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DYNAMIC MODULUS ESTIMATION AND STRUCTURAL VIBRATION ANALYSIS*

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ABSTRACT

Often, the dynamic elastic modulus of a material with frequency dependent properties is difficult to estimate. These uncertainties are compounded in any structural vibration analysis using the material properties. Here, different experimental techniques are used to estimate the properties of a particular elastomeric material over a broad frequency range. Once the properties are determined, various structures incorporating the elastomer are analyzed by an iterative finite element method to determine natural frequencies and mode shapes. Then, the finite element results are correlated with results obtained by experimental modal analysis.

NOMENCLATURE

A = $(f_s/f_n)^2 (2 + DT)(B/2)$
A_c = cross-section of cylindrical specimen, m^2
B = $1/6\,(1 + T)^2$
C_n = coefficient for mode n, of claped-free (Oberst) bar
C_1 = 0.55959
C_2 = 3.5069
C_3 = 9.8194
D = ρ_1/ρ, density ratio
E = Young's modulus of Oberst bar, Pa
E_c = complex modulus = $E + j\omega\eta$
G = shear modulus of viscoelastic material, Pa
H = thickness of Oberst bar, m
H_2 = thickness of viscoelastic material, m
L = length of cylindrical specimen, m
M = mass of metal disk on top of specimen, Kg
M_s = mass of specimen = $\rho_1 A_c L$, Kg
T = H_2/H thickness ratio
η = loss factor of viscoelastic material, dimensionless
η_s = $\Delta f_s/f_s$, loss factor of sandwiched specimen, dimensionless
ρ = density of Oberst bar, kg/m^3
ρ_1 = density of damping material, kg/m^3
Δf_s = half-power bandwidth for mode s of composite bar, Hz
fn = resonance frequency for mode n of Oberst bar, Hz
fs = resonance frequency for mode s of composite bar, Hz
s = index number: 1, 2, 3, ... ($s \equiv n$)
l = length of bar, m

1. INTRODUCTION

Viscoelastic materials due to their low elastic modulus and high loss modulus are used in many cases. However, modulus of a viscoelastic material is highly dependent on many parameters such as frequency, temperature strain amplitude, preload etc. [1]. Due to widespread use of viscoelastic materials in various industries including automotive, bio-mechanical, and structural, various test methods have been proposed to characterize these materials and no single or test method has been adopted universally.

One such method is Oberst bar method [2]. Since modulus of the material being investigated is very low, a sandwich specimen as shown in Figure 1 was tested in a Bruel & Kjaer apparatus [3].

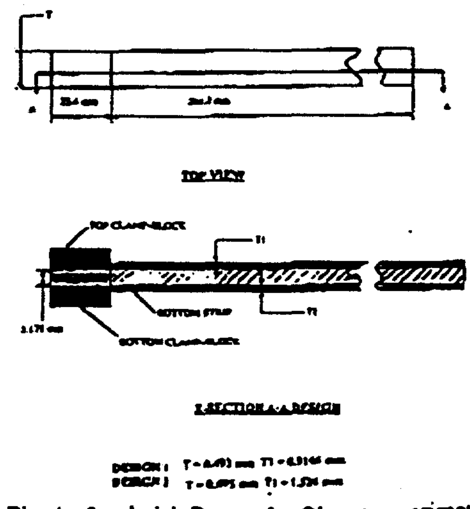

Fig. 1. Sandwich Beams for Oberst experiment

Natural frequencies and damping ratios were measured first for the bare specimen with cantilever end conditions and then for the sandwiched specimen. Magnetic excitors and pickups were used to avoid mass loading of the beam. This method is apparently popular in automotive industry where different materials are explored for their sound and vibration damping applications.

Another method (hereafter referred to as tripod method) has been proposed by Nielson etal [4]. In this method cylindrical test specimens with metal plates at one end is compressed by an inverted shaker as shown in Figure 2. Measuring the force and acceleration (an impedence head was used to measure both) the modulus and loss factor can be calculated using a numerical iterative scheme. Strain gages were bonded to the specimen to monitor strain due to preload as well as dynamic strain. This method (so called non-resonant method) is used to compute stiffness and loss factor continuously over a wide frequency range.

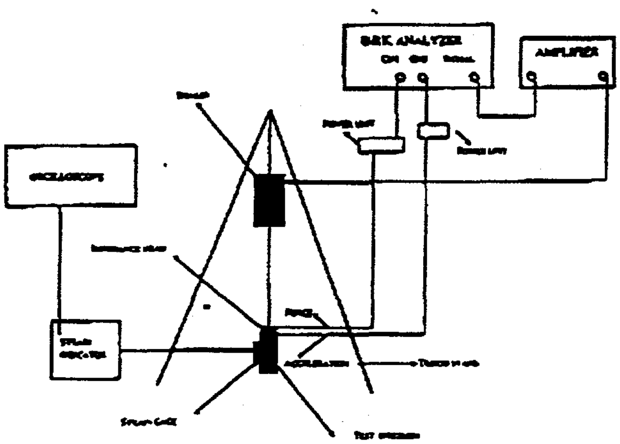

Fig. 2. Tripod experimental setup

It was decided that a structure made of the same viscoelastic material would be tested by experimental modal analysis to determine its natural frequencies. The same structure would also be modelled by finite element method using ANSYS [5] and the stiffness of the viscoelastic material would be adjusted to match the experimental result.

2. THEORY

In the Oberst bar method, for a sandwich specimen shown in Figure 1, the shear modulus is given by

$$G = \frac{[(A-B) - 2(A-B)^2 - 2(A\eta_s)^2]\left[\frac{2\pi C_n E H H_2}{\rho}\right]}{(1-2A+2B)^2 + 4(A\eta_s)^2} \quad (1)$$

and loss factor is given by

$$\eta = \frac{A\eta_s}{(A-B) - 2(A-B)^2 - 2(A\eta_s)^2} \quad (2)$$

For the tripod method shown in Figure 2, the complex modulus Ec can be solved from the measurement of compliance [4] as

$$\text{compliance}(\omega) = \frac{\Delta L(\omega)}{\text{force}(\omega)} = \frac{\text{Sin}(\beta)}{\omega^2(M_e/\beta \times \text{Cos}\beta - M \times \text{Sin}\beta)} \quad (3)$$

where:

$$\beta = \sqrt{\omega^2 L^2 \rho / Ec} \quad (4)$$

RESULTS

The dimensions of two sandwich specimens for the Oberst method are shown in Figure 1. The coefficients used in formulae (1) and (2) are based on cantilever end conditions. In order to verify the end conditions, natural frequencies of the bare bars in Oberst apparatus were measured and are presented in Table 1 along with the theoretical values. It may be noted that the first mode results are ignored as suggested in the test procedure [2].

Beam Thickness (m)	Theoretical Frequency f_n (Hz)	Experimental Frequency f_n (Hz)
0.9144	66.8 187.4 367.3	65.6 184.3 361.3
1.524	111.4 312.4 612.1	111.2 311.8 611.8

Table 1: Natural frequencies of bare bars

So it seems that cantilever boundary conditions is satisfied approximately for the thinner beam whereas agreement is very well for the thicker beam.

Next natural frequencies and damping ratios of the sandwich specimens were measured by using the Oberst apparatus and magnetic sensors, connected to Bruel & Kjaer 3550 frequency analyzer and transferring the data to a computer where STAR MODAL [6] is used to estimate natural frequencies and damping ratios. The values obtained are presented in Table 2.

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Sandwich Specimen #	Natural Frequency f_s (Hz)	Damping Ratio η_s %
1	200.4 418.4	15.0 20.5
2	228.8 493.9 783.7	13.0 13.6 2.24

Table 2: Sandwich specimen

Combining results of both sandwich specimens, based on the information presented in Tables 1 and 2, shear modulus and loss factor (twice the damping ratio) were estimated using formulae (1) and (2). Assuming a Poisson's ratio of 0.45, elastic modulus was also estimated.

Frequency (Hz)	Shear Modulus G (MPa)	Loss Factor η	Elastic Modulus E (MPa)
200.4	2.987	0.408	8.66
228.8	3.691	0.384	10.70
418.4	8.679	0.548	25.17
493.9	10.483	0.454	30.40
783.7	13.4	0.1	38.86

Table 3: Properties of viscoelastic material by Oberst experiment

There were four samples tested to estimate dynamic modulus by tripod method following equation (3). The samples have dimensions as per Table 4.

Sample #	Length L (mm)	Diameter (mm)	Mass of End Plate M (grams)
1	49.68	18.5	2.2
2	45.43	26.54	8.5
3	41.85	34.73	13.3
43	41.3	49.9	16.8

Table 4: Specimens for tripod testing

Modulii were obtained over frequency range typically up to 450 Hz as shown in Figures 3 and 4. Beyond 450 Hz the results were erroneous (probably effected by the resonance). In order to compare this test data with other test methods, the modulii obtained for different specimens with different pre-strains and strain amplitudes at four frequencies (100 Hz, 200 Hz, 300 Hz and 400 Hz) are presented in Table 5.

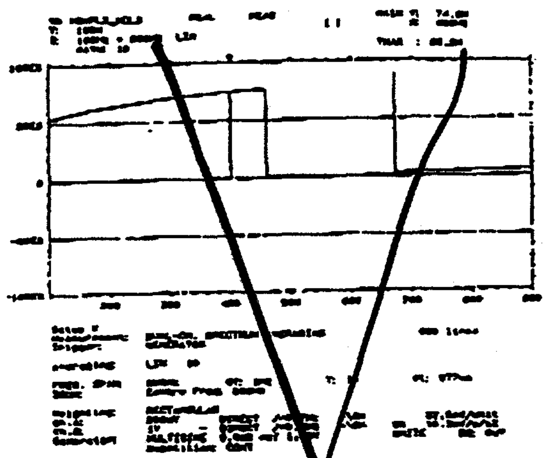


Fig. 3. E for sample 3 at 40E-6 strain amplitude

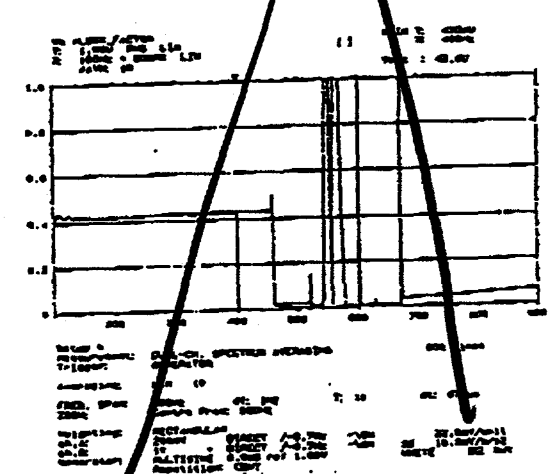
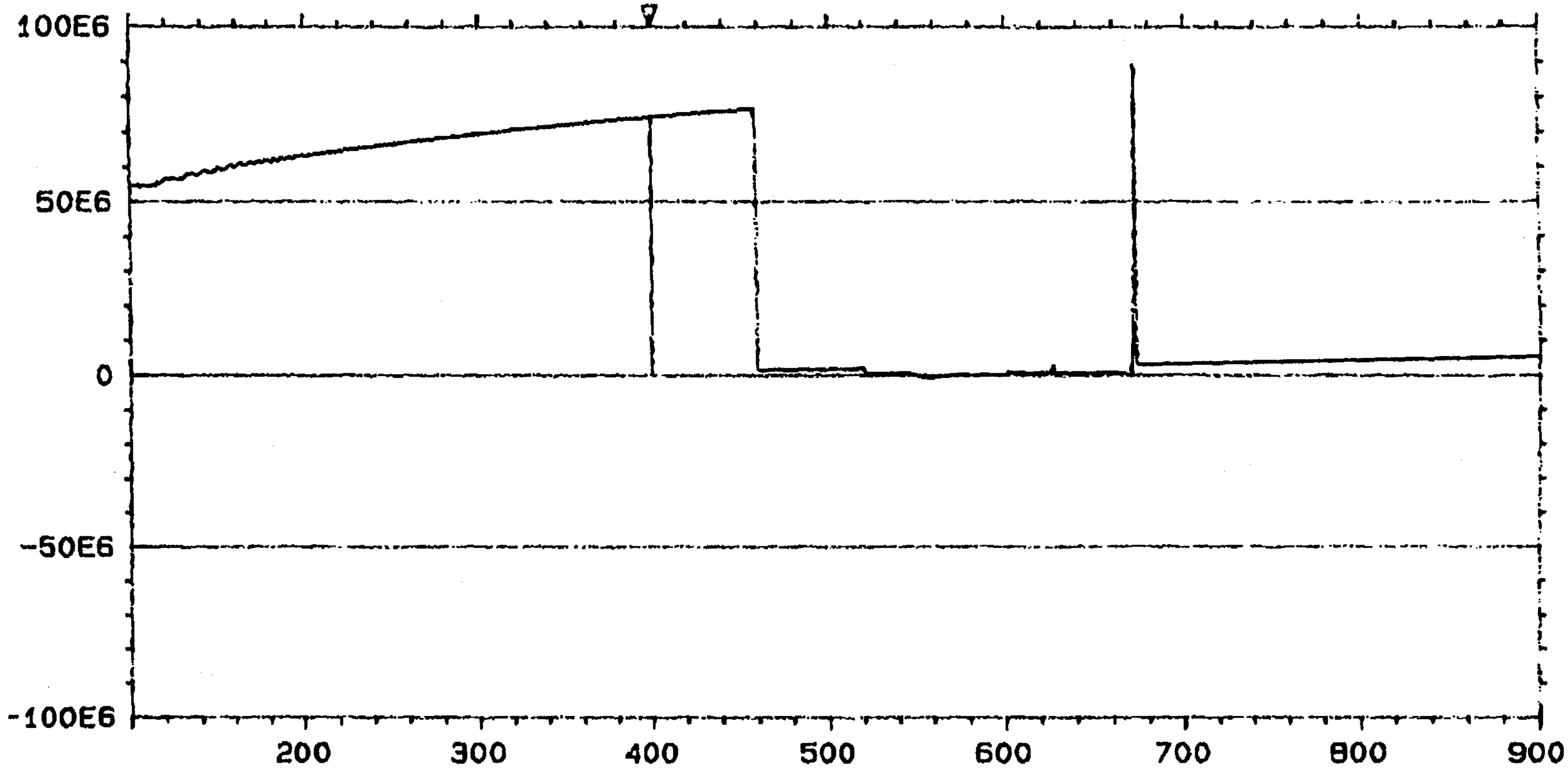


Fig. 4. Loss factor for sample 3 at 40E-6 strain amplitude

It may be mentioned that in the Oberst test method, strain amplitude is not constant and it varies along the length due to cantilever geometry whereas for the tripod test strain is constant throughout the sample which is under compression. In order to compare results from both of these methods, average values of elastic modulus and loss factor (without considering effect of pre-strain and strain amplitude) are presented in Table 6.

Wm >CMPLX_MDLS REAL MEAS MAIN Y: 74.5M
Y: 100M [ ] X: 400Hz
X: 100Hz + 800Hz LIN
#AVM: 10 YMAX : 89.5M



Setup W
Measurement: DUAL-CH. SPECTRUM AVERAGING 800 lines
Trigger: GENERATOR

Averaging: LIN 10

FREQ. SPAN: 800Hz df: 1Hz T: 1s dt: 977us
ZOOM: Centre Freq: 500Hz

Weighting: RECTANGULAR
ch.A: 200mV DIRECT /-0.7Hz -\ON 22.5mV/unit
ch.B: 1V - DIRECT /-0.7Hz -\ON SS 10.2mV/m/s2
Generator: MULTISINE 0.0dB ref 1.00V WHITE DC: 0uV
Repetition: CONT

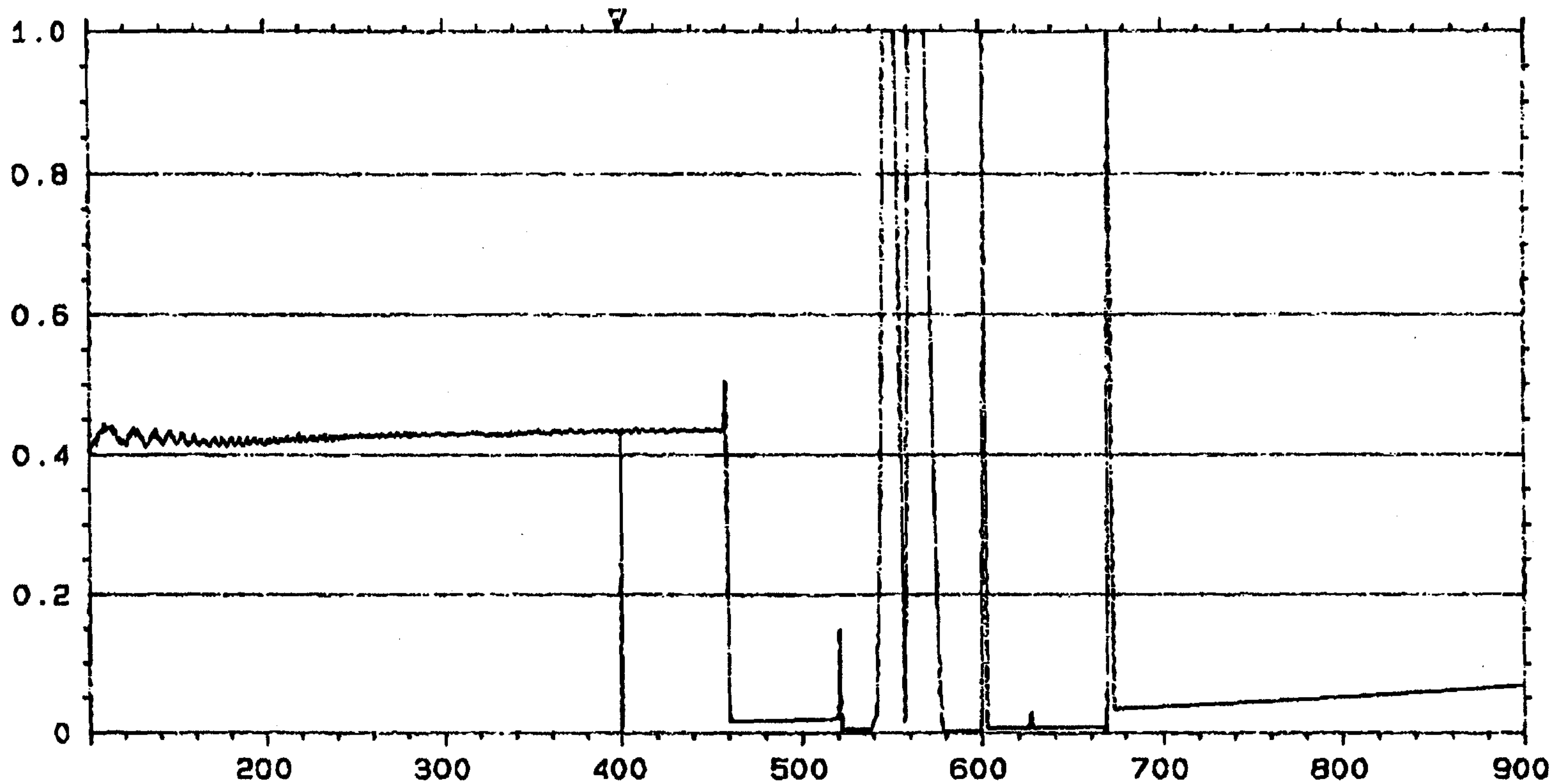
Fig.3

Wm >LOSS_FACTOR
Y: 1.00U RMS LIN
X: 100Hz + 800Hz LIN
#AVM: 10

[ ]

MAIN Y: 435mU
X: 400Hz

YMAX : 43.0U



Setup W
Measurement: DUAL-CH. SPECTRUM AVERAGING 800 lines
Trigger: GENERATOR

Averaging: LIN 10

FREQ. SPAN: 800Hz df: 1Hz T: 1s dt: 977us
ZOOM: Centre Freq: 500Hz

Weighting: RECTANGULAR
ch.A: 200mV DIRECT /-0.7Hz -\ON 22.5mV/unit
ch.B: 1V - DIRECT /-0.7Hz -\ON SS 10.2mV/m/s2
Generator: MULTISINE 0.0dB ref 1.00V WHITE DC: 0uV
Repetition: CONT

Fig. 4

Sample #	Pre-strain $\times 10^{-6}$	Strain Amplitude $\times 10^{-6}$	Elastic Modulus E MPa				Loss Factor η			
			100 Hz	200 Hz	300 Hz	400 Hz	100 Hz	200 Hz	300 Hz	400 Hz
1	60	10	36.9	43.1	45.8	48.0	0.447	0.476	0.519	0.531
	80	40	39.3	46.5	49.2	51.6	0.462	0.474	0.502	0.519
	100	60	39.3	46.5	48.8	51.3	0.449	0.473	0.499	0.517
2	80	10	37.1	44.8	49.2	52.3	0.436	0.457	0.469	0.478
	100	30	43.7	48.6	55.7	58.7	0.439	0.462	0.454	0.466
	100	60	39.5	52.1	55.5	58.1	0.431	0.425	0.432	0.442
3	80	10	52.0	64.1	69.4	72.7	0.409	0.416	0.423	0.434
	100	20	52.6	62.1	68.2	74.0	0.442	0.418	0.421	0.434
	120	40	53.2	62.3	68.7	74.5	0.402	0.419	0.426	0.435
4	60	10	42.3	50.6	55.9	60.7	0.414	0.414	0.409	0.406
	100	20	45.2	55.1	60.2	65.1	0.427	0.440	0.402	0.398
	110	30	43.1	55.0	60.3	65.6	0.499	0.409	0.387	0.382

Table 5: Dynamic modulus from Tripod test

Frequency Hz	Elastic Modulus E (MPa)	Loss Factor
100	43.7	0.438
200	52.6	0.440
300	57.2	0.445
400	61.1	0.454

Table 6. Average value of dynamic modulus from Tripod test

Now a structure made of two aluminum disks are joined by viscoelastic material (as shown in Figure 5) and is tested for natural frequencies and mode shapes. The inner disk is a solid disk of radius 119.38 mm and outer disk is an annular disk of inside radius of 122.17 mm and the outside radius of 152.4 mm. The space in between the disks is filled with the viscoelastic material under consideration. The disk is suspended by an elastic cord and is impacted by a hammer in horizontal direction. The natural frequency for 2 nodal diameter 0 nodal circle mode was observed at 1110 Hz. Next the finite element solution was obtained with modulus of aluminum as 73 GPa, Poisson's ration of 0.33, and density of 2700 Kg/m^3 and. Poisson's ratio of viscoelastic material as 0.45, and density of 1037.1 Kg/m^3. The modulus E of viscoelastic material was varied as shown in Table 7 until the natural frequency of 2 nodal diameter nodal circle mode correspond to the experiment value of 1110 Hz.

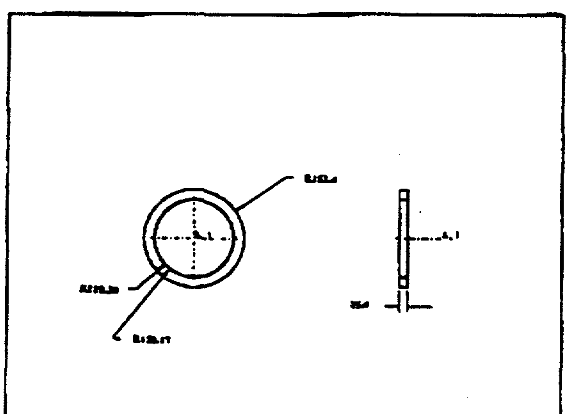


Fig. 5. Aluminum disks bonded by viscoelastic material

Elastic Modulus E (MPa)	Frequency and 2 nodal diameter mode (Hz)
10	919.1
20	995.1
40	1092.4
45	1109.4

Table 7. Elastic modulus of viscoelastic material used in FEM

CONCLUSION

In the Oberst method, the specimens are in cantilever configuration resulting in non-uniform strain. Thus any strain dependence phenomenon cannot be observed in the Oberst method. However, comparing average values, the methods do not agree with each other. Also the tripod method gives data continuously over a range as opposed to the Oberst method which gives data at only a few frequencies. Since a viscoelastic material has different characteristics over different frequence range (glassy, glass rubber transition, rubbery and flow region) any interpolation or extrapolation of data may be questionable. Presently a DMA test is being carried out to determine its characteristics such as transition temperature along with modulus as a function of temperature and frequency. The only natural frequency of the composite aluminum disk that could be excited fall outside the range of measured properties. A new sample is being prepared to have natural frequency within the range for which properties are known. Overall, the tripod method seems the preferable method though it involves a lot more instrumentation than the Oberst method and also only very small pre-strain or strain-amplitude could be applied.

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