SAND2011-2543C

Calculating Path-Dependent Travel Time Prediction Variance and Covariance for the SALSA3D Global Tomographic P-Velocity Model

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Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Project Objective

Establish that locating seismic events using a high-fidelity 3D model of the seismic velocity structure of the Earth leads to locations that are more accurate and precise.

Previous Work

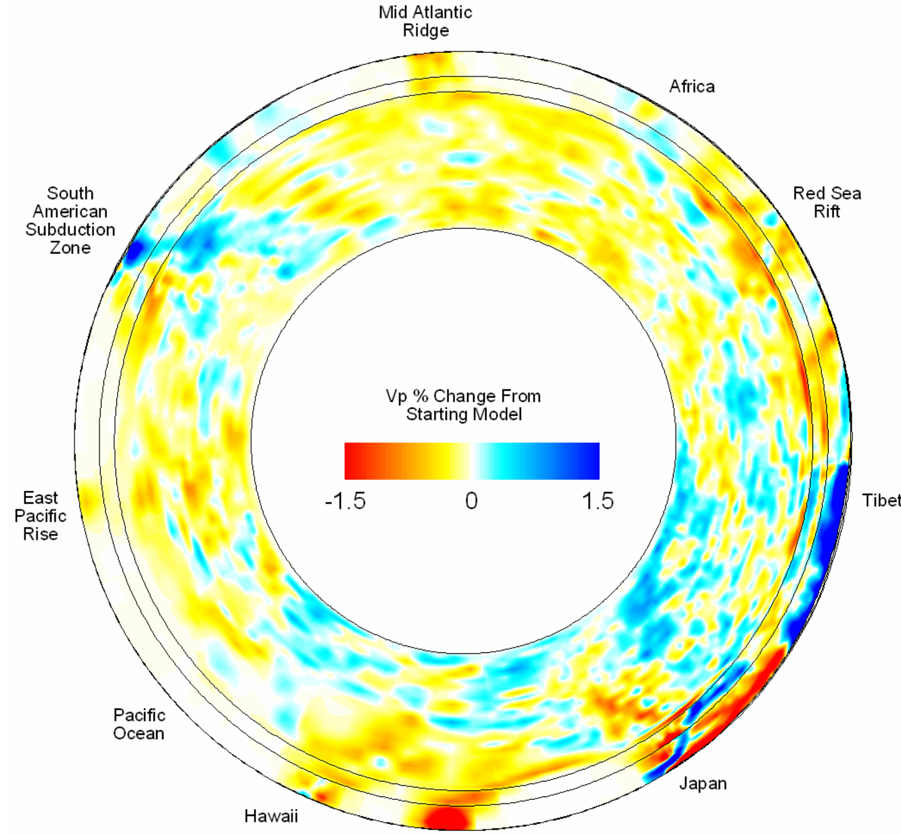
Developed a global, seamless, tomographic 3D earth model for the P wave speed in the Earth's mantle

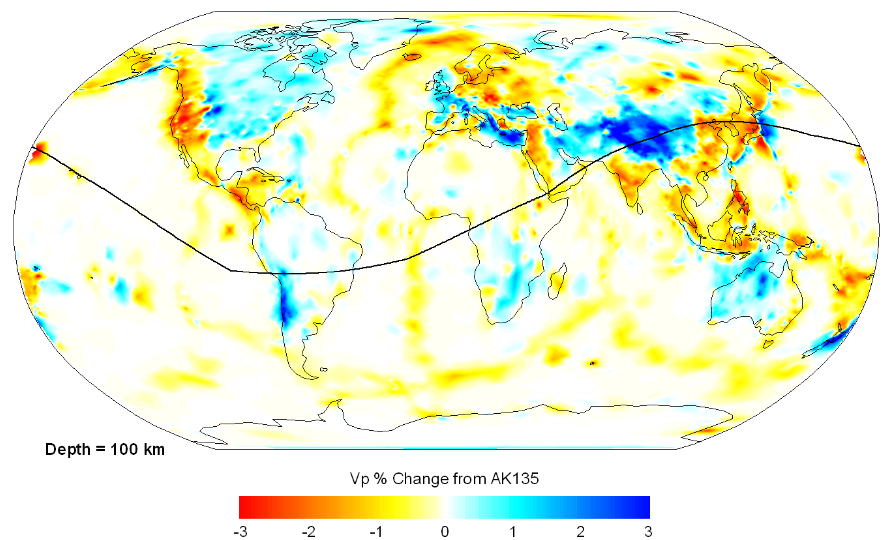
Demonstrated improved P/Pn travel time predictions and event locations

Developed a simple *distance-dependent* prediction uncertainty model

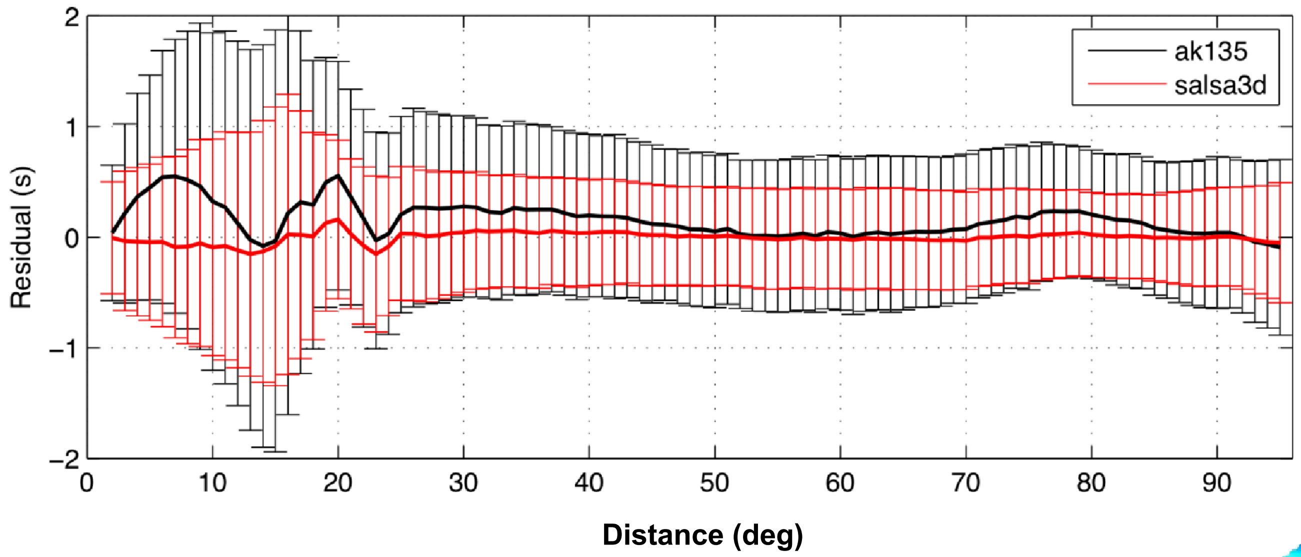
Current Focus

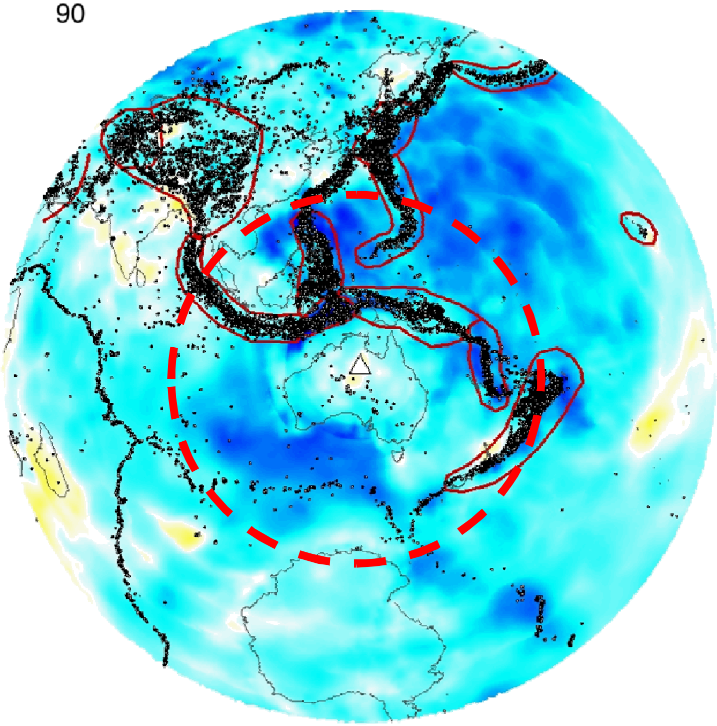
Develop realistic, *path-dependent* travel time uncertainty estimates for P and Pn





Distance Dependent Prediction Uncertainty





- Uncertainty estimates depend only on distance; independent of station, azimuth

Path-Dependent Travel Time Uncertainty

1. Travel Time Tomography Solution:

$$G\Delta s = \Delta d \qquad \text{where} \qquad G = \begin{bmatrix} A \\ \alpha L \end{bmatrix}$$

2. Model Resolution Matrix:

$$R = \left(G^T G\right)^{-1} A^T A$$

3. Model Covariance Matrix:

$$C_{\Delta s} = \left(G^T G\right)^{-1} A^T C_{\Delta d}\, A \left(G^T G\right)^{-1}$$

4. Travel Time Uncertainty:

$$\sigma^2_{TT_{ij}} = P_i\, C_{\Delta s}\, P_j^T = \int_{P_i} \int_{P_j} C_{\Delta s}\, dl_j\, dl_i$$

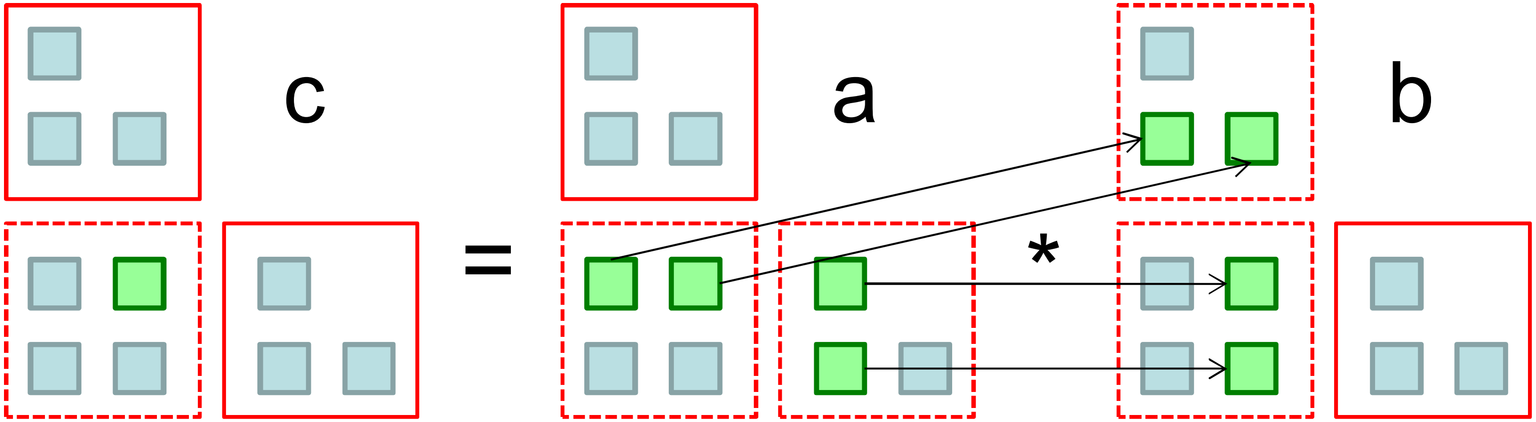
Model Covariance Calculation

Seek to solve for resolution and model covariance using direct methods (Cholesky Decomposition, Forward and Backward Substitution) to obtain inverse $(G^T G)^{-1}$

Current 3D model node count is less than ½ million nodes … implies nearly 1TB storage (memory / disk) for the model covariance/resolution matrix

Use Out-Of-Core (OOC) Cholesky, Forward and Backward Substitution to obtain inverse, OOC matrix multiply for model covariance/resolution

$$c_{i,j} = \sum_{k=0}^{n-1} a_{i,k} b_{k,j}$$



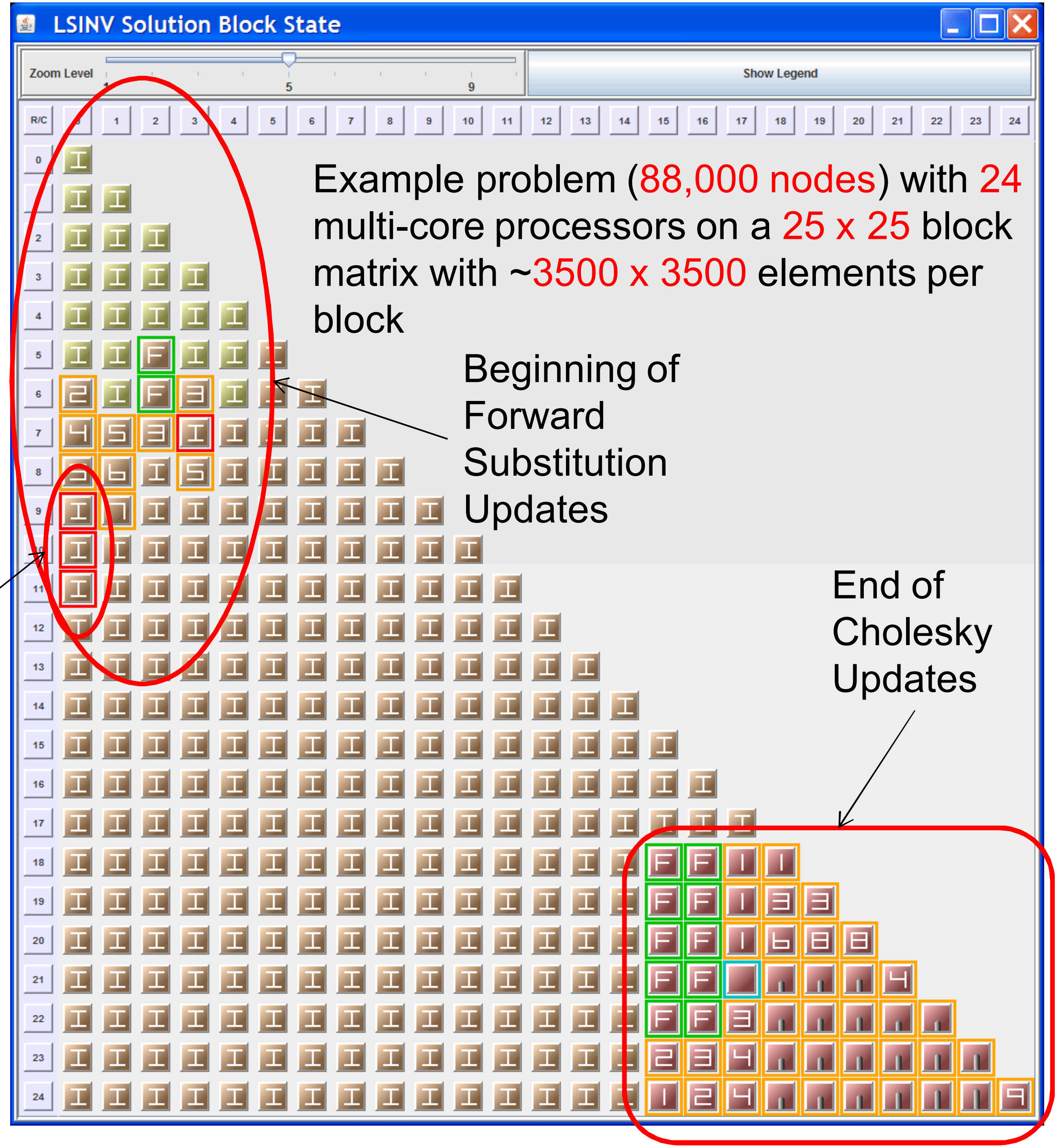
Total solution involves 425,000 nodes

utilizing ~350 multi-core processors

on a 100 x100 block matrix

with ~4300 x 4300 elements per block

Assign multiple blocks per multi-core processor to share blocks avoiding re-reads as much as possible



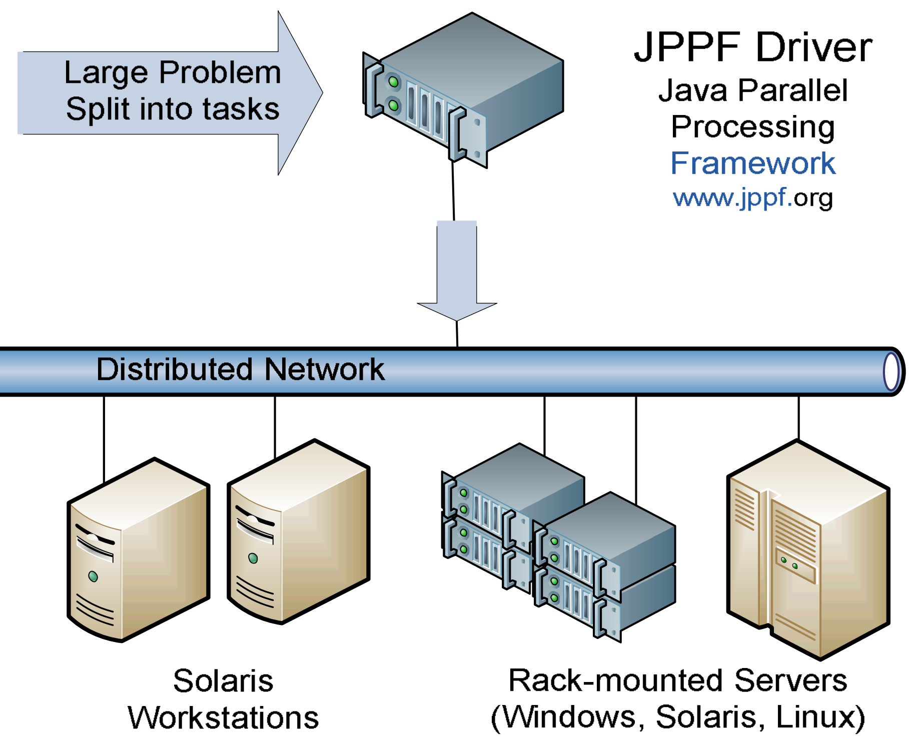
Distributed Computing Infrastructure

Java Parallel Processing Framework (JPPF) and Node Resource Manager (NRM)

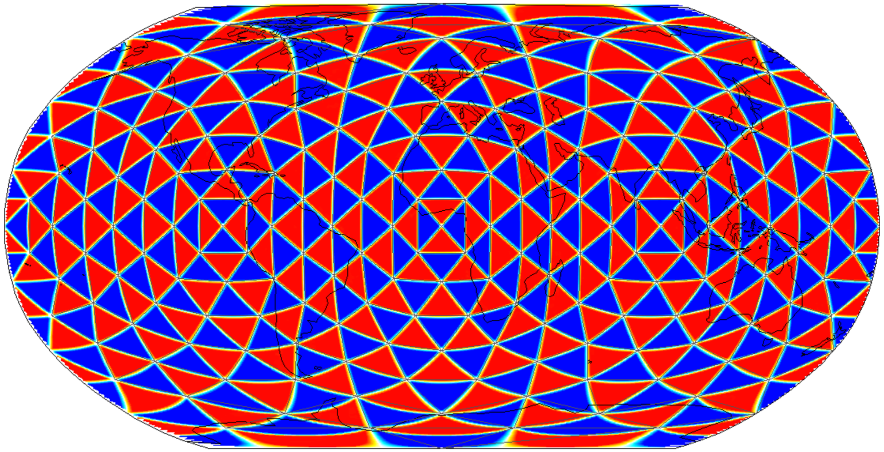
Solve for model covariance using conventional Intel multi-core processors (no super computers)

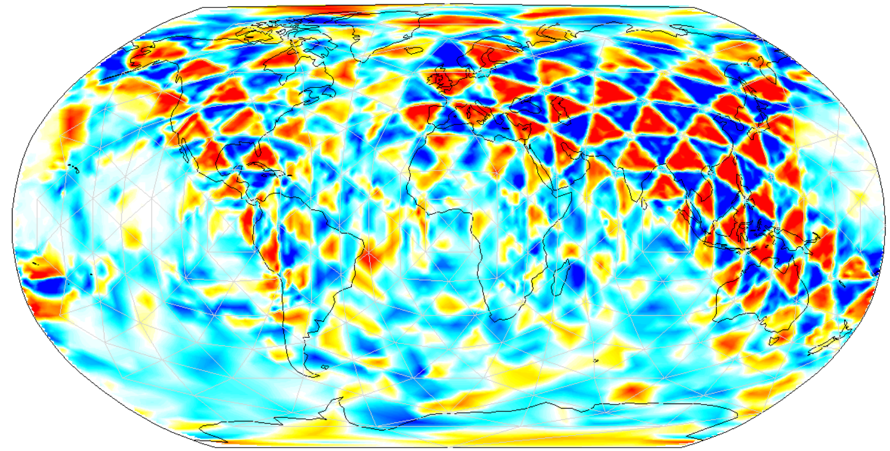
Multi-GPU solution in the works.

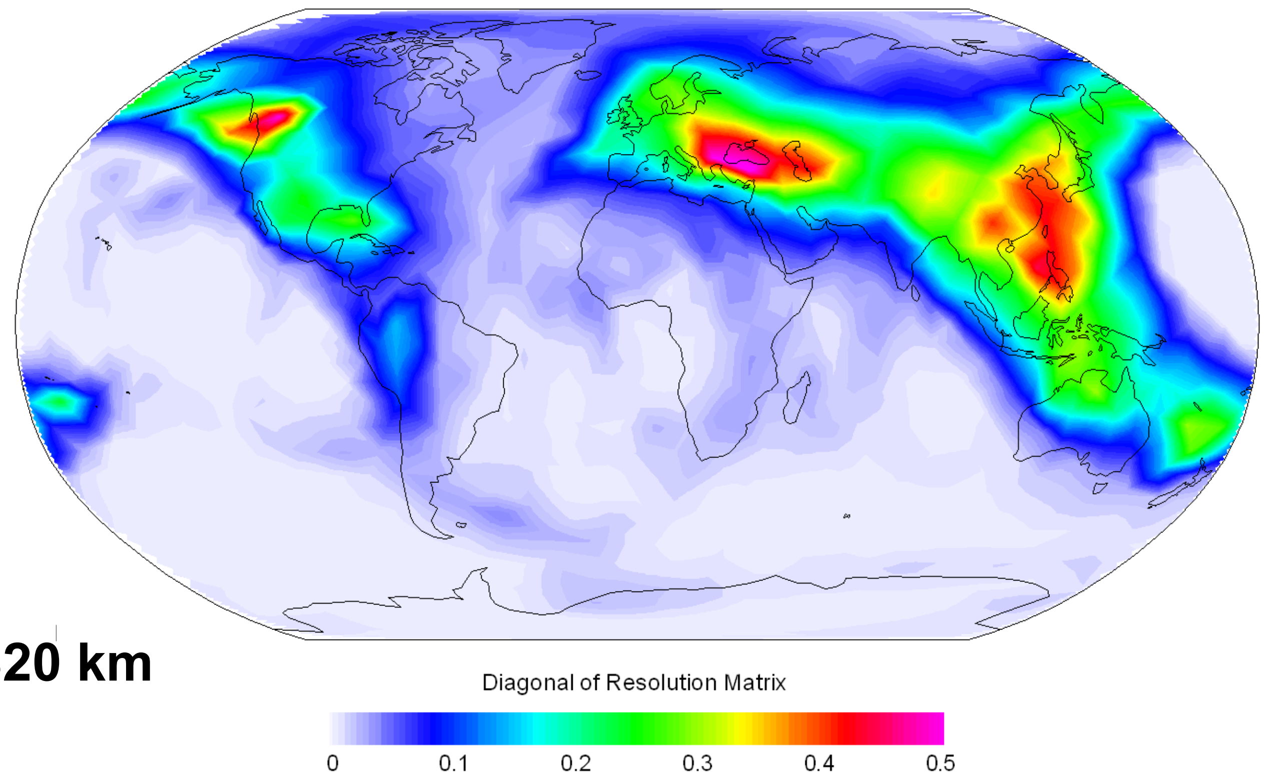
$$C_{\Delta s} = \left(G^T G\right)^{-1} A^T C_{\Delta d} A \left(G^T G\right)^{-1}$$



Model Resolution vs. Checkerboard Test

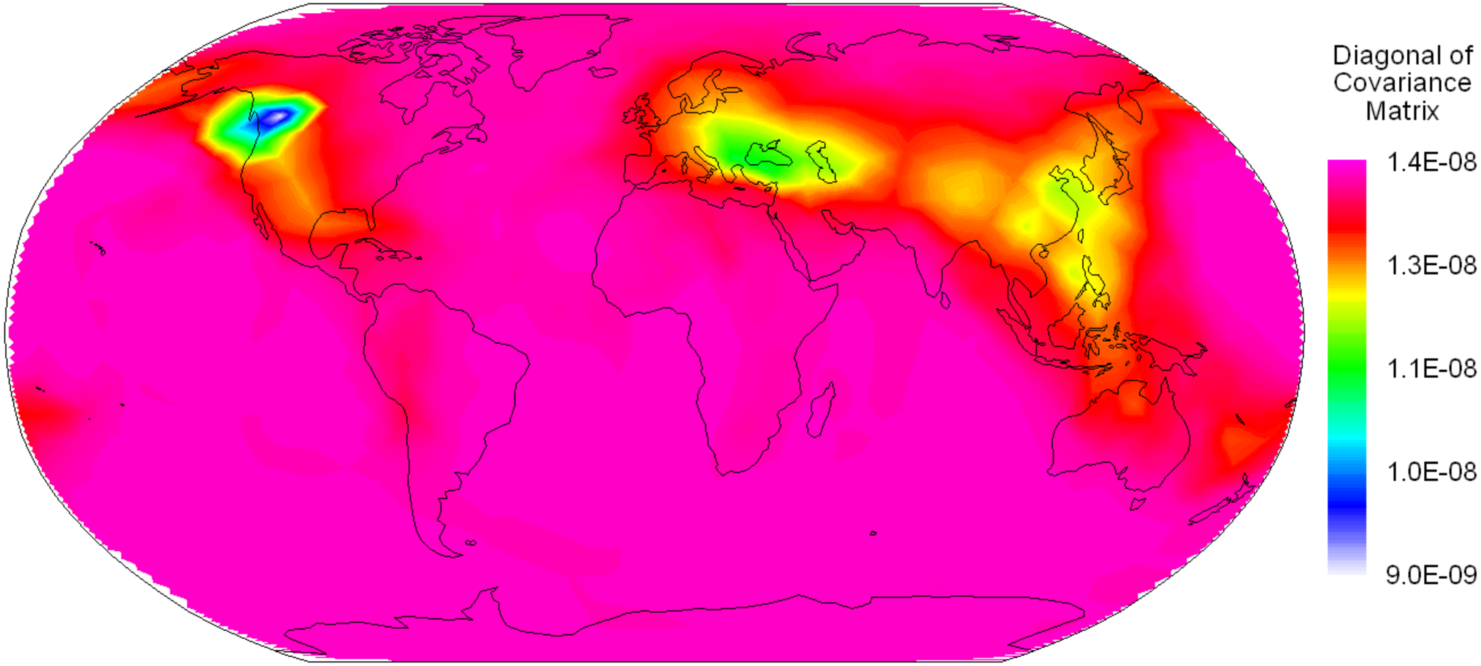




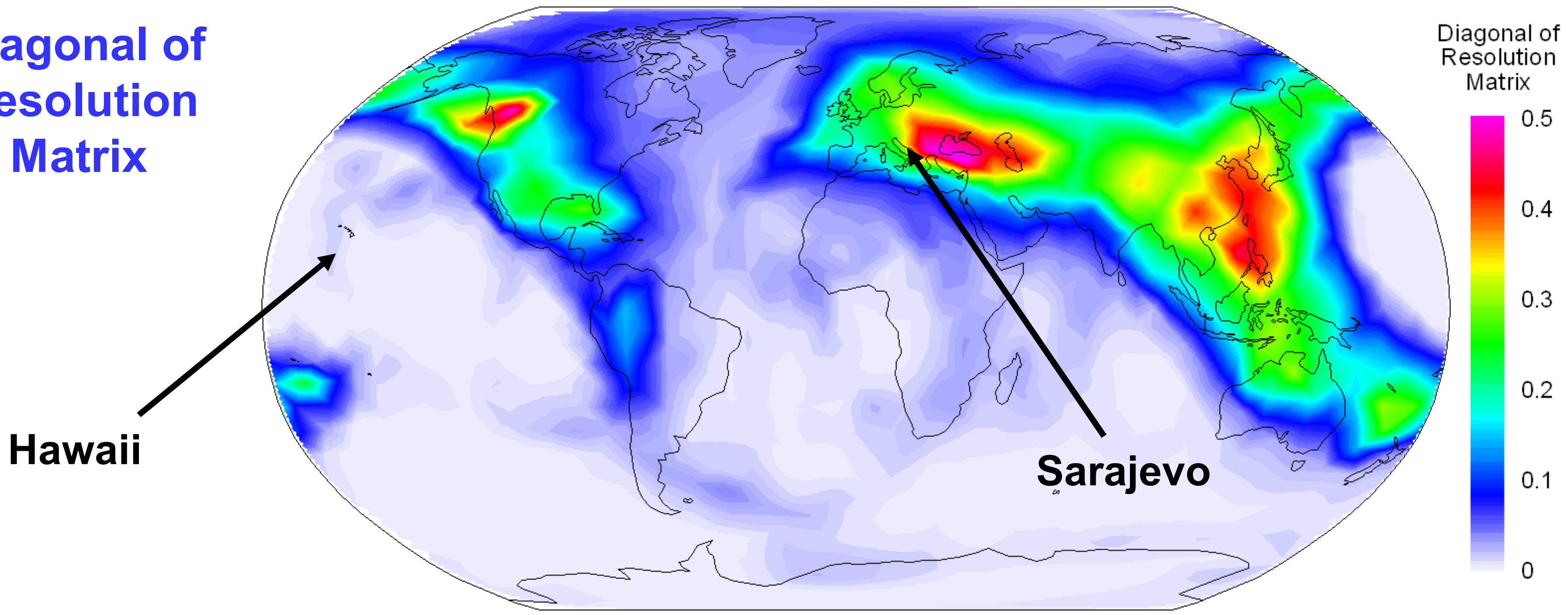


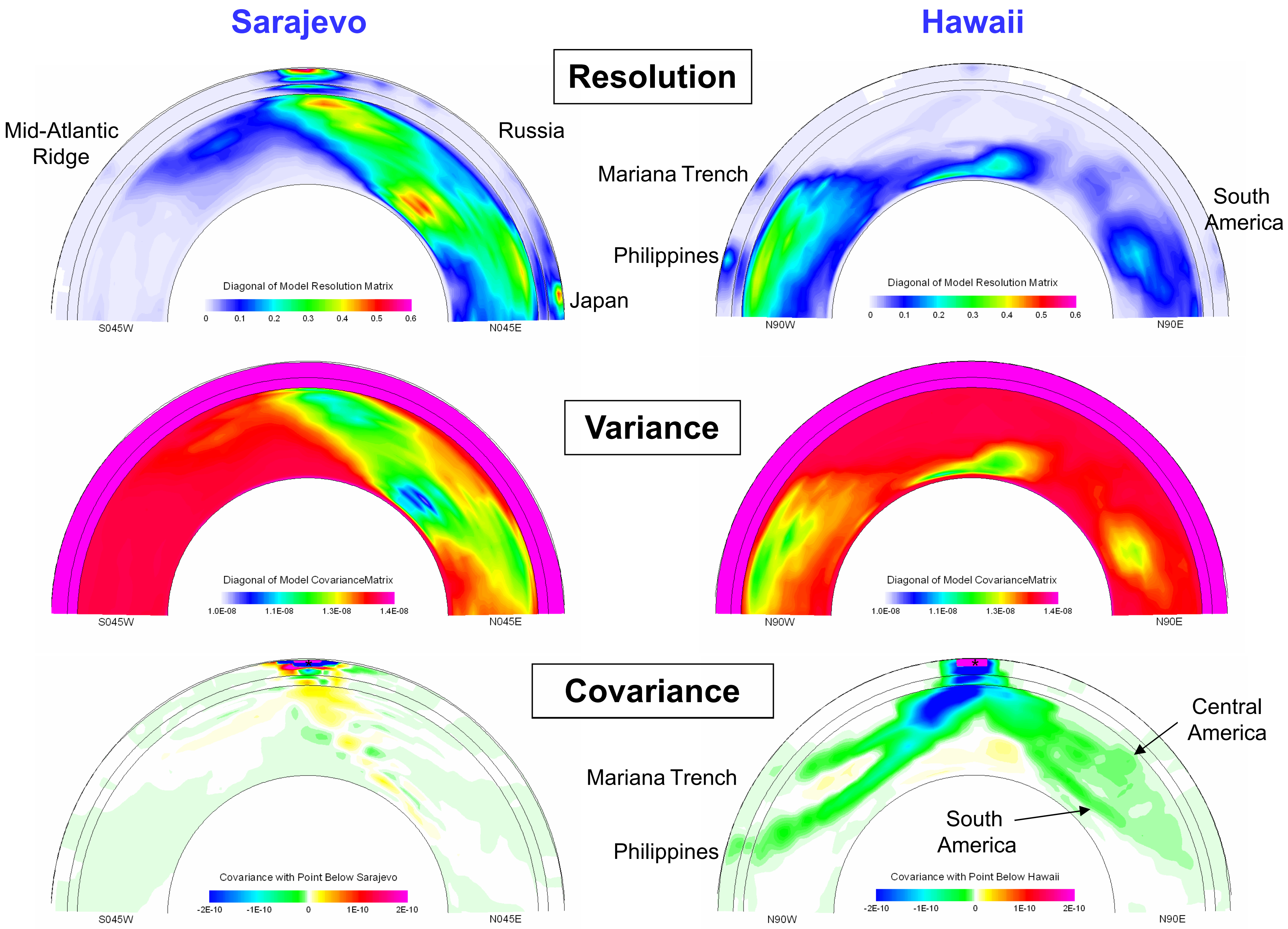
Depth = 820 km

Diagonal of Covariance Matrix (Variance)

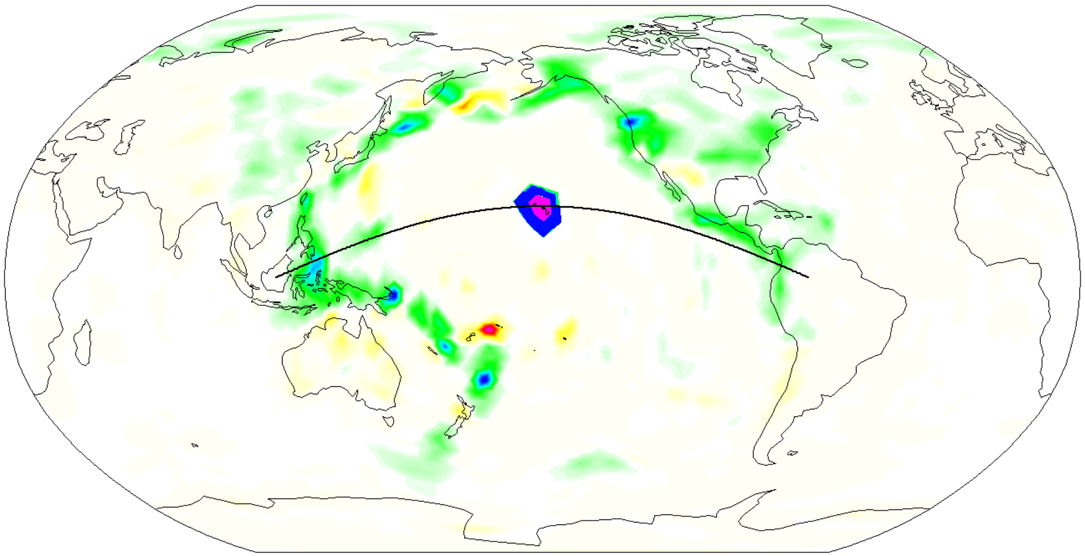


Diagonal of Resolution Matrix



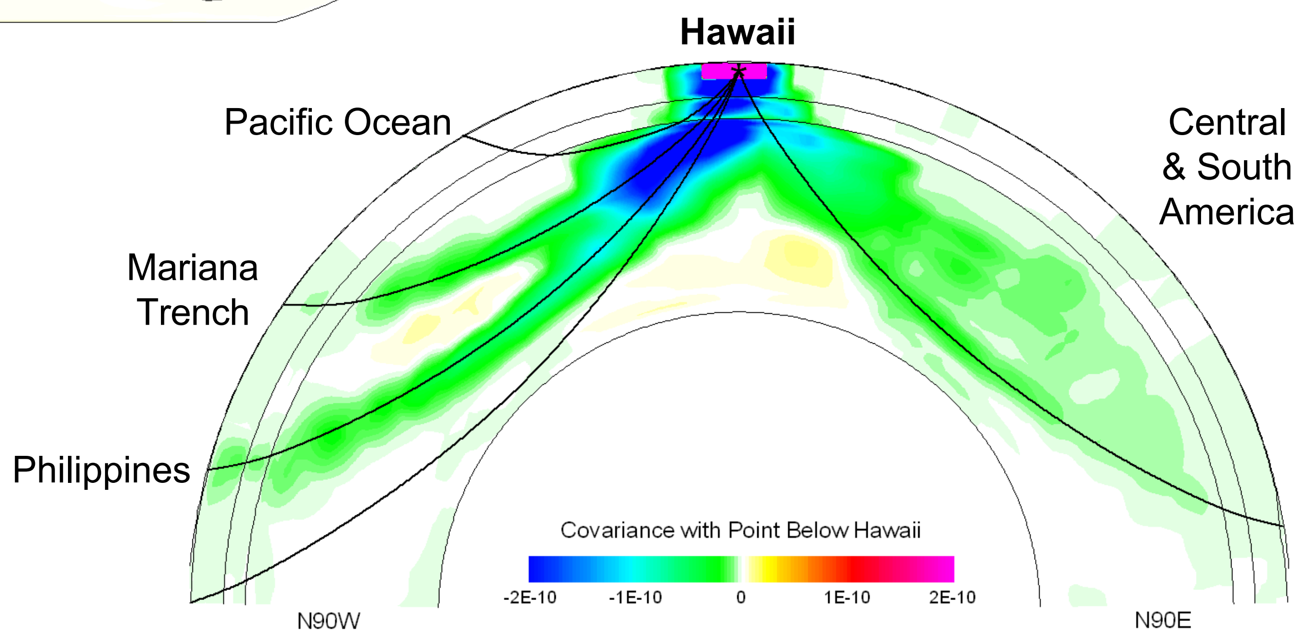

Sarajevo
Hawaii
Resolution
Mid-Atlantic Ridge
Russia
Japan
Diagonal of Model Resolution Matrix
0 0.1 0.2 0.3 0.4 0.5 0.6
S045W
N045E
Mariana Trench
Philippines
South America
N90W
N90E
Variance
Diagonal of Model CovarianceMatrix
1.0E-08 1.1E-08 1.3E-08 1.4E-08
Covariance
Covariance with Point Below Sarajevo
-2E-10 -1E-10 0 1E-10 2E-10
Central America
Covariance with Point Below Hawaii

Path-Dependent Travel Time Uncertainty



Depth = 100 km

$$\sigma^2_{TT_i} = \int_{P_i}\int_{P_i} C_{\Delta s}\, dl_i\, dl_i$$



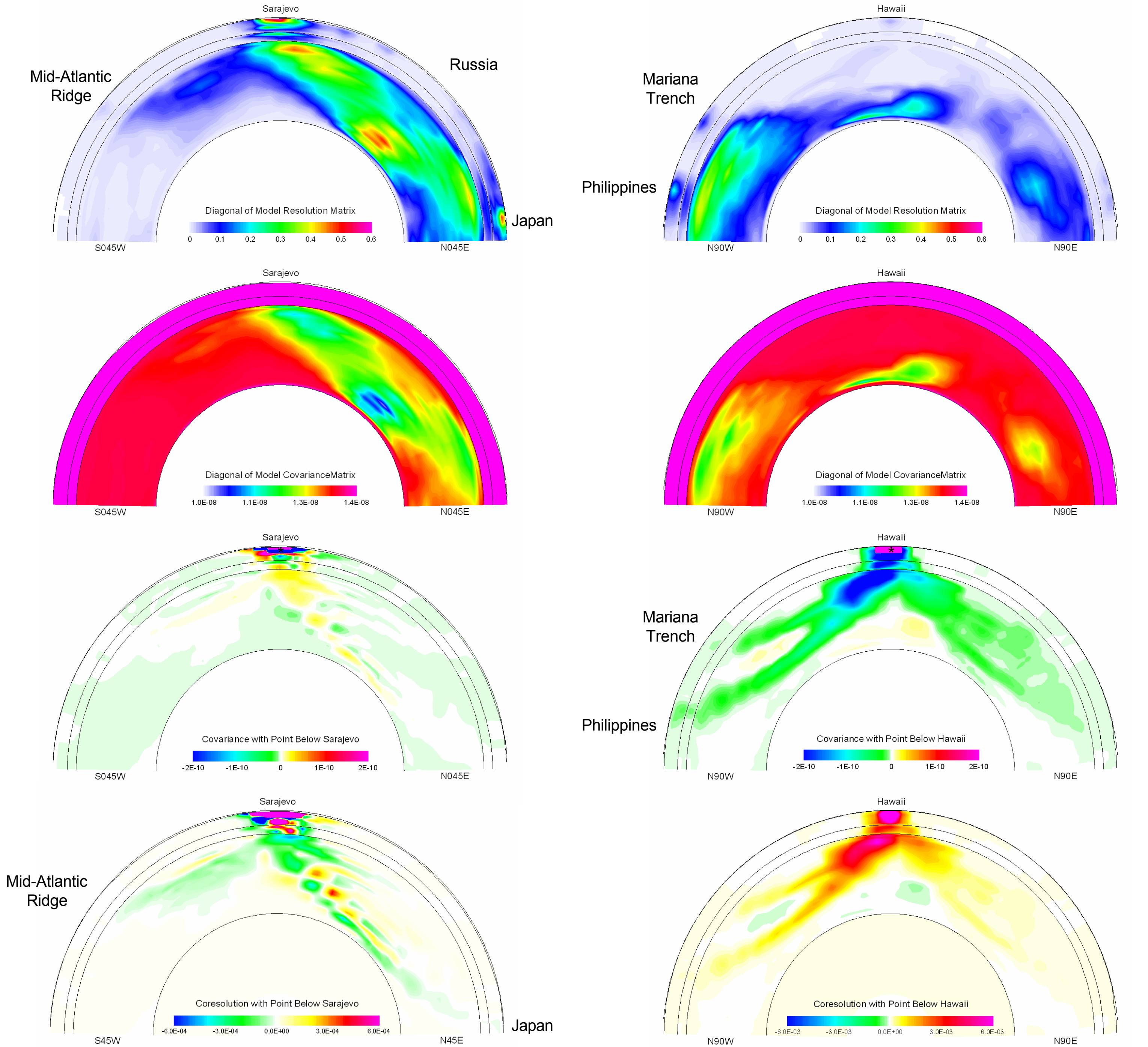
Summary

Previous Work:

- Global, seamless, 3D Earth model capable of computing predicted travel time, azimuth and slowness for phases P and Pn
- Conventional, *distance dependent* prediction uncertainty model
- Demonstrated improved accuracy of seismic locations.

Current Focus:

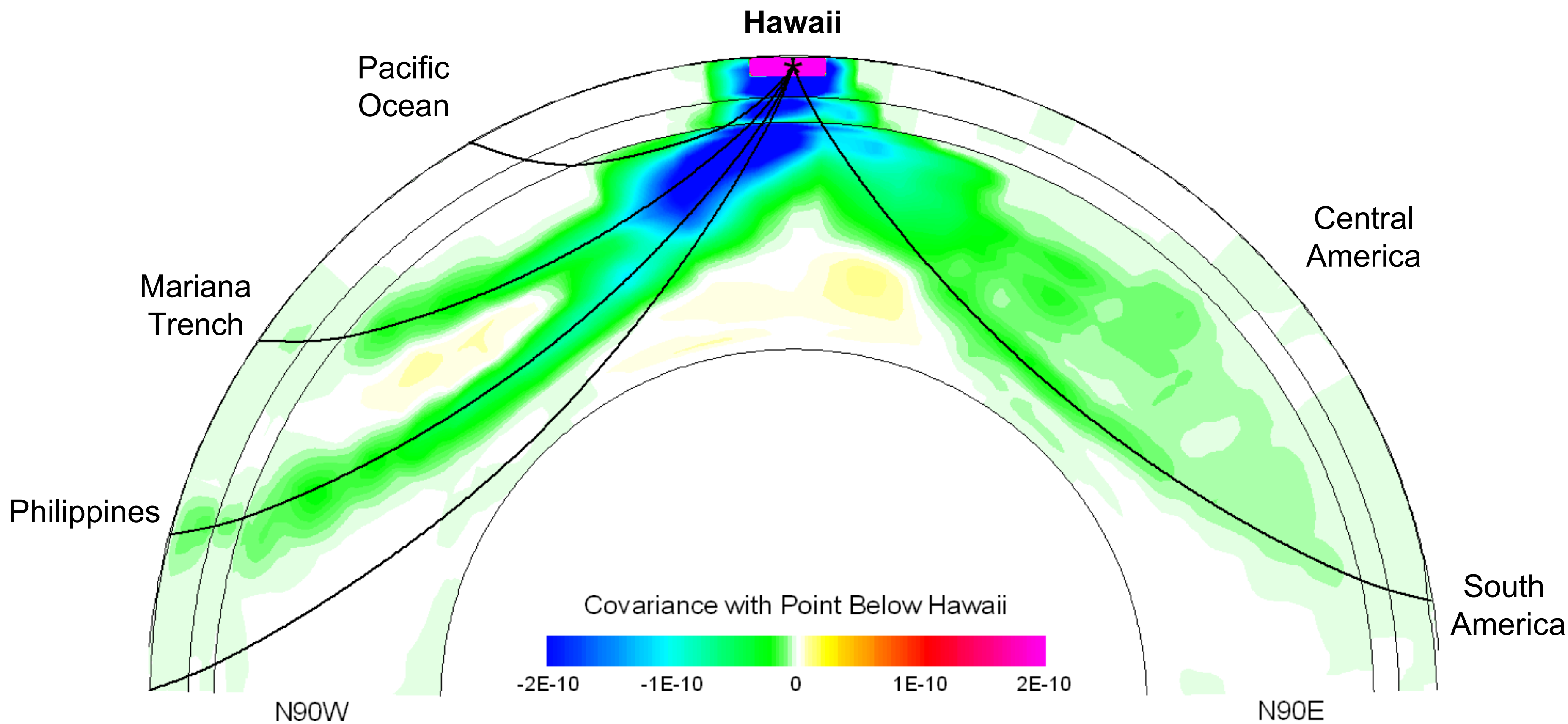
- Model resolution and covariance matrices for our tomographic model
- Realistic, *path-dependent* uncertainty estimates for travel time predictions based on integration along ray paths through the model covariance matrix (in progress).
- Will demonstrate that these uncertainty estimates are more realistic than distance-dependent estimates.



Note different color scales!

Path-Dependent Travel Time Uncertainty

$$\sigma_{TT_i}^2 = \int_{P_i}\int_{P_i} C_{\Delta s}\, dl_i\, dl_i$$



Single Row from Model Covariance Matrix Corresponding to Node Below Hawaii