

Ionic Diffusion Model of Uranium Oxidation

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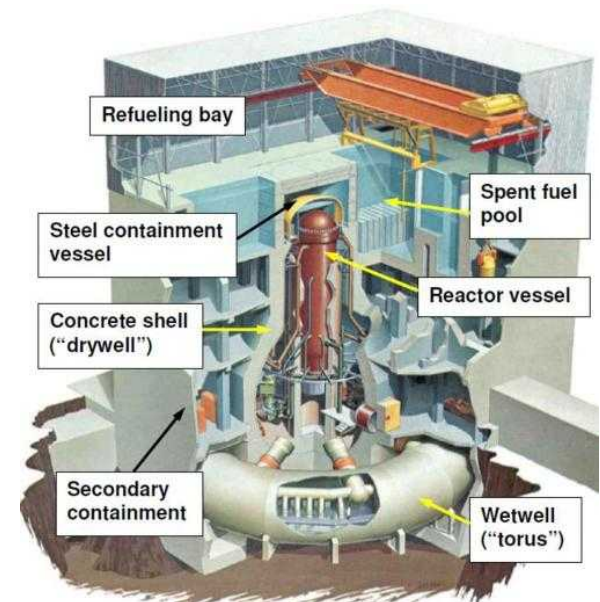


Outline

- **Motivation**
- **Review of Previous Work**
- **Ionic Diffusion Model**
- **Comparison with Oxidation Data**
- **Reaction Rate Constant Formulation**
- **Comparison with Empirical Formulas**

Motivation

- **Applications:**
 - Nuclear Reactors
 - Nuclear Waste Storage
- **Uranium Oxidation Modeling**
 - Current available models of uranium oxidation are empirical
 - Physics based model allows for greater understanding of the underlying causes of oxidation rates



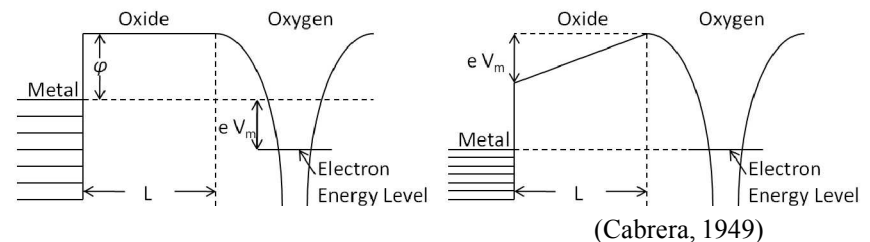
Review of Previous Work

- **Uranium Oxidation:**

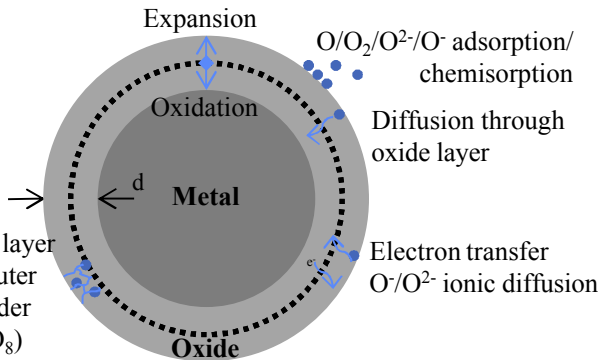
- Empirical growth models (Evans, 1960)
 - Inverse logarithmic
 - Parabolic
 - Linear
 - Paralineer
- Show trends, without need for understanding of physics
- High variability in measurements due to differing material and environmental conditions yield large variations between models

- **Ionic Diffusion:** (Mott, 1947)

- Increased diffusion of ions due to electric potential through oxide
- Accounts for passivation as oxide thickness increases and inverse logarithmic growth trends
- Shown to correctly prediction oxidation of common metals like copper and zinc.



Ionic Diffusion Model



$$J_{ion} = \underbrace{4av \exp\left(\frac{-W}{k_b T}\right)}_{D/a} \sinh\left(\frac{ZeE_0 a}{k_b T}\right) \left[\frac{C(L) - C(0) \exp\left(\frac{ZeE_0 L}{k_b T}\right)}{1 - \exp\left(\frac{ZeE_0 L}{k_b T}\right)} \right]$$

a = distance between U atoms
 v = oxygen atomic vibrational frequency in oxide
 W = energy barrier
 Z = ion charge
 C = ion concentration
 $E_0 = -V_m/L$
 D = diffusion coefficient

Symbol	a	v	W	$C(L)$	$C(0)$	Z
Value	3.87	19	1.24	1×10^{28}	Negl.	-2
Units	Å	THz	eV	$1/\text{m}^3$	$1/\text{m}^3$	—

- Uranium Oxidation modeled based on ionic diffusion of oxygen through oxide film (Fromhold, 1967)
- Model is applicable to oxidation before the oxide layer begins to crack due to stress of expansion from density change.
- Assumptions:
 - Electrons easily transfer from uranium to oxide outer surface
 - Oxygen ions adsorb to oxide surface up to a hyper-stoichiometry of 2.25
 - All ions that reach the uranium/oxide interface react with the uranium to form oxide

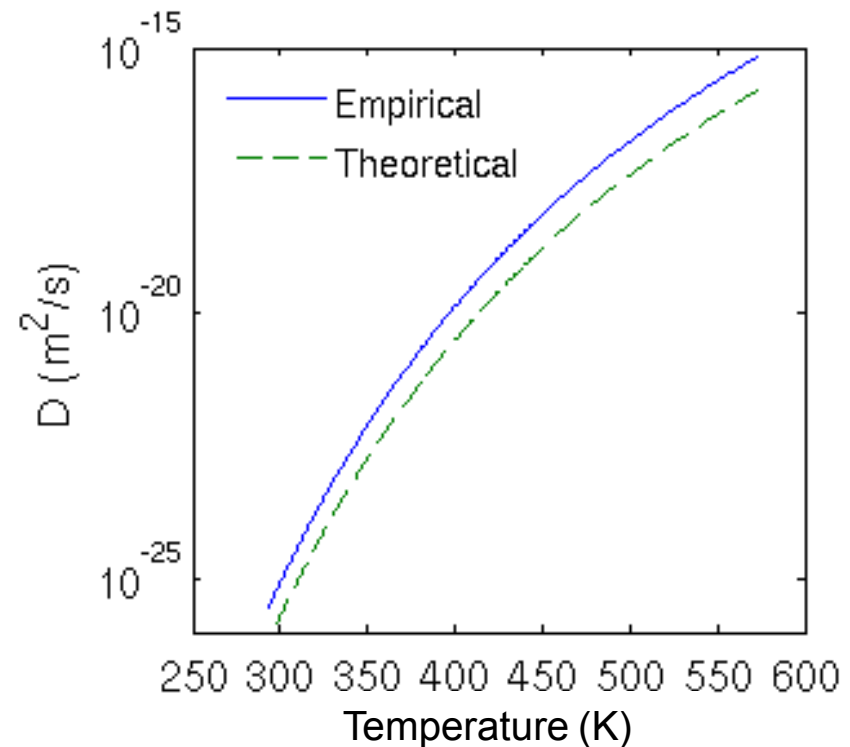
Chemical Diffusion Coefficient Comparison

$$D_{emp} = 5 \times 10^{-5} \left[\frac{m^2}{s} \right] \exp \left(- \frac{119244 [J/mol]}{R_g T} \right)$$

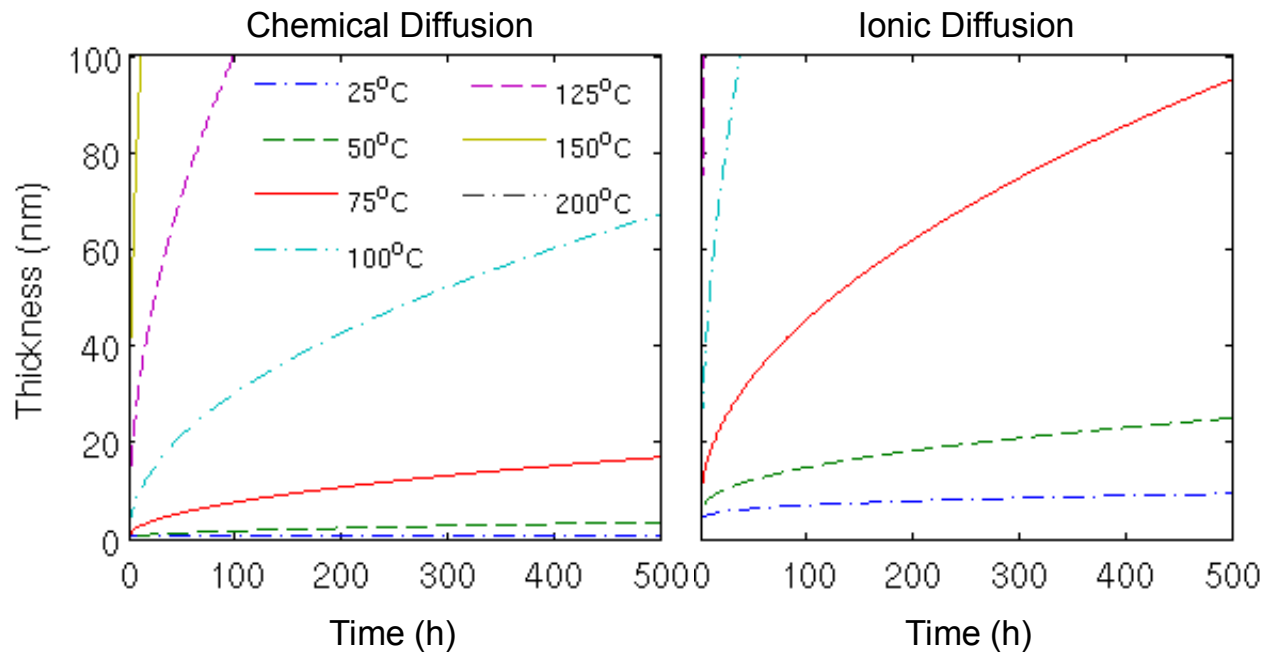
(Lay, 1970)

$$D_{th} = 4a^2\nu \exp \left(- \frac{W}{k_b T} \right)$$

- Reported chemical diffusion activation energy consistent with reported anion migration energy for hyper-stoichiometric uranium dioxide of 1.3 eV
(Catlow, 1977)
- Using reported activation energy, empirical and theoretical chemical diffusion coefficient are of same order of magnitude and differ by only a factor of 4

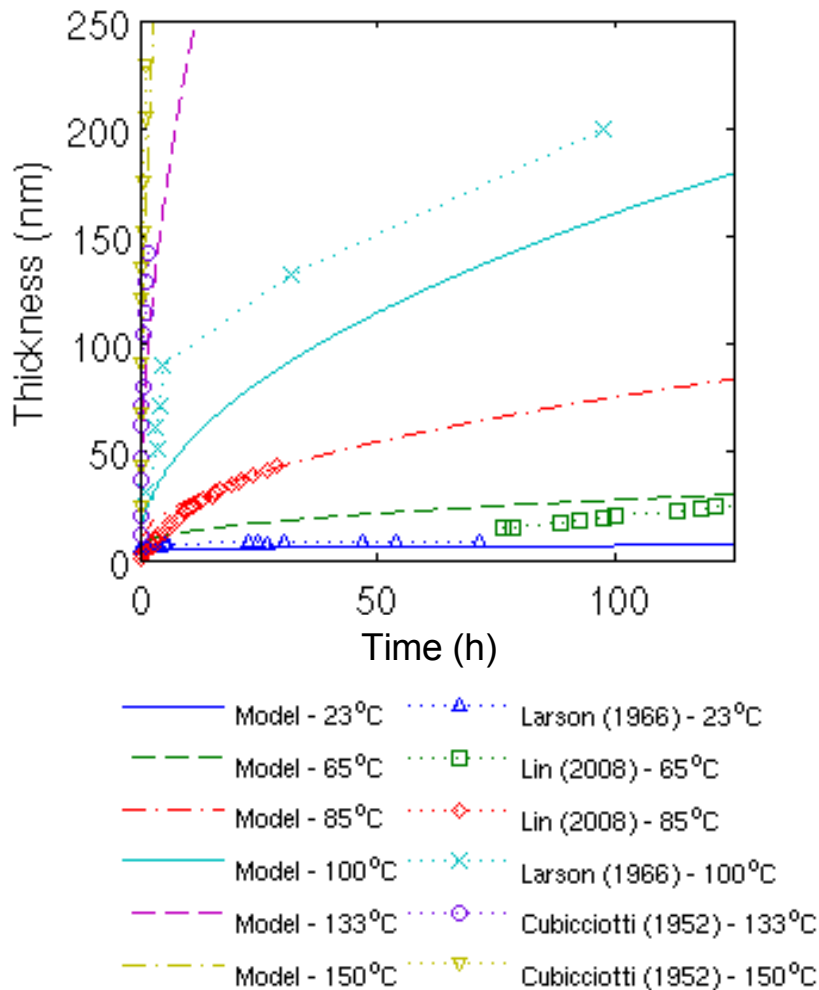


Chemical versus Ionic Diffusion



- Comparison of diffusion based oxidation for chemical diffusion using Fick's Law and ionic diffusion accounting for the added effect of the electric potential
- Electric potential greatly enhances oxide growth
- Ionic diffusion model shows inverse logarithmic growth behavior at low temperatures and transitions to parabolic behavior at higher temperatures

Comparison with Published Data



- Results for modeled oxide thickness over time match previously reported experimental data well at small oxide thicknesses (< 300 nm)
- Agreement persists over multiple decades of data and varying measurement methods
- At larger thicknesses the cracking and spalling of oxide layer allows oxygen to penetrate surface
- At higher temperatures, the oxide thickness grows past 300 nm faster than measurements were conducted

Reaction Rate Constant Formulation

- Equate general empirical equation for thickening rate and ionic diffusion equation:

$$\frac{dL}{dt} = 2K_1 \exp\left(-\frac{W}{k_b T}\right) \sinh\left(\frac{K_2}{Lk_b T}\right) = V_{ion} J_{ion} \approx \underbrace{4V_{ion} a \nu C(L)}_{2K_1} \exp\left(-\frac{W}{k_b T}\right) \sinh\left(\frac{ZeV_m a}{Lk_b T}\right) K_2$$

(Evans, 1960)

- Case of high temperature and moderate oxide thickness yields parabolic growth:

$$\frac{dL}{dt} \approx \frac{2K_1 K_2}{Lk_b T} \exp\left(-\frac{W}{k_b T}\right) \Rightarrow \frac{1}{2}(L^2 - L_0^2) = \left[\frac{2K_1 K_2}{k_b T} \exp\left(-\frac{W}{k_b T}\right) \right] (t - t_0) = k_p (t - t_0)$$

(Ritchie, 1984)

- Case of low temperature and small oxide thickness yields variation of inverse logarithmic:

$$\frac{d\left(\frac{K_2}{Lk_b T}\right)}{dt} = -\frac{K_1 k_b T}{K_2} \left(\frac{K_2}{Lk_b T}\right)^2 \sinh\left(\frac{K_2}{Lk_b T}\right) \exp\left(-\frac{W}{k_b T}\right)$$

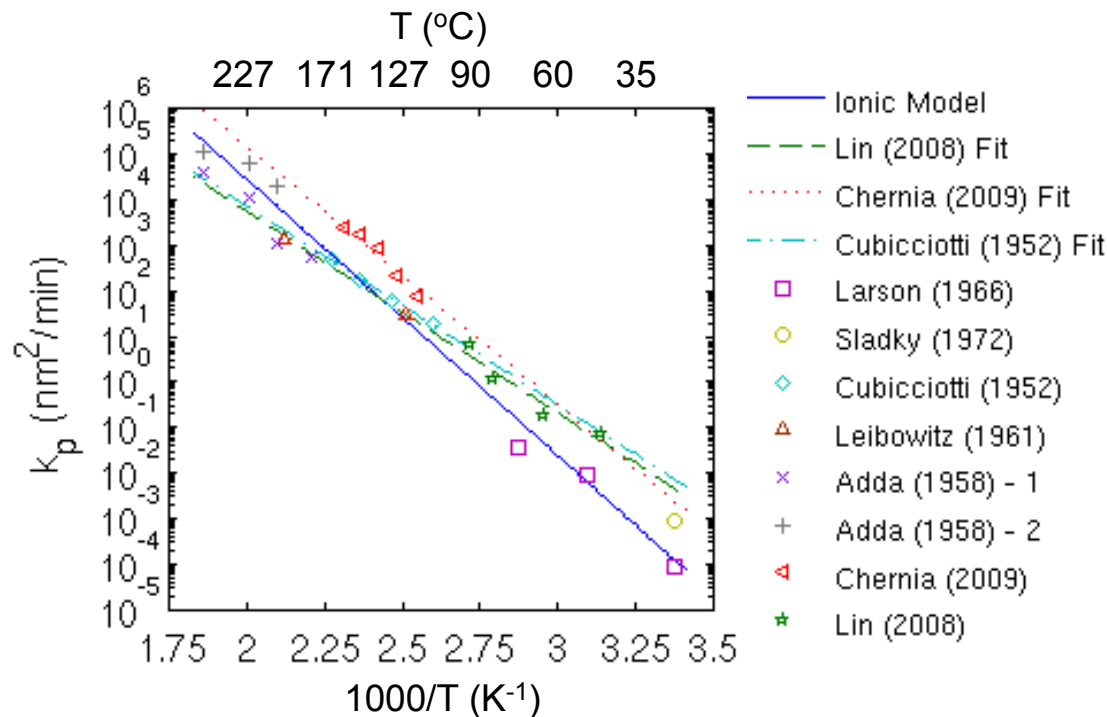
$$\Rightarrow L_0^2 \left[\ln\left(\frac{e^{K_2/L_0 k_b T} - 1}{e^{K_2/L_0 k_b T} + 1}\right) - \ln\left(\frac{e^{K_2/Lk_b T} - 1}{e^{K_2/Lk_b T} + 1}\right) \right] = \left[\frac{2K_1 K_2}{k_b T} \exp\left(-\frac{W}{k_b T}\right) \right] (t - t_0) = k_l (t - t_0)$$

(Ritchie, 1984)

- Both cases yield reaction rate constant:

$$k = \frac{a^5 \nu C(L) ZeV_m}{k_b T} \exp\left(-\frac{W}{k_b T}\right) = \frac{0.641}{T} \left[\frac{m^2 K}{s} \right] \exp\left(-\frac{14338[K]}{T}\right)$$

Reaction Rate Comparison



- Matches calculated values based on measurements well
- Fits data slightly better than empirical models
- Shows a slight non-linearity with temperature not discernable with empirical models



Conclusions

- **An ionic diffusion model was applied to the oxidation of uranium**
- **Ionic diffusion model predicts the transition between inverse logarithmic and parabolic growth trends**
- **The model agrees well with published data**
- **The theoretical reaction rate constant was derived**
- **Derived equation fits compiled data set as good as or better than empirically derived equations**



Questions?
