SAND2011-2777C

Closed Loop Optimal Load Control with Application to Renewable Energy Grid Integration

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***Abstract*— Microgrids are envisioned containing several interconnected sources supplying power to multiple loads. While many technologies may be used for generation, such as fossil fuels or renewables, there will likely be some amount of storage capacity. Efficient use of the stored energy is the motivation for this work. This paper focuses on developing closed loop control laws that define optimal load forms. Optimal will be defined as a combination of energy dissipation rate and power flow. In general, the control laws are nonlinear and thus the optimal load forms are nonlinear functions of the states of the network. A tutorial LC circuit is used to illustrate the method for developing the optimal load that efficiently uses the available stored energy.**

I. INTRODUCTION

A microgrid is a modular version of a traditional power grid containing the key elements of generation, transmission, distribution, controllable loads, storage, and separability from other power grids. Microgrids are envisioned that are scalable, and when connected together will have the same functionality as a large, regional power grid. Introduction of renewable sources poses a particular challenge for microgrids due to their variability (e.g. wind and photovoltaic sources) and potential for causing grid voltage instability when they become a large portion (roughly greater that 20%) of the overall electrical supply. The ability to optimally control the loads will be important as more renewables and dispatchable loads, such as plug-in vehicle fleets, are incorporated into microgrids. In this paper we consider a very simple LC circuit and a controllable load. The capacitor has an initial storage state and an optimal control law is used to transfer the capacitor energy to the load. After introducing the system equations and the optimal control law, simulation results are used to illustrate the competing effects of minimizing load voltage, load power and transfer time.

Pandya and Joshi present a recent review of optimal power flow concepts in reference [3]. The work here differs from the traditional network-centric power flow studies by focusing on transient behavior. Nonlinear control of similar circuits have been investigated previously. For example, Sira-Ramirez et al. [4] developed a sliding mode controller with application to a boost converter. Using a passivity-based approach for the sliding mode controller they showed that the integrated stored energy was bounded. Beccuti et al. also considered a boost converter where the optimal control objective was to minimize inductor current in the presence of changes in the load or the source voltage [1].

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II. SYSTEM DESCRIPTION AND CONTROL LAW FORMULATION

The series LC circuit is described by Eq. 1 where $q(t)$ is the charge on the capacitor, $v_L(t)$ is the load voltage, L is the inductance, and C is the capacitance. The current in the circuit is $\dot{q}(t)$. The initial conditions on the circuit states $[q, \dot{q}]$ are $[q_0, 0]$ and thus the capacitor energy, $E_c = \frac{1}{2c}q^2$, is non zero, whereas $E_L = \frac{L}{2}\dot{q}^2$, the initial inductor energy, is identically zero. The energy in the load is $E_s = E_L + E_c$, or $E_s = \int_0^t v_L \dot{q} d\tau$ where $v_L \dot{q}$ is the load power.

$$L\ddot{q}(t) + \frac{1}{C}q(t) = v_L(t) \tag{1}$$

The cost function is given in Eq. 2 and is a weighted sum of the integral of the load power squared and the integral of the load voltage squared. The values of k_1 and k_2 are constrained to being positive and are simply used to balance the objective function goals of minimizing a power-related quantity to load voltage as the energy is being moved from the capacitor to the load.

$$J = \int_0^\infty L(q, \dot{q}, v_L)\, dt = \int_0^\infty \left[k_1 (v_L \dot{q})^2 + k_2 (v_L)^2\right] dt \tag{2}$$

The second method of Lyapunov is a convenient mechanism for deriving control laws for both linear and nonlinear systems. A classical approach will be used as described in reference [2]. By specifying a Lyapunov candidate function, V, a control law is constructed that minimizes Eq. 2 while ensuring asymptotic stability. The method is summarized below as applied to Eq. 1.

The sufficient condition for the optimal control law, v_L^*, is that

$$\dot{V}(q, \dot{q}, v_L^*) = -L(q, \dot{q}, v_L^*) \tag{3}$$

where V is a radially unbounded, positive definite, Lyapunov candidate function. Thus, a control law can be formed based on the Lyapunov candidate function selected. Consider the V of Eq. 4 which is the total energy of the circuit.

$$V = \frac{1}{2c}q^2 + \frac{L}{2}\dot{q}^2 \tag{4}$$

The derivative of Eq. 4, and substitution of Eq. 1, is shown in Eq. 5.

$$\begin{aligned} \dot{V} &= \frac{1}{C}q\dot{q} + L\dot{q}\ddot{q} \\ &= \frac{1}{C}q\dot{q} + L\dot{q}\left[-\frac{1}{LC} + \frac{1}{L}v_L\right] \\ &= \dot{q}v_L \end{aligned} \tag{5}$$

The control law is found by apply Eq. 3 to Eq. 5 and gives the v_L^* of Eq. 6.

$$\dot{q}v_L^* = -k_1(v_L^*\dot{q})^2 - k_2(v_L^*)^2 \tag{6}$$

Equation 6 gives two possible solutions for v_L^*, with one of them being the trivial $v_L^* = 0$ case. The nonzero solution is shown in Eq. 7

$$v_L^* = -\frac{\dot{q}}{k_1\dot{q}^2 + k_2} \tag{7}$$

where $k_1 = 0$ minimizes load voltage and $k_2 = 0$ minimizes the power during the energy transfer from the capacitor to the load.

Before considering other Lyapunov candidate functions or simulation results, further exploration of the control law of Eq. 7 is insightful. First, let's consider the value of the cost function of Eq. 2 when using this control law.

$$\begin{aligned} J &= \int_0^\infty \left[(k_1\dot{q}^2 + k_2)v_L^2\right] dt \\ &= \int_0^\infty \left[(k_1\dot{q}^2 + k_2)\left(\frac{\dot{q}}{k_1\dot{q}^2 + k_2}\right)^2\right] dt \\ &= \int_0^\infty \dot{q}\; v_L dt = E_s \end{aligned} \tag{8}$$

Regardless of the values of k_1 and k_2 the numerical value of the cost will always be equal to the energy transferred from the capacitor to the load. Next, we'll examine limiting values of k_1 and k_2 and their effect on v_L^*. First consider the case where k_1 is sufficiently small that the term $k_1\dot{q}^2 \ll k_2$. This corresponds to the minimum voltage case and results in an approximately linear control law of equation Eq. 9.

$$v_{L_1}^* \approx -\frac{\dot{q}}{k_2} \tag{9}$$

The resulting closed loop system has the characteristic equation $s^2 + \frac{1}{Lk_2}s + \frac{1}{Lc} = 0$ and its roots can be placed by selecting k_2 while also ensuring the constraint on k_1 is in effect.

Next consider the case where k_2 is small. Certainly if $k_2 = 0$ the control law of Eq. 7 is singular. However, this condition is not approached for $k_2 \ll 1$. As k_2 is made increasingly small the resulting maximum $\dot{q}$ (current) becomes proportionally smaller. Therefore $k_1\dot{q}^2 \ll k_2$ once again and the control law is approximated by Eq. 9. It should also be noted that the resulting characteristic becomes very lightly damped as k_2 is increased and the time required to transfer the energy from the capacitor to the load increases.

The final behavior of interest is when the control law operates in a nonlinear regime. This occurs when $k_1\dot{q}^2$ and k_2 are roughly the same order of magnitude. In that case energy oscillates between the capacitor and the inductor while simultaneously extracting a component for the load. This occurs for the $k_2 \ll 1$ case described above as well, but when operating in the nonlinear regime the rate of transfer can be made significantly faster.

These three phenomena, where the control law is approximately linear, and fully nonlinear, will be investigated in the next section using a numerical simulation of the system.

III. RESULTS

A MATLAB simulation of the system was created to investigate the effect of k_1 and k_2 on the form of the optimal control solution to Eq. 1, and the time required to transfer the initial capacitor energy to the load. The simulation used a fixed-time, fourth order Runge-Kutta solver and a time step of 10^{-6}. The inductance was set to 1 Henry and the capacitance, C, to 10^{-6} farads. In the remainder we'll consider the case where $k_1\dot{q}^2 \ll k_2$, then $k_2 << 1$, and finally a specific example of the nonlinear operating regime of the closed loop control.

A. Case 1: Linear Regime with $k_1\dot{q}^2 \ll k_2$

The values of $k_1 = 1$ and $k_2 = 0.01$ were used. Figure 1 shows q and $\dot{q}$ where it is clear that over the entire simulation $k_1\dot{q}^2 \ll k_2$ is indeed true. The capacitor charge decays with a lightly damped, linear response. This is because the damping introduced by the control law of Eq. 9, $1/k_2$, is significantly smaller than the stiffness of the system, $1/C$. Figure 2 shows the energy of the three devices where the energy primarily moves between the capacitor and inductor and only slowly is accumulated by the load.

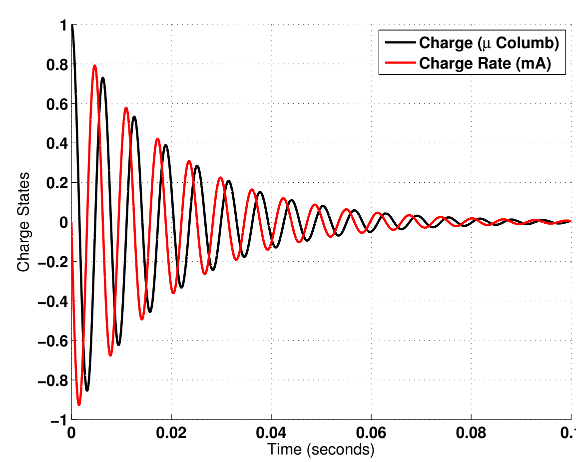

Fig. 1. Case 1, linear control regime, charge and charge rate.

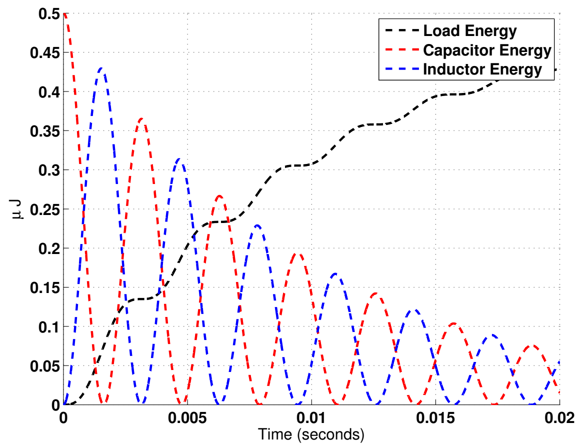

Fig. 2. Case 1, linear control regime, component energies.

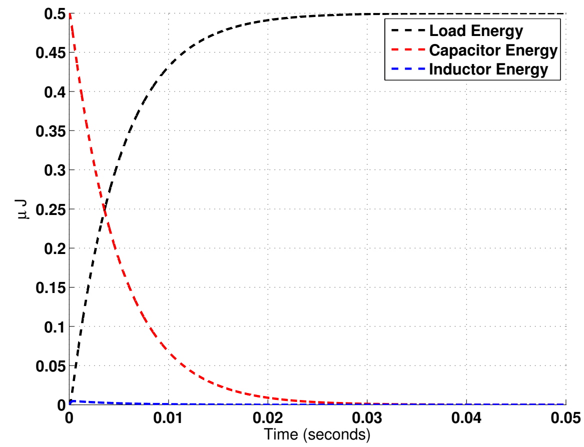

Fig. 4. Case 2, linear control regime, component energies.

B. Case 2: Linear Regime with $k_2 \ll 1$

The values of $k_1 = 1$ and $k_2 = \frac{1}{10000}$ were used. Figure 3 shows the capacitor charge and charge rate. The charge rate, $\dot{q}$, magnitude causes the condition $k_1\dot{q}^2 \ll k_2$ even though $k_2 \ll 1$. The approximate characteristic equation introduced earlier is $s^2 + 10^4 s + 10^6 = 0$ with a dominant root at $s = -100$. Therefore the approximate time constant of the response is 0.01 seconds and is consistent with the q response of Figure 3. Figure 4 compares the energy in all three devices, where the energy of the load is equal to the instantaneous evaluation of the objective function.

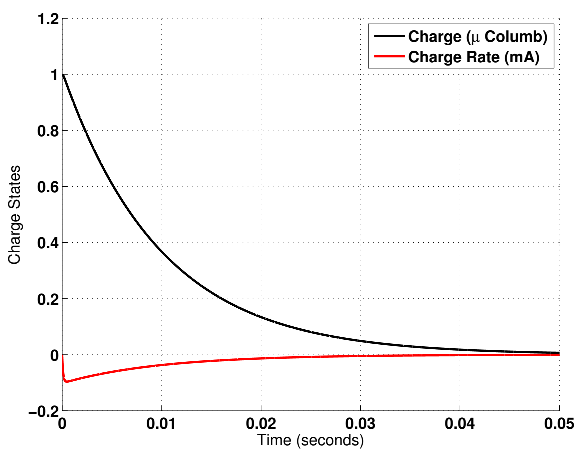

Fig. 3. Case 2, linear control regime, charge and charge rate.

C. Case 3: Nonlinear Regime

The values of $k_1 = 10,000$ and $k_2 = \frac{1}{2000}$ were used so that the magnitudes of $k_1\dot{q}^2$ and k_2 where similar during part of the simulation and the nonlinear behavior of the control law would be present. The charge rate, or current, is shown in Figure 5 along with the capacitor charge. The underdamped responses deviate from that of a linear second order system confirming that a the nonlinear control law effect is present. The energy of all the devices is shown in Figure 6 where energy is being moved between the capacitor and the inductor while incrementally being harvested by the load. The load voltage is shown in Figure 7 and the power in Figure 8. Since this case has significant weight applied to minimizing power, the power level is capped. If k_1 is increased the maximum power reduces. It should be noted that the time for charge transfer is about 0.01 seconds which is much faster than the previous two cases.

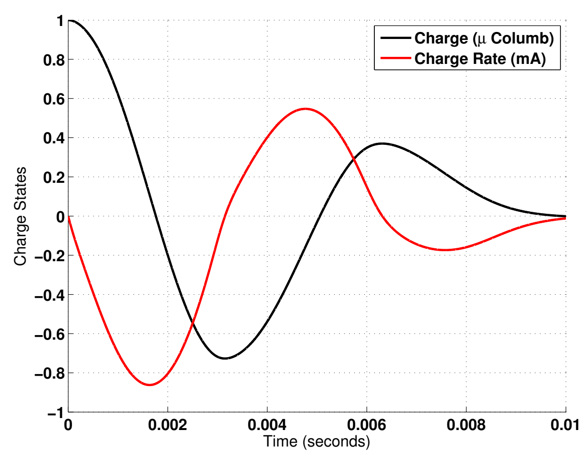

Fig. 5. Case 3, nonlinear control regime, charge and charge rate.

IV. CONCLUSIONS

A very simple, linear LC circuit was considered for the closed loop, optimal control of a load. The goal was to move an initial capacitor charge to the load while maintaining stability and minimizing a specified cost function. The nonlinear control law was based on a particular Lyapunov candidate function, system energy, that facilitated a viable control law solution. For many operating conditions the control law is approximately linear. However, by selecting the weights that trade-off power and voltage penalties, the control law can be made to operate in a nonlinear regime. In this condition

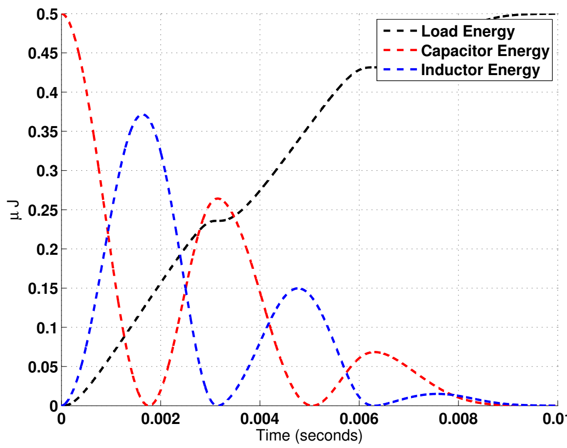

Fig. 6. Case 3, nonlinear control regime, component energies.

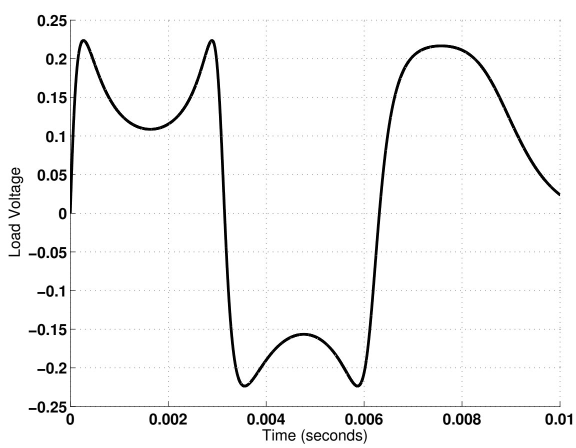

Fig. 7. Case 3, nonlinear control regime, load voltage.

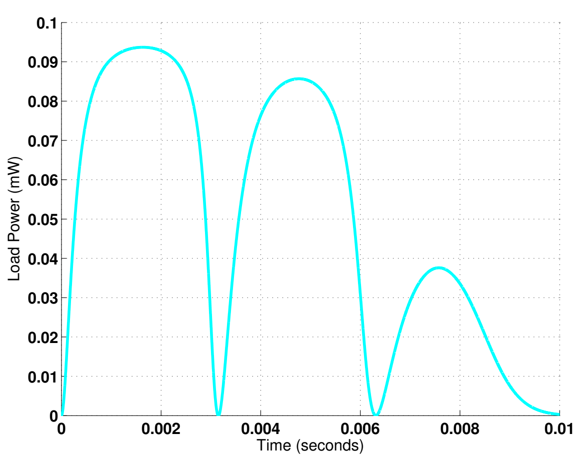

Fig. 8. Case 3, nonlinear control regime, load power.

energy moves between the capacitor and the inductor while being consumed by the load whenever the current is positive.

The advantage of the nonlinear operating regime is that a balance can be enforced between power and voltage. This is not possible when operating in the linear regime since the response is dominated by either a power optimal or voltage optimal scenario.

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