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Mesh generation for modeling and simulation of carbon sequestration process

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Abstract: We introduce a new meshing tool for the computational modeling and simulation of carbon sequestration at specific sites. Fractures created by the insertion of high pressure gasses in carbon sequestration must be included in the geometric model. Our meshing tool meets the meshing requirements imposed by the fracture mechanics simulation codes, namely, to create a good quality Voronoi mesh whose edges have random orientation, conforms to domain boundaries and to internal fractures and voids. Our approach is based on a random cloud of *n* points whose locations are determined by solving a maximal Poisson-disk sampling problem over non-convex domains with holes, required points and multiple regions in contact. A novel constrained Delaunay algorithm is then utilized to generate Poisson-disk triangulations using O(*n*) time and memory. Finally the required Voronoi mesh is constructed by retrieving the dual of the triangular mesh. Each phase (sampling, triangulation, Voronoi meshing) of our algorithm utilizes local operations which facilitates parallel implementations. Examples of the use of our meshes within a fracture simulation are given. The meshing tool can also create hybrid meshes that conform to the geometric complexities that occurs naturally within a regional domain containing long, thin layers with twisting, faulting, and pinch-offs.

1. Poisson-Disk Sampling

Maximal Poisson-disk sampling (MPS) selects random points $\{x_i\} = X$, from a domain, D. There is an exclusion/inclusion radius r: empty disk means no two sample points are closer than r to one another; and maximal means samples are generated until every location is within r of a sample. D_i is the sub-region of D outside the r-disks of the first i samples. For an unbiased sampling procedure, the probability P of selecting a point from a disk-free sub-region **W** is proportional to **W**'s area.

$$\text{Bias-free: } \forall \Omega \subset \mathcal{D}_{i-1} : P(x_i \in \Omega) = \frac{\text{Area}(\Omega)}{\text{Area}(\mathcal{D}_{i-1})} \quad (1a)$$

$$\text{Empty disk: } \forall x_i, x_j \in X, i \neq j : ||x_i - x_j|| \geq r \quad (1b)$$

$$\text{Maximal: } \forall p \in \mathcal{D}, \exists x_i \in X : ||p - x_i|| < r \quad (1c)$$

A maximal r-disk sample (1b) (1c) is equivalent to a maximal sample of non-overlapping $r/2$-disks. These are also known in the literature as random close packings. Sphere packings appear frequently in nature: e.g. sand, atoms in a liquid, trees in a forest. Processes generating packings include random sequential adsorption, hard-core Gibbs process, and the Matern second process. Algorithmically, by successively generating points and rejecting those violating (1b) it is easy to get a near-maximal sample if run-time is unimportant. In recent years the community has developed unbiased MPS algorithms with near linear performance. There are variations based on advancing fronts that have biased point locations, violating (1a), but may be faster and use less memory.

In 2011, We proposed two methods to solve this problem. The first one [1] has a time complexity of O($n \log n$) and satisfies the sampling conditions and achieves maximality independent of the roundoff error by constructing uncovered areas with geometric primitives. The second method [2] works in any d-dimensional space and has a time complexity of O(n). The performance is improved through the use of a finite sequence of uniform grids with increasing resolutions instead of representing the remaining voids via geometrical primitives. The output of our algorithm is illustrated in Figure 1. A comparison with other sampling methods in Figure 2 shows the efficiency of our approach.

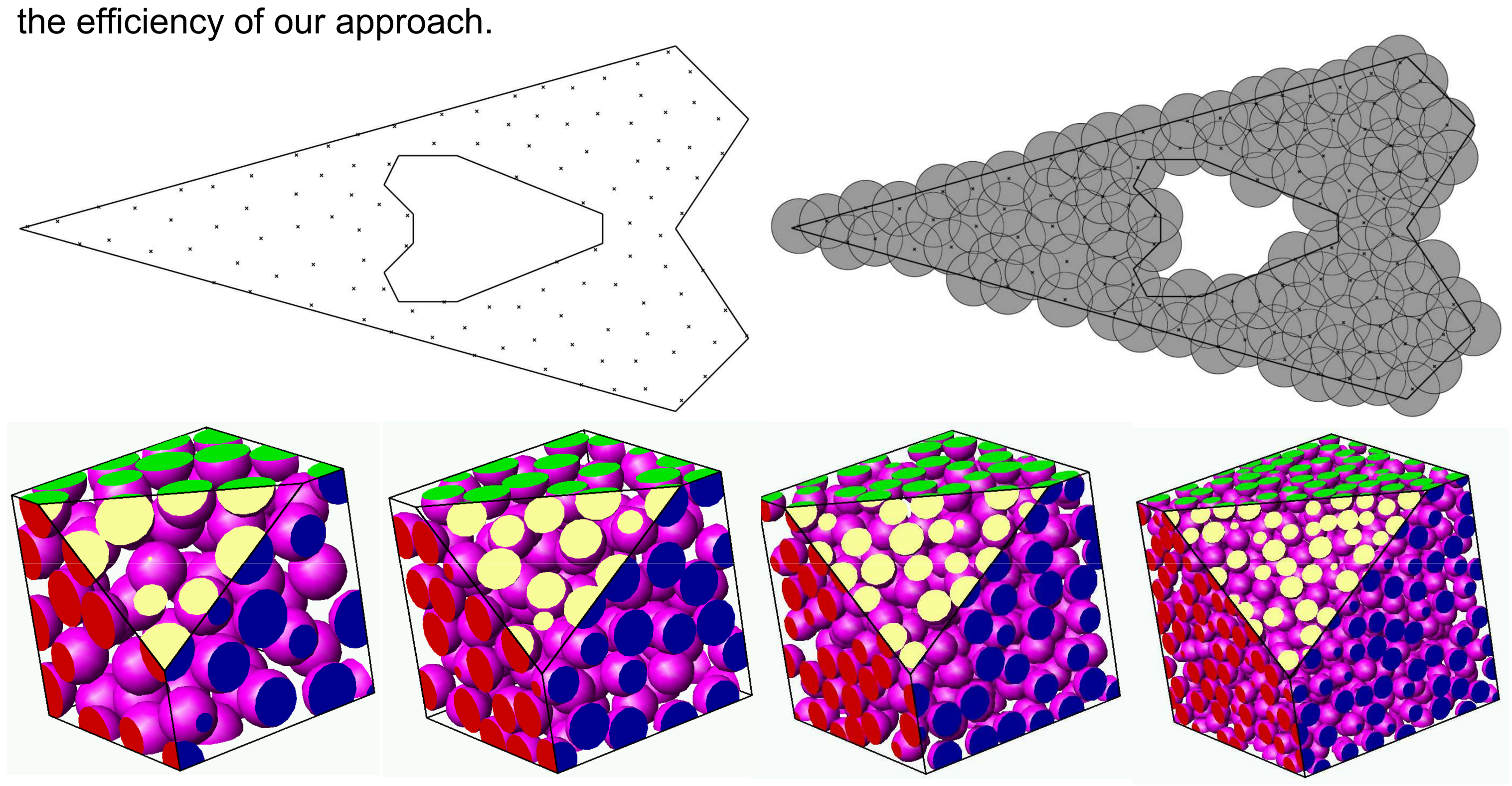
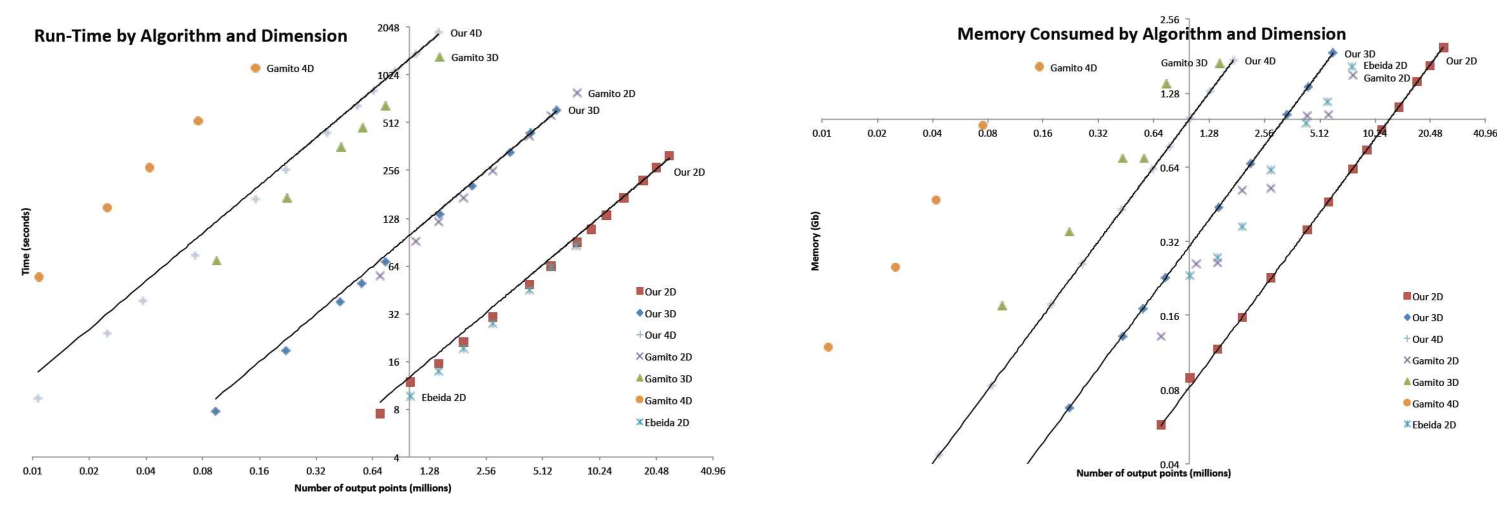


Fig. 2. Poisson-disk sampling of a non-convex domain (top) and unit cube (bottom). For the 3D case we show non-intersecting sphere with radius $r/2$.



(a) Time **(b) Memory**

Fig. 2. Memory and time used by our sequential MPS implementation vs. other sampling codes.

2 Delaunay / Voronoi meshing

The cell structure utilized in our sampling algorithm enables a local, simple, and fast algorithm for constructing the constrained Delaunay triangulation, CDT [3]. This algorithm iterates in constant time over each point p of the maximal Poisson distribution, constructing its star, i.e. the triangles containing it. This results in linear total time. Communication between different points is not required except when a non-unique solution exists. The performance of the sequential implementation illustrate its efficiency in Figure 3.

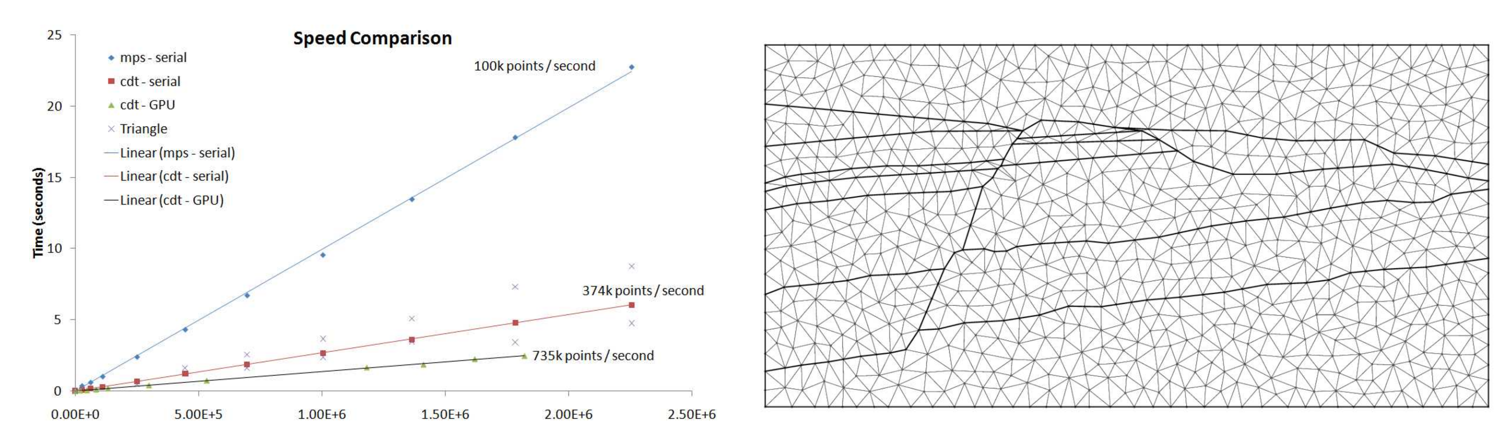


Fig. 3. Our serial CDT implementation shows a linear performance (left). Uniform random CDTs of a seismic domain with internal boundaries (right).

Finally, the required random Voronoi mesh is generated by retrieving the dual of the CDT mesh. This operation has to respect the internal and the external boundaries of the domain. Non-convex Voronoi cells along the boundaries are split into a set of triangles. Moreover, edge collapse operations take place to eliminate all short edges. The capability of our Voronoi meshing tool to handle various domains is illustrated in Figure 4.

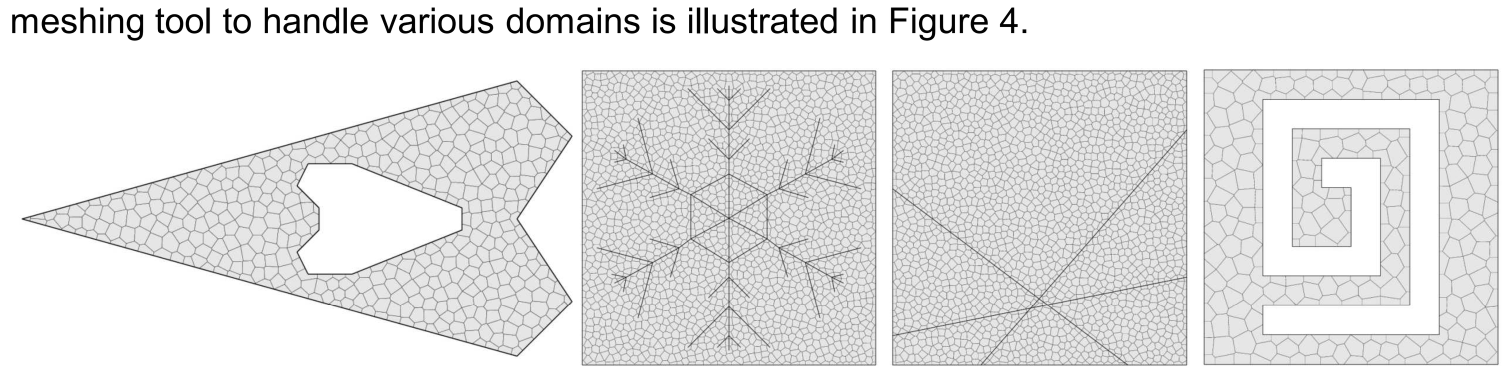


Fig. 4. Our Voronoi mesher is capable of handling non-convex domains with internal boundaries.

3 Hybrid meshing

The hybrid mesher sets up the problem for the Delaunay and Voronoi sub-regions and then calls the algorithms described in the previous subsection. The hybrid capability also contains a simple algebraic method for generating structured quadrilateral meshes on subregions. The hybrid mesher can be viewed as "glue" between different meshing algorithms; alternative meshing algorithms could also be included.

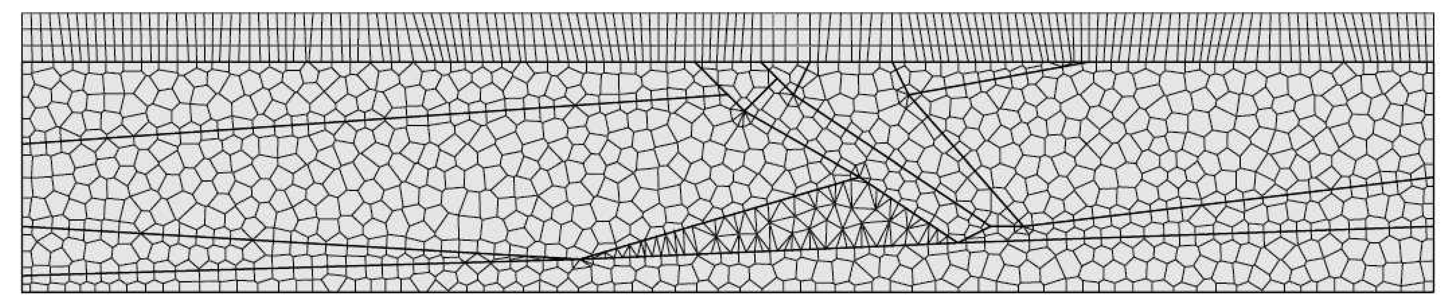


Fig. 5. A hybrid mesh formed of Voronoi cells, structured quadrilaterals and Delaunay triangles

4. Application Example

In this section we present an application example that utilized our meshing tool to create a fracture conforming Voronoi mesh to simulate the injection of CO_2 below a sub-scale model of a caprock layer (1000 m. below the surface). As illustrated in Figure 6, the initial fractures represent joints that are sealed, but are reactivated due to the changing mechanical stress and deformation caused by the injection in the reservoir below the caprock. The nucleation/growth criterion is based on a limit surface of the allowable stress states. A cohesive law is used on these new surfaces and decays as the crack opens. The mesh randomness should be viewed as a subset of the material variability (random field in strength).

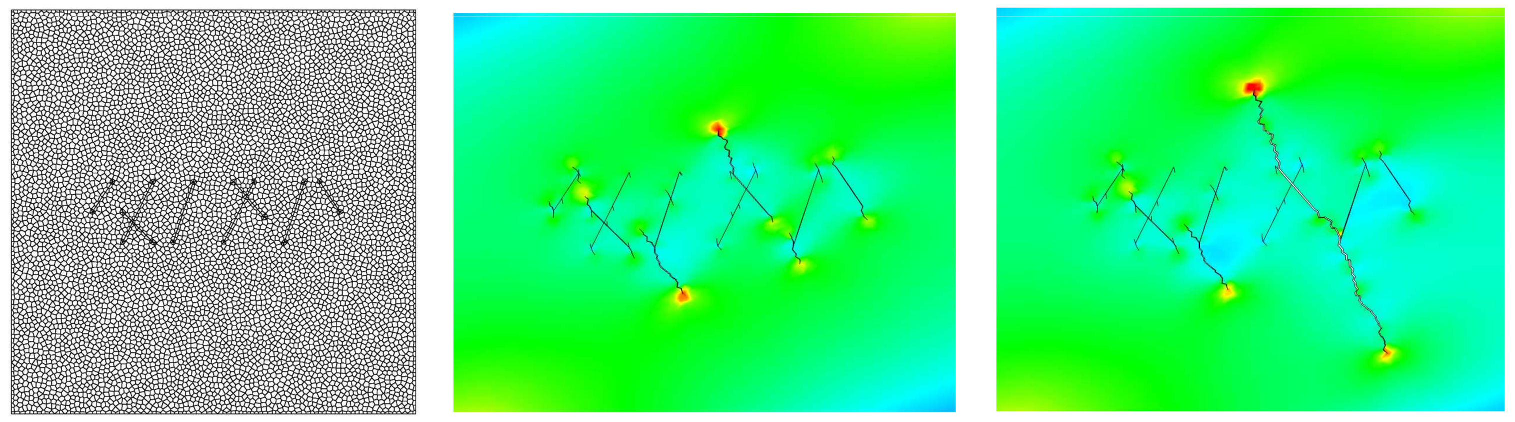
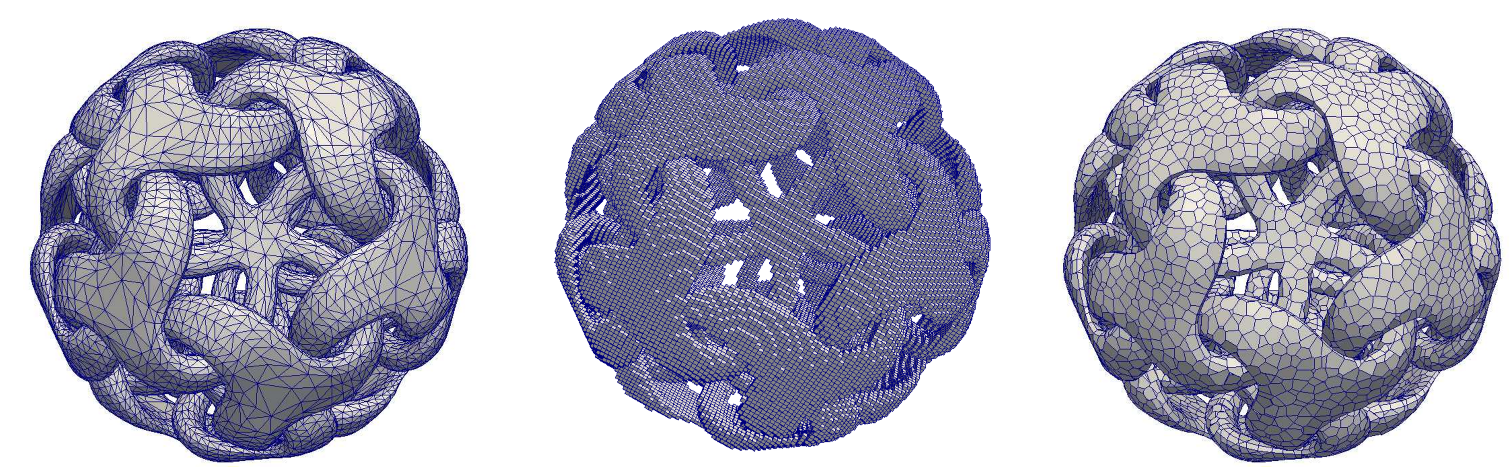


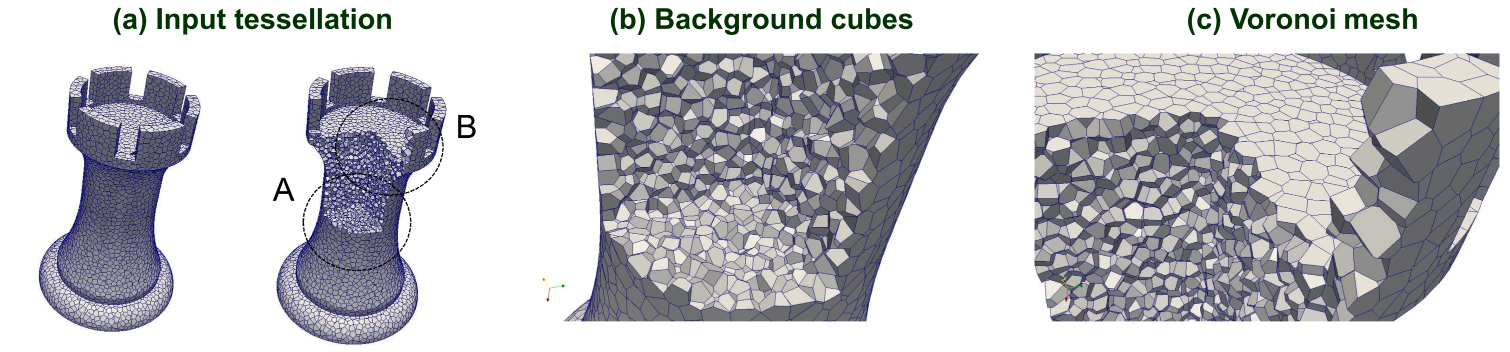
Fig. 6. Injecting CO_2 below a sub-scale model of a caprock layer. The color represents maximum principal stress. Frames are snapshots of the solution as it progresses in time.

5. Recent Developments

We have recently extended our Voronoi mesh tool to handle 3-dimensional domains. The new extension generates Voronoi cells directly without generating CDTs. The execution time is linear in *n*. The final mesh is associated with bounds on some quality measure such the dihedral angles and the aspect ratio. Figure 7 illustrates the output of our algorithm in 3D.



(a) Input tessellation **(b) Background cubes** **(c) Voronoi mesh**



(a) Mesh **(b) Mesh clipped** **(c) zoom-in A** **(d) zoom-in B**

Fig. 7. Voronoi mesh for a model with smooth surface and narrow regions (top) and a non-convex model with many sharp features (bottom)

6. Related publications

1. Mohamed S. Ebeida, Anjul Patney, Scott A. Mitchell, Andrew A. Davidson, Patrick M. Knupp and John D. Owens, "Efficient Maximal Poisson-Disk Sampling", SIGGRAPH2011, August 7-11, Vancouver, Canada, SAND2011-0260C.
2. Mohamed S. Ebeida, Scott A. Mitchell, Anjul Patney, Andrew A. Davidson, and John D. Owens, "Maximal Poisson-disk sampling with finite precision and linear complexity in fixed dimensions", SIGGRAPH Asia 2011, December 12-15, 2011, Hong Kong, SAND2011- 3606C. (under review)
3. Mohamed S. Ebeida,Scott A. Mitchell, Andrew A. Davidson, Anjul Patney, Patrick M. Knupp and John D. Owens, "Efficient and Good Delaunay Meshes From Random Points", SIAM conference on Geometric and Physical modeling, October 24-27, 2011, Orlando, Florida, SAND2011- 0519C.
4. 4. Mohamed S. Ebeida, Patrick M. Knupp, Vitus J. Leung, Joseph E. Bishop, and Mario J. Martinez, "Mesh generation for modeling and simulation of carbon sequestration process", SciDAC2011, July 10-14, 2011, Denver, CO, SAND2011-3771A.
5. Mohamed S. Ebeida, Patrick M. Knupp, Scott A. Mitchell, and Vitus J. Leung, Uniform Random Voronoi Meshes for Ensembles of Lagrangian Fracture Simulations, 20th International meshing roundtable, October 23-26, Paris, France 2011. (under review)

SciDAC 2011, July 10-14, Denver, Colorado.

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