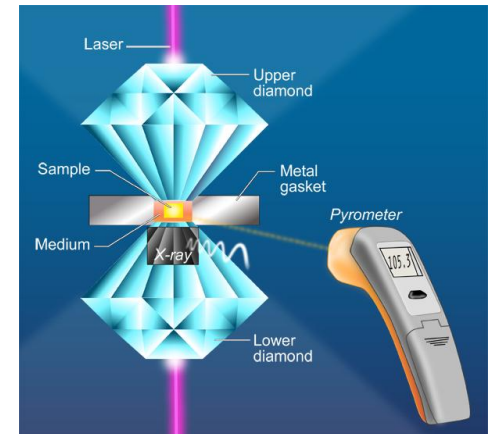


Figure 3 | Model structure of $\text{Xe}(\text{H}_2)_7$. The xenon atoms are surrounded by dumbbell-shaped, freely rotating hydrogen molecules represented by the spherical shells.



Xe melting: disagreement between DAC and DFT

- Disagreement between melting under pressure between DAC, ab initio calculations and shock measurements is common
 - See for example Ta, Fe, MgO and Xe
- Many sources of uncertainty
 - DAC
 - *Anisotropic Stress, Reactivity, Ambiguous Phase Assignment*
 - Shock
 - *Temperature measurements*
 - Ab Initio
 - *DFT Approximations, convergence*
- Xe : Demanding for ab initio
 - Van der Waals effects in DFT
 - Low number density requires large simulation cells
 - But the Hugoniot was calculated and measured with great accuracy and agreement
 - *Root et al. PRL **105**, 085501 (2010)*
 - *Constrained EOS at high temperatures and pressures*

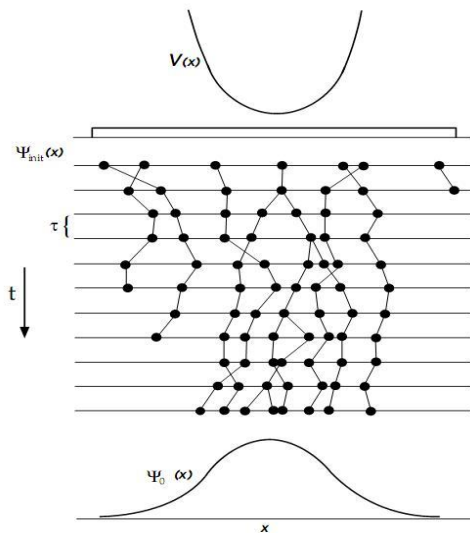


Klug, Physics. **3**, 52 (2010)

Assessing Quantum MD melting with Quantum Monte Carlo

- **Quantum Monte Carlo is an appealing alternative**

- Use stochastic projection to solve many body Schrodinger equation exactly
- Only uncontrolled approximation, fixed node approximation, does not involve Hamiltonian (interactions)
- **Limitations**
 - *Significantly more expensive than DFT*
 - *Forces not currently available*
 - *Only norm-conserving pseudopotentials*



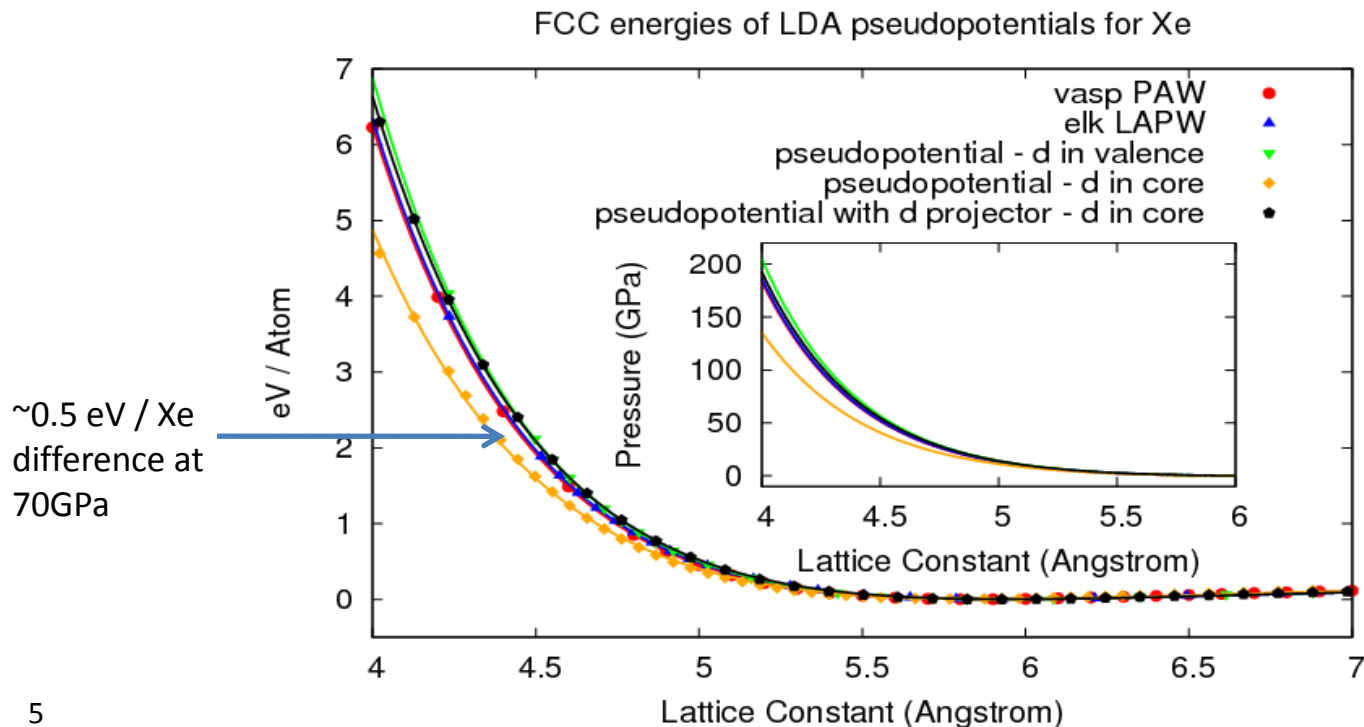
J. Needs, M. D. Towler, N. D. Drummond,
and P. Lopez-Rios, Casino Version 2.2
User Manual, University of Cambridge,
Cambridge (2008)

- **Use snapshots from quantum MD calculations**

- Assess the ability of DFT to determine proper energy landscape
- Use thermodynamic integration approach of Sola et al to estimate change in melting temperature
 - Sola and Alfe, PRL. 130, 078501 (2009)
- Assume electronic excitations are treated appropriately within DFT

Importance of pseudopotential in QMC

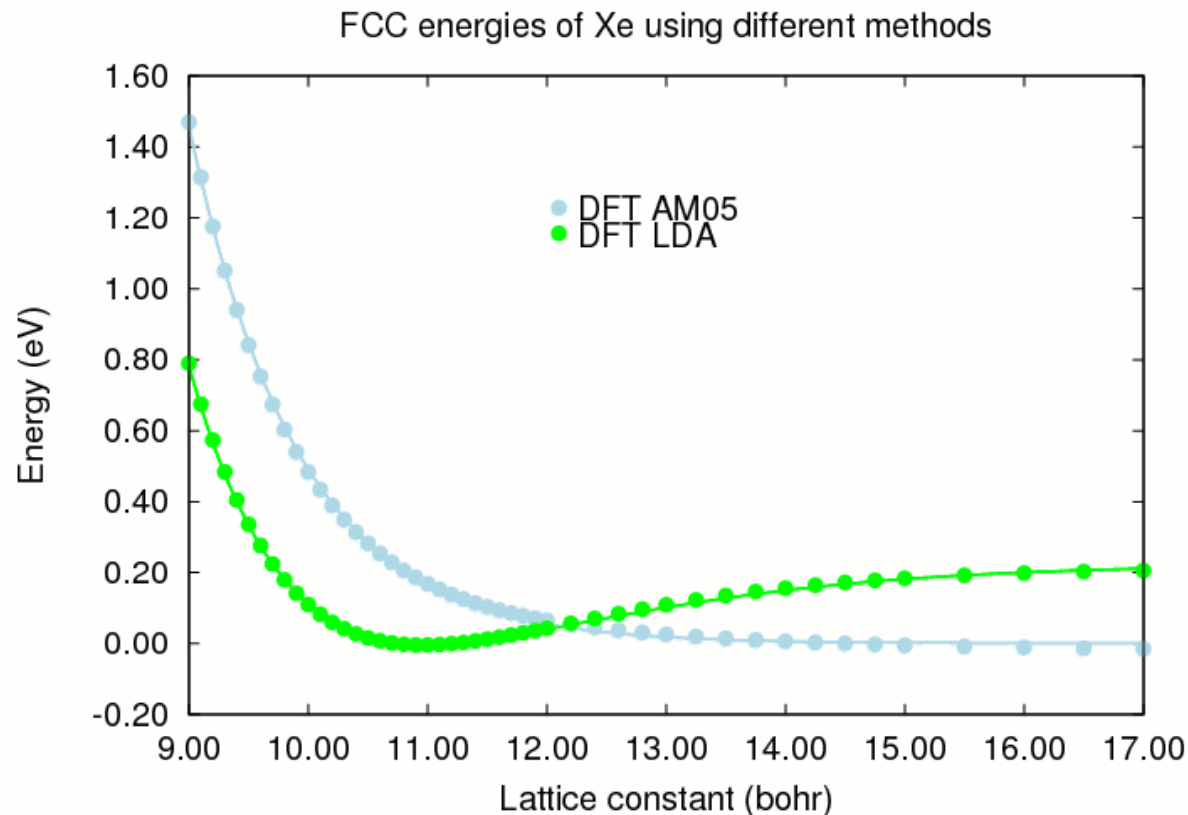
- **Must strip out core electrons to make problem computationally tractable**
 - Core does not contribute to chemistry at these pressures
- **Validated norm conserving Xe pseudopotentials not widely available**
- **D-states well removed from valence, but d-projector is crucial**
 - Increasing d-hybridization suggested as cause of flat melt line
 - *Ross et al. PRL 95. 257801 (2005)*



Fixed node approximation and DFT Functional

- **FCC equation of state**

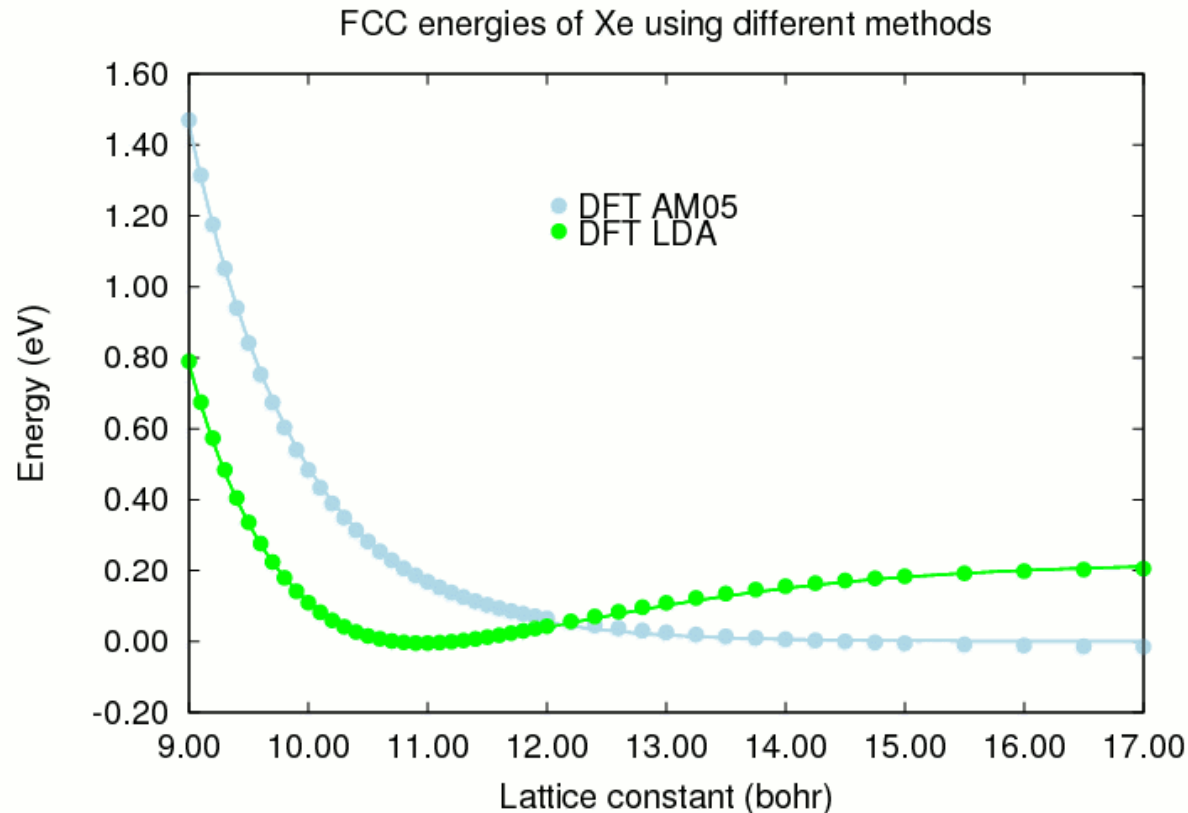
- LDA → no long range correlation, but self interaction in low density regions
- AM05 → subsystem based functional, van der Waals is completely absent



Fixed node approximation and DFT Functional

- **FCC equation of state**

- LDA → no long range correlation, but self interaction in low density regions
- AM05 → subsystem based functional, van der Waals is completely absent
- DMC with nodes and pseudopotentials taken from above calculations
 - *Very small dependence on DFT trial wavefunction*



Thermodynamic Integration approach to melting

- Use thermodynamic integration to calculate relative change in Helmholtz free energy going from DFT to QMC

$$\Delta F = \int_0^1 d\lambda \langle \Delta U \rangle_\lambda \approx \langle \Delta U \rangle_{\lambda=0} - \frac{1}{2k_B T} \langle (\Delta U - \langle \Delta U \rangle_{\lambda=0})^2 \rangle_{\lambda=0}$$

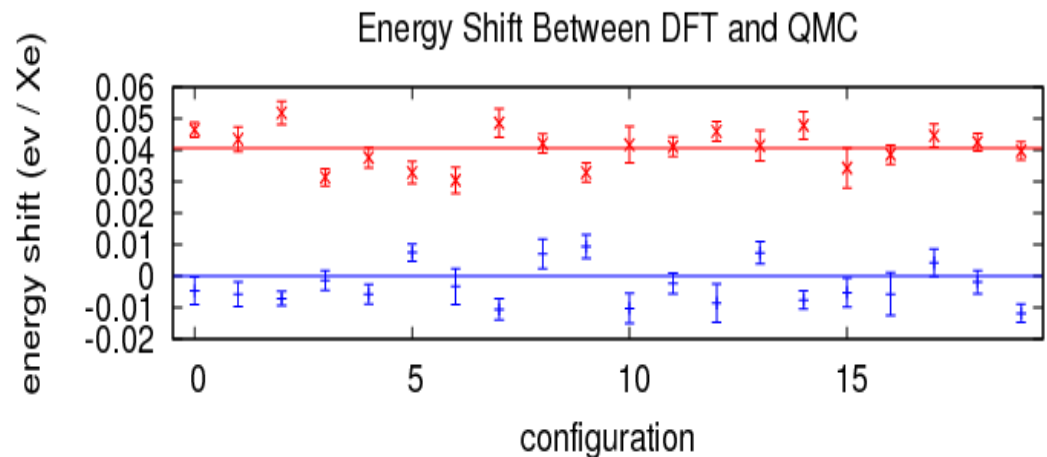
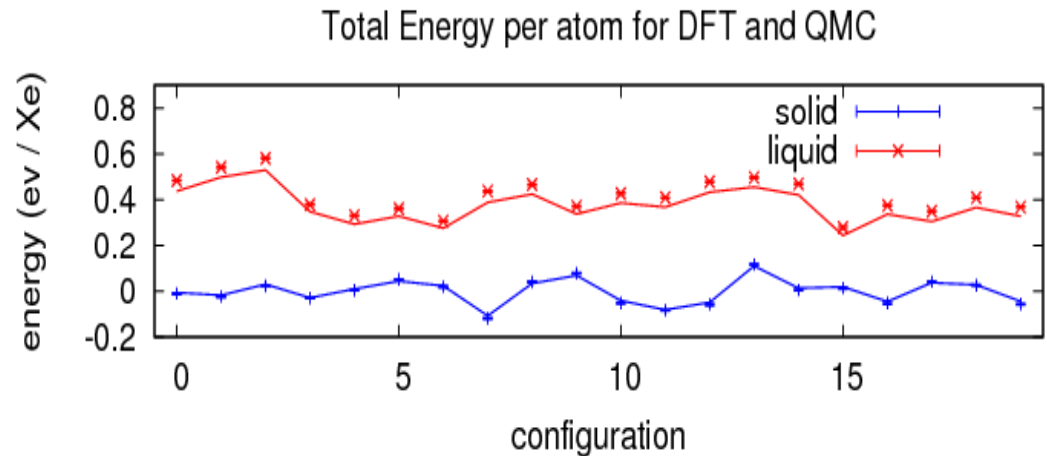
- The change in melting temperature between DFT and DMC is

$$\Delta T_m \approx \frac{\Delta G^{ls}}{S_{DFT}^{ls}} \quad \Delta G \approx \Delta F - V \Delta p^2 / 2 K_T$$

- Assume that difference in dynamics between DFT and DMC is small (fluctuation terms above are small)
- Take two long molecular dynamics calculations at coexistence with electronic temperature equal to ionic temperature
- Extract snapshots from liquid and solid phase and calculate change in melting temperature

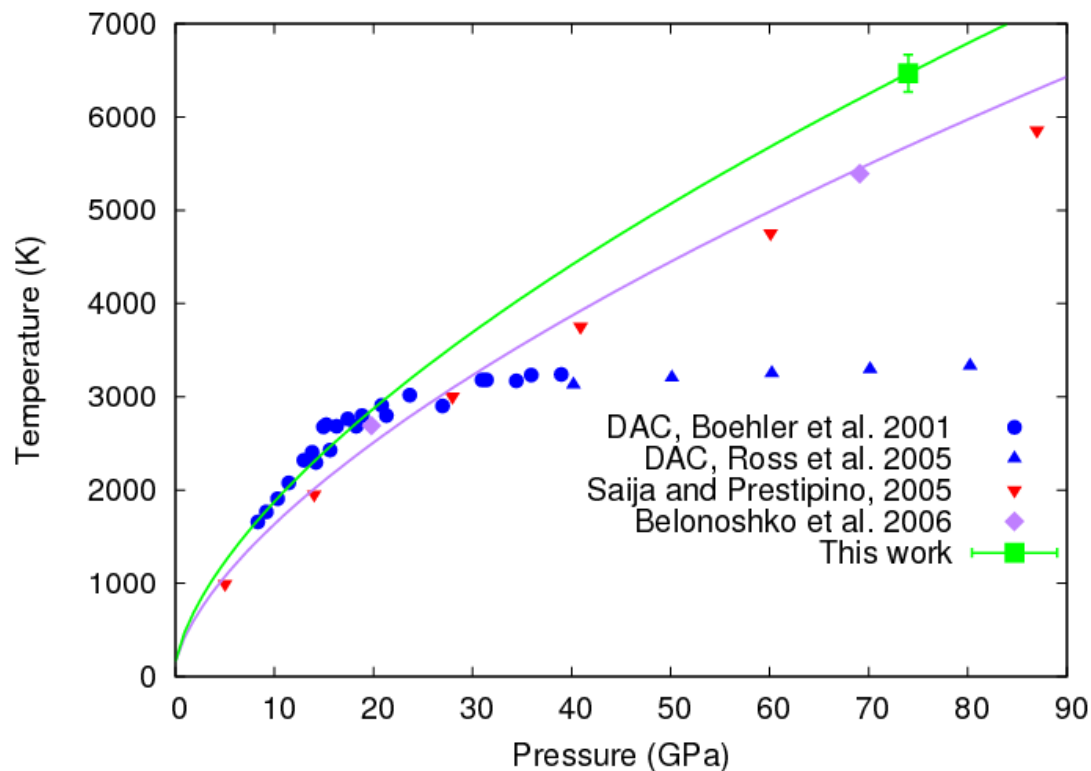
DMC calculations of solid and liquid snapshots

- 20 snapshots from 108 atom solid and liquid LDA runs at 6000 K
- Fluctuations of QMC energy about LDA energies are small
- Energy difference between liquid and solid 0.0406 ± 0.0027 eV / Xe greater in DMC
- Assuming a rigid shift of the enthalpy curves \rightarrow **Increase** in melting temperature by 470 ± 30 K at 73 GPa
- Magnitude of correction similar to Sola and Alfe. PRL **130**, 078501 (2009)



Conclusions

- **Diffusion Monte Carlo can accurately treat Xe under pressure**
 - Pseudopotential Approximation is small
 - Fixed node approximation is likely a small error
- **Relative energies from DFT/LDA are accurate near 1 Mbar**
- **Errors in total energies from DFT/LDA will increase melting temperature**



QMC Calculation Details

- Trial wavefunctions for qmcpack using hybrid LMTO like and b-spline representation - 22 GB of memory per node for the wavefunction
- Slater-Jastrow form used with independent one body jastrow factors for each Xe and an overall two body jastrow
- Each DMC calculation required 15,000 CPU hours
- Timestep (0.01 Ha) converged to within 0.0001 Ha / Xe
- Finite size correction using MPC and Chiesa corrections