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Title: Computing & Verifying Compressible Fluid Dynamics: The Good, The Bad and The Ugly

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Title:
Computing & Verifying Compressible Fluid Dynamics: The Good, The Bad and The Ugly

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Abstract:
Many practical engineering calculations utilize the inviscid compressible Euler equations of fluid mechanics. And although many methodologies for constructing consistent numerical methods for these equations have been successful, there remain many outstanding issues. Several examples of "The Good," "The Bad," and "The Ugly" will be presented and discussed. The focus here will be on solutions containing discontinuities, and the variety of schemes built to handle them. For a single numerical method, one may observe high order, low order and zero order convergence for depending solely on the chosen initial conditions.

Computing & Verifying Compressible Fluid Dynamics: The Good, The Bad and The Ugly

Tariq Aslam

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WX-9: **Shock** and Detonation Physics

Background:

Verification: Solving the Equations "Right"

- Comparison with Exact/Manufactured Solutions
- Error Norms & Rates of Convergence
- Truncation Error, Round Off Error
- Are the above ***representative*** of the solutions we are really interested in?

Validation: Solving the "Right" Equations

- Finite Error must be tolerated in both the model and experiment
- How good is good enough?

Focus on the Euler Equations:

$$(\rho)_t + (\rho u)_x + (\rho v)_y = 0$$

$$(\rho u)_t + (\rho u^2 + p)_x + (\rho u v)_y = 0$$

$$(\rho v)_t + (\rho u v)_x + (\rho v^2 + p)_y = 0$$

$$(E)_t + (uE + up)_x + (vE + vp)_y = 0$$

$$E = \rho e + \frac{\rho}{2}(u^2 + v^2)$$

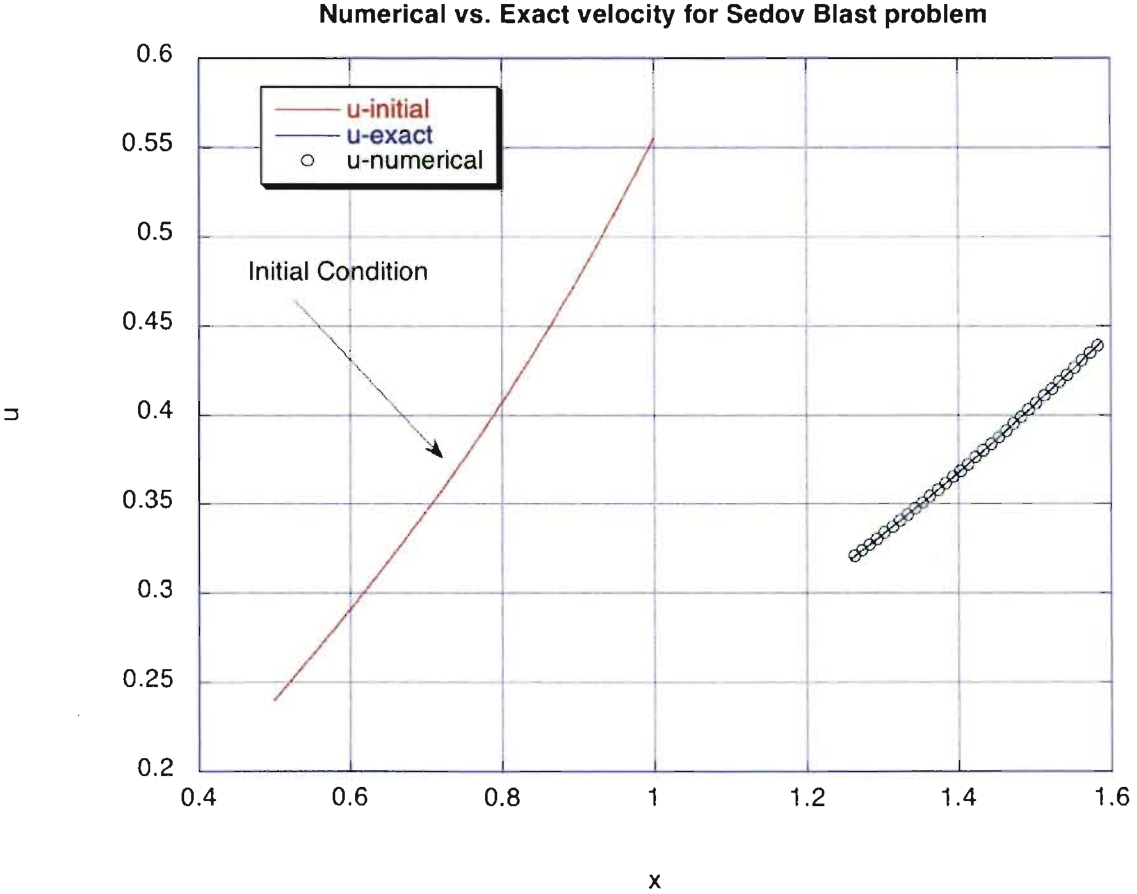
$$e = e(p, \rho)$$

A rich set of discontinuous waves can exist (shock, slip, contact, etc.)

The Good:

High Order Shock Fitting/Tracking

Although nontrivial, shock fitting/tracking techniques can be used to obtain “as advertised” high rates of convergence.



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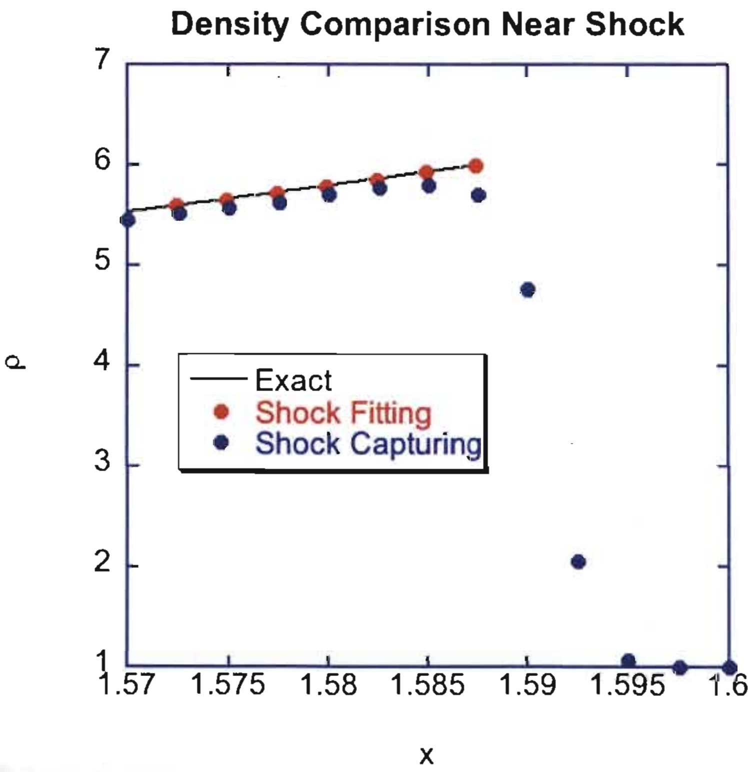
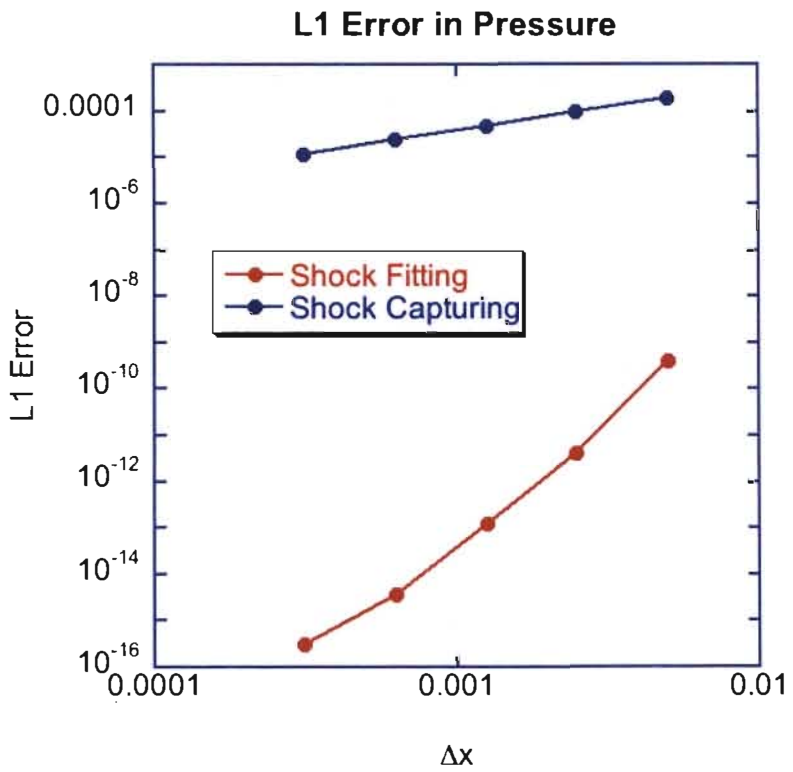




The Good:

Not so Good…

From a practical point of view, it is nearly impossible to use fitting/ tracking techniques for general problems in multiple dimensions.

We generally are relegated to shock capturing techniques…

The Bad:

Shock (Contact, Slip) Capturing

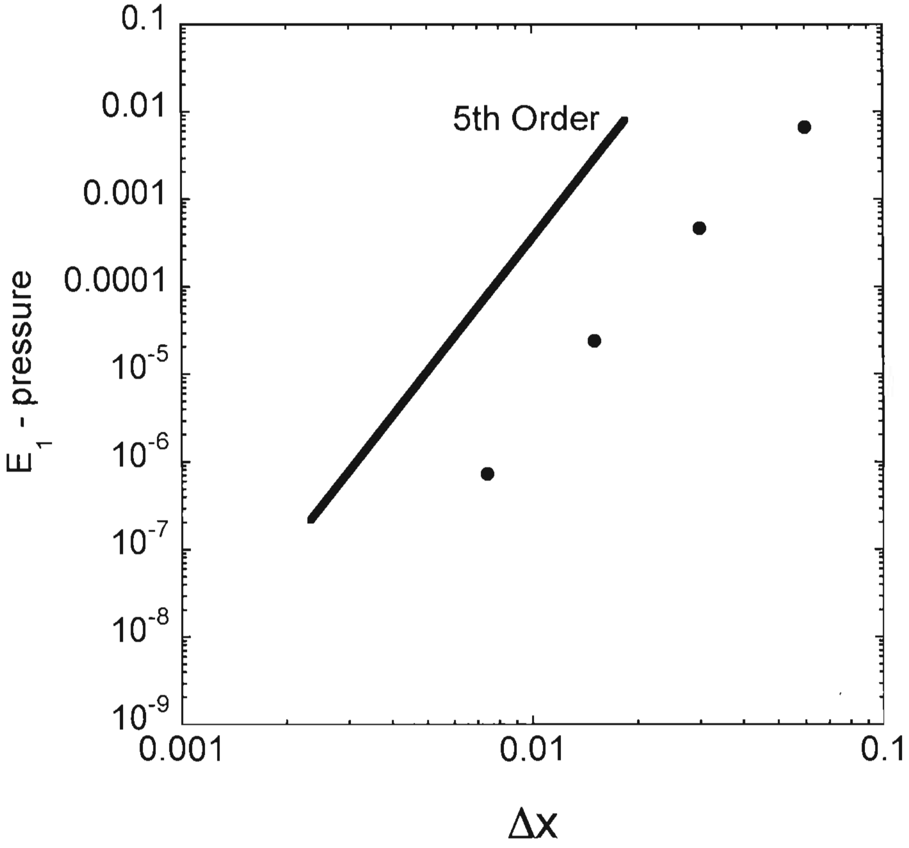
A rather simplistic approach to discretizing the Euler Equations…

…That often leads to unexpected convergence behavior!

The Bad:

Shock Capturing works well with no Shocks

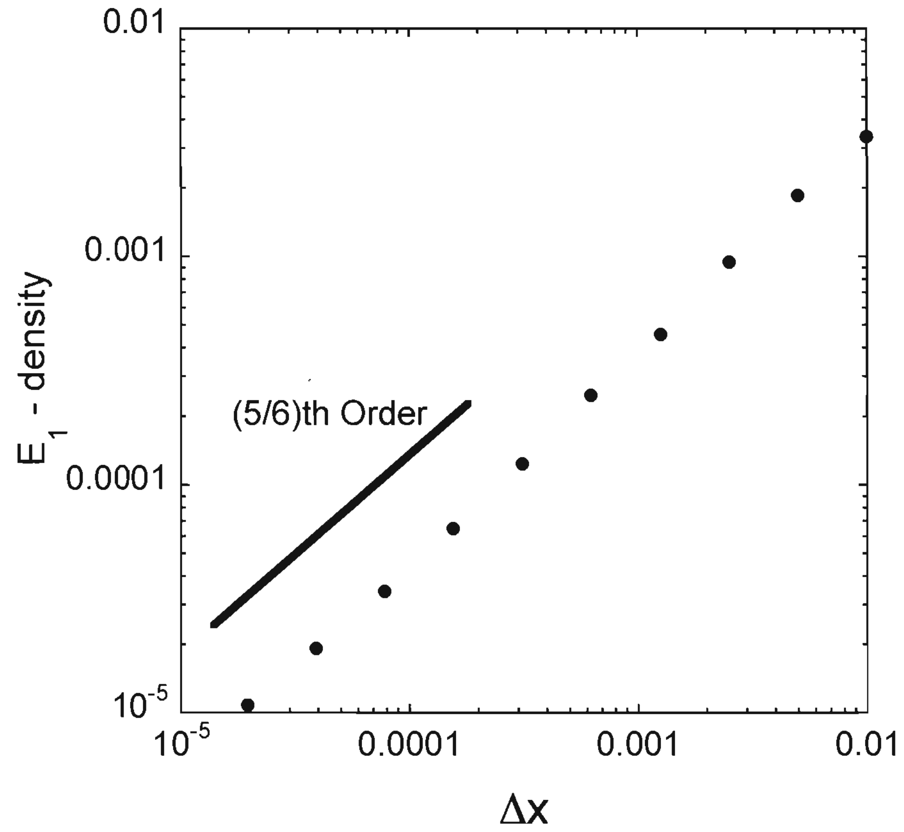
WENO5 combustion example from Xu, Aslam, Stewart (1997):



The Bad:

Shock Capturing works poorly with Discontinuities

WENO5M Sod Riemann Problem from Henrick, Aslam, Powers (2006):



The Ugly:

Without some "regularization," many multidimensional Euler simulations do not converge "nicely"

Reasoning:

K-H Instabilities

R-M Instabilities

R-T Instabilities

Growth rate ∝ Wave Number

Conclusions:

- Many different kinds of convergence can be observed for the same “verified” solution methodology

- For many interesting problems, esp. in multi-D, instabilities can plague inviscid Euler solutions

- Some regularization is then required (viscosity, thermal/ mass/diffusion, surface tension, material strength, etc.)

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