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SALINAS: A Massively Parallel Finite Element Code for Structural Dynamics and Acoustics Analysis

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Introduction / Outline

- History
- Domain Decomposition
- Solution Methods
- Element Types
- Structural Acoustics Formulation
- Quadratic Eigenvalue Problem
- Structural Acoustic Tying/Mortars
- Infinite Elements
- Inverse Methods
- Conclusions

History and Intent

- **Created in 1990's as part of Accelerated Strategic Computing Initiative (ASCI) of the US Dept. of Energy**
- **Intended for *extremely* complex finite element analysis**
  - Models with 10s or 100s of millions of DOF
- **Scalability**
  - Ability to solve *n*-times larger problem using *n*-times larger number of processors in nearly constant CPU time
- **Code portability**

To Meet ASCI Requirements

- **Massively Parallel**
  - Distribution of processors (nodes), each with own memory, linked together by a specialized network communication system
- **Employ Domain Decomposition Methods**
  - First performed by Schwarz in the 1870s
- **Began First Using FETI-DP solver**
  - "Finite Element Tearing and Interconnecting" *(C. Farhat, et al., 2000)*
  - Versatile iterative solver
- **Current Solvers:**
  - FETI-DP and FETI-DPH
  - GDSW *(C. Dohrmann, et al., 2007)*
  - Others

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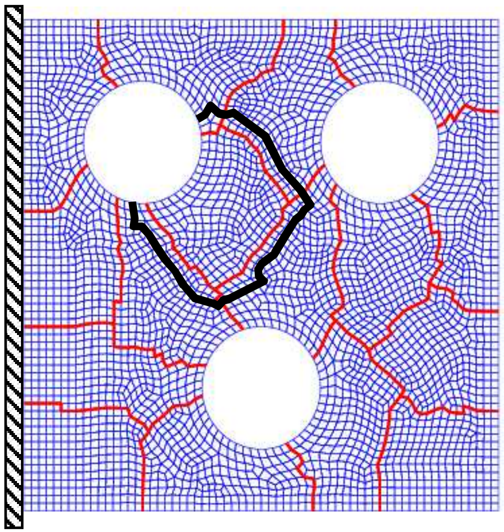
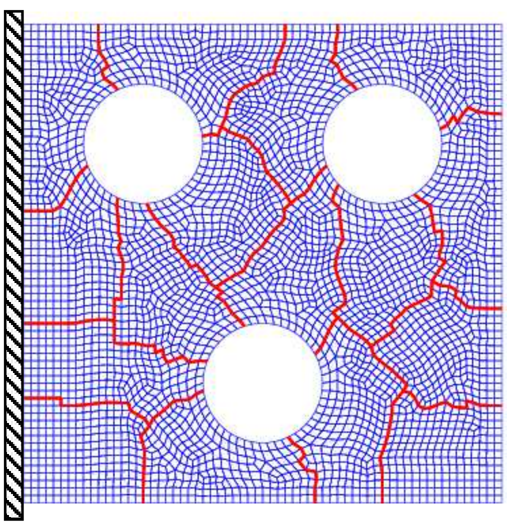
Domain Decomposition



Schwarz Methods
(Overlapping)



Schur Complement
Methods
(Iterative
Substructuring)

- Decompose model into smaller subdomains
- Each subdomain is often assigned to one processor
- Two-level methods have “local” subdomain solves and “global” coarse solve
- Solve using preconditioned conjugate gradients or GMRES

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Solution Methods

- **Linear and Nonlinear Statics and Transient Dynamics**
- **Eigenanalysis**
  - Real and complex (quadratic)
- **Direct Frequency Response**
- **Random Vibration Analysis**
- **Modal Based Solutions for Transient Dynamics, SRS, Frequency Response**
- **Coupled Nonlinear-Linear Analysis**
  - With Adagio/Presto *(Sandia in-house codes)*

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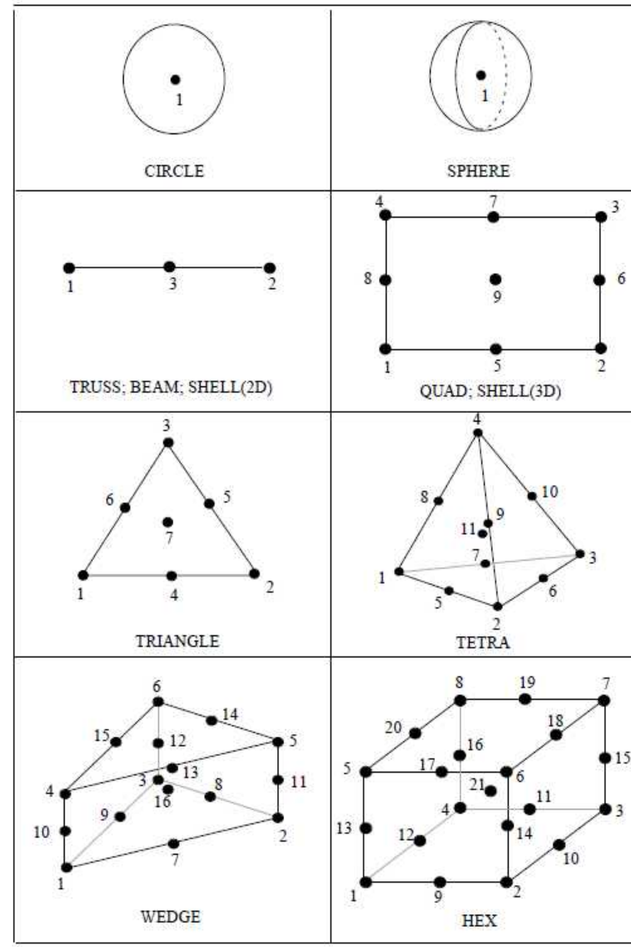
Large Element Library

- **Solid Elements**
  - Hexahedral, Tetrahedral, Wedge
- **Shell Elements**
  - Triangle, Quadrilateral, HexShell (hybrid)
- **Bar/Beam Elements**
  - Beam, Truss, Spring, Dashpot
- **Point Elements**
  - Conmass (concentrated mass)
- **Specialty Elements**
  - Iwan, Hys, Shys, Joint2G, Gap



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Structural Acoustics

- **Formulations for Structural Acoustics:**

Scalar Based

  - Velocity potential formulation *(Everstine, 1981, 1997)*
  - Mixed pressure-potential symmetric formulation *(Felippa & Ohayon, 1990; Pinsky, 1991; Ohayon 1996)*

Vector Based

  - Displacement-based formulation *(Hamdi & Ousset 1978; Belytschko, 1980; Wilson, 1983; Chen 1990; Bermudez 1994)*
  - Space-time formulation *(Harari et al., 1996; Thompson and Pinsky, 1996)*
  - Others …

- **All fully-coupled formulations (monolithic)**

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Structural Acoustics Formulation

- **Applied two-field formulation of Everstine[1]**
  - Structural displacement
  - Fluid velocity potential
- **Exterior problems straightforward**
  - Compared to other formulations
- **Symmetric, indefinite matrices**
  - Best suited for domain decomposition-based solvers
- **Results in 2nd order equations**
  - Compatible with Newmark beta and alpha time integration
- **Added by Tim Walsh beginning in 2003**

[1] G. C. Everstine, "Finite Element Formulations For Structural Acoustics Problems," *Computers & Structures* **65**: 307-321, (1997).

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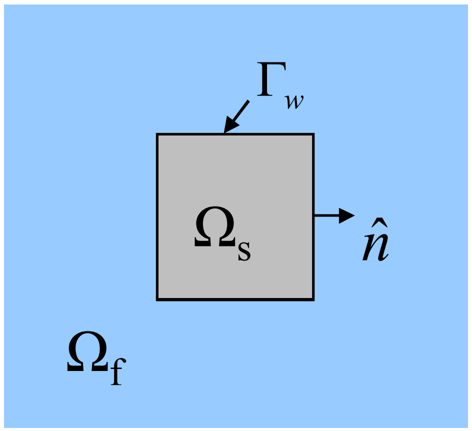
Structural Acoustics Formulation



Structure: $\rho_s \dfrac{\partial^2 \vec{u}}{\partial t^2} - \vec{\nabla} \cdot \tau = \vec{f}(\vec{x}, t) \quad \Omega_s \times [0, T]$

Fluid: $\nabla^2 \varphi - \dfrac{1}{c^2} \dfrac{\partial^2 \varphi}{\partial t^2} = 0 \quad \Omega_f \times [0, T]$

where $p = \dfrac{\partial \varphi}{\partial t}, \quad \vec{v}_f = -\dfrac{\vec{\nabla}\varphi}{\rho_f}$

Fluid-Structure B.C.'s: $\tau \cdot \hat{n} = -\dfrac{\partial \varphi}{\partial t} \qquad \rho_f \dfrac{\partial \vec{u}}{\partial t} \cdot \hat{n} = -\vec{\nabla}\varphi \cdot \hat{n}$

Weak form with applied B.C.'s:

Structure: $\int_{\Omega_s} \rho_s \dfrac{\partial^2 \vec{u}}{\partial t^2} \cdot \vec{w}\, d\Omega + \int_{\Gamma_w} \vec{w} \dfrac{\partial \varphi}{\partial t} d\Gamma + \int_{\Omega_s} \left(\vec{\nabla}^s \vec{w}\right) \tau \, d\Omega = \int_{\Omega_s} \vec{f} \cdot \vec{w}\, d\Omega$

Fluid: $\dfrac{1}{c^2} \int_{\Omega_f} \dfrac{\partial^2 \varphi}{\partial t^2} \Psi \, d\Omega - \rho_f \int_{\Gamma_w} \left(\dfrac{\partial \vec{u}}{\partial t} \cdot \hat{n} \right) \Psi \, d\Gamma + \int_{\Omega_f} \vec{\nabla}\varphi \cdot \vec{\nabla}\Psi \, d\Omega = 0$

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Structural Acoustics Formulation

- **Weak form with B.C.'s :**

Structure: $$\int_{\Omega_s} \rho_s \frac{\partial^2 \vec{u}}{\partial t^2} \cdot \vec{w}\, d\Omega + \int_{\Gamma_w} \vec{w} \frac{\partial \varphi}{\partial t} d\Gamma + \int_{\Omega_s} \left(\vec{\nabla}^s \vec{w}\right) \tau \, d\Omega = \int_{\Omega_s} \vec{f} \cdot \vec{w}\, d\Omega$$

Fluid: $$\frac{1}{c^2} \int_{\Omega_f} \frac{\partial^2 \varphi}{\partial t^2} \Psi \, d\Omega - \rho_f \int_{\Gamma_w} \left(\frac{\partial \vec{u}}{\partial t} \cdot \hat{n} \right) \Psi \, d\Gamma + \int_{\Omega_f} \vec{\nabla} \varphi \cdot \vec{\nabla} \Psi \, d\Omega = 0$$

- **Applying spatial discretizations (finite element basis):**

$$\begin{bmatrix} M_s & 0 \\ 0 & \tilde{M}_f \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\varphi} \end{Bmatrix} + \begin{bmatrix} C_s & L \\ L^T & \tilde{C}_f \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{\varphi} \end{Bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & \tilde{K}_f \end{bmatrix} \begin{Bmatrix} u \\ \varphi \end{Bmatrix} = \begin{Bmatrix} f_s \\ \tilde{f}_f \end{Bmatrix}$$

Coupling

- **Divided fluid equation by acoustic density**
  - Symmetric, indefinite system (but nonsingular)

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Structural Acoustics Solvers/Capabilities

- **Full massively parallel functionality**
- **Hex, wedge, and tetra acoustic elements**
- **Acoustic coupling with both 3D and shell (2D) structural elements**
- **Allows for mismatched acoustic/solid meshes**
  - Inconsistent Tying
  - Standard Mortars
- **Solvers: FETI-DP, GDSW**
- **Solution Procedures:**
  - Frequency Response (frequency-domain)
  - Transient (time-domain)
  - Eigenvalue Analysis (real and quadratic)
- **Nonlinear Acoustics – Kuznetsov Equation**

Quadratic Eigenvalue Problem

- **Eigenanalysis formulation:**

$$\lambda^2 \begin{bmatrix} M_s & 0 \\ 0 & \tilde{M}_f \end{bmatrix} \begin{Bmatrix} u \\ \varphi \end{Bmatrix} + \lambda \begin{bmatrix} C_s & L \\ L^T & \tilde{C}_f \end{bmatrix} \begin{Bmatrix} u \\ \varphi \end{Bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & \tilde{K}_f \end{bmatrix} \begin{Bmatrix} u \\ \varphi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

  – Coupling within damping matrix brings about complex eigenvalues for structural acoustics (non-diagonalizable)

- **State-Space form of time domain:**

$$\underbrace{\begin{bmatrix} M_s & 0 \\ 0 & \tilde{M}_f \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\varphi} \end{Bmatrix}}_{[M]\{\ddot{r}\}} + \underbrace{\begin{bmatrix} C_s & L \\ L^T & \tilde{C}_f \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{\varphi} \end{Bmatrix}}_{[C]\{\dot{r}\}} + \underbrace{\begin{bmatrix} K_s & 0 \\ 0 & \tilde{K}_f \end{bmatrix} \begin{Bmatrix} u \\ \varphi \end{Bmatrix}}_{[K]\{r\}} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \{w\} = \begin{bmatrix} 0 & M \\ -M & -C \end{bmatrix} \{\dot{w}\} \quad \text{where } w = \begin{Bmatrix} \dot{r} \\ r \end{Bmatrix}$$

Quadratic Eigenvalue Problem

- **Linear eigenvalue problem :**

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix}\{w\} = \begin{bmatrix} 0 & M \\ -M & -C \end{bmatrix}\{\dot{w}\} \quad \text{where} \quad w = \begin{Bmatrix} \dot{r} \\ r \end{Bmatrix}$$

- **Assume time decay and a complex configuration space:**

$$w = \varphi\, e^{\lambda t} + \varphi^{*}\, e^{\lambda^{*} t}$$

  – φ are complex mode shapes; *w* is real

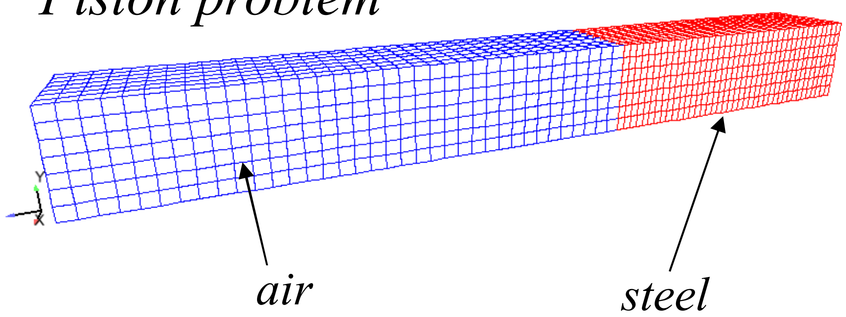
- **Solve right *and* left eigenvalue problem**

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix}[\varphi] = \lambda \begin{bmatrix} 0 & M \\ -M & -C \end{bmatrix}[\varphi] \quad [\varphi]^{*}\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} = \lambda\,[\varphi]^{*}\begin{bmatrix} 0 & M \\ -M & -C \end{bmatrix}$$

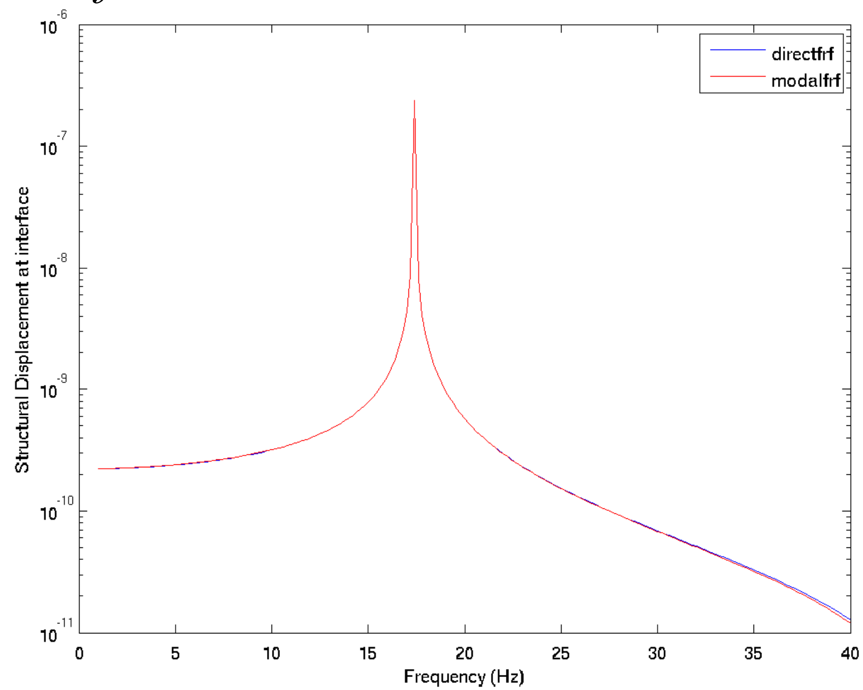
- **Resulting complex modes used for frequency response function predictions**

Quadratic Eigenvalue Verification

Piston problem



A comparison of structural displacement from directFRF vs modalFRF



- **DirectFRF:**

$$u(\omega) = \frac{F(\omega)}{-\omega^2 [M] + i\omega [C] + [K]}$$

- **ModalFRF:**
  - Use complex modes from quadratic eigenvalue solution

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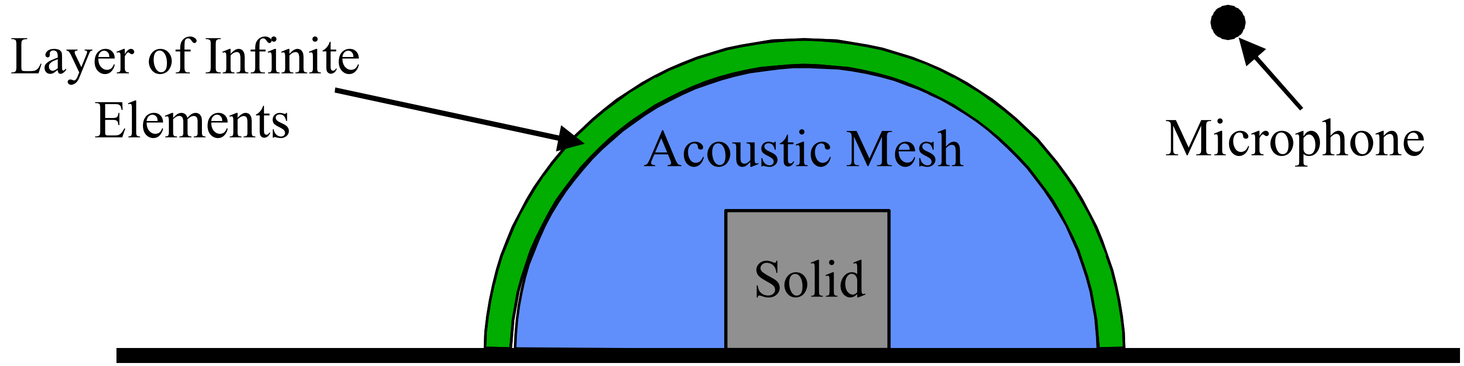
Mismatched Acoustic/Solid Meshes

- **Mesh density requirement inconsistency**
  - Acoustic phase speed < structural (typically)
- **Solution: tying/mortars**
  - Use ghost acoustic d.o.f. on solid nodes at interface, conforming coupling to solid
  - Couple the acoustic d.o.f. now on both sides of wet interface using constraint equations

Infinite Elements Capability



- **Provides an asymptotically exact boundary condition for exterior problems**
- **Allows for computing response at far-field points outside of acoustic mesh**
- **Currently implementing time-domain, conjugated version of "mapped wave envelope" elements of Astley et al.**

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Inverse Material Identification Capabilities

- **Joint work with Wilkins Aquino, Cornell University**
- **Determination of material properties given measured experimental displacement data**
  - Allows for different time steps between experimental data and transient dynamics prediction
- **Time and frequency domain**
  - Based on Error in Constitutive Equations (ECE) method
- **Performed at the block level (i.e. homogeneous block) or at the element level (i.e. heterogeneous block)**
- **Solvers: FETI-DP and GDSW**

Conclusions

- **Massively Parallel FEM**
- **Fully Coupled Structural Acoustics**
- **Quadratic Eigenvalue Solver**
- **Structural Acoustic Tying/Mortars**
- **Infinite Elements**
- **Inverse Methods**

- **Salinas is an export controlled code. Shared with other US Government Labs for use.**
- **For Inquiries:**

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