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# PARAMETER UNCERTAINTY IN THE SANDIA ARRAY PERFORMANCE MODEL FOR FLAT-PLATE CRYSTALINE SILICON MODULES

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## ABSTRACT

The Sandia Array Performance Model (SAPM) [1] describes the power performance of photovoltaic (PV) modules under variable irradiance and temperature conditions. Model parameters are estimated by regressions involving measured module voltage and current, module and air temperature, and solar irradiance. Measurements are made under test conditions chosen to isolate subsets of parameters and which improve the quality of the regression estimates. Uncertainty in model parameters results from uncertainty in each measurement as well as from the number of measurements. Uncertainty in model parameters can be propagated through the model to determine its effect on model output. In this paper we summarize the process for estimating uncertainty in model parameters for flat-plate, crystalline silicon (cSi) modules from measurements, present example results, and illustrate the effect of parameter uncertainty on model output. Finally, we comment on how analysis of parameter uncertainty can inform model developers about the presence and impacts of model uncertainty.

## INTRODUCTION

The Sandia Array Performance Model [1] relates module voltage and current at five points on the IV curve to effective irradiance and cell temperature. Model parameters are estimated by regression of measurements of current and voltage to measured irradiance and temperature and are available for a large number of PV modules.

Uncertainty in model output may result from: (1) uncertainty in model parameters, termed parameter uncertainty; (2) variability in irradiance and temperature, (3) variability among modules of the same manufacturer; and (4) misspecification of the model itself, referred to as model uncertainty. Uncertainty analysis of model output involves characterizing uncertainty and variability in model inputs and propagating uncertainties through the model. Uncertainty analysis is complemented by sensitivity analysis which identifies the contribution to output uncertainty of each uncertain input. Quantifying uncertainty in model output is currently of interest because such uncertainty informs decisions about investment risk for large-scale PV power plants.

When a single module is available for testing, parameter error is quantified by the estimation error in each parameter. This estimation error results from measurement error and from the necessarily limited number of measurements. In this analysis, we consider only the contribution to parameter uncertainty resulting from the number of measurements, and defer consideration of measurement error to later investigation. Moreover, we consider tests of a single module taken as a representative of a production batch of modules. These limitations significantly understate the range of uncertainty in each parameter, compared to that which would result if measurement error were propagated and several modules were tested. However, we are able, from a single module test, to examine correlations between parameter estimates and to perform uncertainty and sensitivity analysis to identify which parameters influence uncertainty in the performance model output.

Methods for determining uncertainty in the SAPM model parameters have recently been codified [2]. These methods are summarized here and are illustrated by estimation of model parameters for a 230W module characterized at Sandia. Uncertainty and sensitivity analyses are reported for this module. Finally, we examine the results of these analyses to determine if model uncertainty is present to any significant degree.

## PARAMETER ESTIMATION

For flat-plate cSi modules, the fundamental equations in the SAPM are:

$$V_{OC} = V_{OC0} + N_S \delta(T_C) \ln(E_e) + \beta_{OC}(T_C - T_0) \quad (1)$$

$$\begin{aligned} V_{MP} &= V_{MP0} + C_2 N_S \delta(T_C) \ln(E_e) \\ &\quad + C_3 N_S \left(\delta(T_C) \ln(E_e)\right)^2 + \beta_{MP}(T_C - T_0) \end{aligned} \quad (2)$$

$$I_{SC} = I_{SC0} f_1(AM_a) E_e \left(1 + \alpha_{SC}(T_C - T_0)\right) \quad (3)$$

$$I_{MP} = I_{MP0}\left(C_0 E_e + C_1 E_e^2\right)\left(1 + \alpha_{MP}(T_C - T_0)\right) \quad (4)$$

where:

$V_{OC}$ is open-circuit voltage (V);

$V_{MP}$ is voltage (V) at maximum power;

$I_{SC}$ is short-circuit current (A);

$I_{MP}$ is current (A) at maximum power;

$T_C$ is cell temperature (K);

$E_e$ is effective irradiance (suns);

$N_S$ is the number of cells in series;

$\delta(T_C) = \frac{nkT_C}{q}$ is the thermal voltage;

$n$ is the empirical diode factor;

$k/q$ is the ratio of Boltzmann's constant to the elementary charge.

The remaining terms (e.g., $\beta_{OC}$) involve parameters that are to be estimated. The subscript $\sim_0$ indicates a constant evaluated at reference irradiance of 1000 W/m$^2$ (i.e., 1 sun) and reference cell temperature $T_0$ (typically 25°C). The empirical function $f_1(AM_a)$ of absolute air mass $AM_a$ corrects $I_{SC}$ to account for atmospheric attenuation of the solar spectrum.

## PARAMETER ESTIMATION

Parameters are estimated in three stages, with values estimated at earlier stages being used in the estimation of parameters at later stages.

### Temperature coefficients

Module testing begins with a thermal performance test designed to support estimation of the parameters $\beta_{OC}$, $\beta_{MP}$, $\alpha_{SC}$ and $\alpha_{MP}$. Measurements are made at midday when $E_e \approx 1$ and during clear-sky, low wind conditions. The module is first cooled by being covered, then exposed to sunlight; IV curves are recorded while the module returns to equilibrium temperature. Cell temperature is estimated from thermocouple measurements at the module backplane; irradiance is measured by a reference cell. Measured voltage and current are adjusted to one-sun conditions by assuming module response to irradiance similar to that of the reference cell, and $f_1(AM_a)$ is assumed to equal 1. These assumptions are justified because test conditions are chosen so that $E_e \approx 1$ and $AM_a \approx 1.5$ (and $f_1(1.5) = 1$ by design). With these adjustments and assumptions and an assumed generic value for the diode factor $n$, Eq. (1) through Eq. (4) reduce to linear equations in cell temperature $T_C$.

Significant correlations are present between voltage and current measurements as illustrated in Figure 1. Because measurements are simultaneous and are paired with measured irradiance and temperature, correlations between measured voltage and current induce correlations between parameters estimated by regression. Consequently, values of $\beta_{OC}$, $\beta_{MP}$, $\alpha_{SC}$ and $\alpha_{MP}$ are estimated by multivariate linear regression on temperature. Figure 2 illustrates correlations between estimated parameter values resulting from correlations in the dependent regression variables. The scatterplots in Figure 2 were created by randomly sampling from the multivariate normal distribution $N(\mu, \Sigma)$ where $\mu$ is the vector of ordinary least squares estimates of $\beta_{OC}$, $\beta_{MP}$, $\alpha_{SC}$ and $\alpha_{MP}$, and $\Sigma$ is the covariance matrix associated with the parameter estimates.

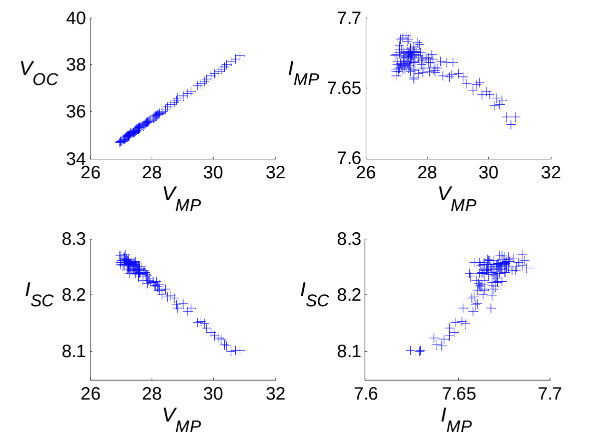


**Figure 1. Correlations between $V_{MP}$, $V_{OC}$, $I_{SC}$ and $I_{MP}$ adjusted to one-sun conditions for a 230W module.**

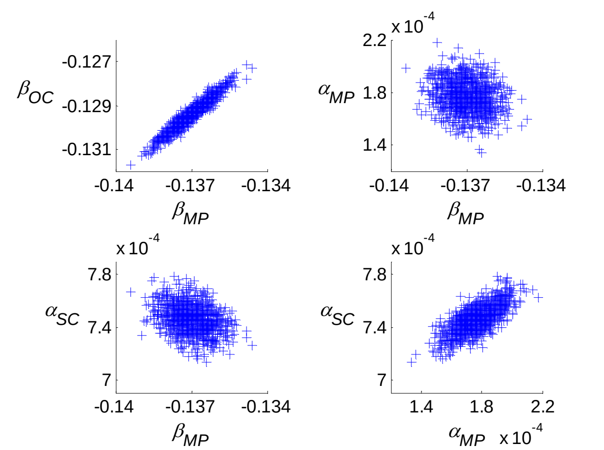


**Figure 2. Values for $\beta_{OC}$, $\beta_{MP}$, $\alpha_{SC}$ and $\alpha_{MP}$ sampled from distribution of estimation error for a 230W module.**

### Intercepts and spectral correction function

Values for the intercepts $I_{SC0}$, $V_{OC0}$, $V_{MP0}$,. and $I_{MP0}$ are estimated from measurements obtained at midday during clear sky conditions during which module temperature is relatively stable, $E_e \approx 1$ and $AM_a < 2$. Values for these parameters could be obtained simultaneous with the regression onto thermal test data; however, current practice is to first estimate the temperature coefficients $\beta_{OC}$, $\beta_{MP}$, $\alpha_{SC}$ and $\alpha_{MP}$ and to use these values to correct measured voltage and current to reference temperature conditions before obtaining the intercept values. The spectral correction function $f_1(AM_a)$ is characterized by a 4th degree polynomial; coefficients are estimated from measurements during clear-sky conditions.

The value for $I_{SC0}$ is obtained first. Measured values of voltage and current are adjusted to one-sun conditions by assuming module response similar to that of the reference cell. By choice of test conditions, $AM_a < 2.0$, and over this range of $AM_a$, $f_1(AM_a)$ is approximated by a linear expression: $f_1(AM) \approx b_0 + b_1 AM_a$. Eq. 3 is rearranged to a linear expression in $AM_a$:

$$\frac{I_{SC}}{1+\alpha_{SC}(T_C - T_0)} = I_{SC0}(b_0 + b_1 AM_a) \qquad (5)$$

Regression obtains values for the products $I_{SC0} b_0$ and $I_{SC0} b_1$; by design, $1 = f_1(1.5) \approx b_0 + b_1 1.5$, and this expression is used to obtain the value of $I_{SC0}$.

The coefficients for the full 4th degree polynomial for $f_1(AM_a)$ are obtained by regression using Eq. 3 and correcting measured $I_{SC}$ to a reference cell temperature of 50°C:

$$I_{SC,50} = I_{SC} / (1 + \alpha_{SC}(T_C - 50)) \qquad (6)$$

$$f_1(AM_a) = I_{SC,50} / E_e I^*_{SC0} = \text{Rel. Norm. Isc} \qquad (7)$$

The term $I^*_{SC0}$ in Eq. 7 is determined in a similar manner as $I_{SC0}$ (Eq. 5) by evaluating a linear equation at $AM_a = 1.5$ obtained by regressing $I_{SC,50}$ to $AM_a$.

Finally, the value for $I_{SC0}$ is used to obtain the other three intercepts. The effective irradiance on the module, $E_e$, is adjusted by using the measured short-circuit current to improve the estimated values:

$$\hat{E}_e = \frac{I_{SC}}{I_{SC0}(1 + \alpha_{SC}(T_C - T_0))} \qquad (8)$$

The adjusted values $\hat{E}_e$ remove the discrepancy between the spectral response of the reference cell and the module. Eq. 1, 2, and 4 are then used to obtain the corresponding intercepts from the constant terms in regressions with $\hat{E}_e$. Test conditions are chosen so that $\hat{E}_e \approx 1$ to minimize the influence of the irradiance terms in Eq. 1, 2, and 4.

As was the case with the temperature coefficients, correlations between measured current, voltage and environmental quantities result in correlations between estimated parameter values (illustrated in Fig. 3).

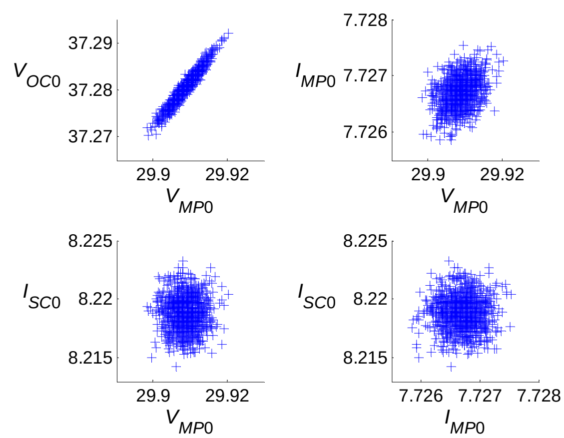


**Figure 3. Values for $V_{OC0}$, $V_{MP0}$, $I_{SC0}$ and $I_{MP0}$ sampled from distribution of estimation error for a 230W module.**

### Diode factor and irradiance-related coefficients

The final set of parameters to be estimated include the diode factor $n$ and the coefficients describing the effects of irradiance on voltage and current, i.e., $C_0$ through $C_3$. Test conditions are chosen to ensure a wide range of irradiance conditions are observed. Diode factor $n$ is first determined by a regression using Eq. 1:

$$V_{OC} - \beta_{OC}\left(T_C - T_0\right) = b_0 + N_S \ln\left(\hat{E}_e\right)\frac{kT_C}{q}n \qquad (9)$$

The constant $b_0$ in Eq. 9 is discarded. With a value for $n$ in hand, Eq. 2, 3 and 4 are expressed as polynomials in $\ln\left(\hat{E}_e\right)$ or $\hat{E}_e$ and the corresponding coefficients are determined, using the additional condition that $C_0 + C_1 = 1$ Figure 4 illustrates values for these parameters obtained by sampling from the multivariate normal distribution describing error in the parameter estimates.

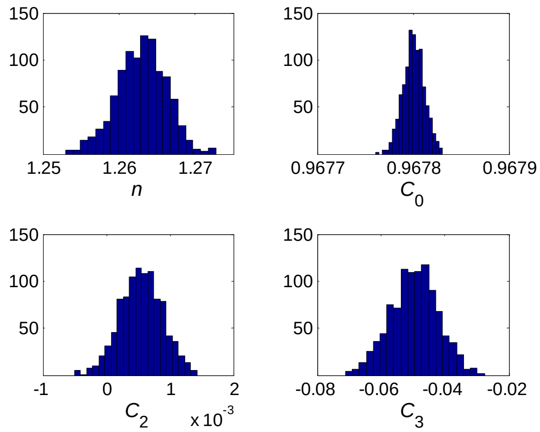


**Figure 4. Values for $n$, $C_0$, $C_2$ and $C_3$ sampled from distribution of estimation error for a 230W module.**

## UNCERTAINTY AND SENSITIVITY ANALYSIS

Uncertainty in model output is quantified by propagating uncertainty in model parameters through the array performance model. A random sample $\mathbf{X}$ of parameter values is drawn consistent with correlations between parameter values as illustrated in Fig. 2, 3 and 4; for each element $\mathbf{X}_i$ of this sample (where $\mathbf{X}_i$ is a vector of parameter values), model output $y_i$ is calculated. To isolate the effects of parameter uncertainty on model output, model inputs characterizing the solar resource (i.e., $E_e$) and environmental effects (i.e., $AM$ and $T_C$) are fixed using the typical meteorological year (TMY2) data for Albuquerque, NM.

Figure 5 illustrates the range of values for total energy obtained from the sample $\mathbf{X}$. Uncertainty in annual energy resulting from parameter uncertainty is less than 1%. The small effect on the magnitude of annual energy results because of the relatively small estimation error in each parameter, which in turn results from testing only one module and by excluding consideration of measurement error. These findings are consistent with previous analyses of uncertainty in predictions of total energy (e.g., [3]).

It is certain that the effects on annual energy of variability in meteorological quantities, specifically irradiance, will dwarf the effects of parameter uncertainty. We opine that, if several modules were tested, uncertainty ranges for parameters would increase by a factor of 2 or more, and that consideration of measurement uncertainty would increase parameter uncertainty to a lesser extent. However, we do not believe that the effects of greater parameter uncertainty, resulting from tests of multiple modules and consideration of measurement error, would approach that of variability in irradiance.

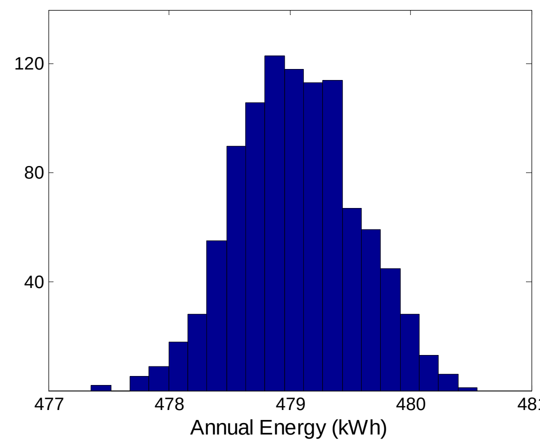


**Figure 5. Histogram of annual energy obtained using TMY2 data for Albuquerque, NM, and sampled values for performance model coefficients.**

The mapping between $\mathbf{X}_i$ and $y_i$ forms the basis for sensitivity analysis to determine the relative contribution of each uncertain input to uncertainty in model output. Identifying model sensitivities informs efforts to improve models and to reduce uncertainties through improvements to module characterization. We performed stepwise ranked regression, which adds variables sequentially to a regression model; at each step, the method adds the variable that explains the greatest fraction of remaining variance. Ranked regression transforms monotonic relationships to linear relationships, removing the effects of disparate parameter scales on the sensitivity analysis results. Before conducting the regression, we examined the sample $\mathbf{X}$ to remove elements from consideration where significant correlations (i.e., correlation coefficient exceeding 0.97) were present. Stepwise regressions are unreliable when highly correlated predictors are included. The final regression included: temperature coefficients, $\beta_{MP}$, $\alpha_{SC}$ and $\alpha_{MP}$; intercepts $I_{SC0}$, $V_{MP0}$ and $I_{MP0}$

; diode factor $n$; irradiance coefficients $C_0$, $C_2$ and $C_3$; and the cubic coefficient of $f_1(AM_a)$, denoted $a_3$. Parameters $\beta_{OC}$ and $V_{OC0}$ exhibited strong correlations with $\beta_{MP}$ and $V_{MP0}$, respectively; coefficient $C_1$ is redundant because $C_0 + C_1 = 1$. The coefficients of $f_1$ were found to be highly correlated; the cubic term was selected as representative because it showed a strong negative correlation with the values of $f_1(AM_a)$ for $AM < 2$, implying an overall inverse relationship with total energy.

Table 1 illustrates the results of the stepwise ranked regression for total energy. The temperature coefficient for voltage maximum power, $\beta_{MP}$, explains roughly half of the uncertainty in total energy; the positive value for the SRRC indicates that as $\beta_{MP}$ increases, total energy also increases. The counter-intuitive increase in energy with increasing $\beta_{MP}$ occurs because $\beta_{MP}$ is negative; increases in $\beta_{MP}$ result in less reduction of energy as temperature increases. The representative coefficient $a_3$ from the function $f_1(AM_a)$ appears next, indicating a substantial contribution to uncertainty in total energy from uncertainty in the correction for atmospheric spectral effects, which is present through the conversion of measured short-circuit current to effective irradiance (Eq. 8). Roughly half of the remaining uncertainty in total energy results from uncertainty in the temperature coefficient for current at maximum power $\alpha_{MP}$ and from the intercept term for current at maximum power at reference conditions, $I_{MP0}$, and the balance of uncertainty in total energy arising from the uncertainty in other coefficients.

**Table 1. Stepwise ranked regression for total energy.**

| Step | Variable | $R^2$ [a] | SRRC [b] |
|---|---|---|---|
| 1 | $\beta_{MP}$ | 0.42 | 0.64 |
| 2 | $a_3$ | 0.77 | -0.57 |
| 3 | $\alpha_{MP}$ | 0.86 | 0.17 |
| 4 | $I_{MP0}$ | 0.88 | 0.13 |

a: Cumulative R2 with entry of each variable into model
b: Standardized rank regression coefficient in final model

## MODEL UNCERTAINTY

Model uncertainty is present when model results differ systematically from measurements. Model uncertainty results from the absence of important parameters from the model, or from missing or incorrect combinations of parameters in the model's equations. Although the formulation of a model determines which parameters are present, model uncertainty is distinct from parameter uncertainty. Parameter uncertainty involves the values assigned to parameters rather than the use of those parameters in the model's equations.

We investigated whether model uncertainty is present by examining two parameter estimates to which the model results are sensitive, and the agreement between model results and measurements. Figure 6 illustrates the data supporting the estimation of $\beta_{MP}$ and the regression used to determine its value. Correction to one sun nearly linearizes the measurements, resulting in a relatively small estimation error in $\beta_{MP}$ which is the slope of the line fit to the data. Consequently, sampled slope values differ little and in general all provide a reasonable fit to the underlying data. We conclude that there is little evidence of model uncertainty in the representation of $\beta_{MP}$.

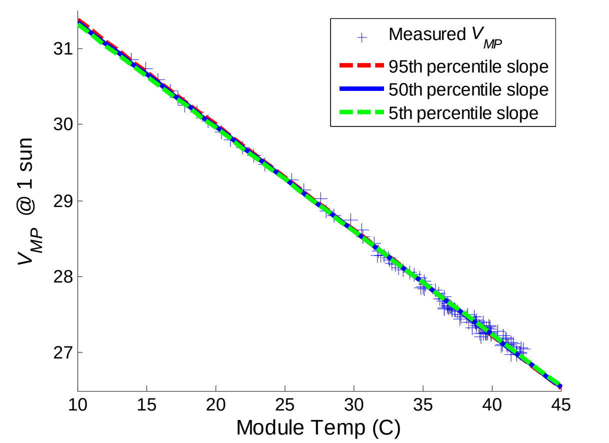


**Figure 6. Estimation of $\beta_{MP}$ from measured $V_{MP}$ scaled to one sun.**

Figure 7 illustrates the fit between measured data and the polynomial $f_1(AM_a)$ (percentiles determined by sampled values for the coefficient $a_3$). Measurements are taken across three days as indicated by the colors in Fig. 7. Parameters (or fitted equations) determined by regression often cannot represent the range of variation present in the underlying data, due to the smoothing inherent in regression. Such is the case with the estimation of $f_1(AM_a)$ where the equation fit to the aggregate of all three days does not represent the variability apparent across the days. We conclude that model uncertainty is present in the correction for atmospheric spectral effects; the sensitivity of model results to this quantity indicates that further development of this part of the model may be warranted if more precise estimates of uncertainty in model results are desired. Extension of the spectral correction to encompass the range of variability in measurements would slightly increase the uncertainty in model results.

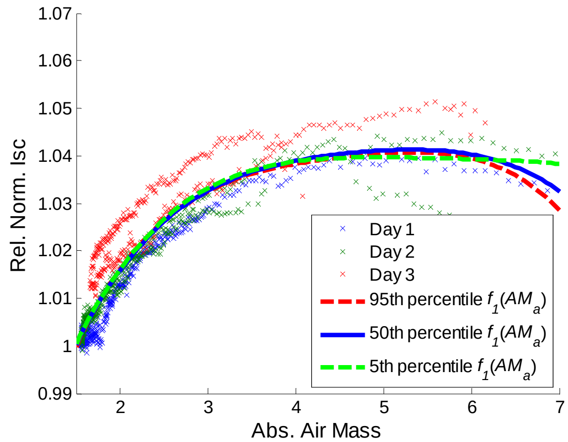


**Figure 7. Estimation of $f_1(AM_a)$ from measured $I_{SC}$, scaled to one sun and normalized to 50°C.**

Figure 8 compares model predictions of power (at the maximum power point) to observations. Comparison is shown as a function of voltage at maximum power $V_{MP}$; variation in $V_{MP}$ reflects a wide range of atmospheric conditions experienced during the tests. The systematic agreement between measured and modeled results indicates that, overall, the model successfully reproduces the measured quantities, despite the presence of model uncertainty in several components of the model.

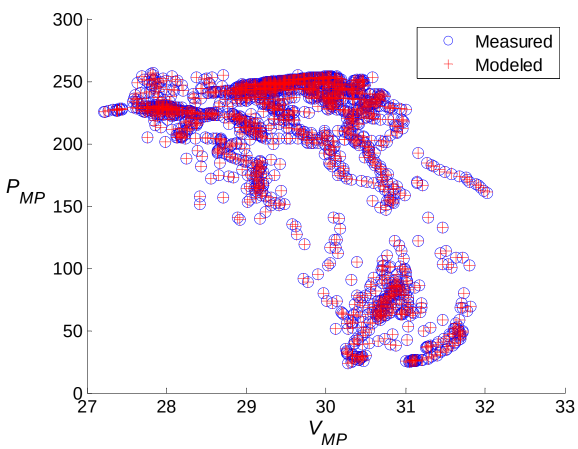


**Figure 8. Comparison of measured and modeled maximum power $P_{MP}$.**

## CONCLUSIONS

We quantified uncertainty in the parameters for the Sandia Array Performance Model considering only estimation error resulting from regressions. We found significant correlations between parameter values due to correlations in the underlying measurements of voltage, current, irradiance and temperature. We propagated the uncertainty in parameters through the model to quantity the resulting uncertainty in total power. We also identified, using sensitivity analysis, the parameters making the largest contributions to uncertainty in total power, and investigated the presence and effects of model uncertainty.

We found that error in parameter estimation does not lead to a significant amount of error in total energy projected by the model. We observe that several components of the model do not reproduce the full variability in the underlying measurements due to the smoothing inherent when estimating parameters by regression. However, we did not find that the model uncertainty contributes significantly to differences between estimated and measured power.

We note that this work is based on testing of a single cSi module, and thus, the uncertainty in total power does not consider variability between modules of the same production lot, nor the effects of measurements error. We opine that, if quantified and propagated, variability between similar modules would lead to significantly greater uncertainty in total power, by roughly a factor of 2 or more. We defer quantification of the effects of measurement uncertainty to further analyses.

## ACKNOWLEDGEMENTS

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