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Measuring and Modeling Impact Rebound

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Overview

- Rebound between two colliding solids is often critical in the performance of mechanisms.
- A theory exists to calculate the coefficient of restitution. However, the theory still depends on some measurement.
- A test setup was developed with a sphere suspended as a pendulum, impacts a wall, and rebounds.
- The motion of the sphere was recorded with a high-speed camera and traced with an image-processing program.
- From the trace, the coefficient of restitution was computed, and shown to confirm the trend predicted by the visco-elastic theory.
- A better, predictive elasto-plastic theory is being developed.

Coefficient of Restitution (C_R) greatly affects the performance of mechanisms.

A Pawl-and-ratchet device is dropped on the floor.

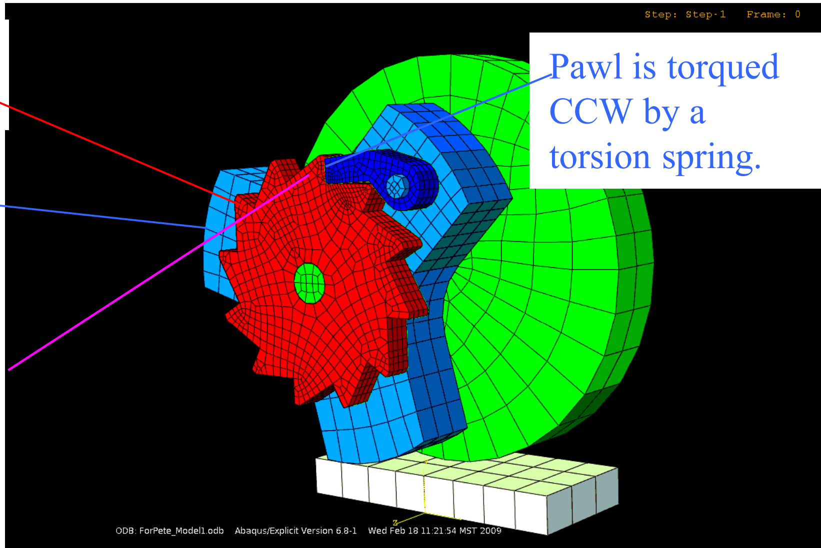
Simulation with C_R=0:

- Device does not bounce off the floor.
- Pawl stays engaged with the ratchet wheel
- Neither realistic nor conservative.

Ratchet wheel is confined relative to housing.

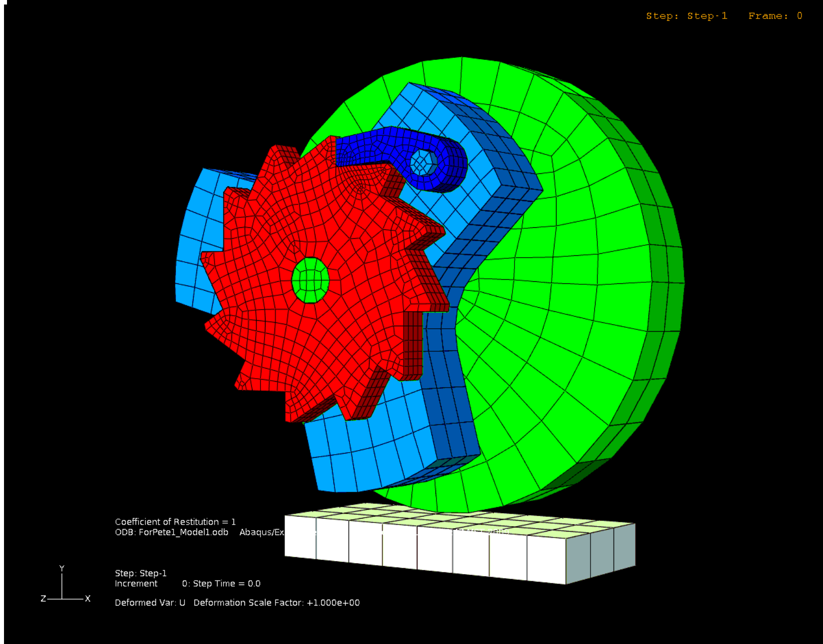
Rotor is torqued counterclockwise by a spring.

Friction helps hold the pawl against the ratchet wheel.



Simulation with CR of almost 1:

- The bounce off the floor is the highest.
- The pawl skips a tooth on the ratchet wheel.
- May be conservative but neither desired nor realistic.

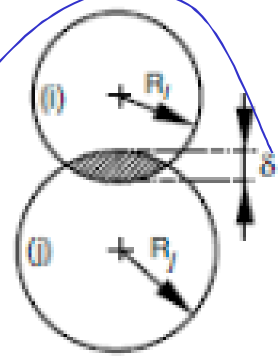


Predictive simulation requires a correct CR.

Hertzian contact theory is the basis of the theories investigated here.

- Sphere-sphere is the geometry studied most.
- Hertzian (elastostatic) contact is the basis for most impact theories.
- Elastostatics says nothing about coefficient of restitution, but is important in deriving theories to predict CR.

Fig. 3.2 Relative penetration depth during the impact between two spheres



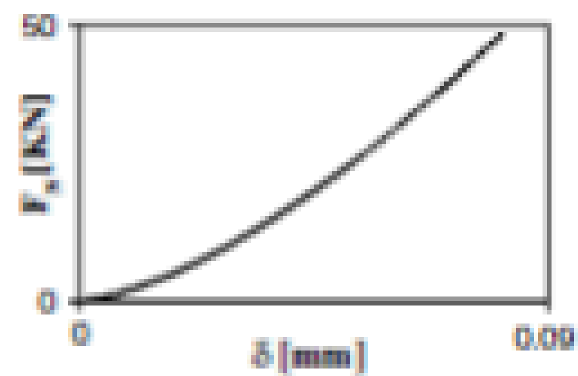
Normal force $F_n = K\delta^n$

where K is the generalized stiffness constant and δ is the relative normal indentation between the spheres. The exponent n is set to 1.5 for the cases where there is a parabolic distribution of contact stresses, as in the original work by Hertz. Although for metallic materials $n = 1.5$ for other materials such as glass or polymer it can be either higher or lower, leading to a convenient contact force expression that is based on experimental work but that should not be confused with Hertz theory. The generalized parameter K is dependent on the material properties and the shape of the contact surfaces. For two spheres in contact the generalized stiffness coefficient is a function of the radii of the spheres i and j and the material properties as (Goldsmith 1960)

$$K = \frac{4}{3(\sigma_i + \sigma_j)}\left[\frac{R_i R_j}{R_i + R_j}\right]^{\frac{1}{2}} \tag{3.3}$$

where the material parameters σ_i and σ_j are given by

$$\sigma_k = \frac{1 - \nu_k^2}{E_k}, \quad (k = i, j) \tag{3.4}$$



P. Flores et al., *Kinematics and Dynamics of Multibody Systems with Imperfect Joints.* 47

A purpose of the experiment is to assess a theory

- Schwager and Pöschel (1999) combined Hertzian theory with dynamics to develop a model for the coefficient of restitution for colliding spheres.
- They assume that the sphere materials behave visco-elastically.

- Define a scaled speed number

$$V^* = \left[2Y\sqrt{R}/\left\{3m\left(1-\gamma^2\right)\right\}\right]^2 V$$

- The coefficient of restitution is

$$\boxed{C_R = \frac{v_s}{v_a} = 1 - C_1 A V^{*1/5} - C_2\left(AV^{*1/5}\right)^2 - C_3\left(AV^{*1/5}\right)^3 - C_4\left(AV^{*1/5}\right)^4 - \ldots}$$

$C_1 = 1.153449$, $C_2 = -0.798267$, $C_3 = 0.483582$, and $C_4 = -0.285279$.

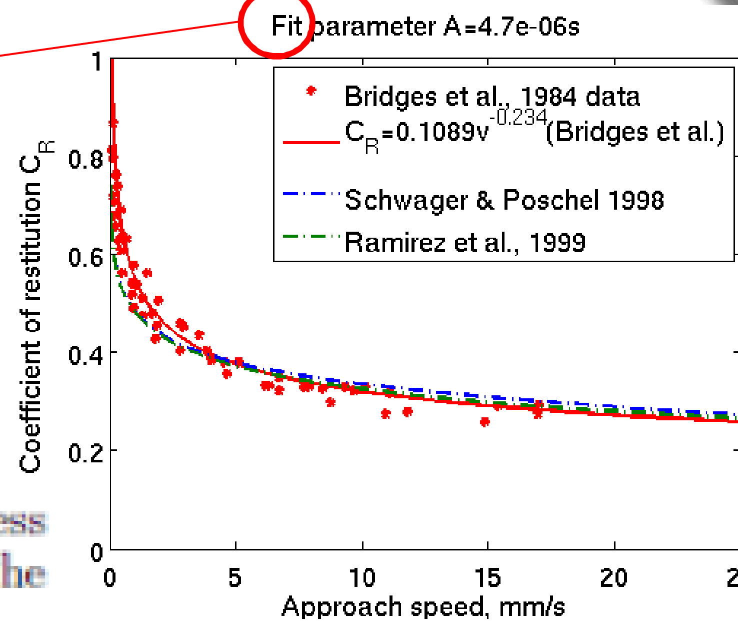
- Y = Young's modulus
- m = effective mass
- γ = Poisson's ratio
- v_a = the normal component of the speed of approach
- v_s = normal speed of separation.
- $R = R_1R_2/(R_1+R_2)$; R_1= radius of sphere1
- $m = m_1m_2/(m_1+m_2)$; m_1= mass of sphere1

The theory assumes (solid) viscosity to be the loss mechanism

A ***validated, predictive*** theory is still a quest!

$$A = \frac{1}{3}\frac{(3\eta_2 - \eta_1)^2}{(3\eta_2 + 2\eta_1)}\left[\frac{(1-\nu^2)(1-2\nu)}{Y\nu^2}\right].$$

The $\boxed{\text{viscous constants } \eta_1, \eta_2}$ relate the dissipative stress tensor to the deformation rate tensor [25,26,28]. The



Ramirez,R., Poschel,T., Brilliantov,N.V., and Schwager, T., 1999, "Coefficient of restitution of colliding viscoelastic spheres", *Phys. Rev. E*, 60, 4465.

The parameter A depends on viscous energy loss parameters η_1 and η_2.

η's can be obtained from the elastic moduli and the ***viscosity tensor***

$$\eta_{iklm} = \eta_{lmik} = \eta_{kilm} = \eta_{ikml}$$

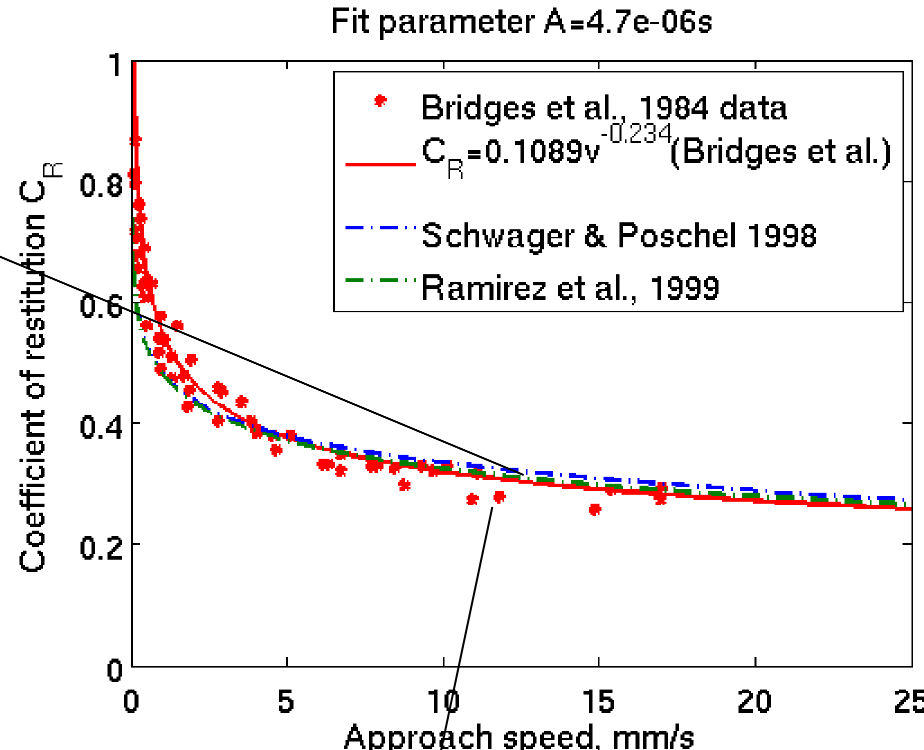
The viscosity tensor determines the viscous energy loss $R = \eta\left(v_{ik} - \delta_{ik}v_{ll}\right)^2 + \tfrac{1}{2}\zeta v_{ll}{}^2$

[28] Landau, L.D., and Lifschitz,E.M., 1965, *Theory of Elasticity*, Oxford University Press.

Experimental comparison is available only for ice balls.

Curves calculated from

Ramirez,R., Poschel,T., Brilliantov,N.V., and Schwager, T., 1999, "Coefficient of restitution of colliding viscoelastic spheres", Physical Review E, 60, 4465.



Test data from

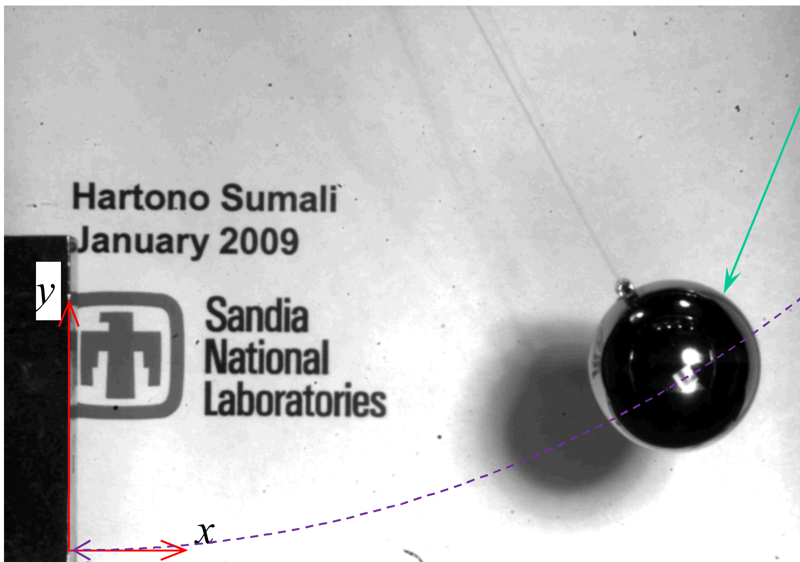
Bridges et al, 1984, *Nature*, 309, p 333-5.

The purpose of the work is to:

- Determine if the theory is valid in **steel-to-steel impact**.
- Develop a method to **obtain the fit parameter *A*** from measurement.
- Develop a **method to measure coefficient of restitution using digital high-speed photography**.

The test setup is based on rebound of a pendulum ball.

"Front" view:

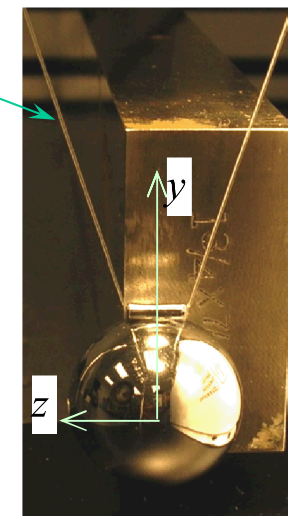


Sphere suspended on two strings

Manually lifted and gravity-released

Circular arc path

"Side" view:



Sphere impacts and rebounds on flat wall.

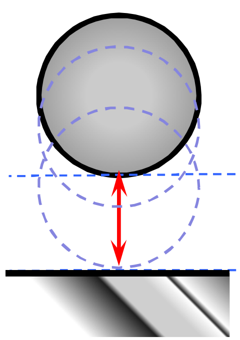
- The motion of the sphere was recorded with a high-speed camera looking into the negative *z* direction.
- An image-processing program was used to track the *x* and *y* positions of a marker on the sphere.

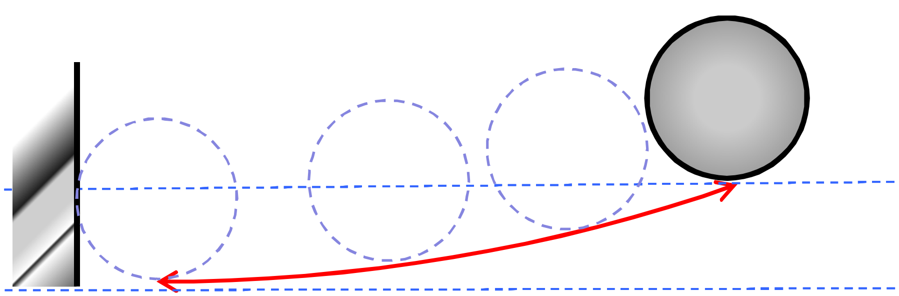
Pendulum setup allows large motion at low speeds.

- We want to examine a theory based on viscoelasticity (no plasticity) for metal-to-metal impacts.
- For a steel sphere impacting a steel wall, the maximum velocity that allows no plasticity is (Johnson, 1985) only 0.014 m/s.
- From potential energy before dropping = kinetic energy upon impact, the vertical drop is only 9.6 mm!

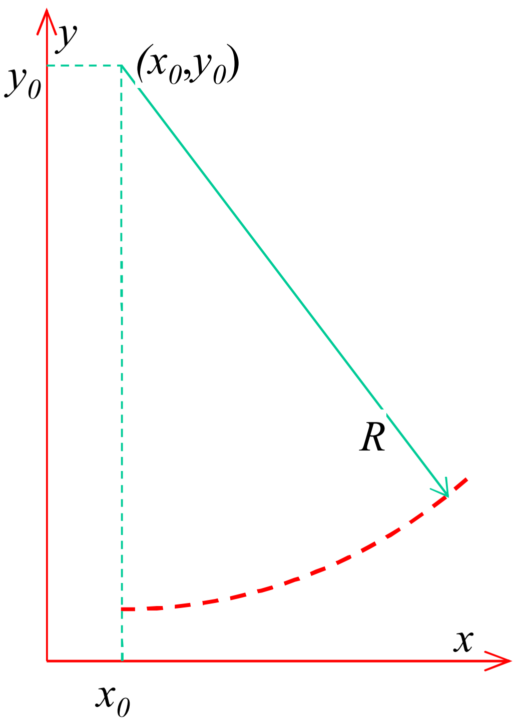
Vertical drop allows little motion to capture.

Pendulum allows much more motion to capture. (Many more frames)





The circular trajectory of the ball can be obtained by circle-fitting.



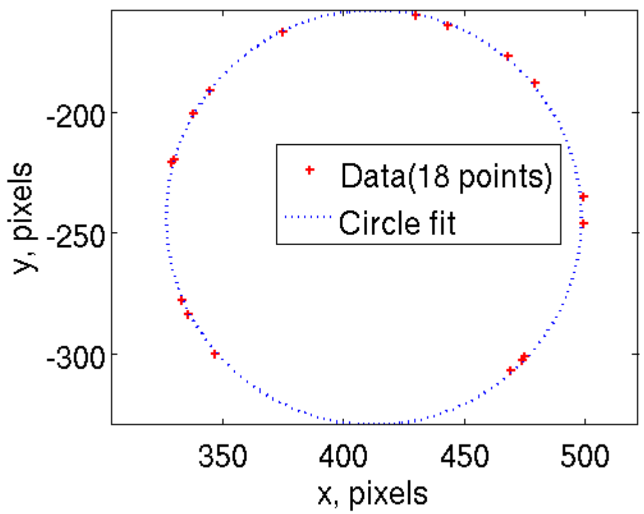
- Position tracking algorithm tracked the trajectory of the ball.
- Most of the motion was in the x direction.
- Motion in the y direction was recorded with less accuracy.
- Motion in the y direction can be computed easily because the path is a circular arc.
- The circle that contains the path can be described completely by the center location (x_0,y_0) and the radius R.
- Input the positions (x_n,y_n) of the ball from the recorded motion into a least-squares fit routine (Maia et al, 1998):

$$\begin{bmatrix} \sum_{n=1}^{N} x_n^2 & \sum_{n=1}^{N} x_n y_n & -\sum_{n=1}^{N} x_n \\ \sum_{n=1}^{N} x_n y_n & \sum_{n=1}^{N} y_n^2 & -\sum_{n=1}^{N} y_n \\ -\sum_{n=1}^{N} x_n & -\sum_{n=1}^{N} y_n & N \end{bmatrix} \begin{Bmatrix} -2x_0 \\ -2x_0 \\ R^2 - x_0^2 - y_0^2 \end{Bmatrix} = \begin{Bmatrix} -\sum_{n=1}^{N} x_n^3 - \sum_{n=1}^{N} x_n y_n^2 \\ -\sum_{n=1}^{N} x_n y_n^2 - \sum_{n=1}^{N} x_n^2 y_n \\ \sum_{n=1}^{N} x_n^2 + \sum_{n=1}^{N} y_n^2 \end{Bmatrix}$$

- Solve the above for the circle parameters x_0,y_0 and R.

Circle fit also converts pixel units to mm

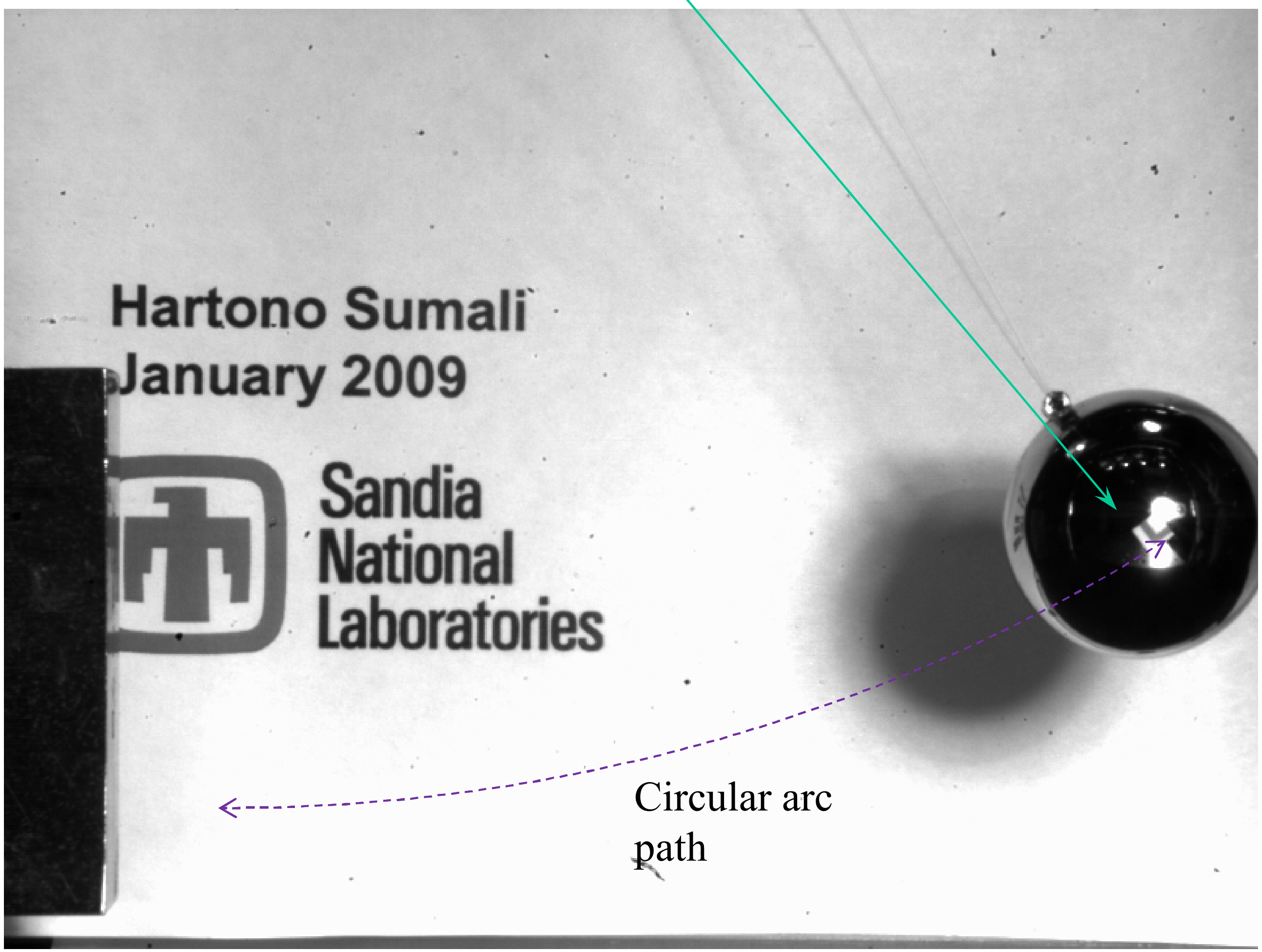
- Length unit in the movie frames are in pixels.
- Needed conversion from pixel to mm.
- Use a snapshot of the ball:
    - Circle-fit to obtain the diameter of the ball in pixels.
    - Measure the diameter in mm.
    - Obtain pixel - mm conversion.
- The circular outline of the ball in a frame was used for calculating pixel size. Using 18 data points, the circle-fit routine



The routine gave $R = 85.95$ pixels. Separate measurement with a vernier calliper gave $2R = 22.25$mm. Therefore, the images had a scale of $2*85.95/22.25 = 7.727$ pixels/mm.

The path of the mark on the ball was very nearly circular.

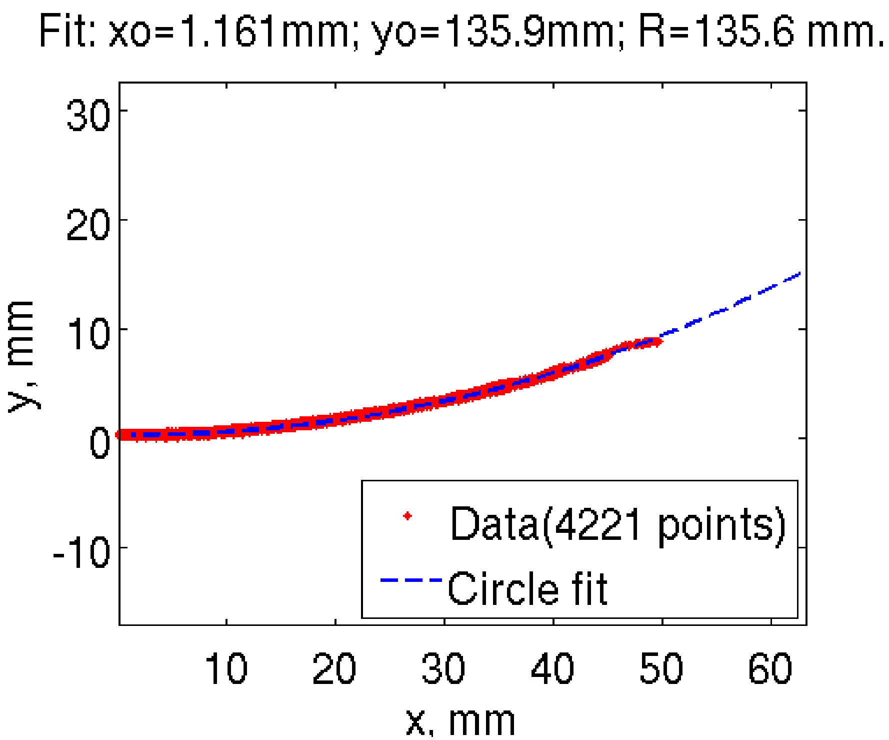
- The path traverse by a marker on the ball is a nearly perfect circular arc.



- However, the initial release condition of the ball was imperfect.
- As a result, the circular arc path has a little ripple.

Circle fitting obtains circular arc trajectory.

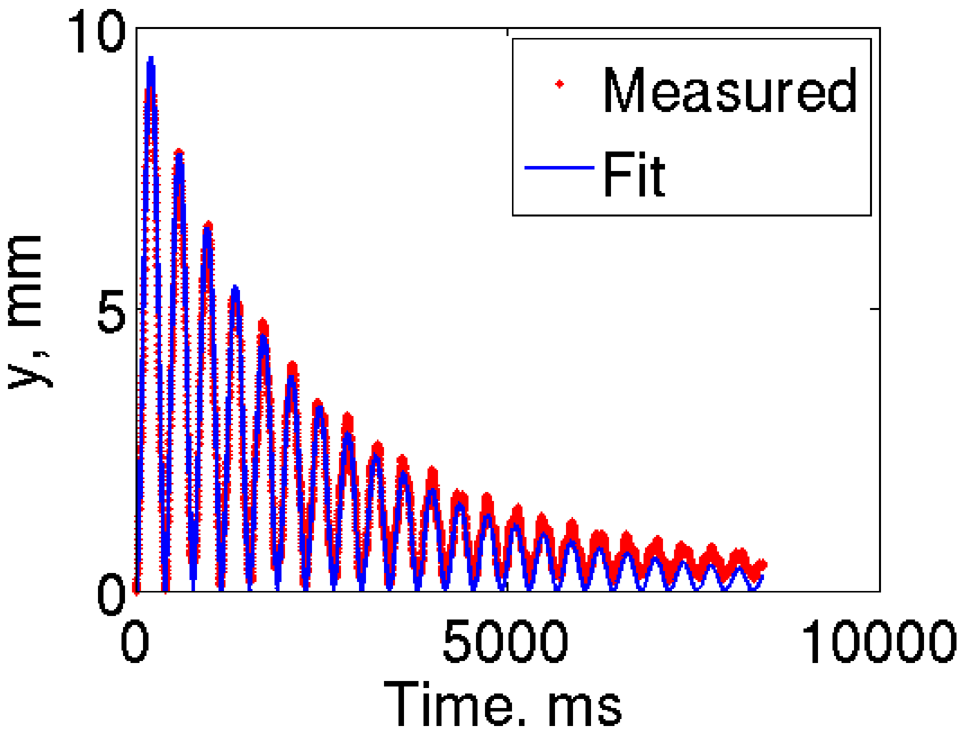
- Using 4221 measured position of a marker on the ball, the circle-fit routine gave the circular arc traversed by the ball as below:



- The trajectory was indeed close to a circular arc as expected.

Knowledge of the trajectory circle improves the measured y position.

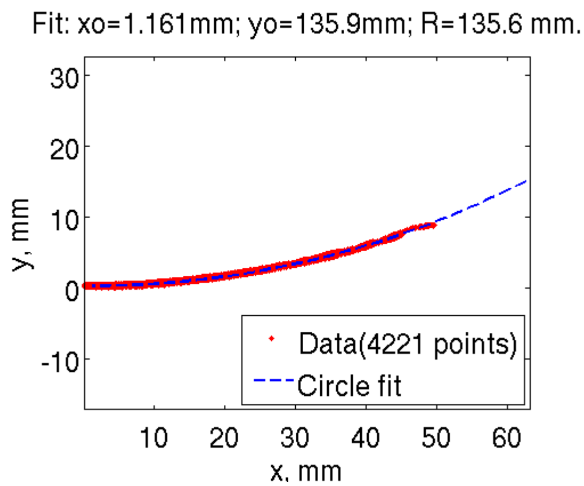
- The trajectory was indeed close to a circular arc as expected.
- However, the ball oscilated about the x axis because the initial release condition was imperfect.
- As a result, the circular arc path has a little ripple.
- The ripple distorts the measured y position.

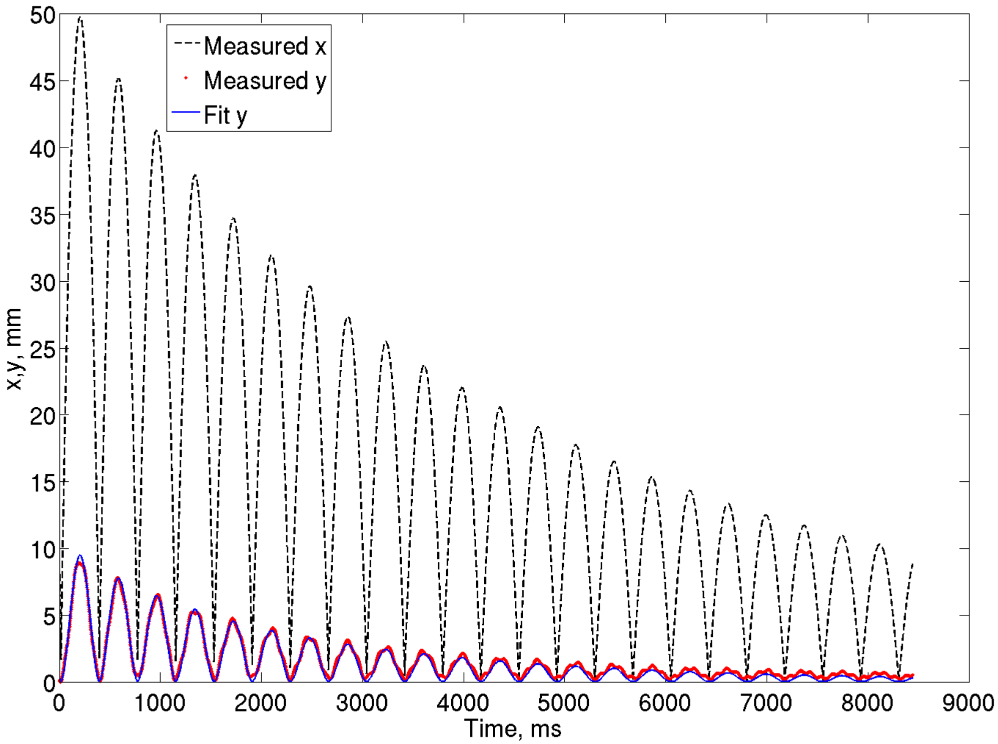


- Since the trajectory circle has been obtained by circle fit, the y position can be recovered from the measured x position.

The motion of the ball in the x and y directions are plotted vs time.

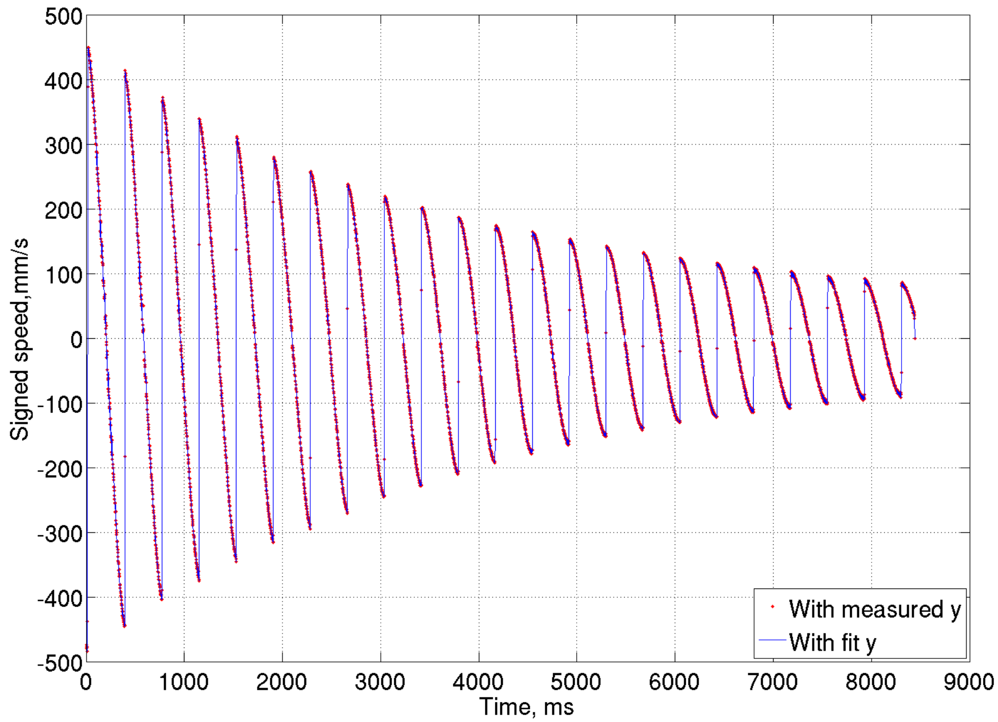
- The measured x and y positions are plotted versus time for 4221 points.
- The motion of the ball is mainly in the x direction.
- The motion in the y direction shows less precision.
- However, the y motion contributes little to the total motion.
- The y motion used for further calculation is the y motion from the fit.





The speed is obtained by time-differentiation of the position.

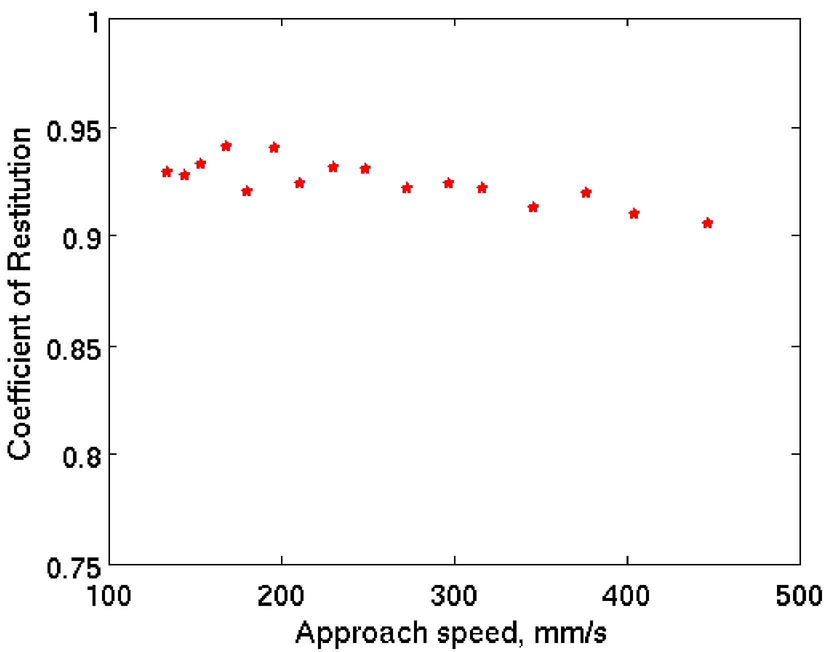
- Signed speed v = sign(dx/dt) × d(position)/dt.

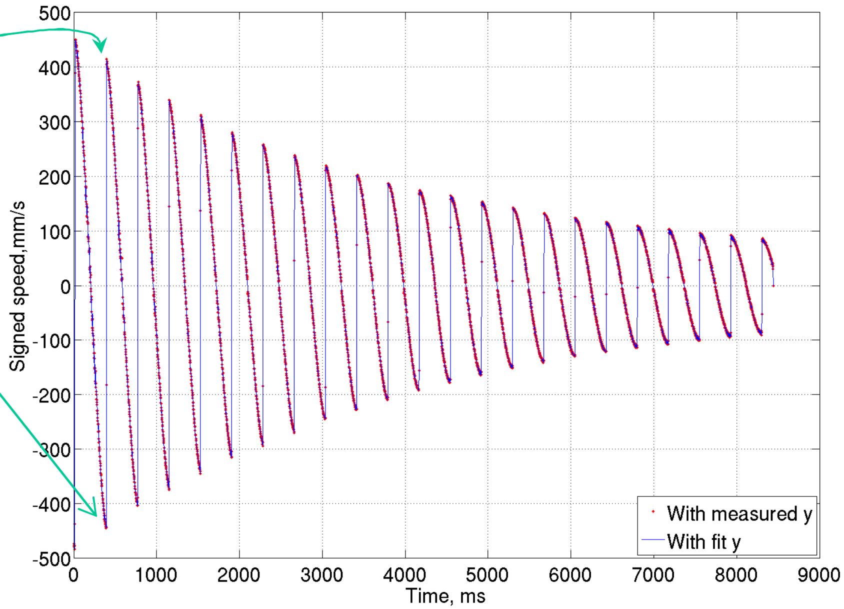


- The y position is noisy. The derivative dy/dt is worse.
- However, the signed speed is almost noise-free because the y motion contributes to d(position)/dt much less than the x motion.

The coefficient of restitution is the ratio of the rebound speed to the approach speeds.

- The plot of signed speed vs time gives Coefficient of restitution $C_R = -v_{rebound}/v_{approach}$.





Recall that the purpose of the experiment is to examine a theory

- The coefficient of restitution is

$$C_R = \frac{v_s}{v_a} = 1 - C_1 AV^{*1/5} - C_2\left(AV^{*1/5}\right)^2 - C_3\left(AV^{*1/5}\right)^3 - C_4\left(AV^{*1/5}\right)^4 - \ldots$$

$C_1 = 1.153449$, $C_2 = -0.798267$, $C_3 = 0.483582$, and $C_4 = -0.285279$.

$$V^* = \left[2Y\sqrt{R}/\{3m(1-\gamma^2)\}\right]^2 V$$ is a scaled velocity

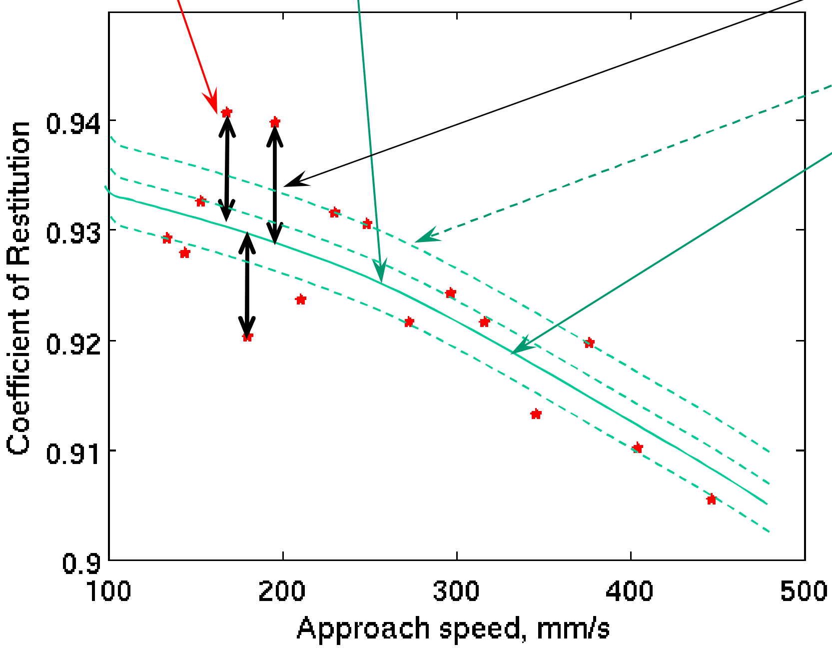
- Y = Young's modulus
- m = effective mass
- γ = Poisson's ratio
- v_a = the normal component of the speed of approach
- v_s = normal speed of separation.
- $R = R_1R_2/(R_1 + R_2)$; R_1 = radius of sphere1
- $m = m_1m_2/(m_1 + m_2)$; m_1 = mass of sphere1

- Fit coefficient A must be obtained from measured data: **How?**
- Assumption: Material is viscoelastic. **Is that valid for the steels?**

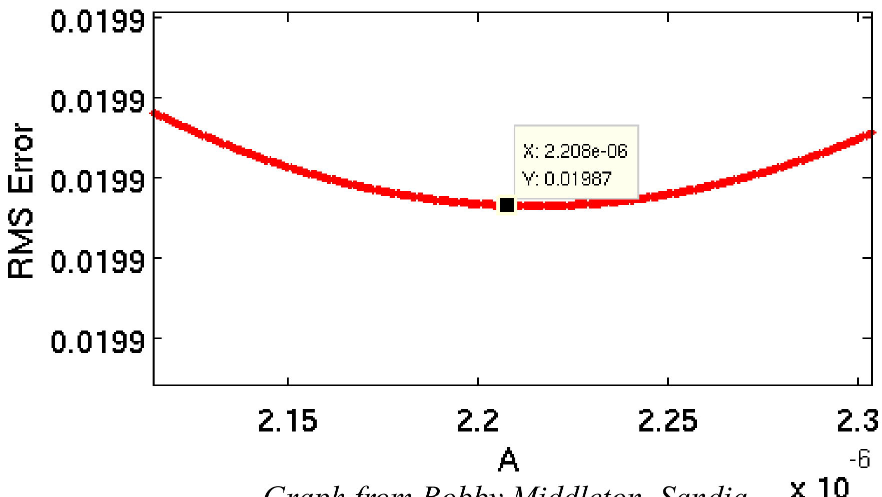
The fit parameter *A* can be obtained from the measured C_R.

- Fit coefficient *A* must be obtained from measured data: **How?**

- To obtain *A* from the measured CR, minimize the RMS difference between the measured and theoretical CR.



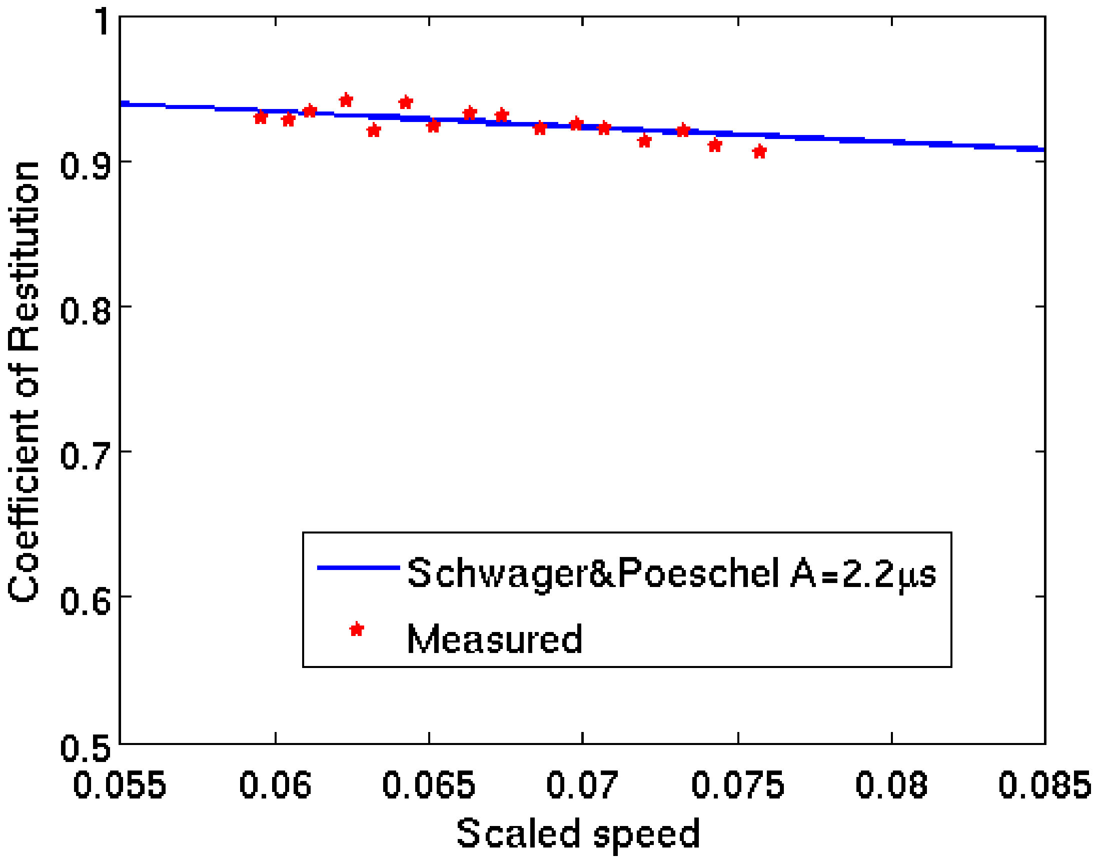
- Theoretical CR depends on *A*.
- Select *A* that minimizes RMS error.



Graph from Bobby Middleton, Sandia National Laboratories, 2010

The measured CR fits the theoretical trend.

- Theory assumption: Material is viscoelastic. **Is that valid for the steels?**
- To examine the theory, C_R is plotted versus scaled speed $V^* = \left[2Y\sqrt{R}/\{3m(1-\gamma^2)\}\right]^2 V$ and compared against the theoretical values.

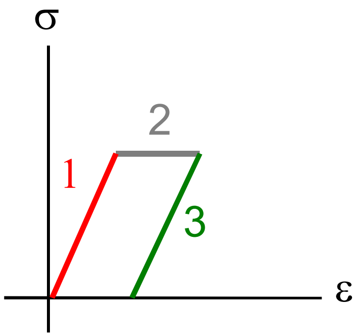


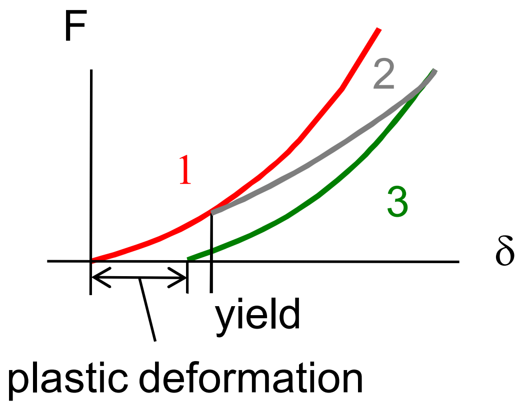
Visco-elastic model does not capture plasticity

- A very low speed, equivalent to a drop from a few millimeters, results in yielding in hard-steel-to-hard-steel impact.
  - Sphere radii do not matter.
- The visco-elastic theory depend very much on the coefficient A.
- A depends on the solid viscosity.
- Solid viscosity is very difficult to measure.
- Thus, the visco-elastic model is not likely to be predictive.
- Need a model that captures yield.
  - Many publications attribute kinetic energy loss to yield.

Simplified Elastic-Perfectly Plastic impact modeling captures yield

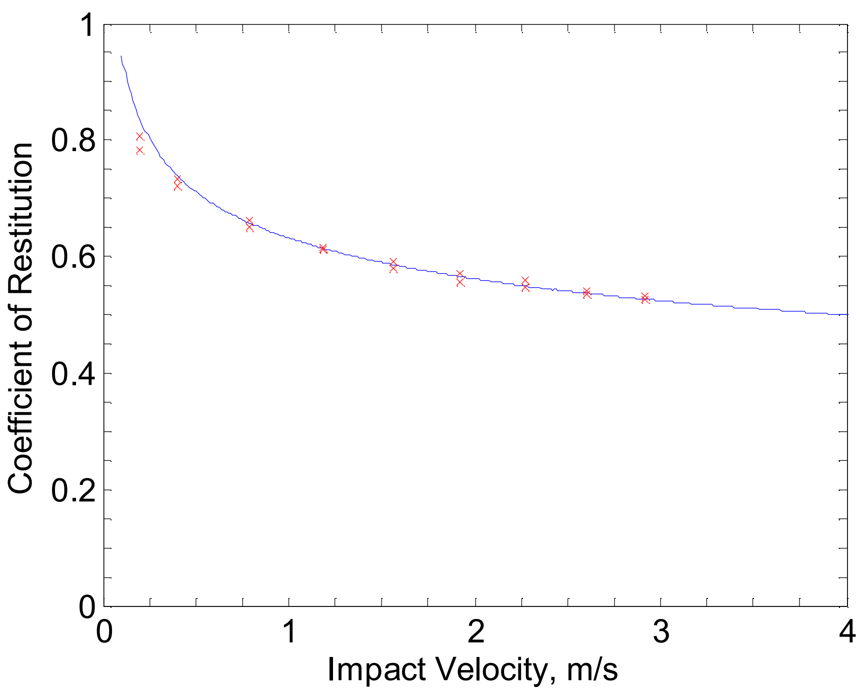
- Deflection divided into three phases:
  - Elastic loading (1)
    - Hertzian force-deflection relationship
    - Spans from the initial contact until the onset of yielding
  - Plastic loading (2)
    - Transition regime from elastic to unconstrained (plastic) flow defined using hardness properties
    - Linear force-deflection relationship in fully plastic regime (elastic-perfectly plastic behavior only…)
  - Elastic unloading (3)
    - Hertzian, but with a different contact radius than for loading
    - A portion of the plastic deflection is unrecoverable



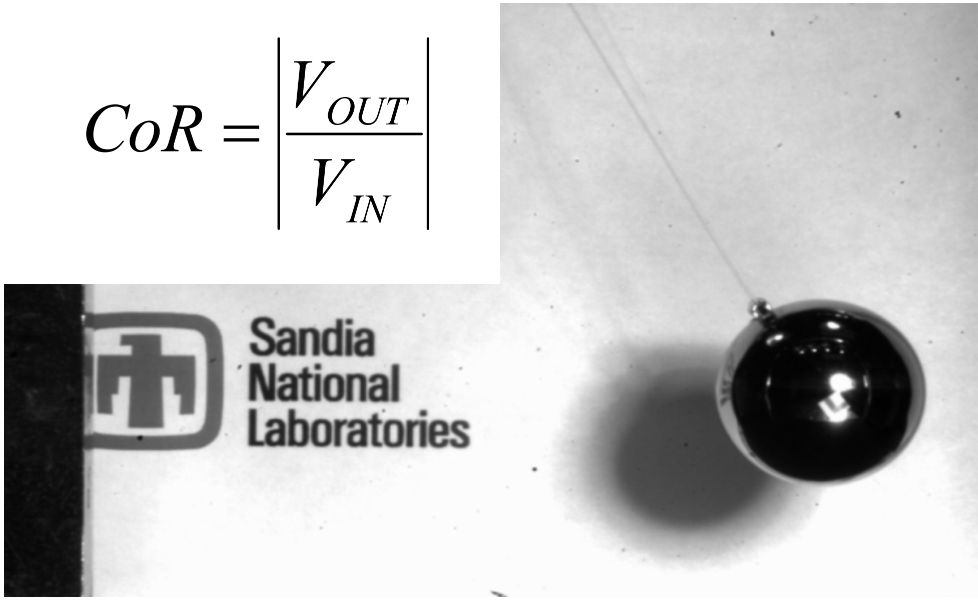


Experimental Efforts to Validate

- Elastic-plastic impact model based on material properties (hardness, yield stress, etc.)
- Experimental validation includes pendulum impact studies
- The measured coefficient of restitution is compared to the predicted results from a simulation of the system:

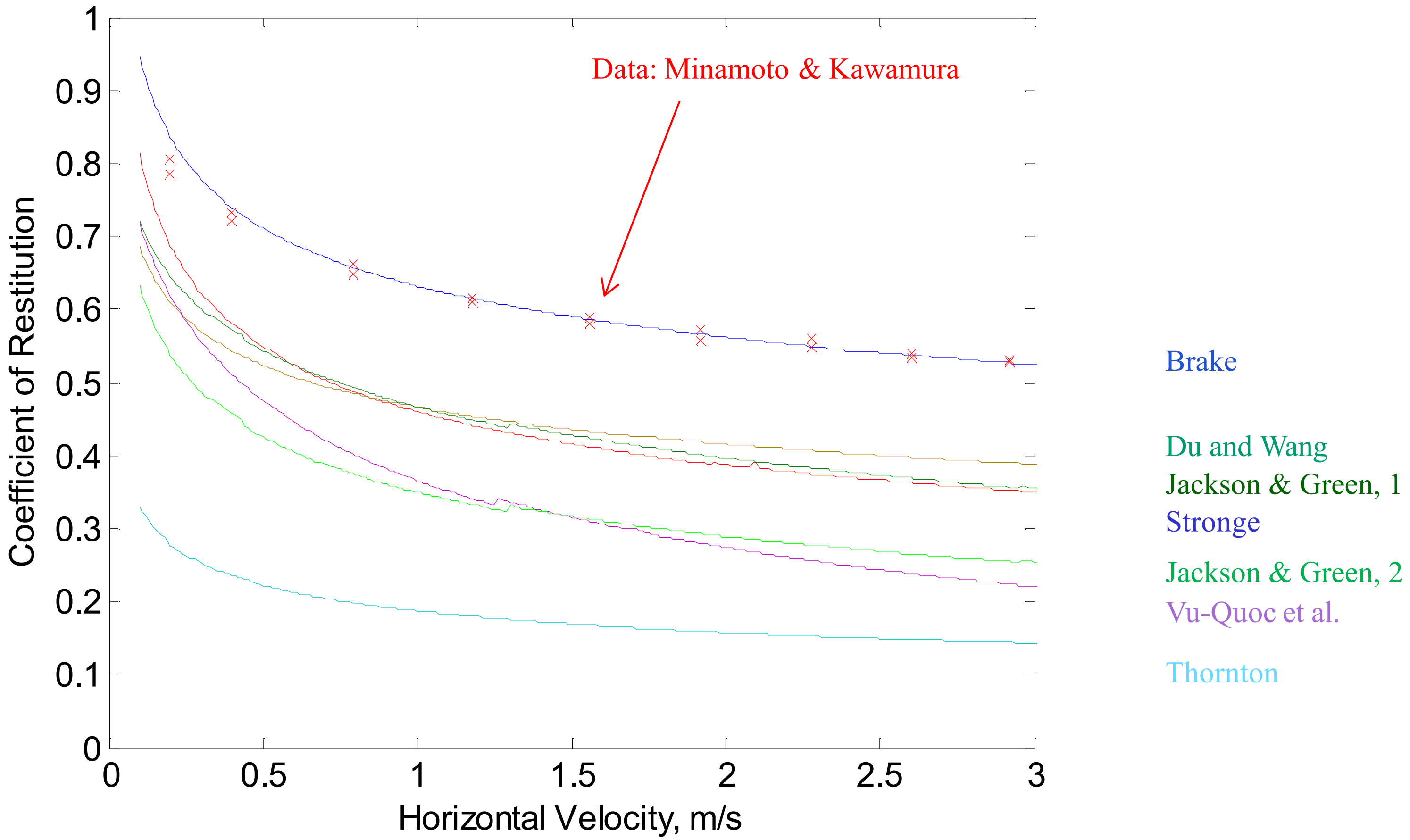


$$CoR = \left| \frac{V_{OUT}}{V_{IN}} \right|$$



Matthew M. Brake, Sandia

Brake's Model fits Minamoto and Kawamura's test data.

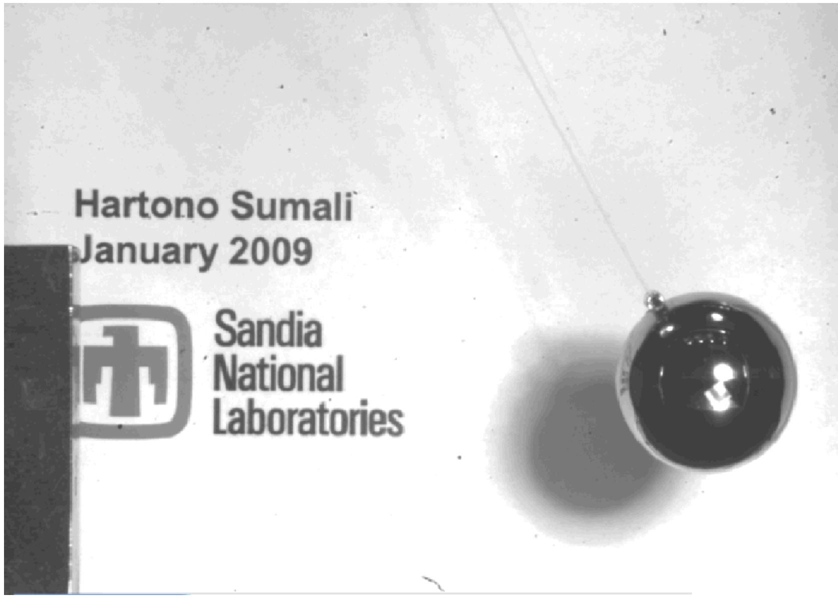


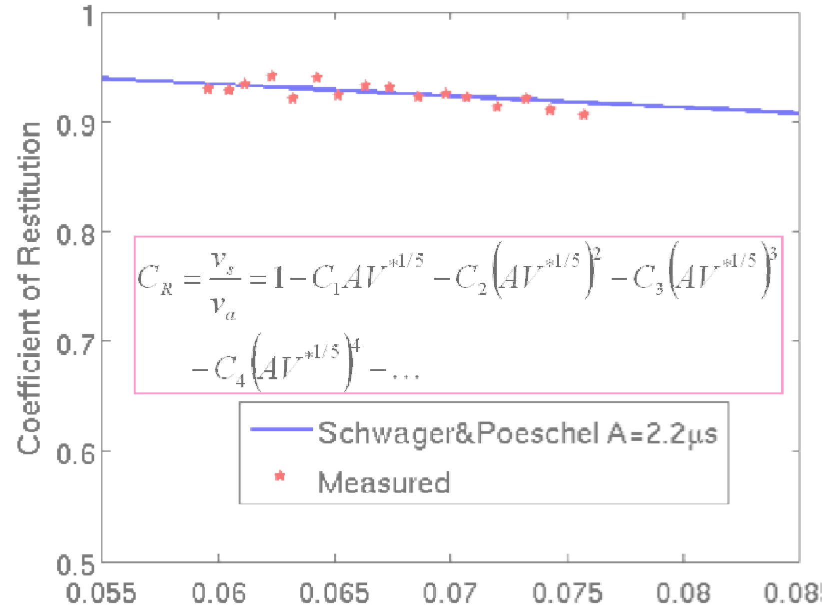
- Similar results for other materials and experiments

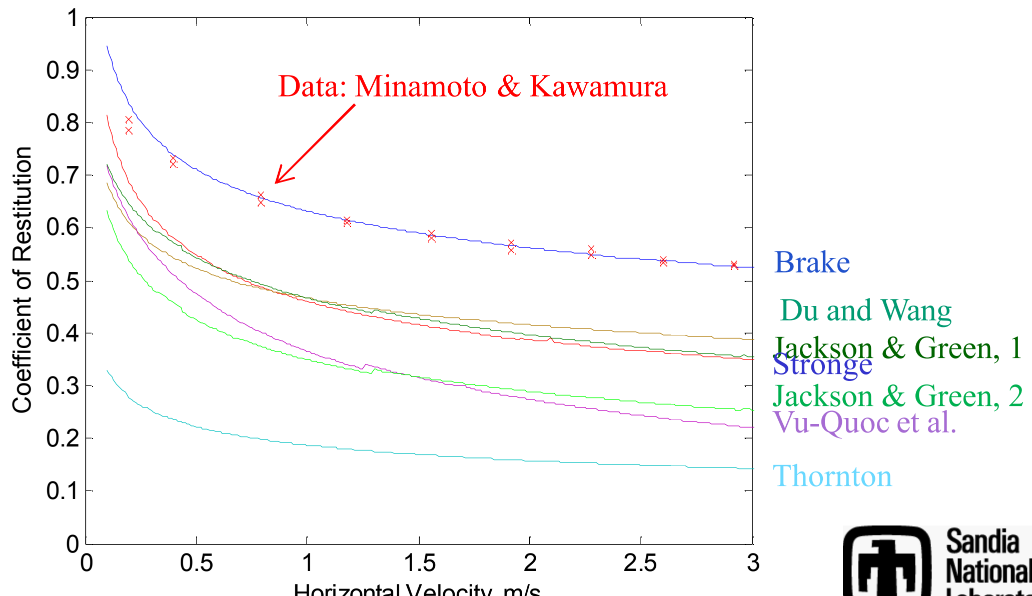
Conclusions and future work

- The pendulum-ball method records the motion of a sphere rebounding from a flat wall.
- The method results in a much slower motion of the ball compared to vertical drop.
- Thus, the method gives many more frames and higher temporal resolution than vertical drop.
- An experiment using the method indicates that the visco-elastic theory (Schwager et al, 1999) is valid for steel-to-steel impact in the speed range of 0.1 - 0.5 m/s.
- The theory depends on a parameter *A*, which can be obtained from the rebound measurement.
- The elastic-perfectly-plastic theory being developed by Brake at Sandia appears to fit experimental data very well.
- Brake's theory will be predictive.
- Future work will include:
  - A methods to obtain the parameter *A* from material property tests. This will predict visco-elastic loss at low velocities.
  - Refinement of Brake's theory to include strain hardening and other material behaviors.

Thank you.







Questions?

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