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Modeling Braided Shields via Multipole Representations for the Braid Charges and Currents

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Abstract — **A first principles calculation for the transfer capacitance of a Beldon cable is carried out by the use of filamentary constant, dipole, quadrupole, and octopole unknown charges placed at the center of each braid wire. Results are compared with full electrostatic simulations and phenomenological models.**

1 INTRODUCTION

A review of the literature for development of phenomenological models for penetration of cable shields has a long history and has been documented in [1]. These models have had considerable success in predicting the penetration through cable shields; however, we occasionally run into modifications of cable topology that call into question the use of these models. It would thus be useful to assemble a first principles model of the shield, not only to handle changes in topology from the standard geometry, but also to form a theoretical underpinning for the existing models. The comercially avaliable Beldon cable of Figure 1 was chosen as a generic test problem. As a simpification the braid is replaced by an infinitely periodic planar (quasi-planar?) braid whose unit cell is given in Figure 1. The relationship between the transfer capacitance (C_T) of the coaxial cable and planar shield is given by

$$C_T = -\frac{\phi_c/E_0}{b} \frac{2\pi\varepsilon_0}{\ln(b_{gnd}/b)\ln\left[(b+\phi_c/E_0)/a\right]}$$

where ϕ_c/E_0 is the solution to the planar problem, a is the inner conductor radius, b is the outer shield radius, and b_{gnd} is the effective radius to a ground outside of the braid. A similar connection holds between the transfer inductance and the magnetic flux of the magnetostatic planar braid problem.

The present work is an extension of [1] where a planar approximation to the cylindrical (coaxial?) braid was modeled with our electrostatic version of EIGER^{TM}. A unit cell for the two-dimensional infinitely periodic problem is shown in Figure 2 . The

Figure 1: A commercially available Beldon cable.

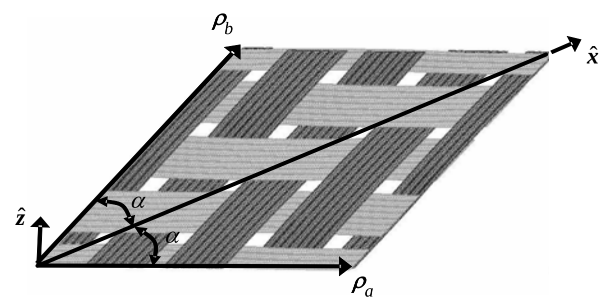

Figure 2: The unit cell of the two-dimensional infinitely periodic braid.

diameter of a single strand is 0.005", the magnitude of both lattice vectors $\boldsymbol{\rho}_a$ and $\boldsymbol{\rho}_b$ is 0.2440474", $\alpha = 24.4000°$, and $\widehat{x}$ is along the cable axis. The unit cell area A is $0.044\,813\,2\ in^2$. In [1] each of the 56 wires of the unit cell was meshed for simulation with Sandia's static version of EIGER^{TM} with 30 segments along the length and 16 segments around the circumference, giving an total triangular unknown count of 53,760. Because of the large number of unknowns and the need to simplify the treatment of the magnetostatic diffusion problem, the use of a modal series for the ϕ variation of the currents on each wire was proposed in [1].

The present work is similar in spirit to the modal

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series solution but is simpler in its implementation. Piecewise constant filament charge densities, as well as their dipole, quadripole, and octipole counterparts, that extend over each wire segment are placed along the center of each ($\rho = 0$) segment.

2 Formulation of the Electro- Static Integral Equation

We formulate the integral equation for the unknown sources in a single unit cell. Ths sources have the form

$$\sigma^{(i,j)}(\mathbf{s}') = \sum_{n=1}^{Nseg} \sigma_n^{(i,j)} u_n(\mathbf{s}')$$

where

$$u_n(s) = \begin{cases} 1, if\ s' \text{ in the } n^{th} segment \\ 0, otherwise \end{cases}$$

We work in the coordinate system of the $n^{th} segment$ which runs from r_{n-1} to r_n in the global coordinate system and points in the local $\widehat{z}_n$ direction. The other unit vectors in the local wire segment coordinate system are denoted by $\widehat{x}_n$, and $\widehat{y}_n$. They are chosen to make a right handed $\widehat{x}_n, \widehat{y}_n, \widehat{z}_n$ triplet. The index pair (i, j) denotes the multipole index. For the charge filament $(i, j) = (0, 0)$, for a dipole charge unknown $(i, j) = (1, 0)$ denotes a charge displacement in the $\widehat{x}_n$ direction where $(0, 1)$ denotes a charge displacement in the $\widehat{y}_n$ direction. For quadripole charge distributions $(i, j) = (2, 0)$ denotes a quadripole charge in the $\widehat{x}_n$direction and $(1, 1)$ denotes a quardipole formed by dipole displacement in both the $\widehat{x}_n$ and $\widehat{y}_n$ directions due to the near linear dependency of the $(2, 0)$ and $(0, 2)$ quadrapole source terms we do not use the $(0, 2)$ quadripole source term. For octipoles we use the $(3, 0)$ and $(2, 1)$ source terms. This gives $N^{u,seg} = 3$, 5, or 7 independent unknowns for each wire segment for dipoles, quadrupoles, and octopoles, respectively. The potential due to each source term is given by

$$\phi^{tot}(\mathbf{r}) = \phi^{inc}(\mathbf{r}) + \phi^{sc}(\mathbf{r}) \tag{1}$$

where the scattered potential is

$$\phi^{sc}(\mathbf{r}) = \frac{1}{4\pi\varepsilon} \sum_{n=1,}^{N_{seg}} \sum_{i,j} \sigma_n^{(i,j)}. \tag{2}$$

$$\int \left(-\frac{\partial}{\partial y_n}\right)^j \left(-\frac{\partial}{\partial x_n}\right)^i G(\mathbf{r} - \mathbf{r}'(s')) u_n(\mathbf{s}') ds'$$

Because the tangential component of the electric field vanishes on the surfacees of the individual wires, the total potential must be a constant. This fact is used to solve for the line multipole moments of the wires. The match points on the surface m^{th} wire segment are described by

$$\mathbf{r}_{(m,j)} = \mathbf{r}_m^c + a(\cos\varphi_j + \sin\varphi_j \widehat{y}_n) \tag{3}$$

where $\mathbf{r}_m^c$ is the centroid of the m^{th} segment, $-\pi < \varphi_j \leq \pi$ is a local coordinate angle at the wire and a is the wire radius. Since we have $N^{u,seg}$ unknowns on each wire segment, we require $N^{u,seg}$ match points per segment to generate the required equations. We place the match points on the surface of the wire at

$$\varphi_{nj} = 2\pi j / N^{u,seg}\,, \tag{4}$$

where φ_n is the polar angle in the coordinate system of the n^{th} wire segment and $j = -\frac{N^{u,seg}-1}{2}, \cdots, \frac{N^{u,seg}-1}{2}$.

In [1] it was shown that the problem of an $E_z^{inc} = -1$ V incident field above the braid and zero incident field below the braid may solved by the superposition of two problems. The first has a $\widehat{z}$ directed uniform incident field of 0.5 V/m with zero total potential on the braid surface, the total charge q^{UF} on that problem is computed. The second problem is that of zero incident field and a unit potential on the braid q^{1V} is computed. The total problem is then solved by setting the braid voltage to be $\phi_{braid} = -(A\varepsilon_0(1V/m) + q^{UF})/q^{1V}$ and incident uniform field excitation $E_z^{inc} = -0.5$ V/m to yield a net field $E_z = -1$ far below the braid and $E_z = 0$ V/m far above the braid.

2.1 Uniform field and zero braid potential problem

We now focus attention on the problem of the planar braid excited by a uniform field

$$\phi^{tot}(\mathbf{r}) = \phi^{inc}(\mathbf{r}) + \phi^{s}(\mathbf{r})$$

where the scattered potential $\phi^s(\mathbf{r})$ is is given by 1 and the incident potential is

$$\phi^{inc}(\mathbf{r}) = -z/2$$

We set the $\phi^{tot}(\mathbf{r})$ to zero on the braid surface 1 to obtain

$$\phi^s\left(r_{(m,j)}\right) = \frac{\widehat{z} \cdot r_{(m,j)}}{2}\, m = 1, ..., N_{seg} \tag{5}$$

a square linear system for the $Nseg \times N^{u,seg}$ unknowns. Once this symsem is solved for all the charge sources

1 is used to obtain the scattered potential $\phi^{UF}(\mathbf{r}_0)$ at the observation point $\mathbf{r}_0$ above the braid. The charge σ^{UF} is obtained by summing the product of the charges $\sigma_n^{(0,0)}$ and each segment length Δ_n.

2.2 Zero uniform field and unit potental on the braid problem

Now a 1 *volt* potential applied to the braid with zero incident field to yield

$$\phi\left(x_{nj}, y_{nj}, z_n\right) = 1\,,\ n = 1, ..., N_{seg} \tag{6}$$

which is again a square linear system for the $Nseg \times N^{u,seg}$ unknowns. Again 1 is used to obtain $\phi^{1V}(r_0)$. The total charge for this excitation q^{1v} is again obtained by summing $\sigma_n^{(0,0)}\Delta_n$ for this excitation.

2.3 Planar transfer Capacitance

The total problem has

$$\phi^{braid} = -(\varepsilon_0\ A\ *1v + q^{UF})/q^{1v} \tag{7}$$

$$\phi^{tot}(r) = (\phi^{sc,UF}(r) - \frac{r_z}{2} + (\phi^{1V} - 1.0)\ \phi^{braid}) \tag{8}$$

The final $\phi^{tot}(r)$ is scaled by 0.0254 m/inch since the units of the braid geometry are provided in inches.

3 Evaluation of the Green's functions by Ewald methods

If $|z - z\prime| << \max\,(\rho_a, \rho_b)$ as occurs for evaluation of the system matrix elements the evaluation of the two-dimensionally infinitely periodic Green's function is carried out by a static modification to the Ewald methods discussed in [2]. These techniques are used to obtain the doubly-infinite, periodic Green's function for the three-dimensional "planar" braid $G(\mathbf{r}, \mathbf{r}')$ and its gradients $\nabla G(\mathbf{r}, \mathbf{r}')$, $\nabla\nabla G(\mathbf{r}, \mathbf{r}')$,and $\nabla\nabla\nabla G(\mathbf{r}, \mathbf{r}')$ that are needed for the dipole, quadrupole, and octopole sources, respectively. Due to space limitations only a brief summary of the results are presented here.

$$G(\mathbf{r}) = \frac{1}{4\pi}\frac{1}{R_{m_0,n_0}} + \widetilde{G}^{E,spatial}(\mathbf{r}) + \widetilde{G}^{E,spectral}(\mathbf{r})$$
$$-\widetilde{G}^{E,spatial}(0,0,0) - \widetilde{G}^{E,spectral}(0,0,0) \tag{9}$$

$$\widetilde{G}^{E,spectral}(\mathbf{r}) = \frac{1}{4A}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\cos(\mathbf{k}_{tmn}\cdot\rho) S_{m,n}^{spectral}$$

$$S_{m,n}^{spectral} = \left(e^{-k_{tmn}|z|}\operatorname{erfc}(\frac{k_{tmn}}{2E} - |z|\,E)\right.$$
$$\left.+e^{k_{tmn}|z|}\operatorname{erfc}(\frac{k_{tmn}}{2E} + |z|\,E)\right)/k_{tmn}$$

$$S_{0,0}^{spectral} = -2\left[|z|\operatorname{erf}(|z|\,E) + \frac{e^{-(|z|E)^2}}{E\sqrt{\pi}}\right]$$

$$\mathbf{k}_{tmn} = \frac{2\pi}{A}m\mathbf{s}_2 \times \widehat{z} + n\widehat{z}\times\mathbf{s}_1$$

$$\widetilde{G}^{E,spatial}(\mathbf{r}) = \frac{1}{4\pi}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} S_{m,n}^{spatial} \tag{10}$$

$$S_{m,n}^{spatial;} = \frac{\operatorname{erfc}(R_{m,n}E)}{R_{m,n}}$$

$$S_{0,0}^{spatial} = \frac{-\operatorname{erf}(R_{m_0,n_0}\ E)}{R_{m_0,n_0}}$$

$$\nabla G(\mathbf{r}) = \frac{1}{4\pi}\nabla\frac{1}{R_{m_0,n_0}} \tag{11}$$
$$+\nabla\widetilde{G}^{E,spatial}(\mathbf{r}) + \nabla\widetilde{G}^{E,spectral}(\mathbf{r})$$

$$\nabla\widetilde{G}^{E,spectral}(\mathbf{r}) = \frac{1}{4A}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}([k_{tmn,x}\widehat{x} + k_{tmn,y}\widehat{y}]$$
$$\sin(\mathbf{k}_{tmn}\cdot\rho)S_{m,n}^{spectral} + sgn(z)\widehat{z}\cos(\mathbf{k}_{tmn}\cdot\rho)T_{m,n})$$

$$S_{m,n}^{spectral} = -\left[e^{-k_{tmn}|z|}\operatorname{erfc}(\frac{k_{tmn}}{2E} - |z|\,E)\right.$$
$$\left.+e^{k_{tmn}|z|}\operatorname{erfc}(\frac{k_{tmn}}{2E} + |z|\,E)\right]/k_{tmn}$$

$$T_{m,n}^{spectral} = -e^{-k_{tmn}|z|}\operatorname{erf}c(\frac{k_{tmn}}{2E} - |z|\,E)$$
$$+e^{k_{tmn}|z|}\operatorname{erf}c(\frac{k_{tmn}}{2E} + |z|\,E)$$

$$+\frac{2E}{k_{tmn}\sqrt{\pi}}e^{-k_{tmn}|z|}e^{-\left[\frac{k_{tmn}}{2E}-|z|E\right]^2}$$

$$-\frac{2E}{k^s_{tmn}\sqrt{\pi}}e^{k_{tmn}|z|}e^{-\left[\frac{k_{tmn}}{2E}+|z|E\right]^2}$$

$$T^{spectral}_{0,0}=-\frac{\operatorname{erf}(|z|\,E)}{2A}$$

$$\nabla\widetilde{G}^{E,spatial}(\mathbf{r})=\frac{1}{4\pi}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}S^{spatial}_{m,n}\nabla R_{m,n}$$

$$S^{spatial}_{m,n}=\left[-\frac{\operatorname{erfc}(R_{m,n}E)}{R^2_{m,n}}-\frac{2Ee^{-[R_{m,n}E]^2}}{\sqrt{\pi}R_{m,n}}\right]$$

$$S^{spatial}_{0,0}=\left[\frac{\operatorname{erf}(R_{m_0,n_0}\ \ E)}{R^2_{m_0,n_0}}-\frac{2Ee^{-\left[R_{m_0,n_0}\ \ E\right]^2}}{\sqrt{\pi}\ R_{m_0,n_0}}\right]$$

where

$$R_{m,n}=\sqrt{\left[x-(m\boldsymbol{\rho}_a+n\boldsymbol{\rho}_b)\cdot\widehat{x}\right]^2+\left[y-(m\boldsymbol{\rho}_a+n\boldsymbol{\rho}_b)\cdot\widehat{y}\right]^2+z^2}$$

$$\nabla R_{m,n}=\frac{\left[x-(m\boldsymbol{\rho}_a+n\boldsymbol{\rho}_b)\cdot\widehat{x}\right]\widehat{x}+\left[y-(m\boldsymbol{\rho}_a+n\boldsymbol{\rho}_b)\cdot\widehat{y}\right]\widehat{y}+z\widehat{z}}{R_{m,n}}$$

results for $\nabla\nabla\nabla G(\mathbf{r}), \nabla\nabla\nabla G(\mathbf{r})$ have also been obtained but are not given due to space limitations.

If $|z-z\prime|$ $>>$ max (ρ_a,ρ_b) as occurs for evaluation of the potential far above the braid a static spectral series approach is used to evaluate $G(\mathbf{r}-\mathbf{r}), \nabla G(\mathbf{r}-\mathbf{r}), \nabla\nabla G(\mathbf{r}-\mathbf{r})$, and $\nabla\nabla G(\mathbf{r}-\mathbf{r})$.

4 Results

We choose the point $r_0=(0,0,z_0)$ above the braid and use 7 and 8 to obtain the transfer capacitance for the case sources up to dipoles, quadrapoles and octipoles.

	q^{UF}	$\phi^{sc,UF}$	q^{1v}	ϕ^{1V}	ϕ^{tot}
dipoles					
quadripoles	-2.543880*10^{-18}	3.752849280*10^{-3}	6.4023434*10^{-12}	-0.95632523	1.19*10^{-7}
octipoles	3.48704559*10^{-3}	3.64155666*10^{-3}	6.4258314*10^{-12}	-0.96533436	2.65*10^{-6}

(Note the dipole simulations are still running , if it agres with the quardapole I may lave out the octipole results)

The full EIGER simulation [1] with ‘12, and 16 elements around the circumference gave $\phi^{tot}/E_0 =$ 9.11*$10^{-8}m$ *and* 8.67* $10^{-8}m$ *respectively*.We contiune to work on the octipole case .

Similar results for the magneto-static transer inductance proble will be adressed in the talk but are omitted here due to space considerations.

References

[1] W. A. Johnson, L. K. Warne, L. I. Basilio, R. S. Coats, J. D. Kotulski, and R. E. Jorgenson, “Modeling of Braided Shields”, Proceedings of Joint 9th International Conference on Electromagnetics in Advanced Applications ICEEA 2005 and 11th European Electromagnetic Structures Conference EESE 2005, Torino, Italy, Sept, 12-15, 2005, pp. 881-884.

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