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Title: Ejecta Transport Distribution Functions

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Ejecta Transport Distribution Functions

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Abstract

A simple analysis of ejecta transport distributions functions will be presented. Solutions for separable distributions will be derived. These solutions may be used as code test problems and for analyses of transport experiments such as the recent experiments of W. Buttler, D. Oro et al.

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Distribution functions for ejecta transport

Distribution function for number of particles in element drdvdm

$$f(\mathbf{r},\mathbf{v},m;t)d\mathbf{r}d\mathbf{v}dm$$

Relaxation time approximation

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}+\frac{\mathbf{F}}{m}\cdot\frac{\partial f}{\partial \mathbf{v}}=\left.\frac{df}{dt}\right|_{scat}=\frac{1}{\tau}df$$

Solution for no scattering

$$f(\mathbf{r},\mathbf{v},m;t)=\int\delta(\mathbf{r}-\mathbf{r}(t))\delta(\mathbf{v}-\mathbf{v}(t))f_0(\mathbf{r}_0,\mathbf{v}_0,m)d\mathbf{r}_0d\mathbf{v}_0$$

Distribution functions for ejecta transport

Equations of motion

$$\frac{d}{dt}\mathbf{r}(t)=\mathbf{v}(t)$$

$$\frac{d}{dt}\mathbf{v}(t)=\frac{\mathbf{F}}{m}$$

$$\mathbf{r}(0)=\mathbf{r}_0, \text{ and } \mathbf{v}(0)=\mathbf{v}_0$$

Connection to piezo-probe experiments

$$P(t)=\int f(z,v,m;t)mv^2dmdv$$

Distribution functions for ejecta transport

Vacuum solutions

$$f(z,v,m;t)=\int\delta(z-vt)f_0(z_0,v,m)dz_0$$

$$P_0(t)=(\frac{z}{t})^2\left[\frac{1}{t}\int f_0(z_0,\frac{z}{t},m)mdmdz_0\right]$$

$$f_0(z_0,v_0,m)=\frac{1}{A}\delta(z_0)f_0(v_0,m)$$

$$P_0(t)=(\frac{z}{t})^2\frac{1}{t}\frac{1}{A}\int f_0(\frac{z}{t},m)mdm$$

$$\rho_0(z;t)=\frac{1}{t}\int f_0(z_0,\frac{z}{t},m)mdmdz_0$$

Distribution functions for ejecta transport

Zero external field relationship between probe pressure and particle density

$$\rho_0(z;t) = \left(\frac{z}{t}\right)^{-2} P_0(t)$$

For non-zero external forces (e.g. drag forces) this relation no longer holds

Solutions for no-zero external field: particle drag

Particle drag force

$$\mathbf{F} = -\frac{1}{2}\rho_g C_D A \left|\mathbf{v} - \mathbf{v}_g\right| (\mathbf{v} - \mathbf{v}_g)$$

General spherical

$$C_D = \frac{24}{Re}(1 + 0.167 Re^{2/3}) \qquad (Re \le 10^3)$$

$$C_D = 0.42 \qquad (Re \ge 10^3)$$

Irregular shape

$$C_D = 1.4$$

Jet-like

$$C_D = \frac{2.17}{\sqrt{Re}}$$

where the Reynolds number is defined as

$$Re = \frac{D_p \left|v_p - v_g\right|}{\nu}$$

Solutions for no-zero external field: particle drag

η is the kinematic viscosity defined in terms of the coefficient of viscosity as $\eta=\rho_g \nu$, and D_p and v_p are the particle diameter and velocity.

Solutions

$$z(t,u_0)=v_{fs}t+u_0\tau(t,u_0)$$

General spherical

$$\tau(t,u_0)=3t_0B^{-\frac{3}{2}}\left\{\sqrt{B}\left[1-\frac{1}{\sqrt{1+B(1-x^2)}}\right]-\left[\sin^{-1}\left(\sqrt{\frac{B}{1+B}}\right)-\sin^{-1}\left(x\sqrt{\frac{B}{1+B}}\right)\right]\right\}$$

$$B=0.167(Re^{(0)})^{\frac{2}{3}} \qquad x=\exp(-\tfrac{1}{3}\tfrac{t}{t_0}) \qquad t_0=\frac{1}{18}\frac{\rho_p D_p^2}{\eta}$$

Solutions for no-zero external field: particle drag

Irregular shape

$$\tau(t,u_0)=t_b\log(1+\frac{t}{t_b})$$

$$t_b=\frac{24}{C_D}\frac{1}{Re^{(0)}}t_0=0.95\frac{D_p}{u_0}\left(\frac{\rho_p}{\rho_g}\right)$$

Jet-like

$$\tau(t,u_0)=t_b\left[\frac{t/t_b}{1+t/t_b}\right]$$

$$t_b=\frac{22.1}{\sqrt{Re^{(0)}}}t_0=1.7\left(\frac{\rho_p}{\rho_g}\right)^{\frac{1}{2}}\sqrt{\frac{m_p}{\eta u_0}}$$

Solutions for no-zero external field: particle drag

In the above expressions:

$$Re^{(0)} = \frac{D_p \left| v_p(t=0) - v_g \right|}{\nu}$$

$$m_p = \rho_p \frac{\pi}{6} D_p^3$$

Separable distribution functions

Simplifications occur when the distribution function is separable

$$f_0(\mathbf{v},m)=f_0^{(1)}(\mathbf{v})f_0^{(2)}(m)$$

$$\rho(z,t)=\frac{1}{A}\int f_0^{(1)}(v_0(z,t))\frac{1}{\left|\frac{dz}{du_0}\right|_{u_0(z,t)}}f_0^{(2)}(m)mdm$$

$$=\frac{1}{M}z\int P_0\left[\frac{z}{v_0(z,t)}\right]\frac{v_0(z,t)^{-3}}{\left|\frac{dz}{du_0}\right|_{u_0(z,t)}}f_0^{(2)}(m)mdm$$

$$P(t)=\frac{1}{M}z\int P_0[\frac{z}{v_0(z,t)}]\frac{v_0(z,t)^{-3}}{\left|\frac{dz}{du_0}\right|_{u_0(z,t)}}v(t,v_0(z,t))^2f_0^{(2)}(m)mdm$$

Separable distribution functions

Where

$$v_0(z,t) = v_{fs} + u_0(z,t)$$

And $u_0(z,t)$ is the solution of

$$z - v_{fs}t = u_0(z,t)\tau(t,u_0(z,t))$$

It is noteworthy that:

- The simple vacuum relationship between density and pin pressure no longer holds.
- The expressions for density and pin pressure are siple integrals over the ***measured*** pin response for vacuum, given the particle size distribution

Example I: Sn-He experiments (Buttler et al.)

For Stokes drag the expressions for pin pressure are, assuming a power law size distribution

$$f_0^{(2)}(D) = N_0 \gamma \left(\frac{D}{D_0}\right)^{-(\gamma-1)} \frac{1}{D_0} \qquad (D \geq D_0)$$

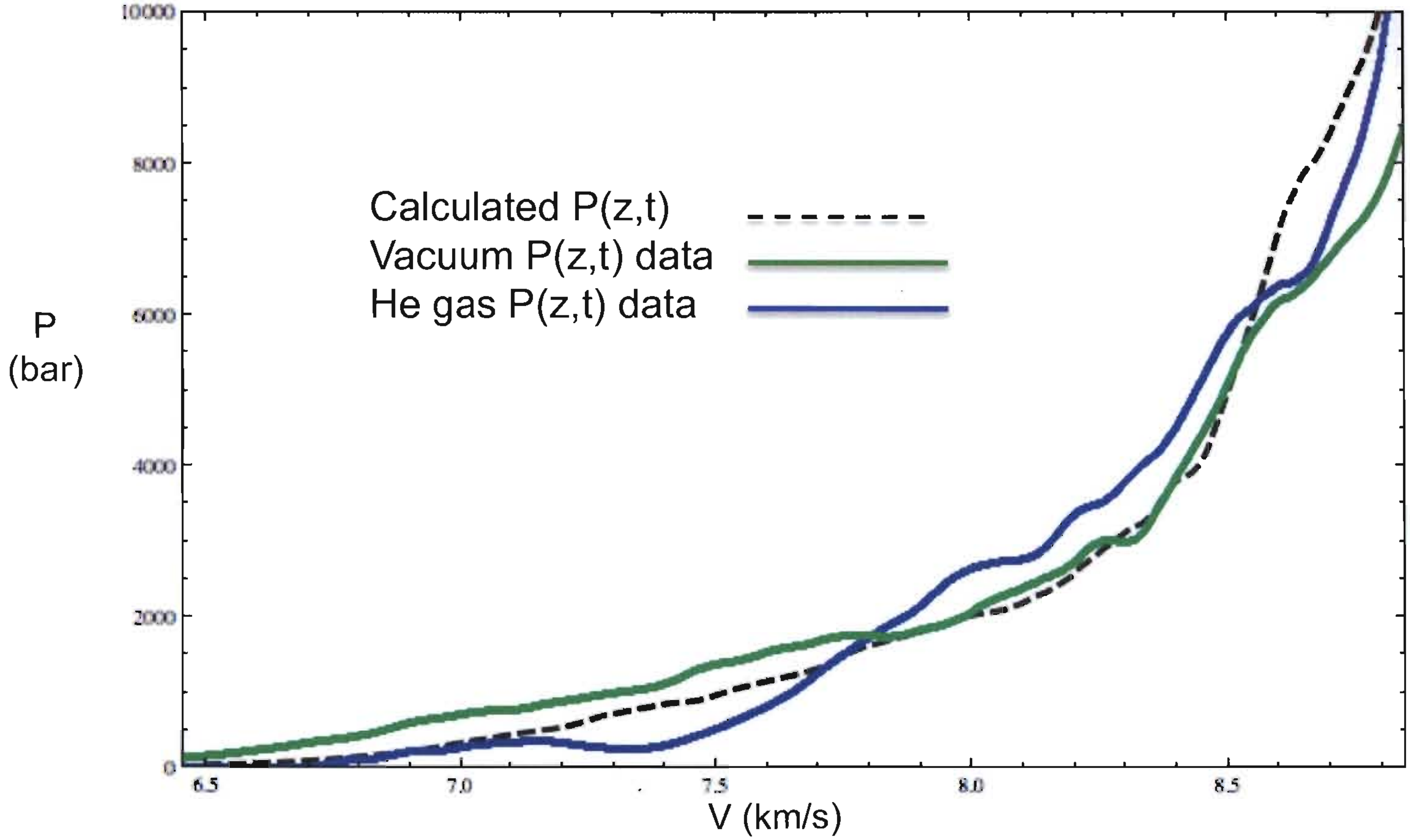
$$P(z;t) = \left(\frac{\gamma-3}{\gamma}\right)(\lambda t)^{-\frac{1}{2}(\gamma-3)} \int_0^{\lambda t} dx \left(\frac{z}{v}\right) P_0\left(\frac{z}{v}\right)\left[1 - \frac{x}{v}\left(\frac{z}{t} - v_{fs}\right)\right]^2 \frac{x^{\frac{1}{2}(\gamma-3)}}{(1-\exp-x)}$$

With

$$v = v_{fs} + \frac{x}{(1-\exp-x)}\left(\frac{z}{t} - v_{fs}\right)$$

$$\lambda^{-1} = t_0^{[min]} = t_0 = \frac{1}{18}\frac{\rho_p D_0^2}{\eta}$$

Example I: Sn-He experiments (Buttler et al.)



Data for Sn with a 60 μ-in finish driven with a Taylor wave profile with a shock pressure of 33 GPa. A value of $\gamma = 6$ was chosen for the comparison. The initial He gas pressure was 4 bar.

Example II: Transport of W surface ejecta in Ar and Xe (Buttler, Oro et al.)

- Nearly mono-disperse 40 μm layer of Tungsten particles with diameters between 0.5 and 1.0 μm
- The initial gas densities were 17 kg/m^3 and the initial temperatures 300 K.

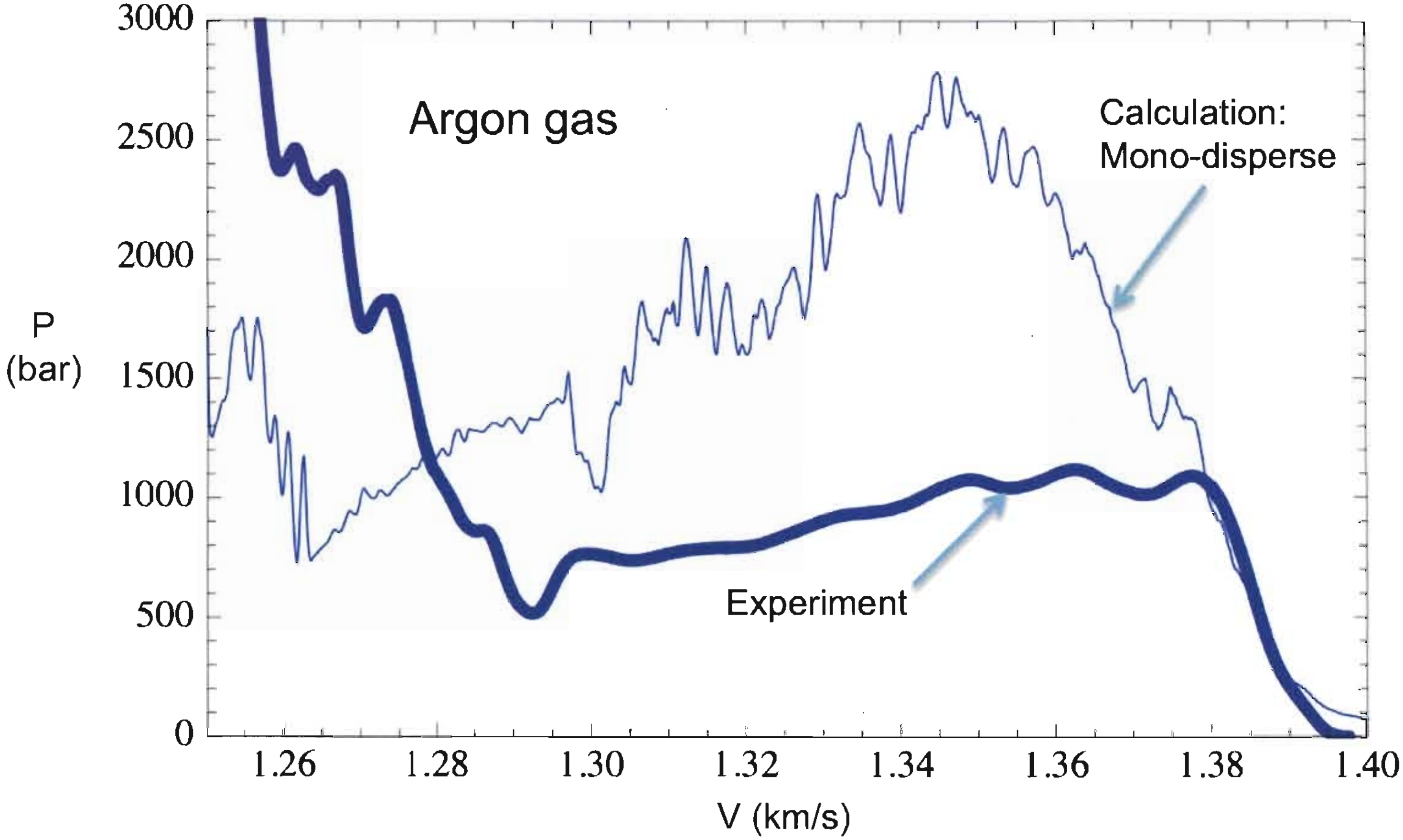
Mono-disperse expressions for P(z,t) in Stokes limit

$$P(z,t)=\frac{t}{\lambda(z,t)\tau(t)}\left\{1-\frac{t}{t_0}\frac{1}{\lambda(z,t)}(1-\frac{v_{fs}t}{z})\right\}^2 P_0(\frac{t}{\lambda(z,t)})$$

$$\lambda(z,t)=1+(\frac{t}{\tau(t)}-1)(1-\frac{v_{fs}t}{z})$$

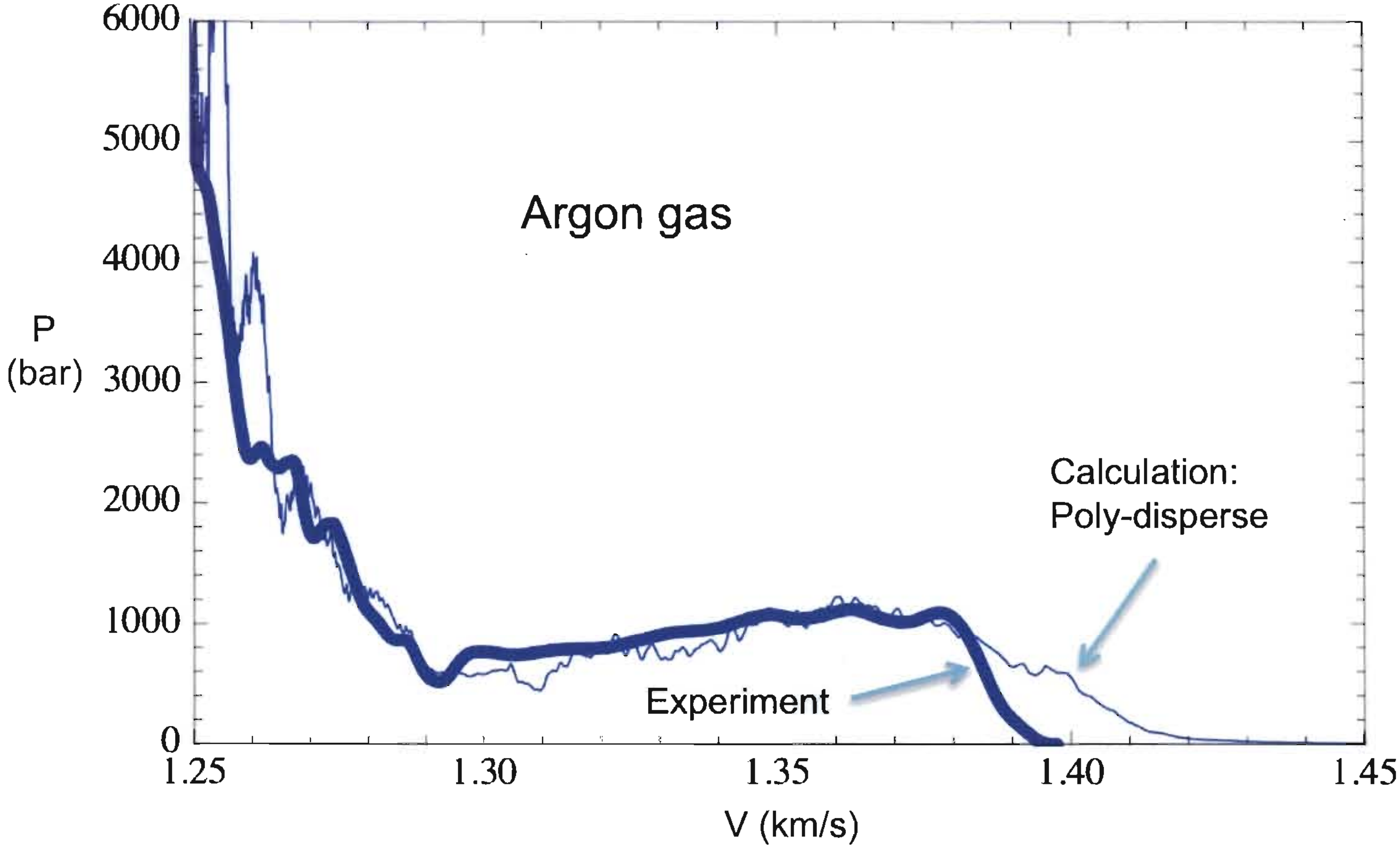
$$\tau(t)=t_0(1-\exp(-\frac{t}{t_0}))$$

Example II: Transport of W surface ejecta in Ar and Xe (Buttler, Oro et al.)



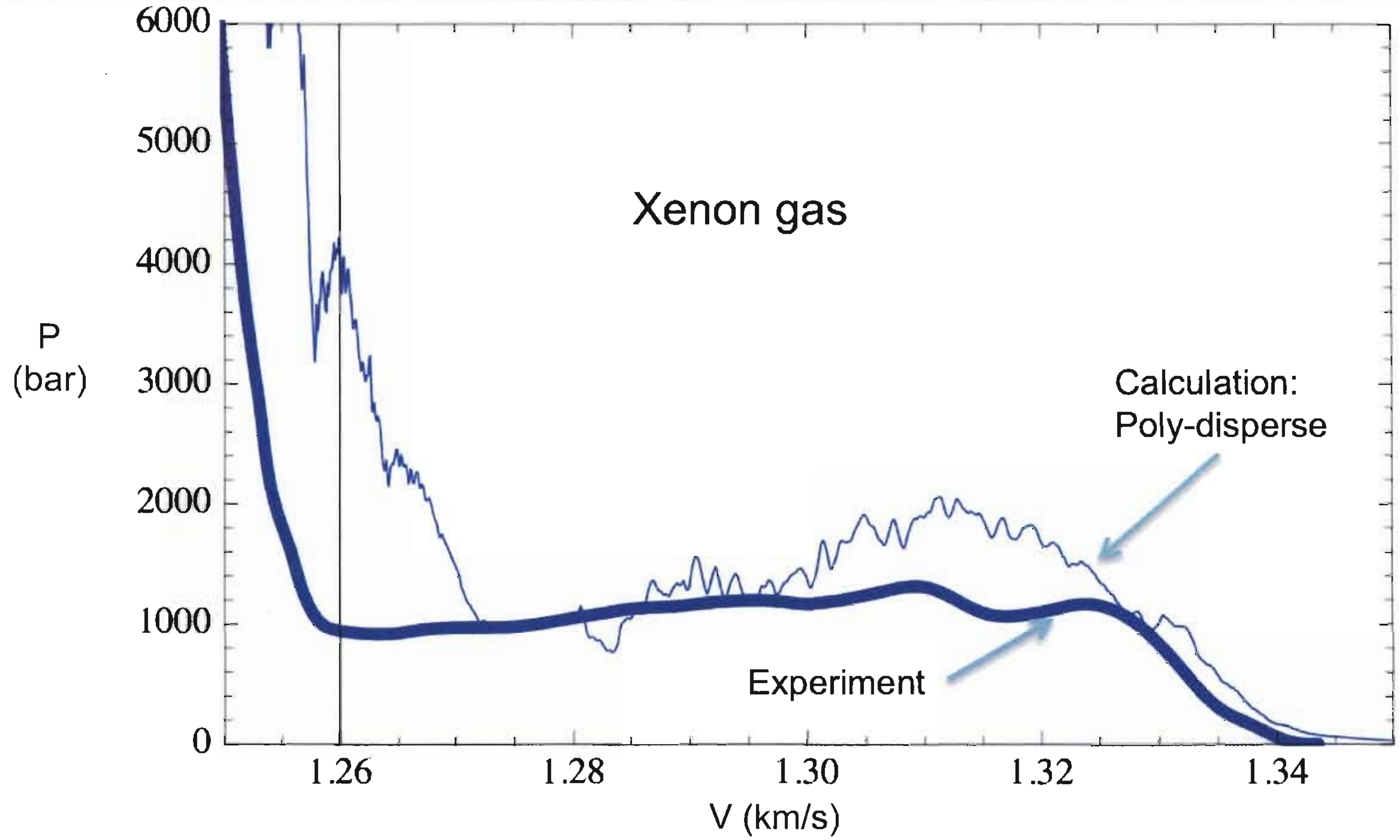
Thick line: Argon gas experimental data. Thin line: Calculation assuming Stokes drag using t_0=9.0 μs
Corresponding to a value of viscosity of 119 μPa-s.

Example II: Transport of W surface ejecta in Ar and Xe (Buttler, Oro et al.)



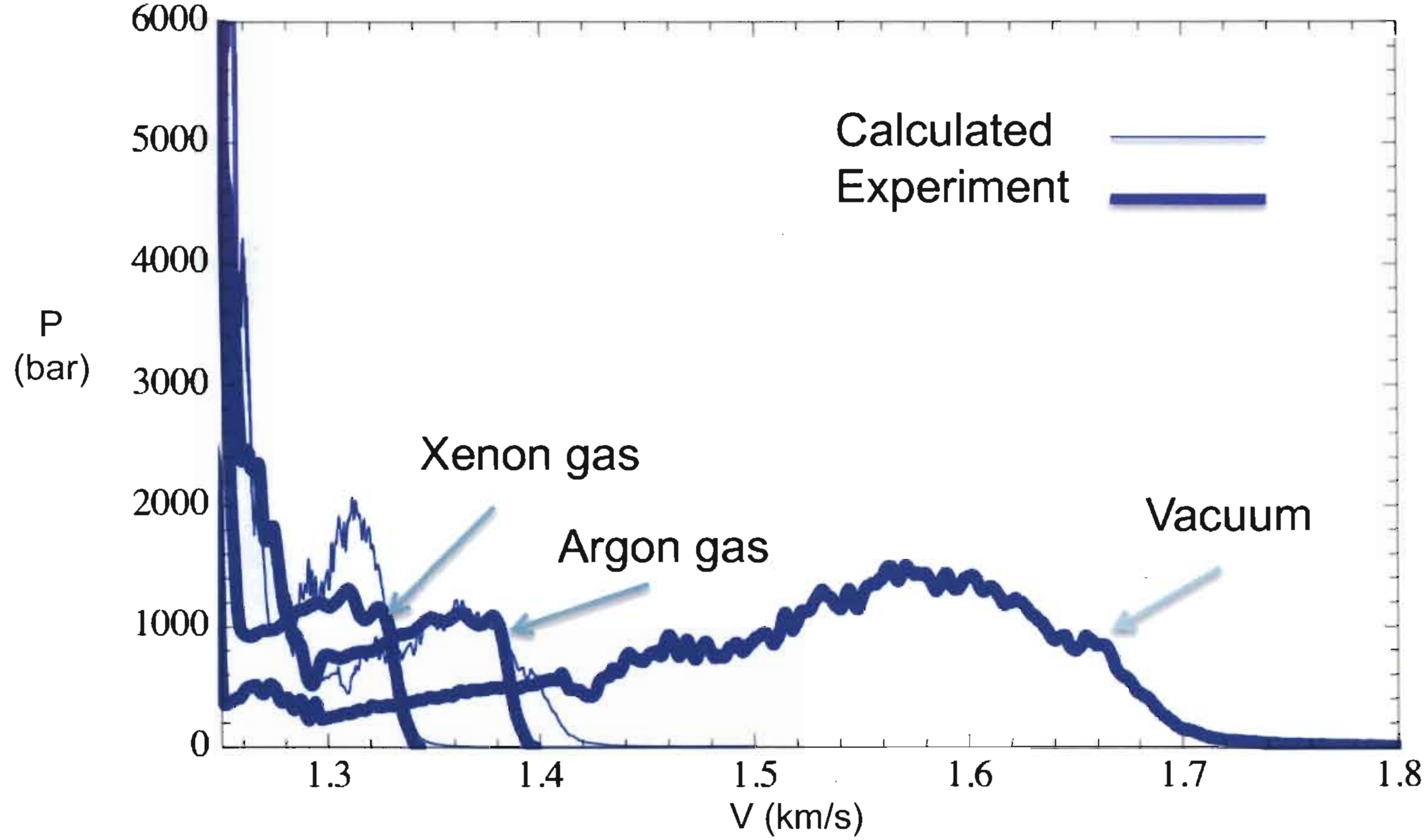
Thick curve: Argon gas experimental data. Thin curve: calculation assuming a viscosity of 100 μPa-s and a Distribution of particle diameters in the rations 0.1[0.2 μm] + 0.1[0.3 μm] + 0.1[0.4 μm] + [0.1[0.5 μm] + 0.5[1.0 μm].

Example II: Transport of W surface ejecta in Ar and Xe (Buttler, Oro et al.)



Thick curve: Xenon gas experimental data. Thin curve: calculation assuming a viscosity of 186 μ Pa-s and a Distribution of particle diameters in the rations 0.1[0.2 μm] + 0.1[0.3 μ m] + 0.1[0.4 μ m] + [0.1[0.5 μm] + 0.5[1.0 μm].

Example II: Transport of W surface ejecta in Ar and Xe (Buttler, Oro et al.)



Summary of W vacuum and gas transport data and calculation assuming Stokes drag and spherical particles. Heavy line, experiment. Thin line, calculation.

Summary

- Expressions for the piezo-probe pressure and density have been derived for ejecta transport in the drag approximation for separable distribution functions.
- These expressions may be used as code test problems and as a means to analyze ejecta transport piezo-probe experiments.
- Two applications have been given for different source mass distributions.
- For the W experiments the calculations have indicated a probable subsequent breakup in the hot Xe gas whose temperature exceeds the melting temperature of W in distinction to the case for Ar where the gas temperature is below the W melt temperature.
- It is possible to include time dependent breakup in the expressions derived.