A Stable and Accurate Method for Tetrahedral Elastic-Plastic Computations

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NNSA National Nuclear Security Administration

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Motivation: Tetrahedral Meshes

- Solid mechanics on hex meshes: mixed staggered Q1/Q0 formulation
  - continuous linear kinematic variables
  - discontinuous piece-wise constant stresses
  - requires various hourglass controls (e.g. Belytschko-Flanagan)
- Tet meshes for solids:
  - use of automated fast meshing
  - ease of use for mesh adaptivity
  - ease of coupling with other physics (thermal, electromagnetic)

Overview of Recent Research

- Swansea: Bonnet, Burton, Marriot, Hassan, (P1/P1-projection)
- SANDIA: Dohrman, Key, Heinstein, Bochev, (P1/P1-projection)
- TU Munich-SANDIA: Gee, Wall, Dohrman, (P1/P1+P1/P0-+proj.)
- LLNL: Puso, Solberg, (P1/P1+P1/P0-+proj.)
- RPI: Maniatty, Klaas, Liu, Shephard, Ramesh, (P1/P1-stabilized)
- Chorin's projection: Onate, Rojek, Taylor, Pastor (P1/P1)
- UPC Barcelona: Chiumenti, Cervera, Valverde, Codina (P1/P1-stabilized)
- UIUC: Nakshatrala, Masud, Hjelmstad, (P1/P1+bubble)
- Swansea II: Bonet, Gil, (P1/P1-stabilized)
- Berkeley/Pavia: Taylor, Auricchio, Lovadina, Reali, (Mixed enhanced)
- UCSD/University of Padua: Krysl, Micheloni, Boccardo (Mixed enhanced)
- Caltech: Thoutireddy, Ortiz, Molinari, Repetto, Belytschko (Composite Tets)

Governing Equations (Mixed Form)

- Solve for $\{d, v, \bar{\sigma}, p\}$ satisfying momentum conservation, Cauchy stress decomposition, and velocity definition:

$$\rho\,\dot{v} = \nabla \cdot \sigma + \rho \cdot b, \quad \sigma = p\,I + \bar{\sigma}, \quad \dot{d} = v. \tag{1}$$

- Assume $\bar{\sigma}$ is a function of the kinematics (strains, strain rates), state variables, the history of $\bar{\sigma}$, etc.
- In the linear case we have the mixed system for displacement (u) and pressure (p):

$$\begin{aligned} \rho\,\ddot{u} - \nabla \cdot \overline{\varepsilon}(u) - \nabla p &= f \\ p - \kappa \nabla \cdot u &= 0 \end{aligned}$$

where $\overline{\varepsilon}(u)$ is the deviatoric strain tensor.

Linear Elasticity: Static Case

- Stabilization for linear elasticity is very similar to Stokes flow
- Incompressible case:
  - P1/P0 locking (as in P1 displacement formulation)
  - P1/P1 checkerboard instability for pressure
- Solution for P1/P1: Hughes/Franca/Balestra stabilization (1986): enrich the velocity/displacement (u) with a residual-based term

$$u = u_h + u', \quad u' = -\tau \frac{h^2}{2\mu}(-\nabla p_h - \nabla \cdot \epsilon(u_h) - f)$$

- Stabilization derives from the additional pressure Laplacian
- This is now called Variational Multiscale (VMS) stabilization

Linear Elasticity: Dynamic Case

- The Hughes/Franca/Balestra stabilization extends naturally to time-dependent Stokes/Navier-Stokes flows
- We could not find an appropriate τ that worked for linear dynamics
- The issue appears to be the different character of the PDEs:
  - elliptic: Stokes and elasticity
  - parabolic: time-dependent Stokes
  - hyperbolic: time-dependent elasticity
- Our solution is to formulate the pressure equation in rate form: which pairs naturally with the momentum equation:

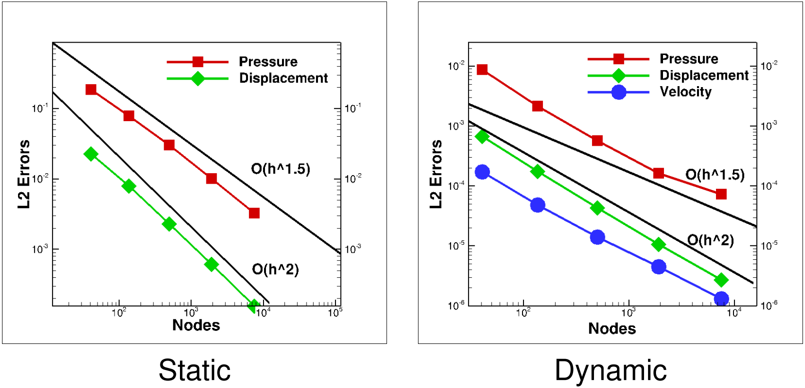
$$
\begin{aligned}
\kappa^{-1}\dot{p} - \nabla \cdot v &= 0 \\
\rho\,\dot{v} - \nabla p &= \nabla \cdot \overline{\varepsilon}(u) + f
\end{aligned}
$$

Linear Elasticity: Dynamic VMS

- The stabilization then is analogous to what is used for the linear acoustic wave equation
- We add in subgrid scales $\{v', p'\}$ defined using residuals
- The resulting pressure Laplacian and velocity div-div terms provide stabilization

Linear Elasticity: Verification

- We verified the linear elastic case under various options: static/dynamic, tri/quad, compressible/incompressible, struct/unstruct grids
- Verification test: analytic pressure/velocity/displacement with valid solution in the incompressible limit (here $\nu = 0.4995$)



Nonlinear Dynamics: Hyper-elasticity

- We concentrate on mixed constitutive models with pressure:

$$\sigma = p\, I + \bar{\sigma}$$

- Pressure is assumed a function of the volumetric part of the deformation gradient:

$$p \equiv \kappa\, U'(J) \tag{2}$$

  where J is the determinant of the deformation gradient F, U is an energy function and κ is a bulk modulus parameter.

- Deviatoric stress is defined in terms of J and $b = F\, F^T$, for example using a neo-Hookean law

$$\bar{\sigma} = \mu J^{-5/3}\, \bar{b}$$

Nonlinear Dynamics: J_2 Plasticity

- For plasticity, the pressure often remains a function of J
- The deviatoric stress is computed through an associative flow rule, with inclusion of constraints and plastic strain
- We have implemented a simple plasticity model (Simo and Hughes, 1998) using linear hardening and a product factorization:

$$F = F^e F^p$$

- Extensions to other models (e.g. hypo-elasticity) should be possible provided that
  - we have separate models for $\bar{\sigma}$ and p, and
  - the pressure remains a function only of J

Pressure Evolution Equation and VMS

- The nonlinear pressure equation in an evolution form:

$$\dot{p} = \kappa\, U''(J)\, J\, \nabla \cdot v = \tilde{\kappa}(J)\, \nabla \cdot v. \tag{3}$$

  where we used the identity

$$\dot{J} = J\, \nabla \cdot v,$$

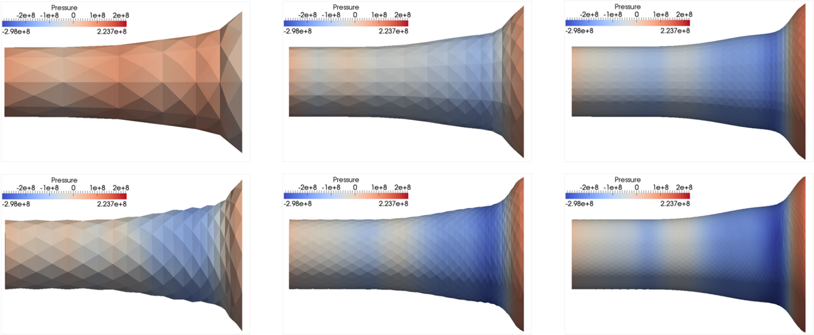
- We have defined an effective bulk modulus that varies as the material undergoes volume change.

$$\tilde{\kappa}(J) \equiv \kappa\, U''(J)\, J$$

- VMS stabilization is as in the linear case using a v' term

Taylor Bar: Pressure

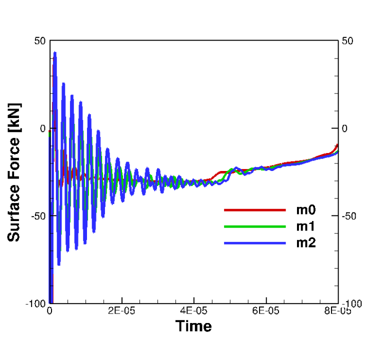
- Length/Radius: 3.24/0.32cm, density: 8930
- Elastic-plastic material (E=117.0e9, ν=0.35, σ_y=0.4e9, H=0.1e9)
- Fixed end, initial uniform x-velocity (smoothed)
- Zero normal velocity at wall, initial velocity 227



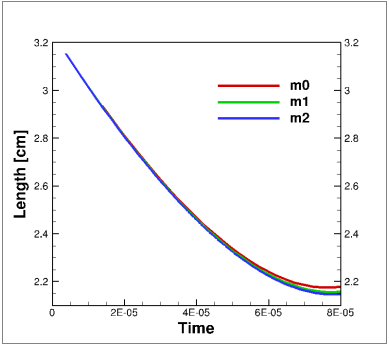
m_0 m_1 m_2

Taylor Bar: Force and Length History

- Convergence of reaction force (x-component) and final bar length:



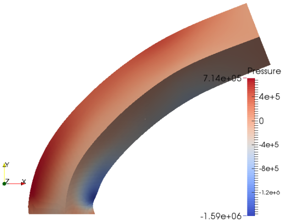
Wall Reaction Force



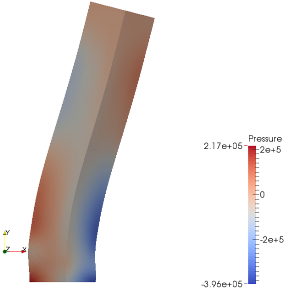
Final Length

Bending Beam: Pressure

- Length/Width: 6/1.4m, density: 1.1e3
- Neo-Hookian material (E=1.7e7, ν=0.45)
- Fixed end, initial uniform x-velocity (smoothed)



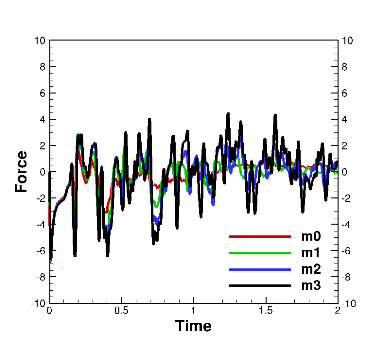
$t = 0.5$



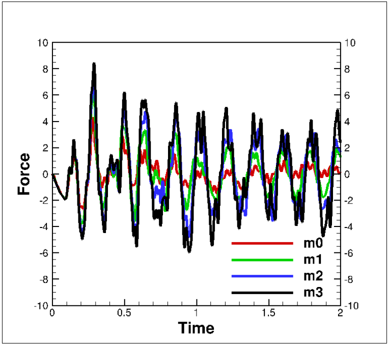
$t = 2.0$

Bending Beam: Force History

- We run on four uniform (unstructured) tet meshes (m_0-m_3)
- Convergence of reaction forces (x, y) at fixed surface:



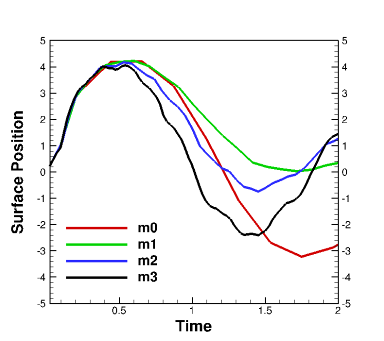
Wall Reaction Force (X)



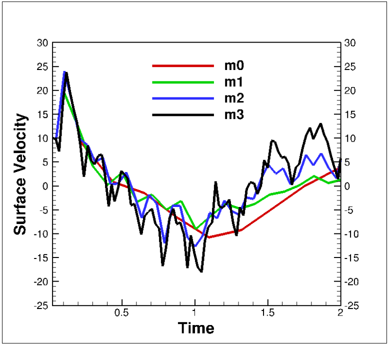
Wall Reaction Force (Y)

Bending Beam: Position/Velocity History

- Convergence of average position/velocity (x) at top surface:



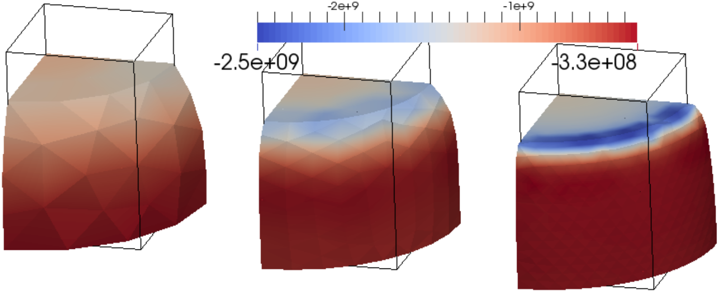
Position (X)



Velocity (X)

Billet in Compression:

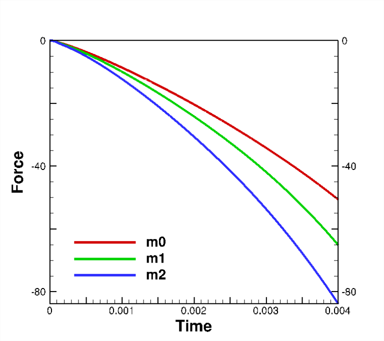
- Length/Radius: 1.5/1.0cm, density: 1e5
- Elastic-plastic material (E=384.62e9, ν=0.423, σ_y=1e9, H=3e9)
- Quasistatic approx. using dynamics (fictitious density, velocity)
- Top: dirichlet uniform velocity, bottom: zero normal displacement



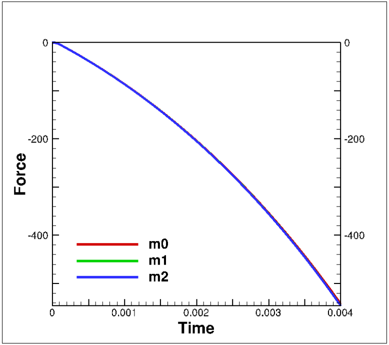
Pressure contours for three meshes (25% compression)

Billet in Compression: Force History

- Convergence of reaction forces (x, z) at moving surface (top):



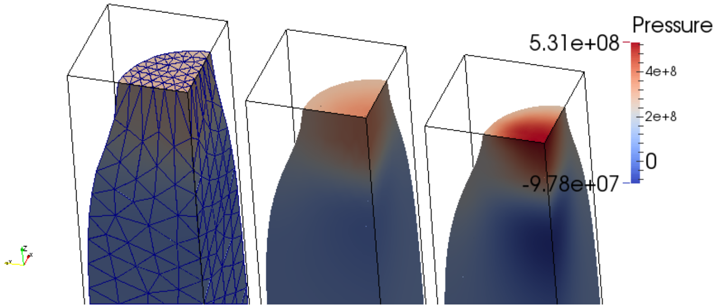
Wall Reaction Force (X)



Wall Reaction Force (Z)

Bar in Tension: Pressure

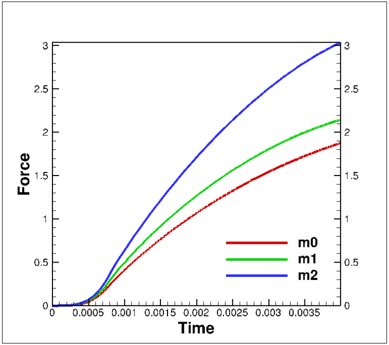
- Length/Radius: 5.33/0.641cm, density: 1e5
- Elastic-plastic material (E=80.2e9, ν=0.29, σ_y=0.45e9, H=0.13e9)
- Quasistatic approx. using dynamics (fictitious density, velocity)
- Top: zero normal displacement, bottom: dirichlet uniform velocity



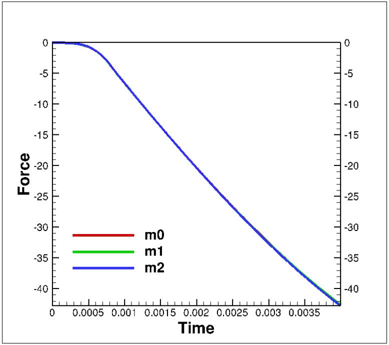
Pressure contours for three meshes (0.4cm extension)

Bar in Tension: Force History

- Convergence of reaction forces (y, z) at moving surface (bottom):



Wall Reaction Force (Y)



Wall Reaction Force (Z)

Summary and Ongoing Work

- Current status
  - Finite deformation solid mechanics capability for tet meshes
  - Method is stable and accurate (based on VMS)
  - Compatible with VMS-based nodal hydrocode (we have a separate fluid/solid coupling module)
- Ongoing work
  - Additional formal code verification
  - Performance improvements
  - Comparisons with hex-based solid mechanics codes
  - Publications on solids and fluid/solid coupling