LA-UR- 11-05865

Approved for public release; distribution is unlimited.

Title: Quantification of Margins and Uncertainties: A Probabilistic ~~Approach~~ Framework

Author(s): Timothy C. Wallstrom, T-4

Intended for: Oral presentation at Nuclear Explosives Design Physics Conference, 2011 (NEDPC 2011)

Los Alamos NATIONAL LABORATORY

EST 1943

Quantification of Margins and Uncertainties: A Probabilistic Framework

Timothy C. Wallstrom
LANL, T-4
tcw@lanl.gov

Abstract

Quantification of Margins and Uncertainties (QMU) was originally introduced as a framework for assessing confidence in nuclear weapons, and has since been extended to more general complex systems. We show that when uncertainties are strictly bounded, QMU is equivalent to a graphical model, provided confidence is identified with reliability one. In the more realistic case that uncertainties have long tails, we find that QMU confidence is not always a good proxy for reliability, as computed from the graphical model. We explore the possibility of defining QMU in terms of the graphical model, rather than through the original procedures. The new formalism, which we call probabilistic QMU, or pQMU, is fully probabilistic and mathematically consistent, and shows how QMU may be interpreted within the framework of system reliability theory.

QUANTIFICATION OF MARGINS AND UNCERTAINTIES: A PROBABILISTIC FRAMEWORK

Timothy C. Wallstrom

Theoretical Division, Los Alamos National Laboratory

NEDPC 2011
October 19, 2011

QMU

What is it?

- Framework for making assertions about confidence in nuclear weapons performance.
- Founding document: Goodwin and Juzaitis (GJ), 2002.
- Many competing interpretations.

Questions:

- Why should nuclear weapons have their own statistical framework? Relation to reliability theories?
- More generally, how is it related to probability theory?
  - No apparent probabilistic content
  - No explicit distributions.
  - Binary output. Does not output a probability.
- Is it sound? (JASONs and NAS/NRC say yes.)

To answer these questions, I have developed pQMU (RESS, 2011).

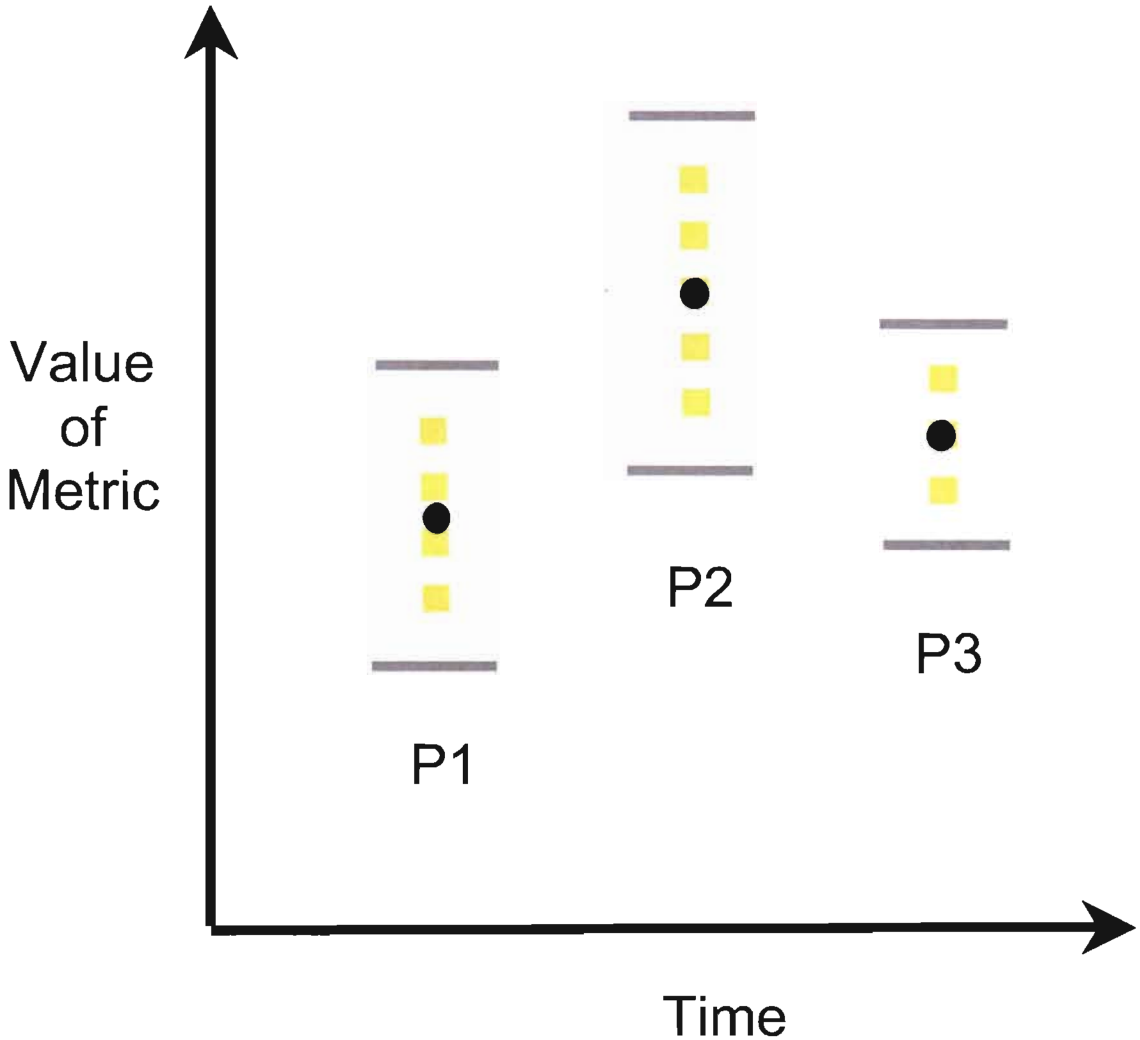
pQMU = probabilistic QMU

- Fully probabilistic formulation of QMU in terms of Graphical Models (GMs).
- Outputs a reliability r, rather than a binary confidence.
- Agrees with QMU when uncertainties are bounded.
  - $r = 1 \Leftrightarrow$ confidence.
  - Ordinary QMU *assumes* bounded distributions.
- But...pQMU also works when uncertainties are unbounded.
- So pQMU is a *generalization* of QMU to the case when uncertainties are unbounded.

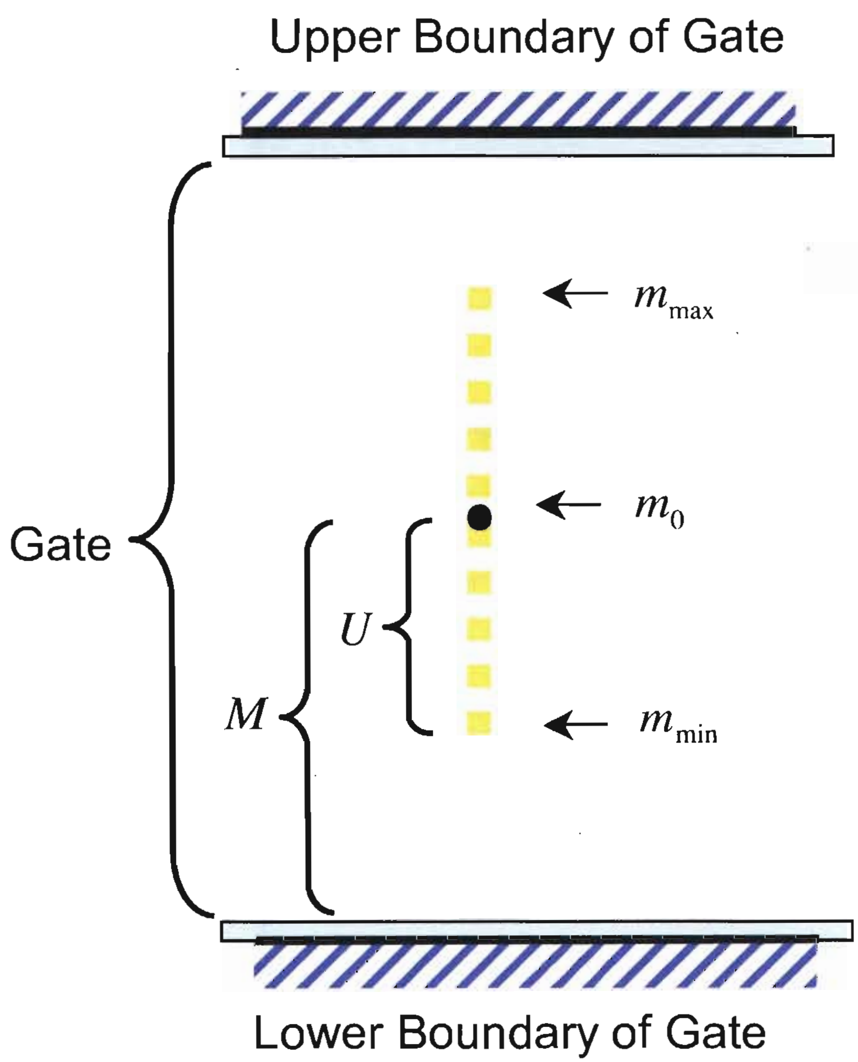
Key issue: extending QMU to unbounded distributions.

- Most realistic distributions are unbounded. (Strict bounds are too large to be useful.)
- Two ways to generalize QMU:
  - Use pQMU.
  - Truncate distributions and use ordinary QMU.
- How do methods compare? Is QMU confidence a good proxy for high reliability?

QMU for system with three performance gates



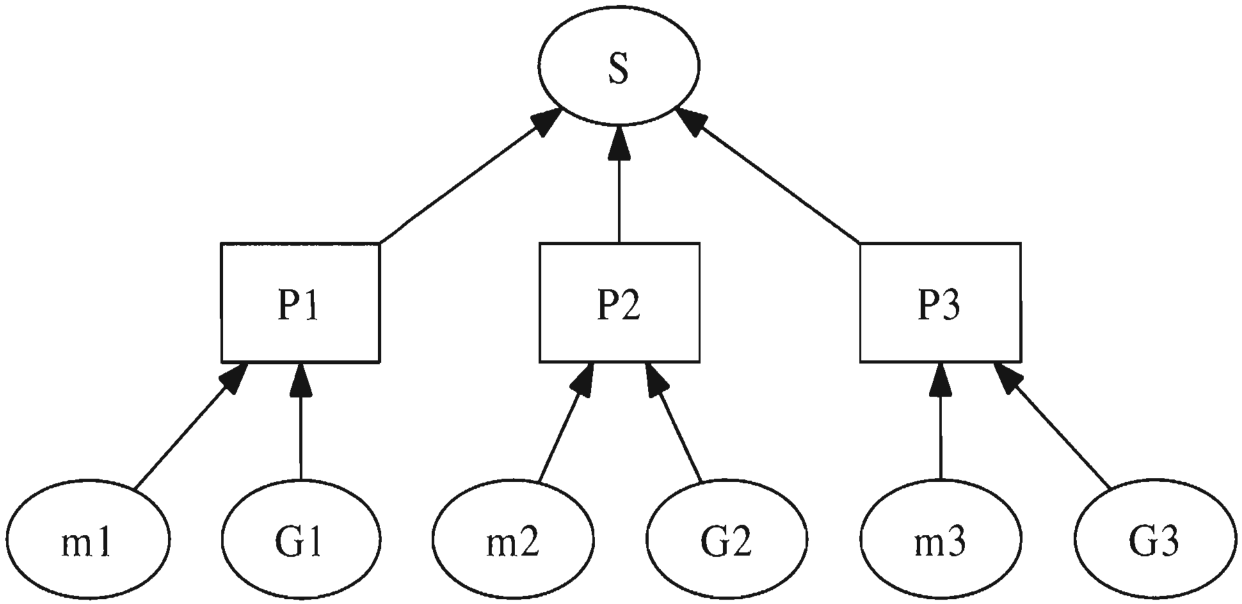
Detailed anatomy of Performance Gate



Key components:

- Metric (m).
- "Performance gate." (Gives "margin" M.)
- Uncertainty U.
  - *Maximum* possible deviation.
- Confidence ratio $CR = M/U$.
- If $CR > 1$ at all gates, then we have "QMU confidence." Otherwise, we don't.

QMU as a Graphical Model



- m_i: random variables.
- $P_i = 1$ if $m_i \in G_i$, zero otherwise.
- $S = 1$ if $P_1 = P_2 = P_3 = 1$; zero otherwise.
- $P(S, P_i, m_i) = P(S|P_i) \cdot \prod_{i=1}^{3} P(P_i|m_i, G_i) \cdot \prod_{i=1}^{3} P(m_i)$.
- Reliability: $r \equiv P(S = 1)$.

Comparison: bounded uncertainties

$$\text{QMU confidence} \Leftrightarrow r = 1.$$

Comparison: unbounded uncertainties, QMU

- U is infinite.
- Must assign finite U by some method
  - E.g., choose fixed quantile.
- In effect, we are replacing $p(m)$ with $p(m \mid |m| < U)$.

QMU confidence means:

> *"100%" certainty that device performs as intended provided model is correct and complete and there are no tail events.*

Very different from usual approach to reliability.

Comparison: unbounded uncertainties, pQMU

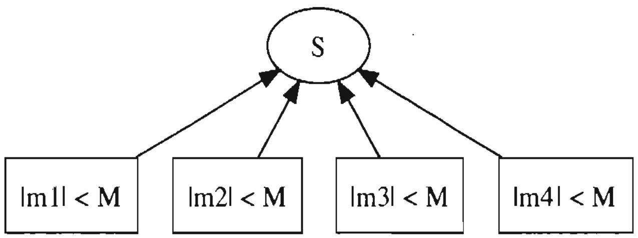
Take GM as primary definition of QMU.

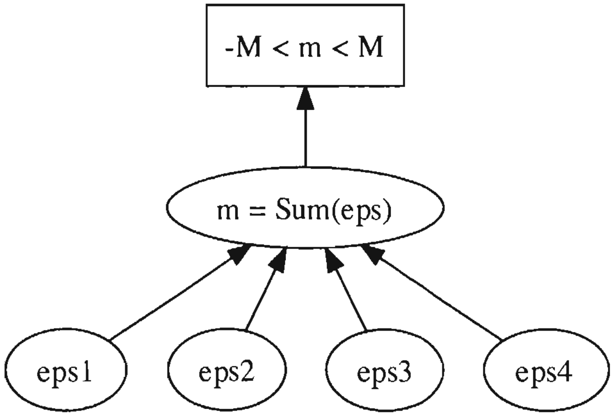
- Consistent, computable formalism, even for unbounded uncertainties.
- Brings QMU fully within fold of reliability theory.
  - GMs ("Bayesian Networks") are playing an increasingly important role in reliability theory over past decade.
- Upgrade to previous approaches.
  - Fault trees and RBDs special cases of GMs.
  - Key new feature: performance gates.
- Permits useful generalizations ("leaky" AND gate.)
- Model can be updated using Bayesian inference.

Comparison: QMU vs. pQMU

Q: Is QMU confidence a good proxy for reliability?

A: No.





Can lead to “confidence” when reliability is low:

- Four gates, with m_i Gaussian.
- Can have QMU system confidence when $r = 0.83$.

Can lead to “no confidence” when reliability is high:

- Metric uncertainty sum of four independent Gaussian uncertainties.
- Can lack QMU confidence when r=0.9999 .

But how do you assign distributions?

Apparently a big difference:

- QMU does not require assignment of distributions.
- pQMU does.

Advantage QMU? Let's look closer.

- Problem is unavoidable.
  - Every formalism must assign tail probabilities, explicitly or implicitly, just in order to get an answer.
- QMU makes no commitment in the main body of the distribution, where it doesn't matter.
- In the tails, QMU makes the strongest possible assumption: it sets the tails to zero.
- Worst possible choice.
  - Thin tails $\Rightarrow$ underestimate of risk.
  - Can't get any thinner than zero!

Solution is to consider many possible uncertainty distributions.

Reference:

Wallstrom TC. "Quantification of margins and uncertainties: A probabilistic framework," Reliability Engineering and System Safety, Vol 96, p. 1053-1062, (2011).