Shock testing accelerometers with a Hopkinson pressure bar

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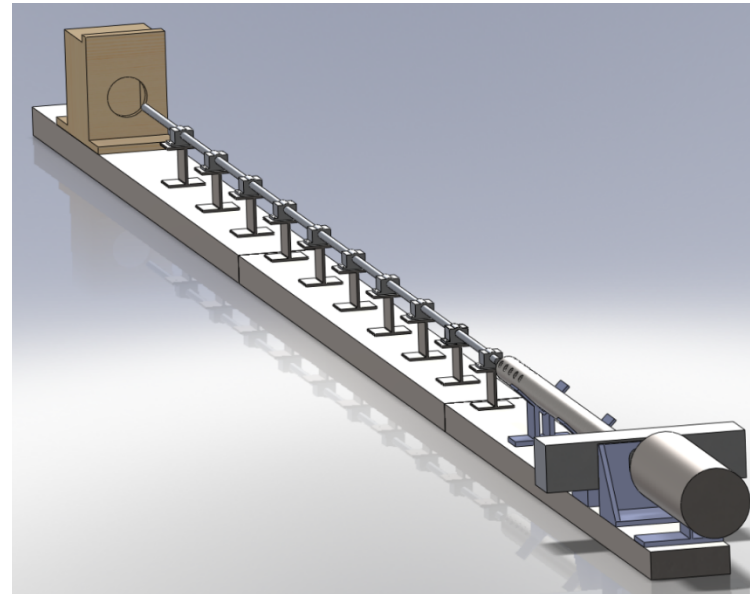
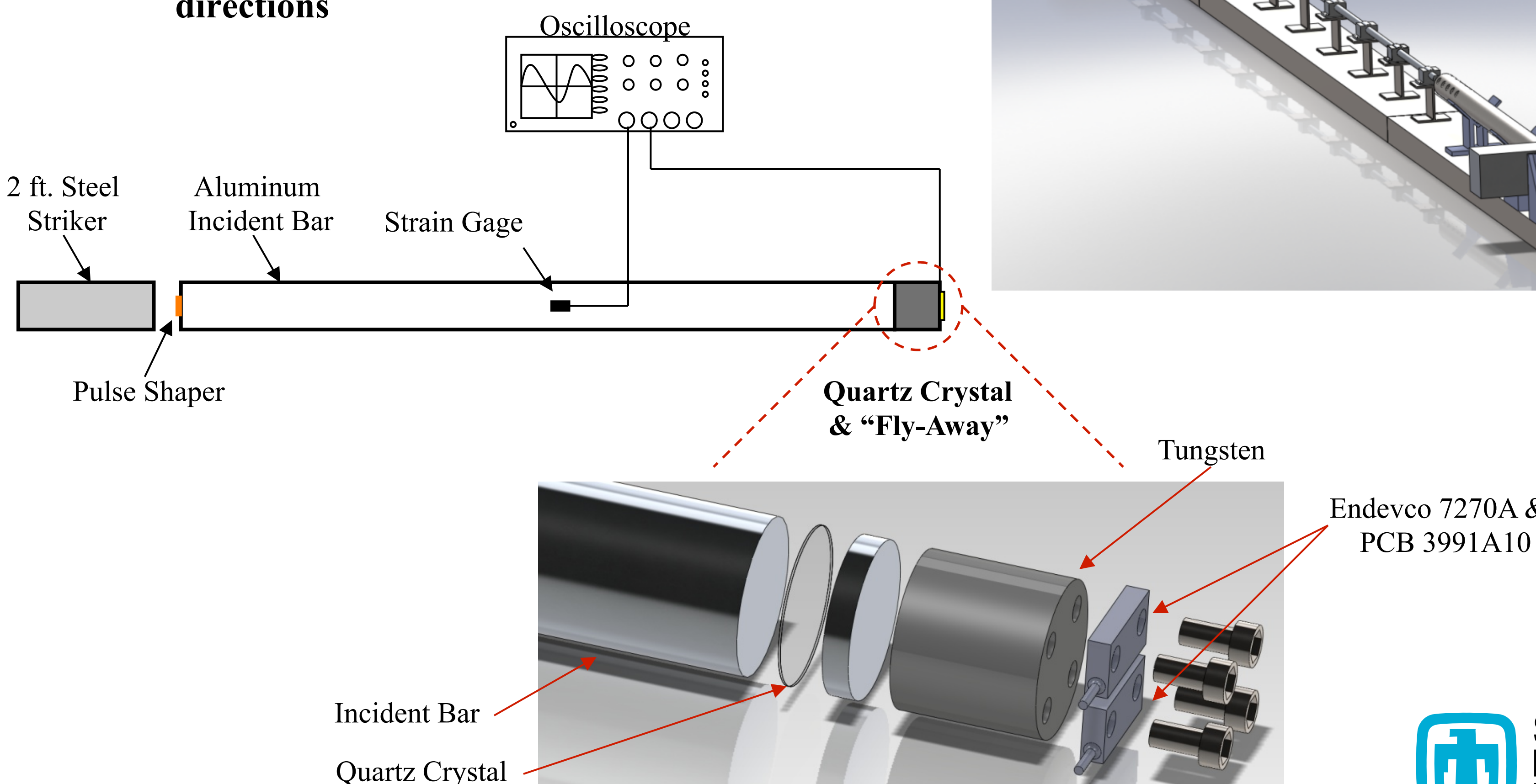
Outline

- Overview of experimental apparatus
- Models utilized in design of experiments
- Model comparisons and results

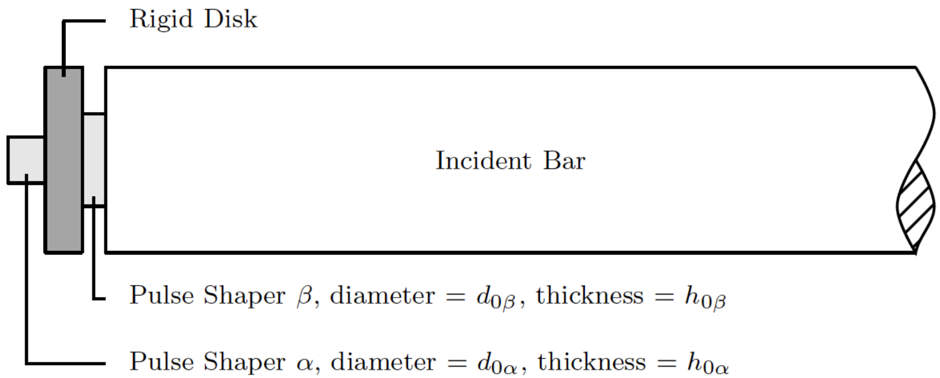
Experimental Apparatus

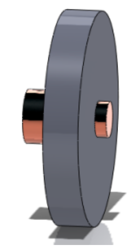
- **Utilize "fly-away" method of characterization**
  - **Controlled pulse shaping**
  - **Single pulse loading**
  - **Evaluation in axial and transverse directions**





Pulse Shaping





Nonlinear ODE

$$\frac{h_{0\alpha}}{V}\dot{\varepsilon}_\alpha(t) = 1 - K\left(\frac{1}{\rho c} + \frac{1}{\rho_{st}c_{st}}\right)\frac{f\left[\varepsilon_\alpha(t)\right]}{1-\varepsilon_\alpha(t)} - \frac{2K}{\rho_{st}c_{st}}\left\{\frac{f\left[\varepsilon_\alpha(t-\tau)\right]}{1-\varepsilon_\alpha(t-\tau)} + \frac{f\left[\varepsilon_\alpha(t-2\tau)\right]}{1-\varepsilon_\alpha(t-2\tau)} + \cdots + \frac{f\left[\varepsilon_\alpha(t-n\tau)\right]}{1-\varepsilon_\alpha(t-n\tau)}\right\} - \frac{h_{0\beta}}{V}\dot{\varepsilon}_\beta(t),$$

$$n\tau \leq t < (n+1)\,\tau,$$

$$K = \frac{\sigma_{0\alpha}a_{0\alpha}}{AV}$$

Algebraic Constraint

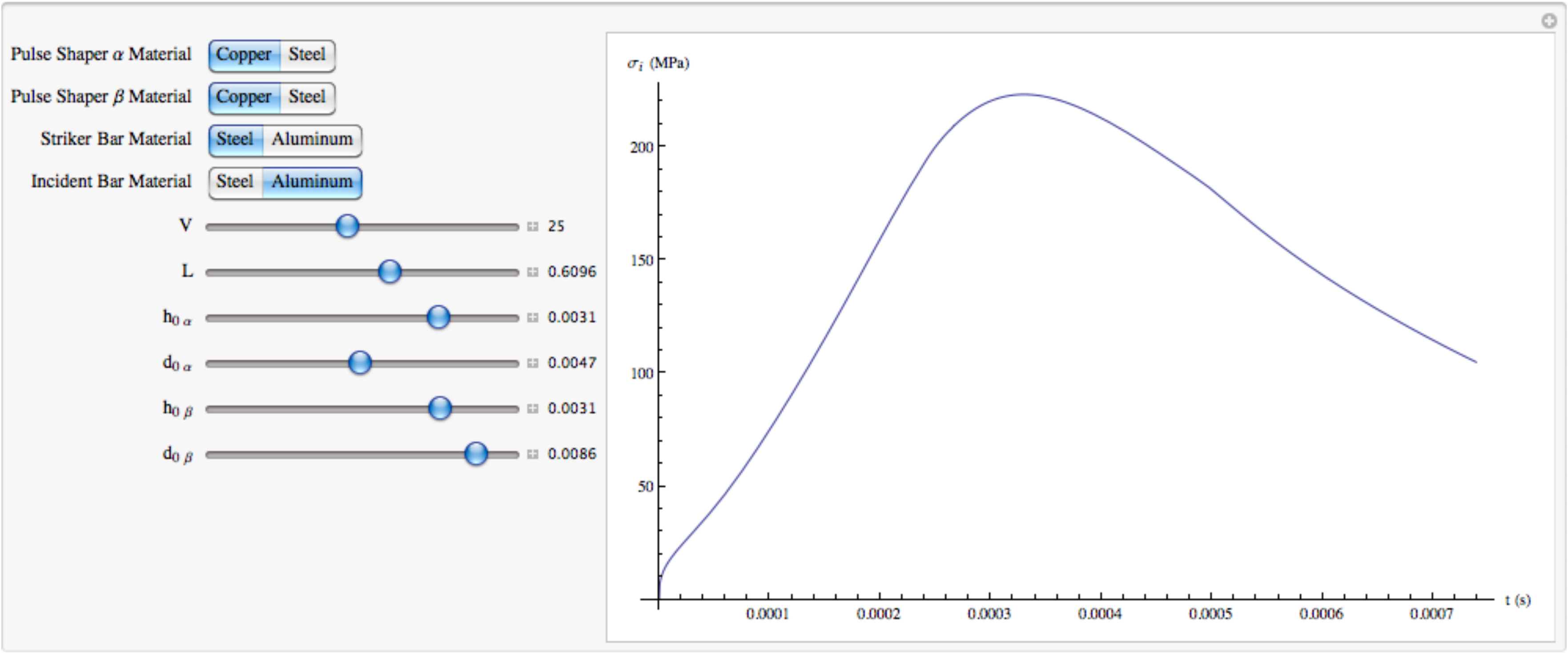
$$\frac{g\left[\varepsilon_\beta(t)\right]}{1-\varepsilon_\beta(t)} = \frac{a_{0\alpha}}{a_{0\beta}}\frac{\sigma_{0\alpha} f\left[\varepsilon_\alpha(t)\right]}{\sigma_{0\beta}\left(1-\varepsilon_\alpha\right)},$$

Response functions

$$f\left[\varepsilon_\alpha(t)\right] = \frac{\varepsilon_\alpha(t)^n}{1-\varepsilon_\alpha(t)^m}, \quad g\left[\varepsilon_\beta(t)\right] = \frac{\varepsilon_\beta(t)^k}{1-\varepsilon_\beta(t)^j},$$

System of equations in solved with implicit time integration scheme to predict incident stress

Pulse Shaper Design Code



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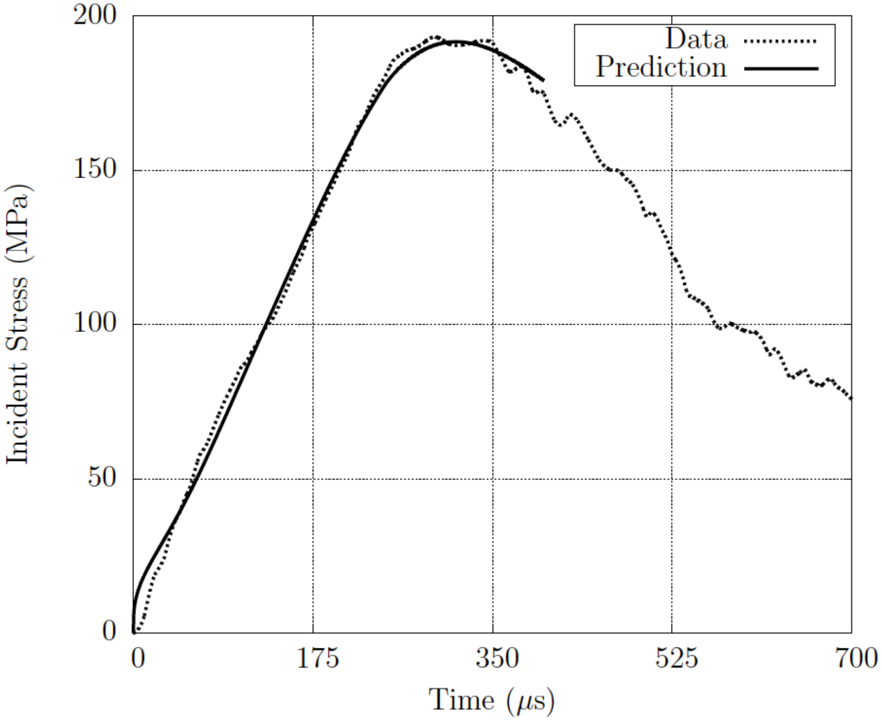
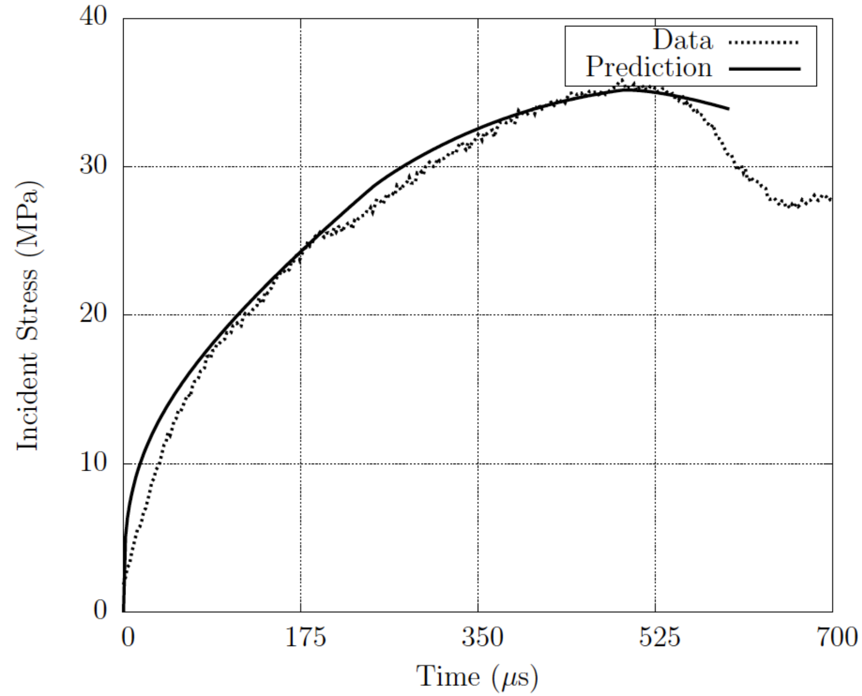
Pulse Shaping Model Comparison



$$V \approx 20 \text{ m/s}$$



$$V \approx 7 \text{ m/s}$$

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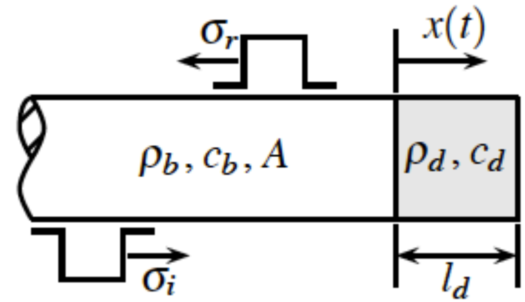
Rigid Body Model

$$A\left(\sigma_r + \sigma_i\right) = A\rho_d l_d \ddot{x}(t) \qquad (1)$$

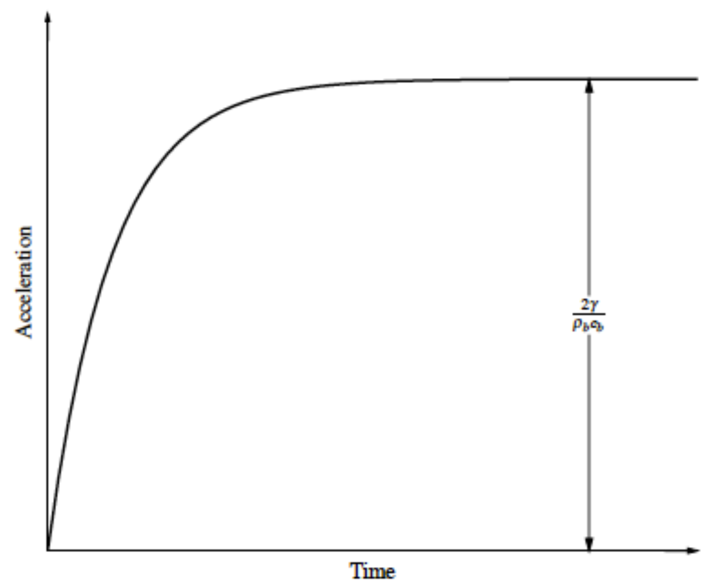


$$\sigma_i - \sigma_r = \rho_b c_b \dot{x}(t) \qquad (2)$$

Combining (1) & (2) by eliminating σ_r, yields:

$$\rho_d l_d \ddot{x}(t) + \rho_b c_b \dot{x}(t) = 2\sigma_i \qquad (3)$$

Let $\sigma_i = \gamma t$, where γ is a constant incident stress rate and solve (3).



Wave analysis

$$a_1(t) = \begin{cases} \frac{2r}{\rho_b c_b (1+r)} \frac{d\sigma_i(t)}{dt} & 0 \le t < 2t_0 \\ \frac{2r}{\rho_b c_b (1+r)} \left\{ \frac{d\sigma_i(t)}{dt} + \left[1 + \frac{1-r}{1+r}\right] \frac{d\sigma_i(t-2t_0)}{dt} \right\} & 2t_0 \le t < 4t_0 \\ \vdots & \\ \frac{2r}{\rho_b c_b (1+r)} \left\{ \frac{d\sigma_i(t)}{dt} + \cdots + \left[\left(\frac{1-r}{1+r}\right)^{\frac{n-2}{2}} + \left(\frac{1-r}{1+r}\right)^{\frac{n}{2}} \right] \frac{d\sigma_i(t-nt_0)}{dt} \right\} & (n-2)t_0 \le t < nt_0 \end{cases}$$

For even n, where $t_0 = l_d / c_d$, $r = \frac{\rho_b c_b}{\rho_d c_d}$

$$a_2(t) = \begin{cases} 0 & 0 \le t < t_0 \\ \frac{4r}{\rho_b c_b (1+r)} \frac{d\sigma_i(t-t_0)}{dt} & t_0 \le t < 3t_0 \\ \vdots & \\ \frac{4r}{\rho_b c_b (1+r)} \left\{ \frac{d\sigma_i(t-t_0)}{dt} + \cdots + \left(\frac{1-r}{1+r}\right)^{\frac{n-1}{2}} \frac{d\sigma_i(t-nt_0)}{dt} \right\} & (n-2)t_0 \le t < nt_0 \end{cases}$$

For odd n, where $t_0 = l_d / c_d$, $r = \frac{\rho_b c_b}{\rho_d c_d}$

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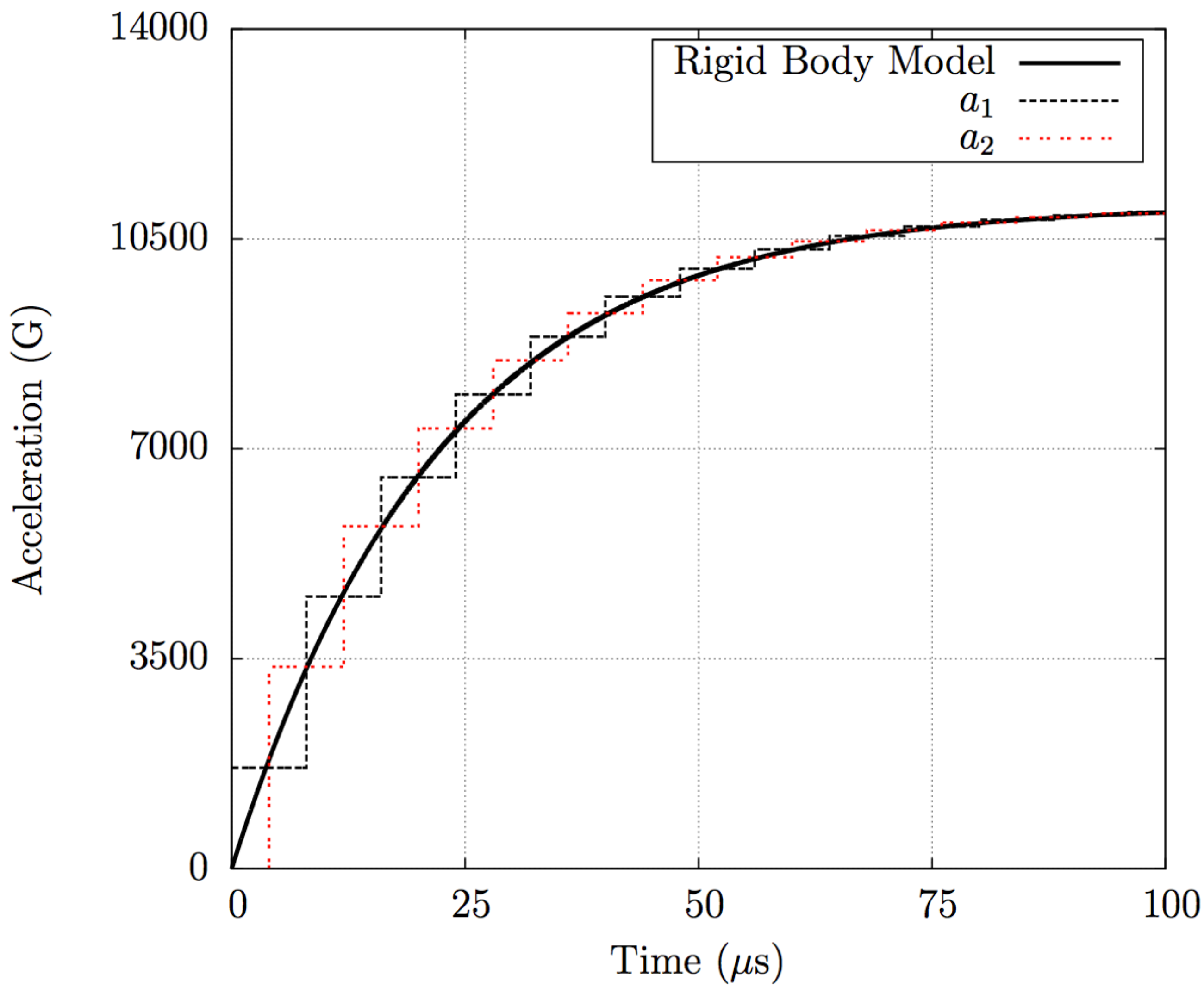
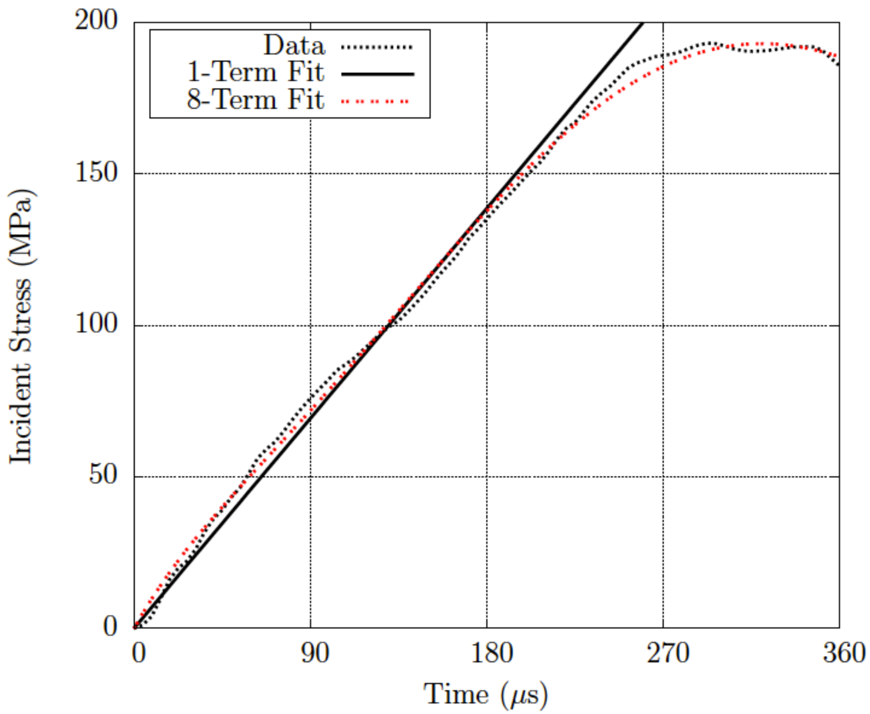
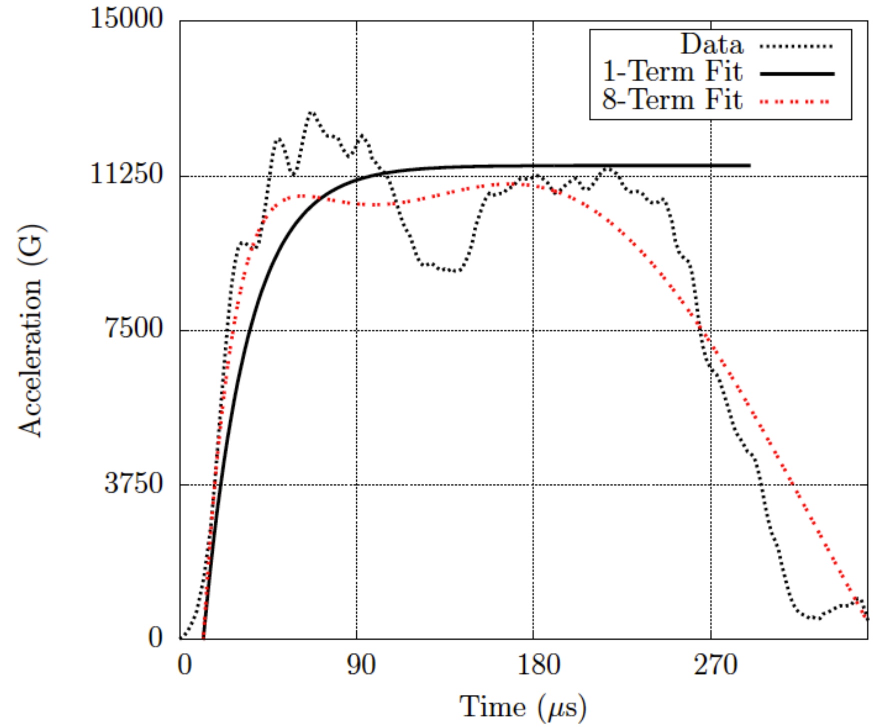
Wave analysis of tungsten disk response to ramp loading



Model for V = 7 m/s



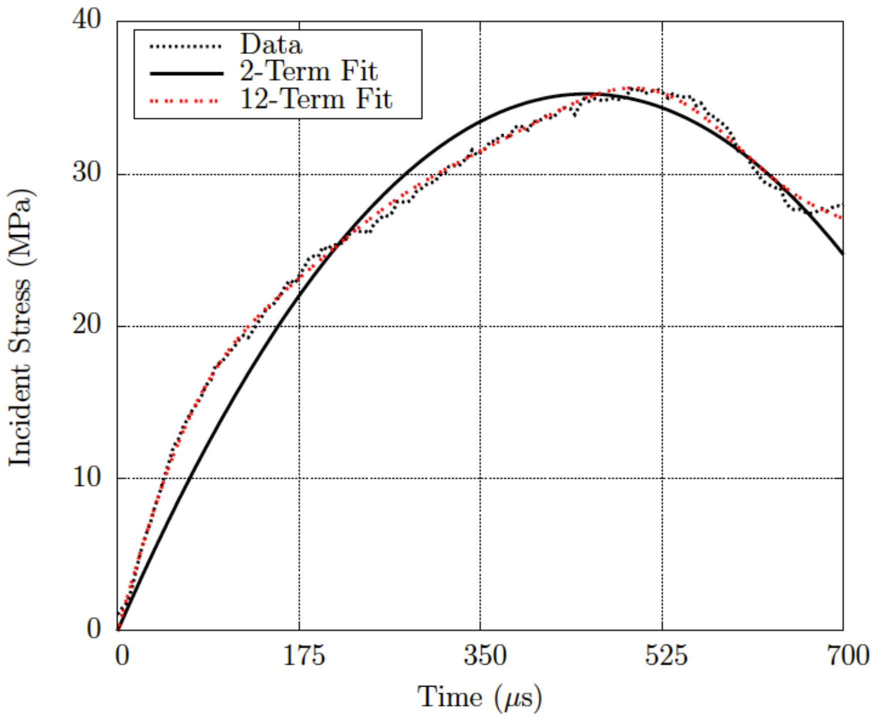


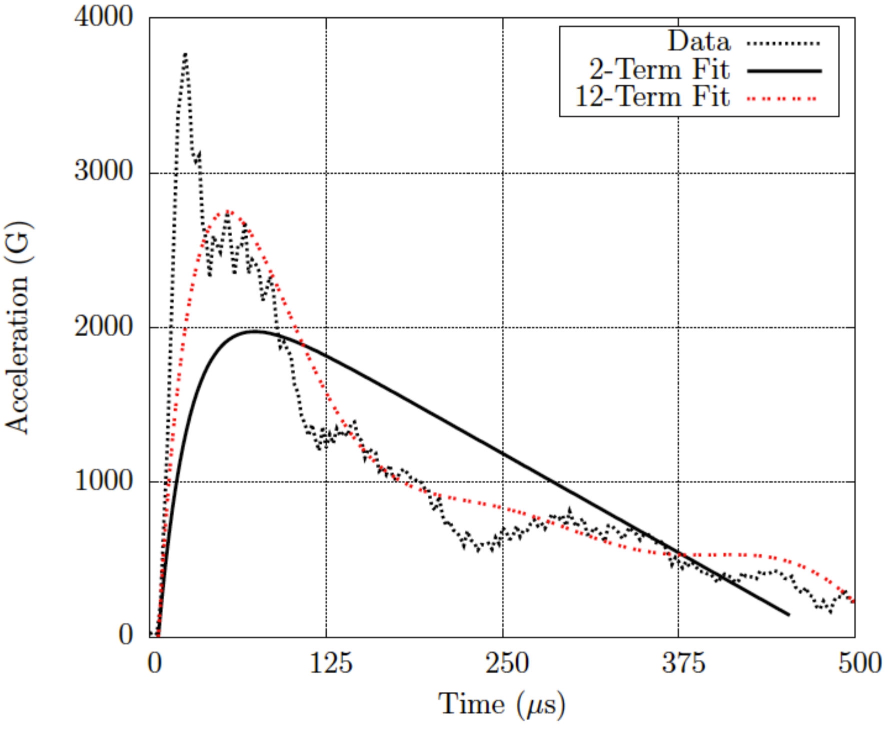
$$\sigma_i = \gamma_1 t + \gamma_2 t^2 + \ldots + \gamma_{\mathcal{N}} t^{\mathcal{N}}$$

$$a(t) = \sum_{n=1}^{\mathcal{N}} \left[(-1)^{n-1} \frac{2n!\gamma_n}{\rho c \lambda^{n-1}} \left(1 - e^{-\lambda t}\right) \right] + \sum_{m=1}^{\mathcal{N}} \sum_{n=1}^{m-1} \left[(-1)^{n+m-1} \frac{2m!\gamma_m}{n!\rho c \lambda^{m-n-1}} t^n \right]$$

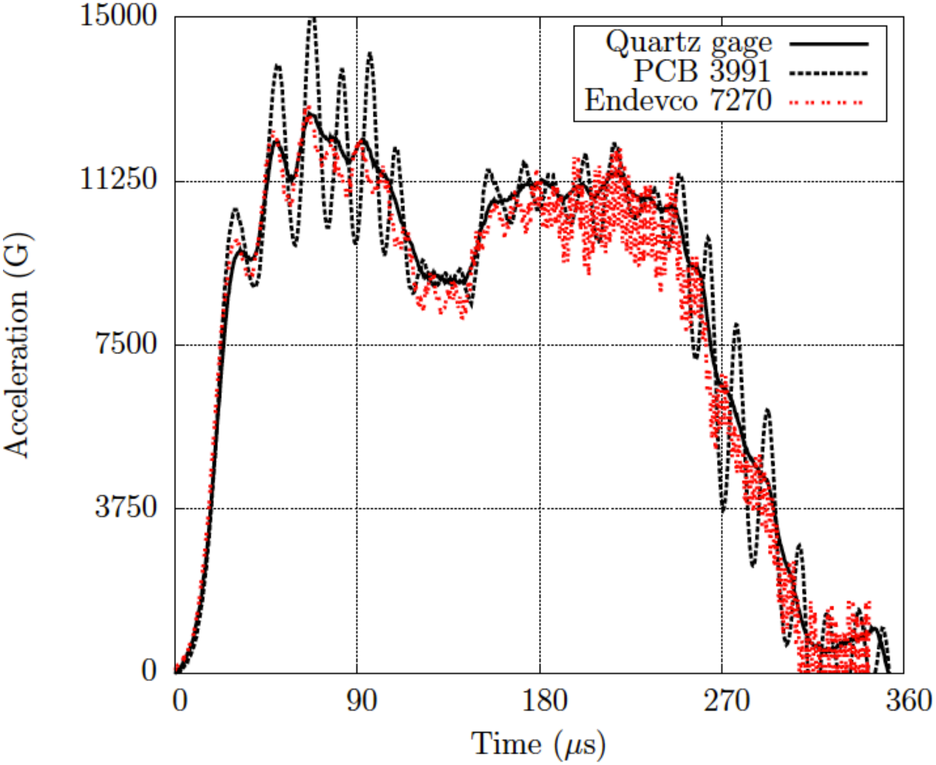
N-term model

Model for V = 20 m/s

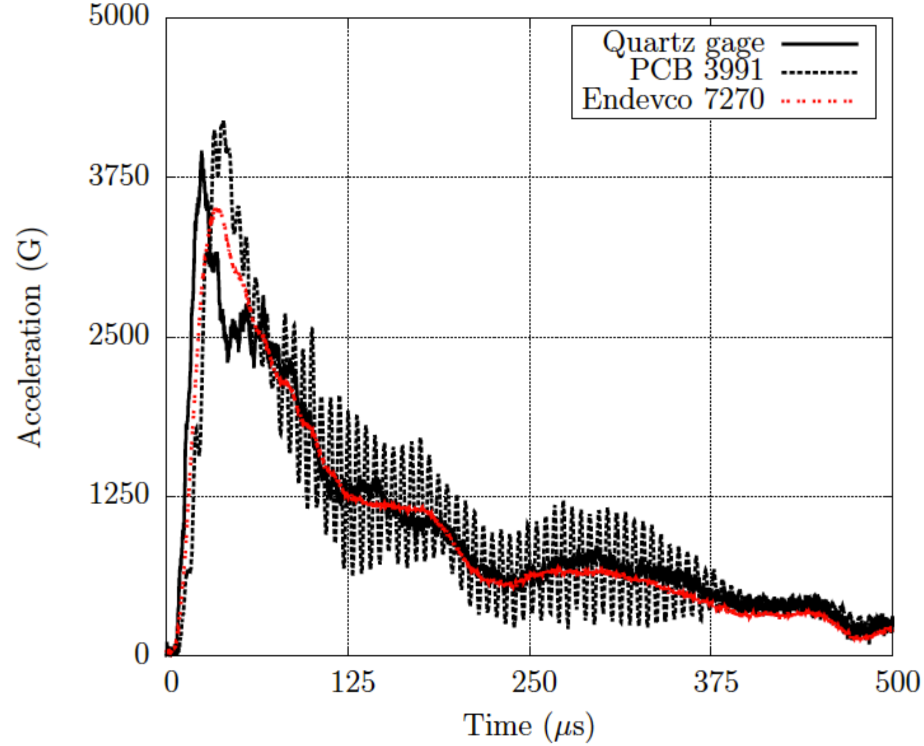




Acceleration Measurements



$V \approx 20$ m/s



$V \approx 7$ m/s

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Summary

- **Presented pulse shaping techniques for extending loading duration of incident stress pulses**
- **Presented both rigid body and wave mechanics models for predicting the acceleration of the "fly-away" disk given an incident stress pulse**
- **Comparison of models to experimental measurements from quartz stress gage and two accelerometers are in good agreement.**