LA-UR- 11-05653

Approved for public release;
distribution is unlimited.

Title: LDRD-DR: "Turbulence By Design"

Author(s): Malcolm J. Andrews

Intended for: JWOG32M
Lawrence Livermore National Laboratory
October 10, 2011

Los Alamos
NATIONAL LABORATORY
EST. 1943

LDRD-DR: "Turbulence By Design"

Malcolm J. Andrews

Abstract

This presentation reviews the three years of research performed under the LDRD-Directed Research titled "Turbulence By Design". The presentation will focus on initial condition modeling for Rayleigh-Taylor mixing.

"Turbulence By Design": LDRD-DR # 20090058

- **Title:** "Turbulence By Design"
- **Start Date:** October 1, 2008
- **Finish Date:** September 30, 2011
- **PI:** Malcolm Andrews (XCP-4), Z-138959, 6-1430
- **Co-PI's:** Daniel Livescu (CCS-2), Z-179815 (DNS)
  Kathy Prestridge (P-23), Z-149289 (RM Experiments)
  Fernando Grinstein (XCP-4), Z-660029 (ILES)
  Raymond Ristorcelli (XCP-4), Z-143749 (Theory)
- **Collaborating University:** Texas A&M (the RT Water Tunnel)
- **Post-doc's:** B. Rollin, T. Wei, S. Balasubramanian, A. Gowardhan
- **GRA's:** A.J. Wachtor, N. Hjelm, S. Reckinger

LA-UR 11-04897

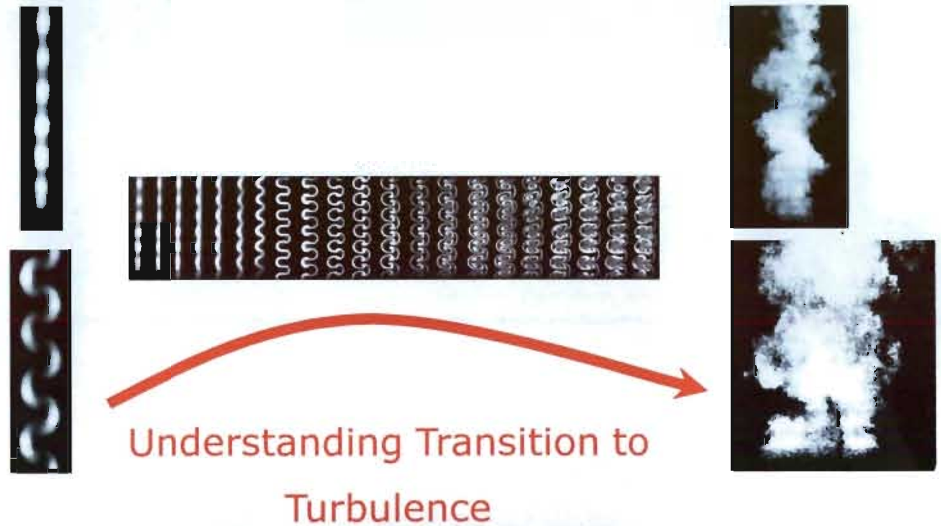
Some Dramatic Effects of Initial Conditions

M.J. Andrews, TAMU water channel experiment

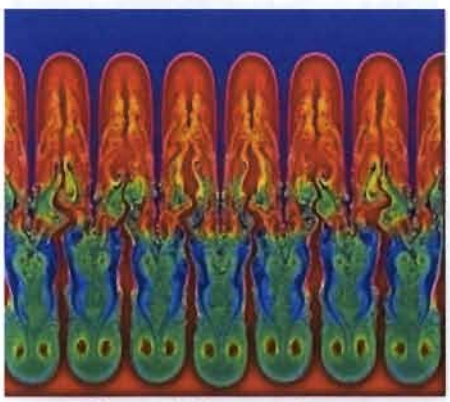
Long wavelength initial conditions | Short wavelength initial conditions

Richtmyer-Meshkov (RM) Transitions From Different Initial Conditions

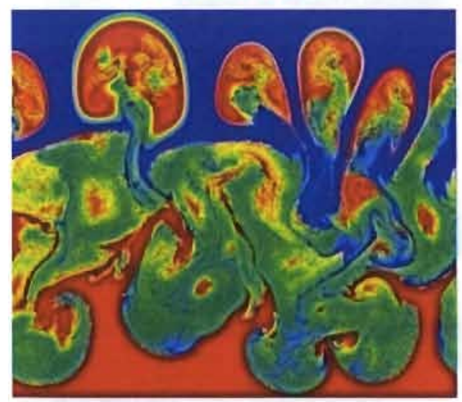
(from the LANL Gas Shock Tube – K. Prestridge)



Credit: Hjelm & Ristorcelli (LBM simulations)



No IC noise



With IC noise

Turbulence "Control" via Initial Conditions

Hypothesis:

Initially seeded small amplitude, long wavelength, perturbations can develop at late-time and be used to control turbulent transport and mixing effectiveness.

Motivation:

Provide a rational basis for setting up initial conditions in turbulence models for Richtmyer-Meshkov and Rayleigh-Taylor driven mixing

Overall Objective:

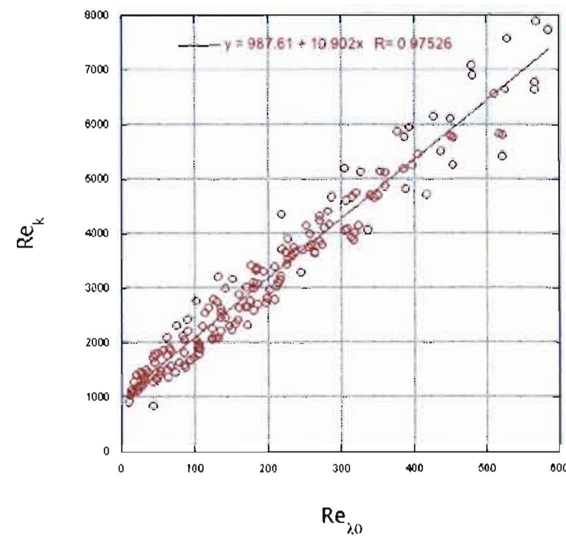
Predict profiles of relevant variables before the fully turbulent regime and use them as initial conditions for a (BHR) turbulence model.

LDRD
innovation for our nation

Initial Conditions for Moments and Mix Growth Collapse with a Taylor Reynolds Number Scaling (Ristorcelli & Hjelm)

Single mode interface: $x_s(x_2,x_3,t)=a_0 e^{i(\kappa_2 x_2+\theta_2)} e^{i(\kappa_3 x_3+\theta_3)} e^{\sigma t}$

$$\mathrm{Re}_k = \frac{\sqrt{k}\,h}{\nu} \ \textit{at centerline}$$



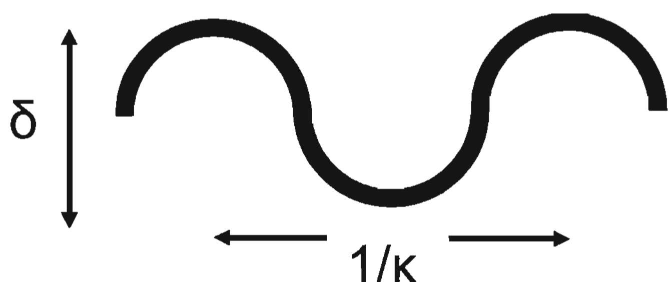
Mean interfacial thickness: $\delta^2 = \langle x_s x_s \rangle = a_0^2 e^{2\sigma t}$

Mean zero crossing wavelength: $\lambda_0^2 = \dfrac{\langle x_s x_s \rangle}{\langle x_{s,k}\, x_{s,k} \rangle} = \dfrac{1}{\kappa_2^2+\kappa_3^2} = \dfrac{1}{\kappa^2}$

Multi-mode: $\delta^2 = \langle x_s x_s \rangle = \sum_n a^2{}_n \qquad (\kappa\delta)^2 = \langle x_{s,j}\, x_{s,j} \rangle = \sum_n a^2{}_n \kappa^2{}_n$

Insert into moment definitions to get initial values, e.g.:

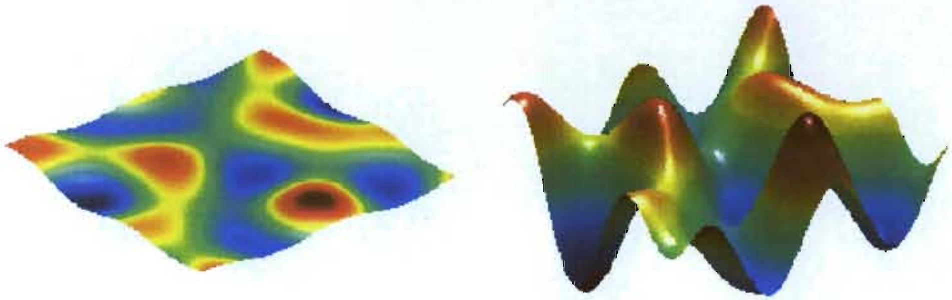
$$k_0 = \tfrac{1}{2} A g \delta (\kappa\delta)[1+\tfrac{2}{3}(\kappa\delta)^2 + ...]$$



$$\mathrm{Re}_{\lambda 0} = \sqrt{\frac{A g \delta^3}{\nu^2 \kappa\delta}}$$

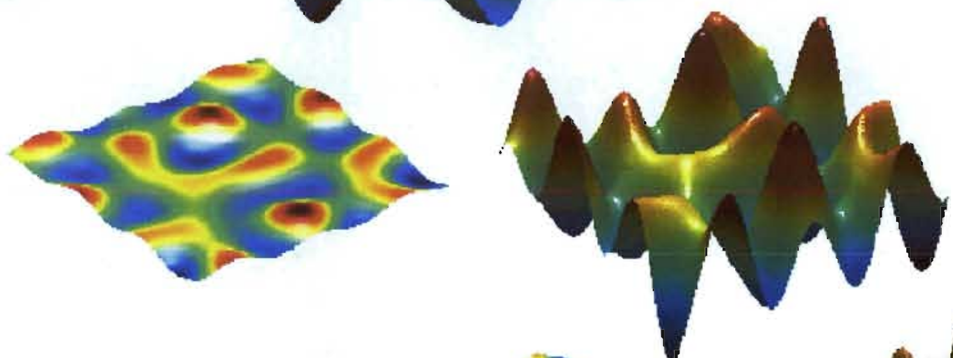
"κδ" metric basis

Initial (single) material interface parameterization



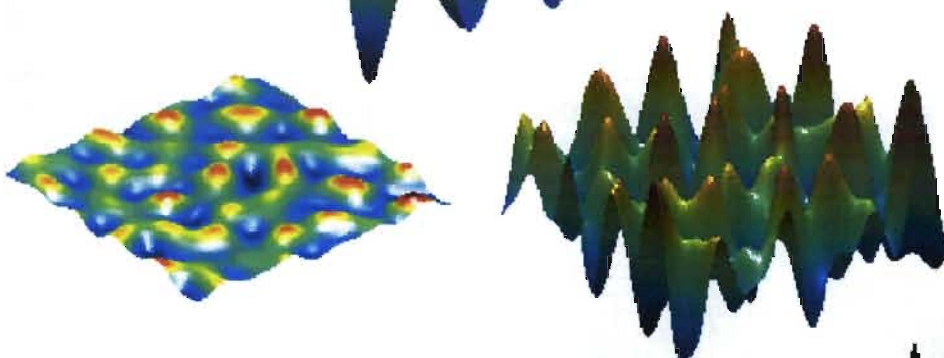
high η_0 class

η_o= **10π/12 , 10π/8, 10π/4 , 10π/2**



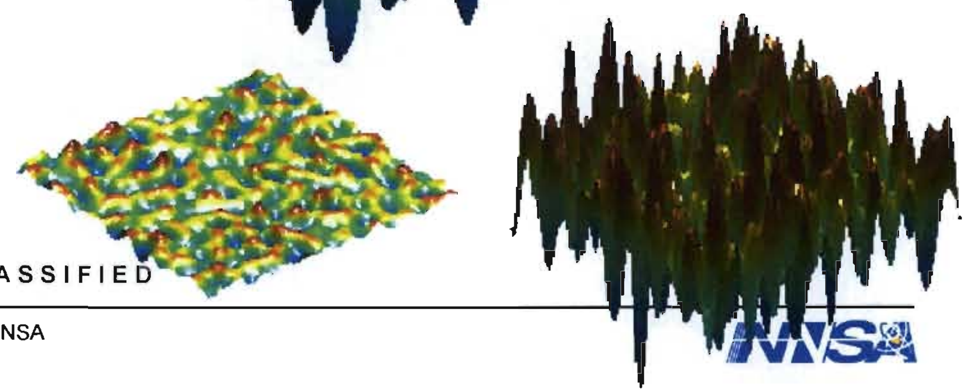
$$\eta_o = \kappa_o \delta_o$$

$$\sim \left\langle \nabla X_s \nabla X_s \right\rangle^{1/2}$$

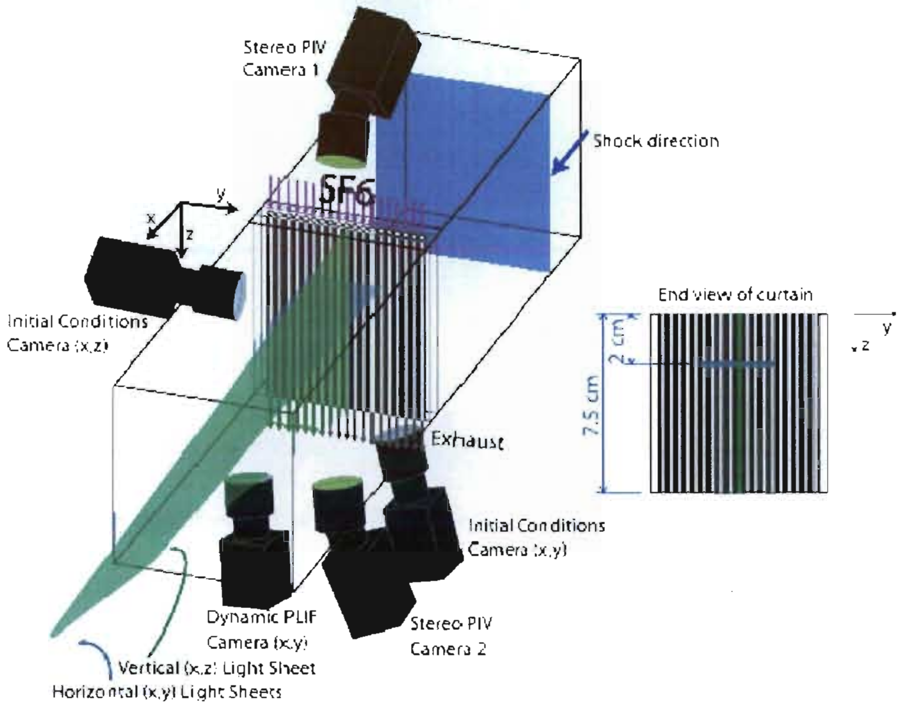


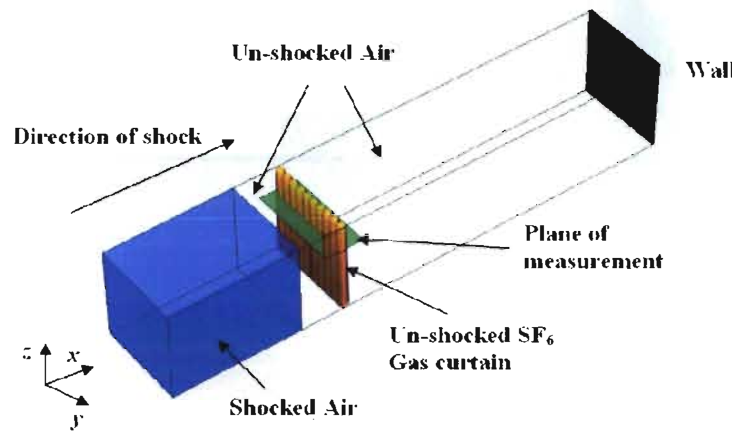
low η_0 class

η_o= **π/12 , π/8 , π/4 , π/2**



Richtmyer-Meshkov Experiments at P-23 (Prestridge et al.)





$$\eta = \kappa\delta \quad \text{where} \quad \kappa = \frac{\pi(z_c)}{y}$$

z_c is the no. of zero crossings

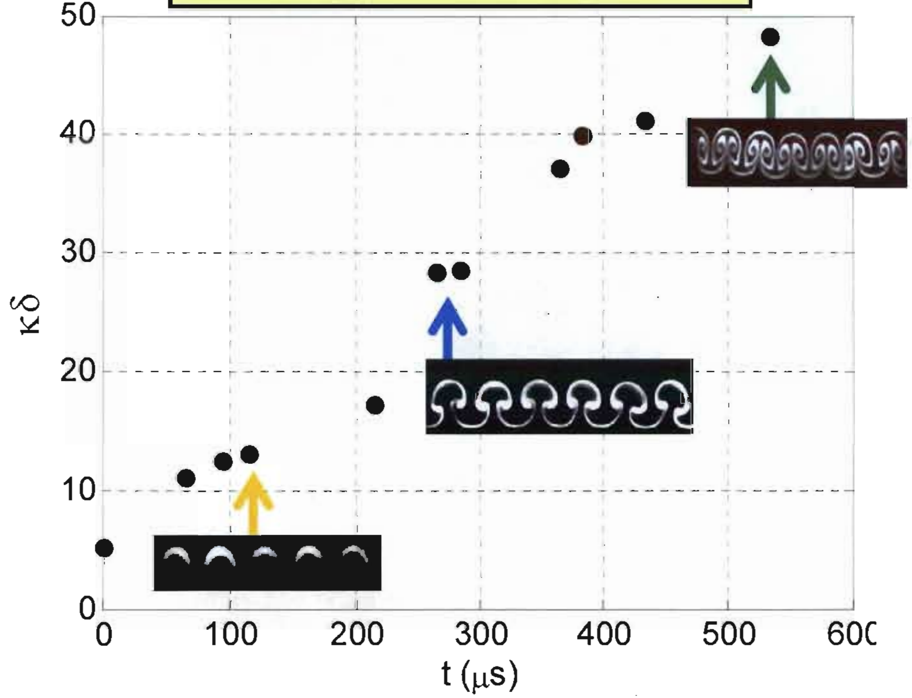
y is the spanwise length

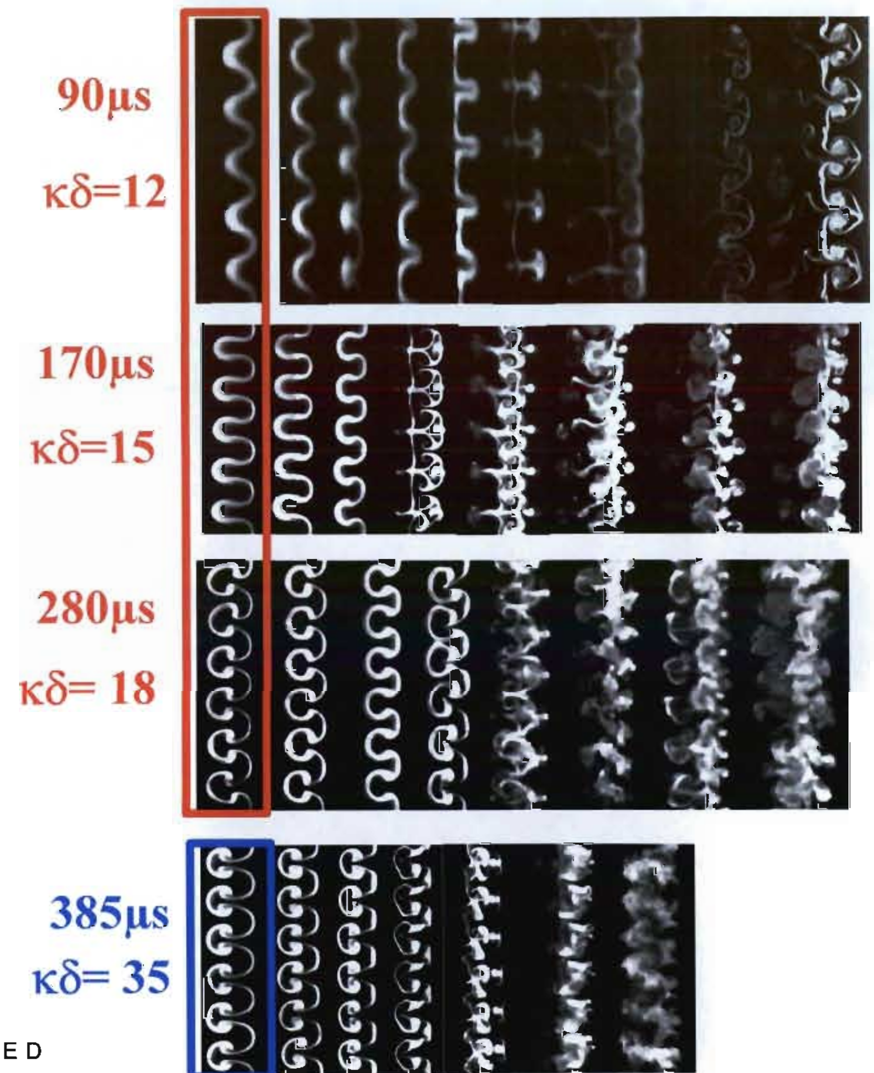
κδ indicates the complexity of the initial conditions

$$\eta = \kappa\delta \quad \text{where} \quad \kappa = \frac{\pi(z_c)}{y}$$

z_c is the no. of zero crossings

y is the spanwise length





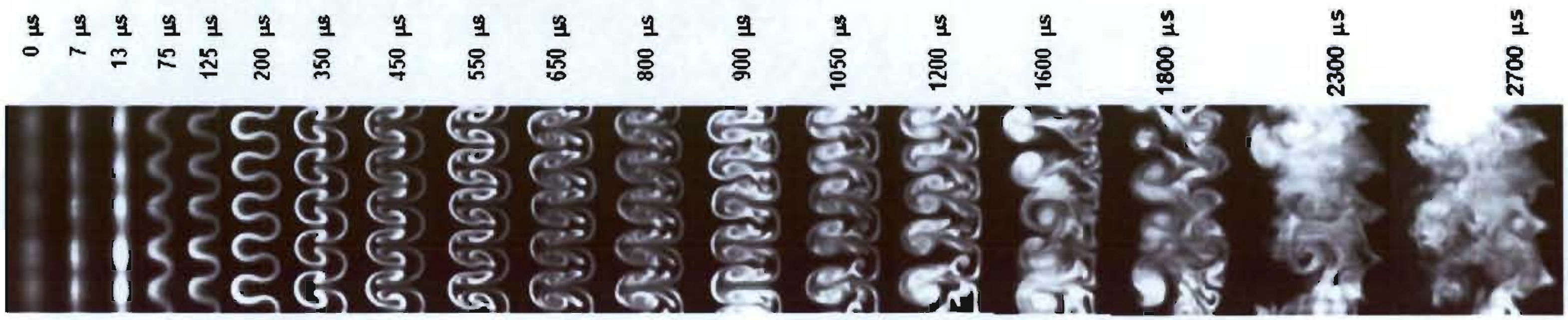
As modes present in initial conditions increase in complexity, the value of $\kappa\delta$ increases. Discontinuities at 300 and 600 μs, indicate possible changes in mixing behavior upon shock.

With similar starting amplitudes, different wavelengths in the initial conditions produce dramatic mixing differences

Incident Shock wave →

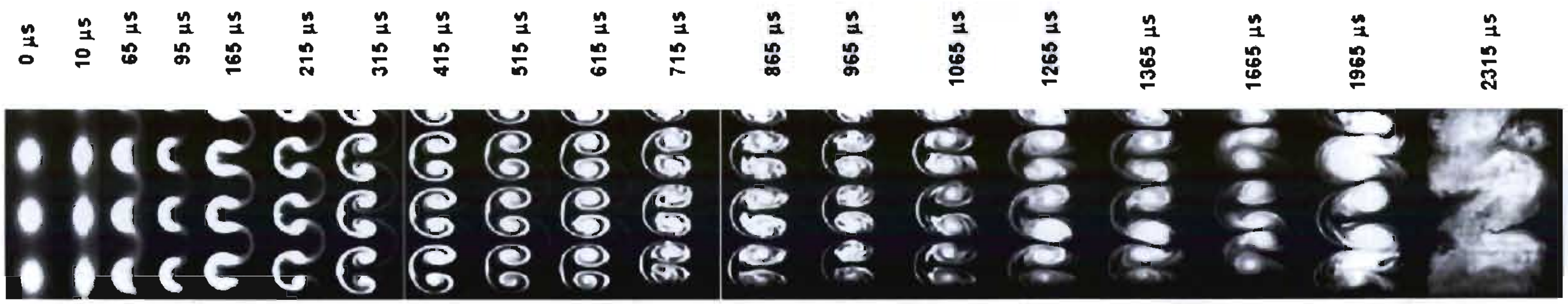
Single mode short wavelength

$\kappa\delta = 5.2$



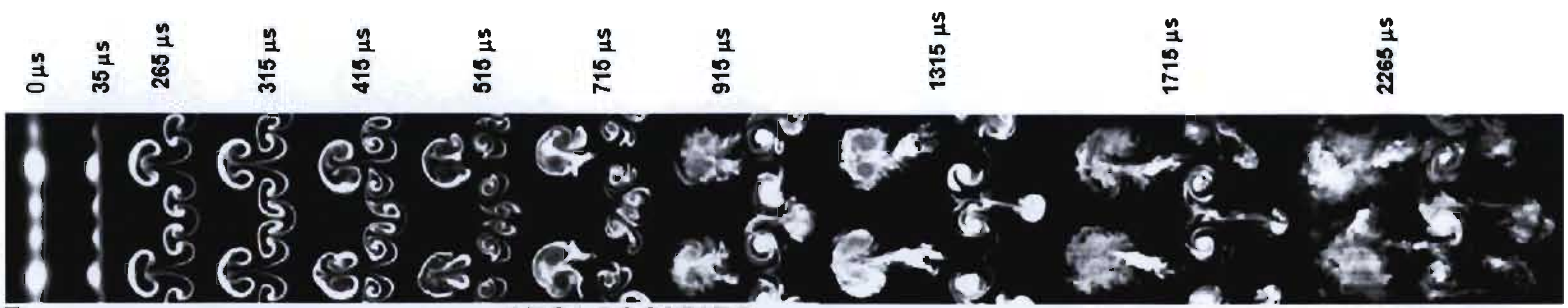
Single mode long wavelength

$\kappa\delta = 2.6$



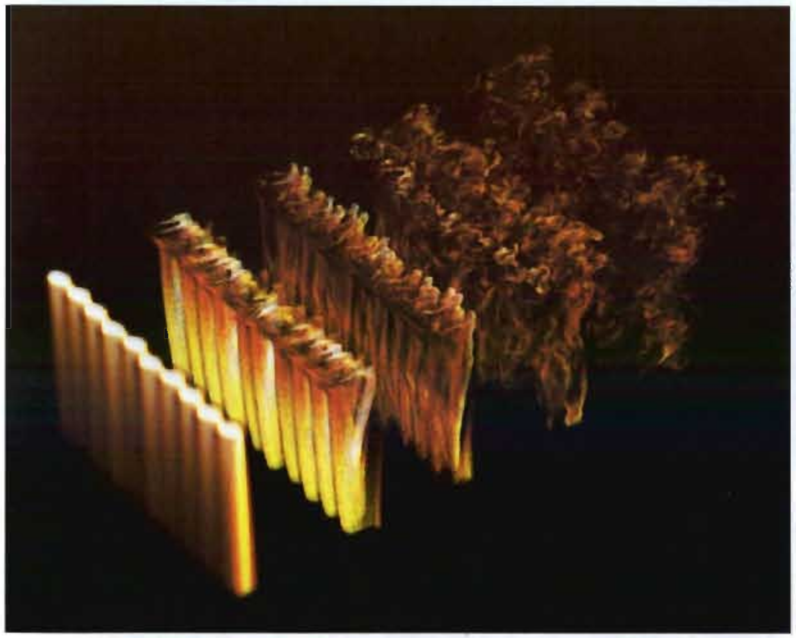
Multi mode

$\kappa\delta = 4.1$



Simulations of Initial Condition Effects on Shock-Driven Turbulent Mixing

Fernando F. Grinstein (XCP-4), Akshay A. Gowardhan (D-4: post-doc), Adam J. Wachtor (XCP-4: GRA student), J. Ray Ristorcelli (CCS-2)



The (single-interface) planar RM experiment

- Challenges to Moment Closures: *the bipolar RM behavior*
- Can reshock effects occur on first-shock ?

The (double-interface) Gas Curtain RM experiment

- Initial 3D GC characterization and modeling
- Sensitivity of turbulence characteristics to ICs
- Data reduction and bipolar RM behavior

Unclassified

ILES RAGE of planar Richtmyer-Meshkov:

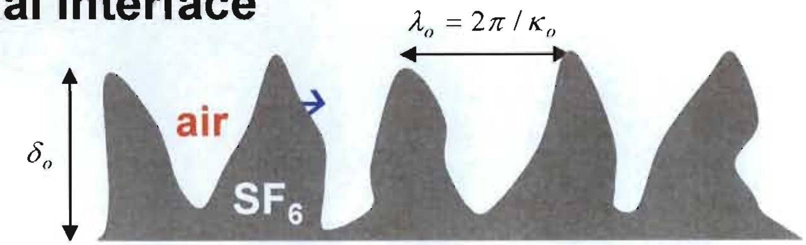
Ma=1.5 shocked air/SF_6 – **no** egg-crate in ICs

Gowardhan, Ristorcelli and Grinstein; *PoF* Letters, 2011

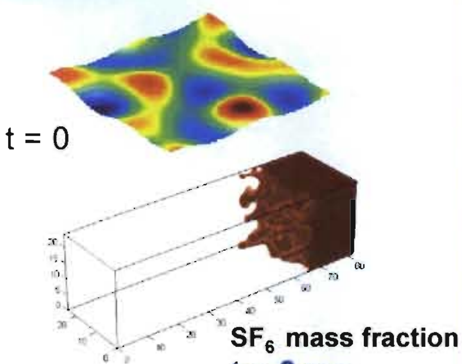
Impact of *rms* slope $\eta_o = \kappa_o \delta_o$ of Initial Material Interface

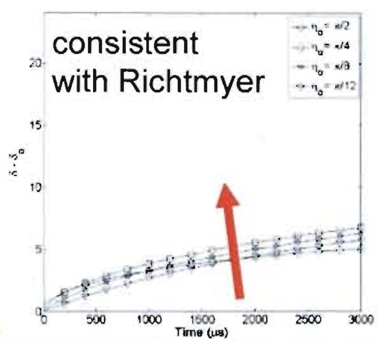
***Beyond Richtmyer** (growth = constant × η_o):*

- → ***bipolar RM behavior vs. IC morphology***
- → ***different instability mechanisms & late-time flow***

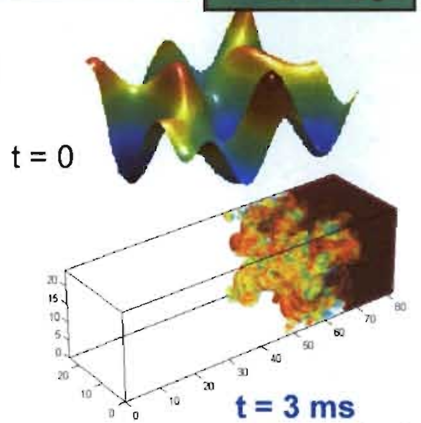


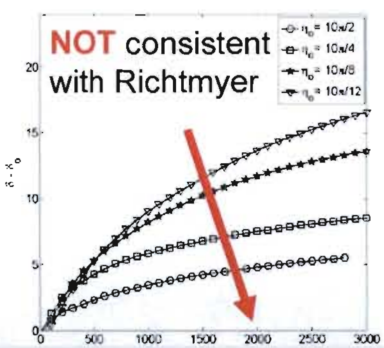
low η_0 | ***Dimensional*** **results** | **high η_0**





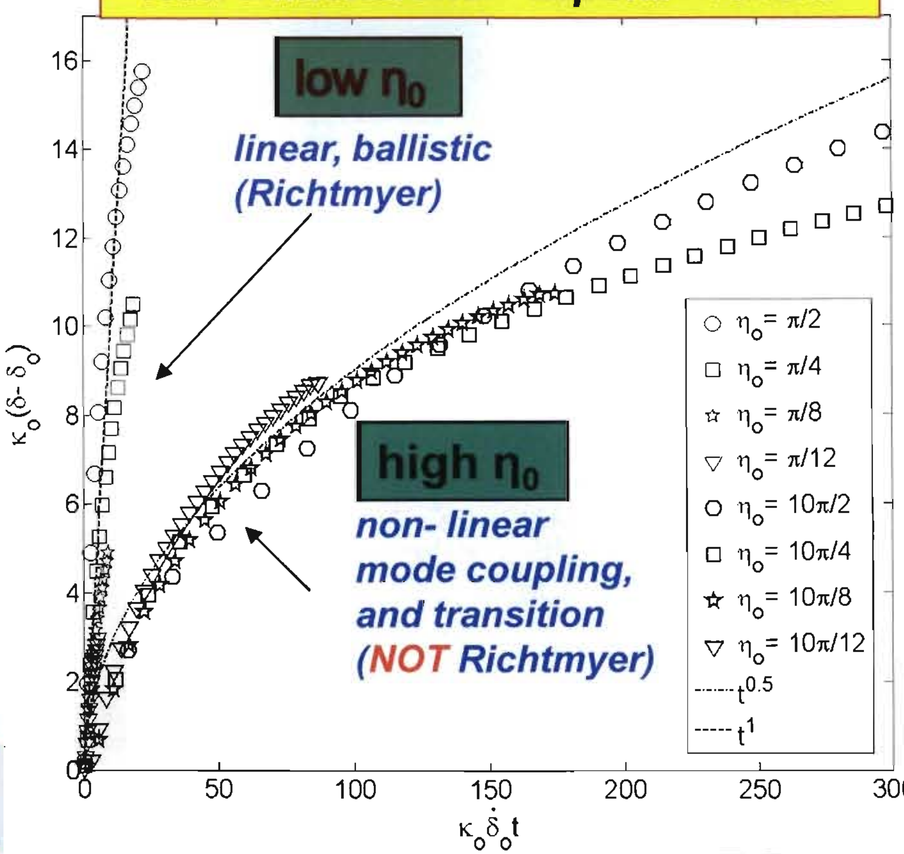
Mixing width **increases** with increasing η_o





Mixing width **decreases** with increasing η_o

Non-Dimensional “Bipolar” **results**



Planar (single interface) RM

Gowardhan, Ristorcelli and Grinstein; in preparation for *PoF* Letters, 2011

- **Initial *rms* slope η_o of the material interface controls RM evolution → bipolar RM behavior:**

low η_0 : *linear, ballistic*

high η_0 : *non-linear, mode coupling*

switch for η_0 ~ 1

→ *transition to turbulence suggested*

→ more material mixing & smaller scales

- **Reshock effects on mixing and transition can be achieved with single shock, if η_o > 1;**

- **The modeler's (initial condition) challenge**
  - **two different instabilities & growth trends**

Unclassified

LDRD

ILES of Shocked SF_6 Gas Curtain
RAGE V&V ; IC characterization / modeling issues

RM instability very sensitive to initial conditions (ICs)

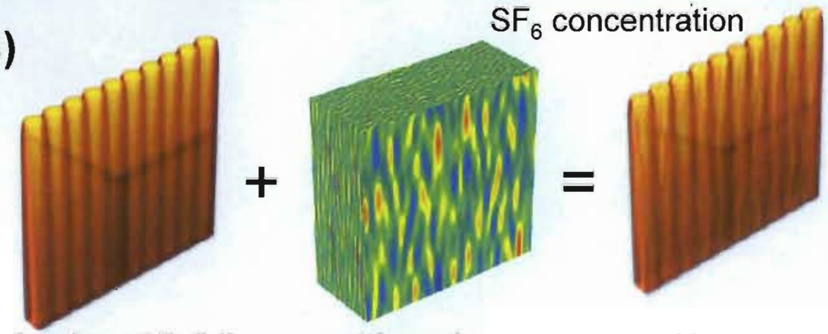
→ insufficiently characterized ICs in lab. experiments

(e.g., SF_6 mixture composition & fluctuations, …)

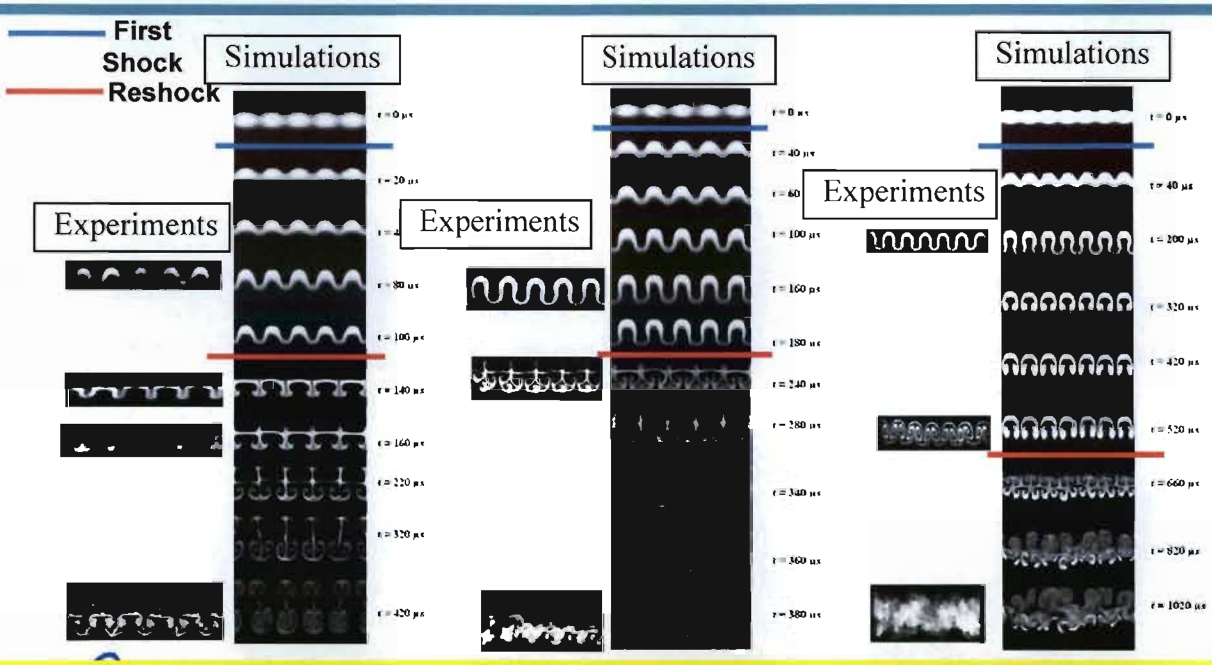
Generate initial 3D Gas Curtain (ICs for RAGE)

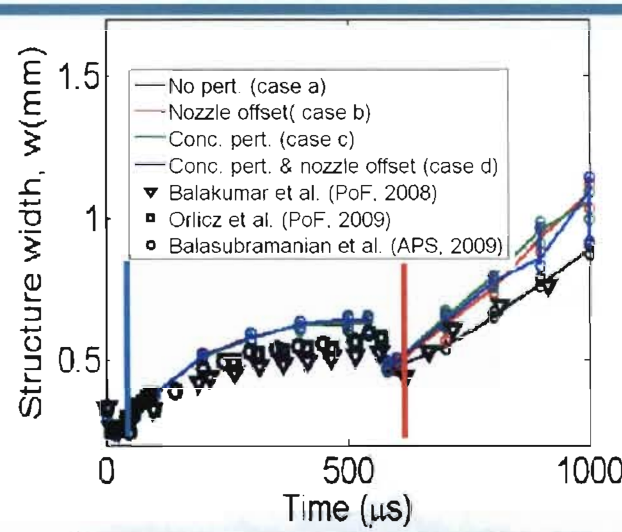
→ use separate 3D (NS–Boussinesq) GC code

→ superimpose multi-mode fluctuations in ICs



laminar 3D GC simulation | **multi-mode fluctuations** | **Initial 3D gas curtain (for RAGE)**





Post-reshock growth suggests issues for LES

very good agreement with lab. flow patterns and growth rates before reshock (insensitive to ICs); late-time results fairly sensitive to ICs **after reshock !**

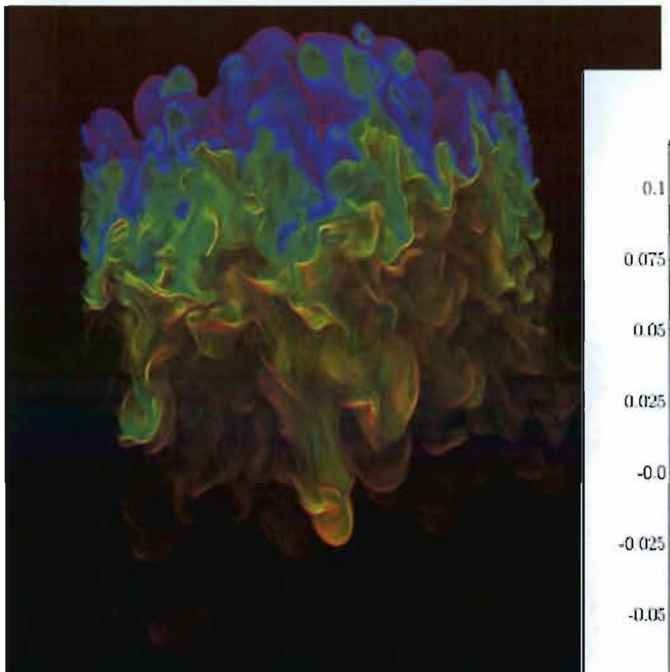
over-predicted early growth reflects lower effective Atwood number in expts. (uncharacterized acetone used with *PLIF* – mixed with SF_6)

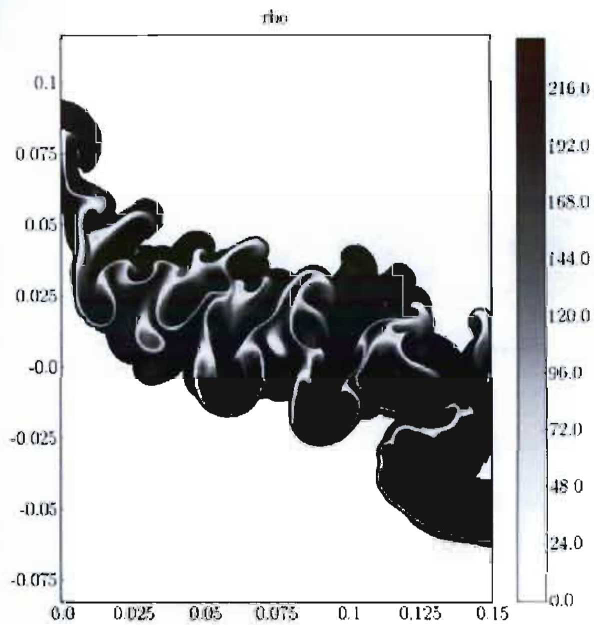
[redacted]

NNSA

DNS for Initial conditions dependence in Rayleigh-Taylor

Daniel Livescu and Tie Wei (CCS-2)





- Code used: CFDNS (Livescu et al LA-CC-09-100).
- 2-D simulations (up to $16{,}384^2$) performed at LANL and on Jaguar, ORNL.
- 3-D simulations (up to $4096^2 \times 4032$ RT) performed on Dawn, LLNL; Jaguar, ORNL; and LANL.

Two-Mode "Leaning" RT Experiments Using the New Computer Controlled Flapper (TAMU + LANL)

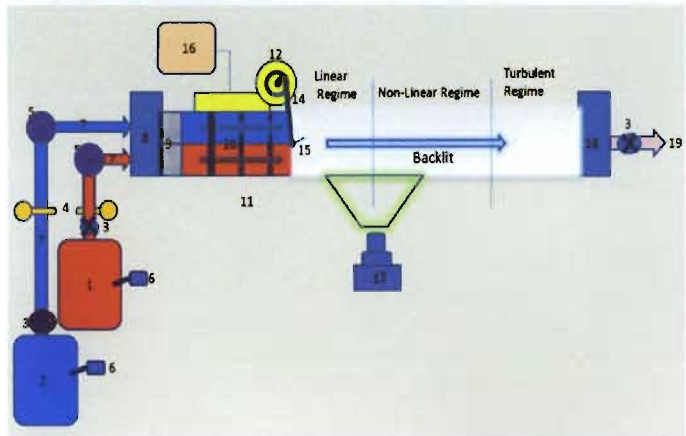
$$A(x) = A_1 \sin\left(\frac{2\pi x}{\lambda_1}\right) + A_2 \sin\left(\frac{2\pi x}{\lambda_2} + \delta\right)$$

$A_1 = 4mm$	$A_2 = 2mm$
$\lambda_1 = 4cm$	$\lambda_2 = 2cm$
$\rho_1 = 997.7 kg/m^3$	$\rho_2 = 99657 kg/m^3$

Phase shift : δ=0, π/2

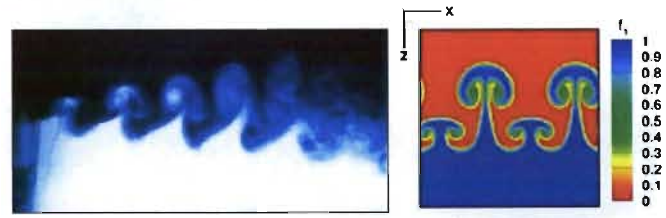
The flapper motion imposes an initial vertical velocity given by:

$$v = \frac{dA}{dt} = \frac{dx}{dt}\frac{dA}{dx} =$$
$$U_0\left(A_1 k_1 \cos(k_1 x) + A_2 k_2 \cos(k_2 x + \delta)\right)$$

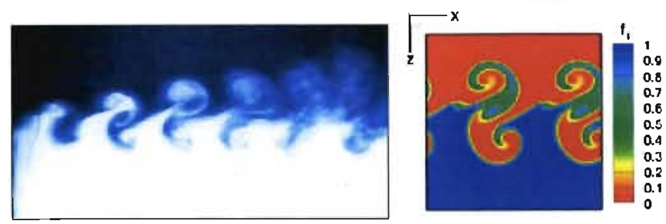


Computer controlled flapper system – up to 16 modes

Binary initial perturbation with δ ~ 0



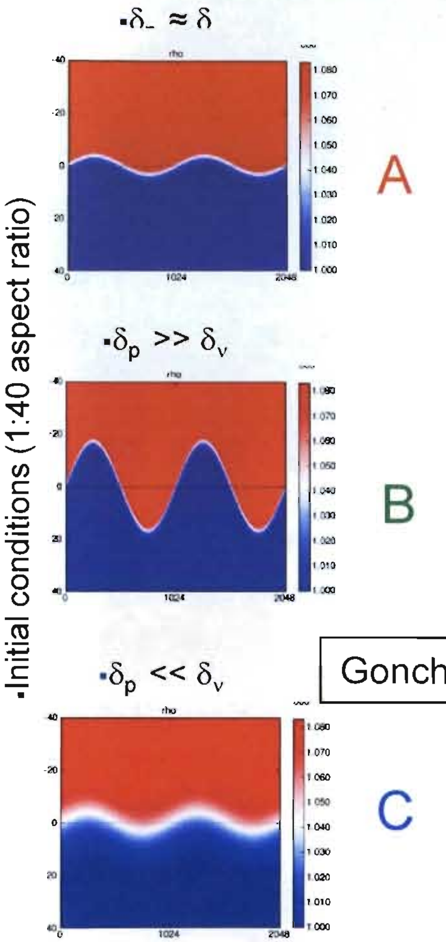
Binary initial perturbation with δ ~ π/2

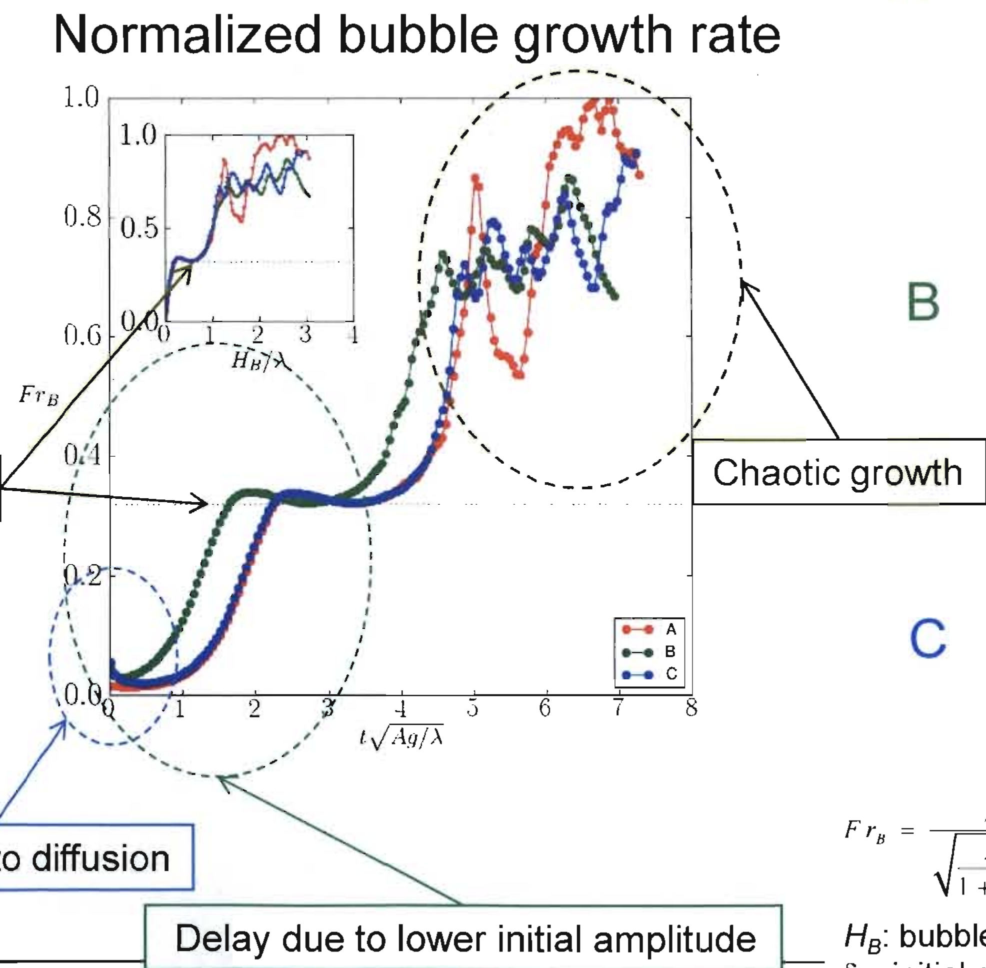


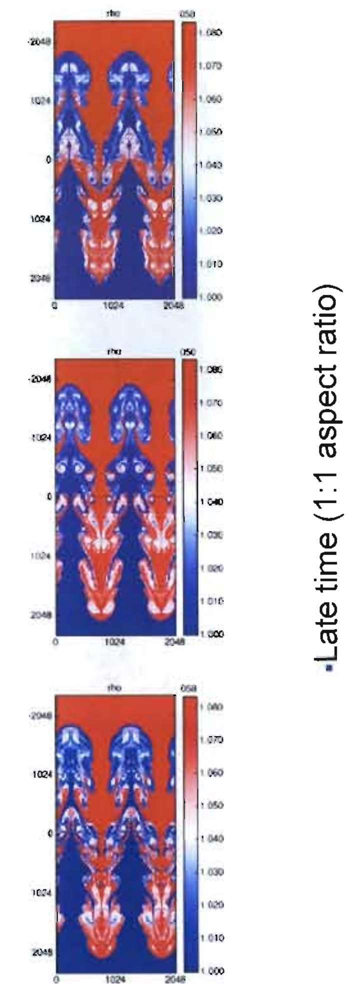
The "leaning" of the growing perturbation with an angle of π/4 observed in the experiment and the simulation is due to mode dynamics, and <u>not</u> mode coupling.

Single-mode RTI: growth stages and initial conditions dependence (A=0.04)

- C-A: change initial diffusion thickness
- B-A: change initial amplitude



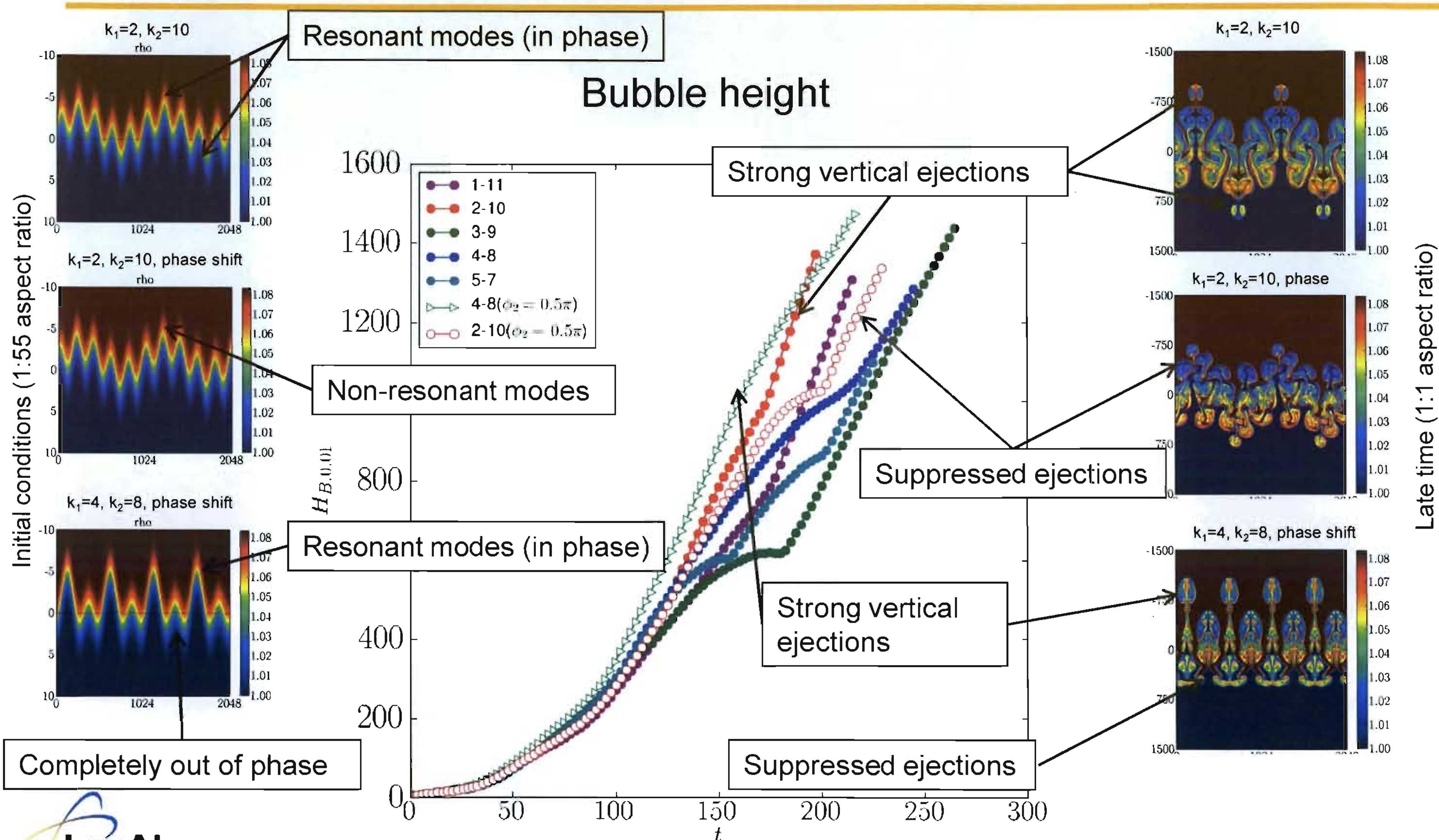




$$Fr_B = \frac{\dot{H}_B}{\sqrt{\frac{A}{1+A} g \lambda}}; \quad A = \frac{\rho_H - \rho_L}{\rho_H + \rho_L}$$

H_B: bubble height
δ_p: initial perturbation amplitude
δ_v: initial diffusion layer thickness

Two-Mode RTI: Layer growth for different mode combinations (A=0.04)



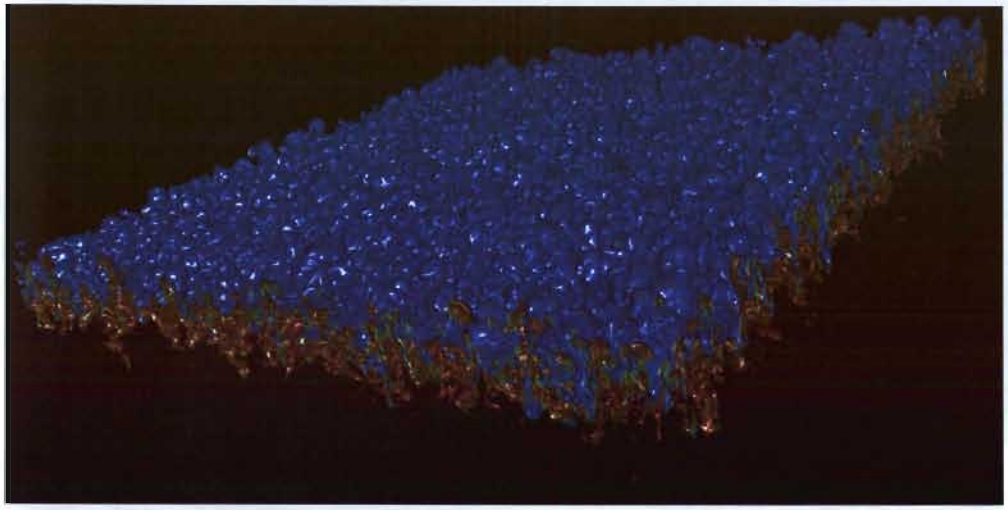
Problem description: New Archival Suite of Direct Numerical Simulations of Rayleigh-Taylor instability

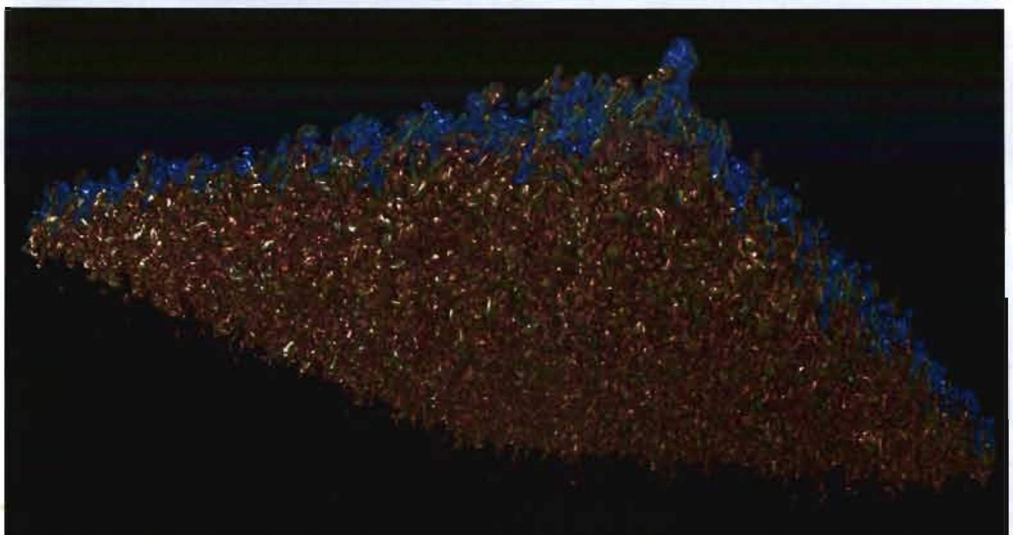
- Suite of 1024^2 x 4608 simulations at A=0.04, 0.5, 0.75, 0.9:
  - Base simulations with initial perturbation peaked around the most unstable mode of the linear problem.
  - After the layer width had developed substantially, the simulations were branched into reversed (g -> -g) and zero (g -> 0) gravity simulations.
  - Different initial perturbation spectra, viscosity and diffusion coefficients to study the effects of various parameters.
- 4096^2 x Nz simulation at A=0.75 (NZ_{max}=4032).
- These have reached Reynolds numbers of:

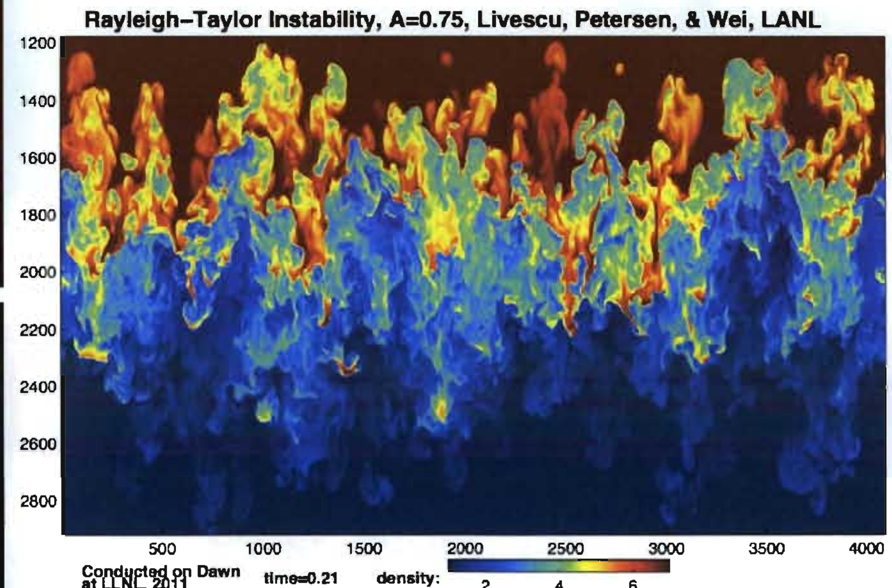
$$\mathrm{Re}_b = h\dot{h}/\nu > 40{,}000$$

$$\mathrm{Re}_T = \widetilde{k}^2/\nu\varepsilon > 5500$$

Multi-mode Rayleigh-Taylor instability: A=0.75, grid size 4096^2 x4032







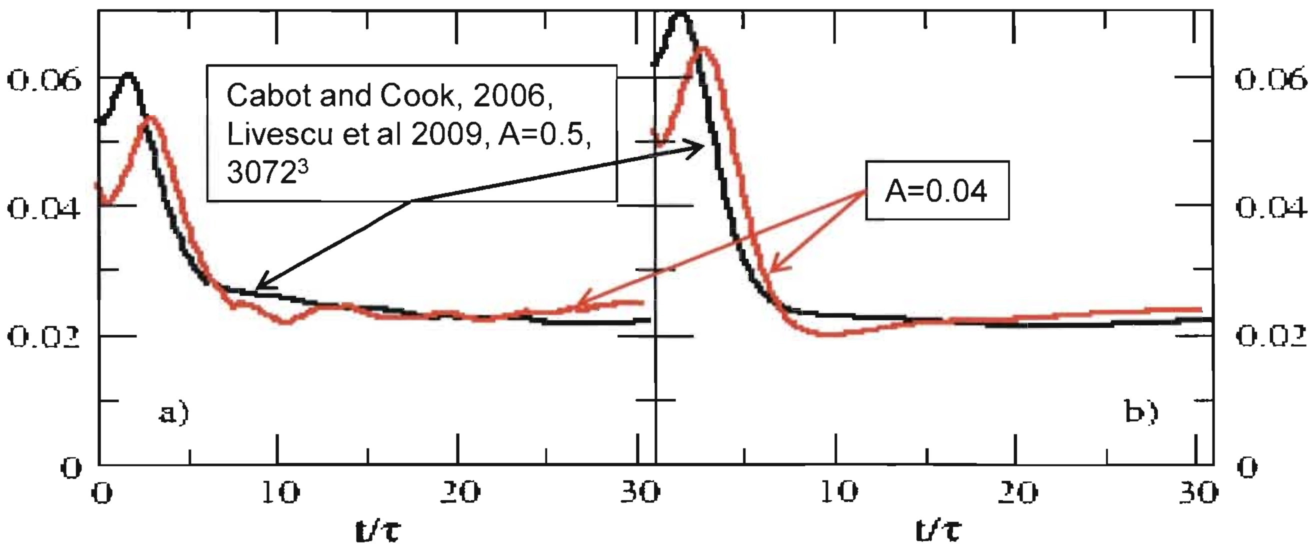
Global measures: mixing layer growth

- Ristorcelli & Clark, (2004) and Cook et al. (2004) showed that self-similarity of the layer width leads to the solution

$$h(t) = \alpha A g t^2 + 2(\alpha A g h_0)^{1/2} t + h_0$$

- Cabot and Cook (2006) measure $\alpha = \dot{h}^2 / 4Agh$
- We introduce a smoother variation that avoids derivatives (left).
- David Youngs uses an integral mix measure, h_{DY} , to define α_{DY} (right).

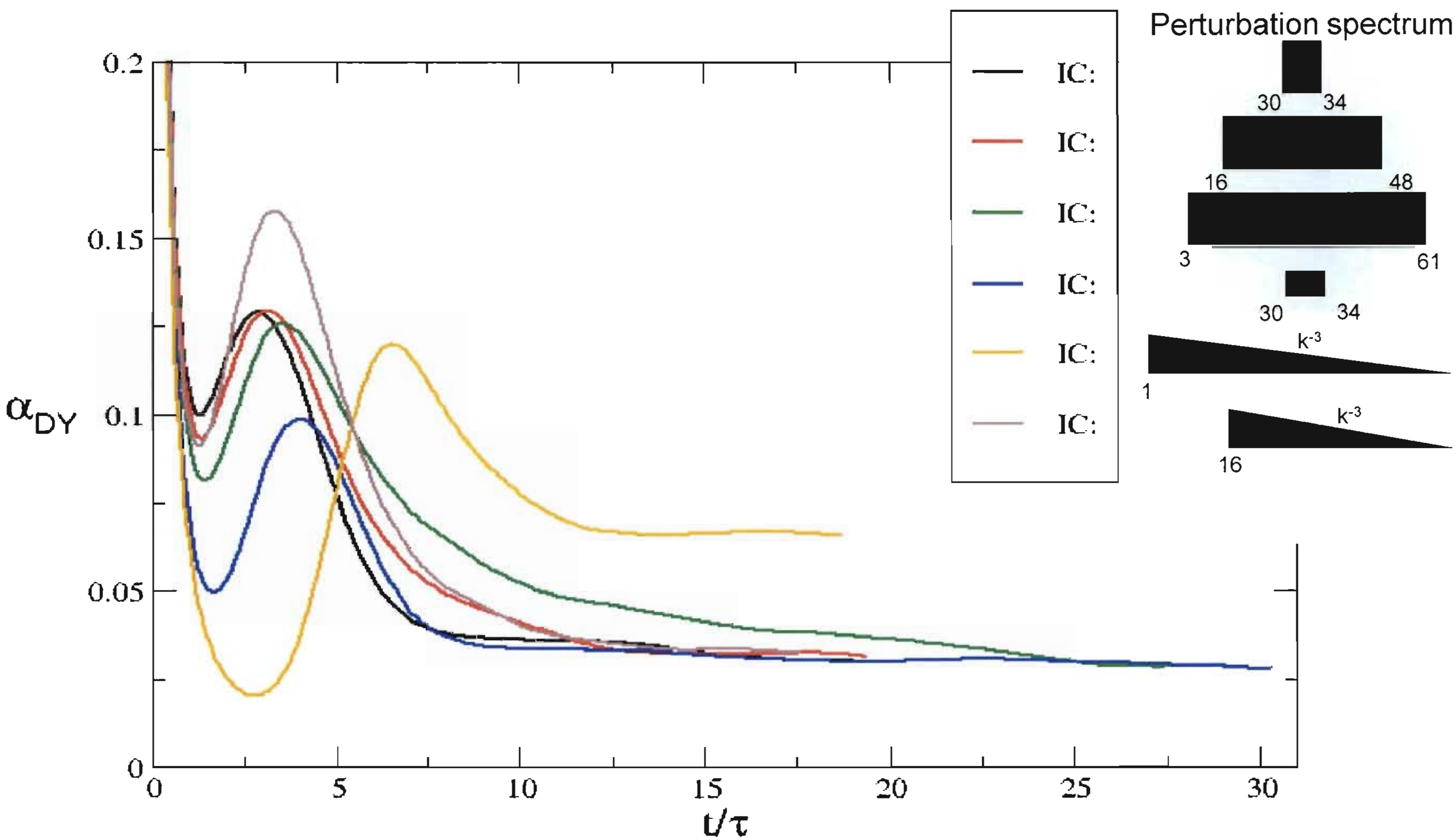
$$\alpha = \frac{\left(\sqrt{h_{0.01}(t)} - \sqrt{h_{0.01}(t_0)}\right)^2}{Ag(t - t_0)^2} \qquad \alpha_{DY} = \frac{\dot{h}_{DY}^{\ 2}}{4Agh_{DY}}$$



$\tau = \sqrt{L_0 / (Ag)}$

L_0= characteristic lengthscale of the initial perturbation

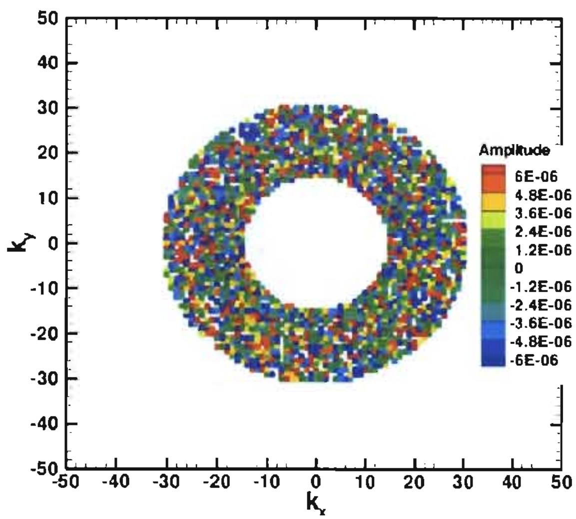
Global measures: mixing layer growth for different perturbation spectra

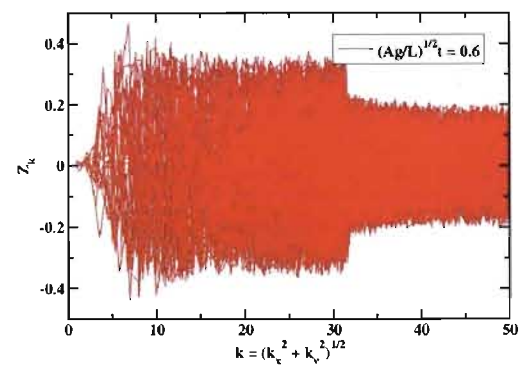


A Modal Model for Rayleigh-Taylor Instability

Bertrand Rollin (CCS-2),

Malcolm J. Andrews (XCP-4)



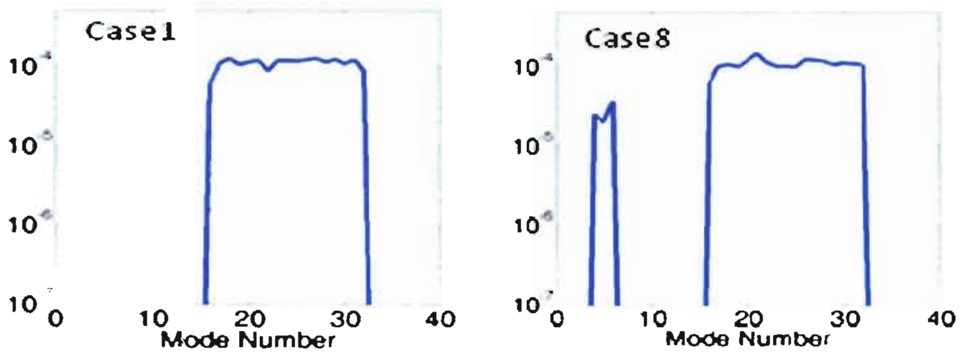


$$\ddot{Z}_k = \frac{4(k-8\eta_2)}{k^2 - 4A_T k\eta_2 - 32A_T\eta_2^2}\left(-\dot{Z}_k^2 k^2 \frac{(5A_T-4)k^2 + 16(2A_T-3)k\eta_2 + 64A_T\eta_2^2}{8(k-8\eta_2)^2} - A_T g\eta_2\right)$$

Why a Modal Model?

A Rayleigh-Taylor Multi-Mode Study with Banded Spectra and Their Effect on Late-Time Mix Growth (Banerjee & Andrews, 2009)

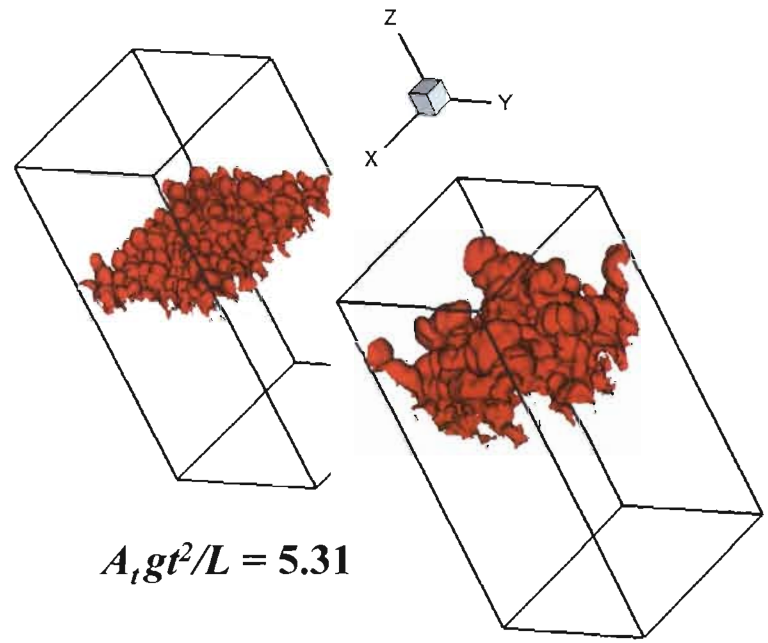
Initial Spectrum



$$\frac{\overline{h_0'^2}}{2} = \int_{k_{min}}^{k_{max}} E_{h0}(k)dk = \int_{k_{min}}^{k_1} E_{h1}(k)dk + \int_{k_2}^{k_{max}} E_{h2}(k)dk = \frac{\overline{h_1'^2}}{2} + \frac{\overline{h_2'^2}}{2}$$

$$\Lambda = \overline{h_2'^2} \Big/ \overline{h_1'^2}$$

3-D ILES Simulations

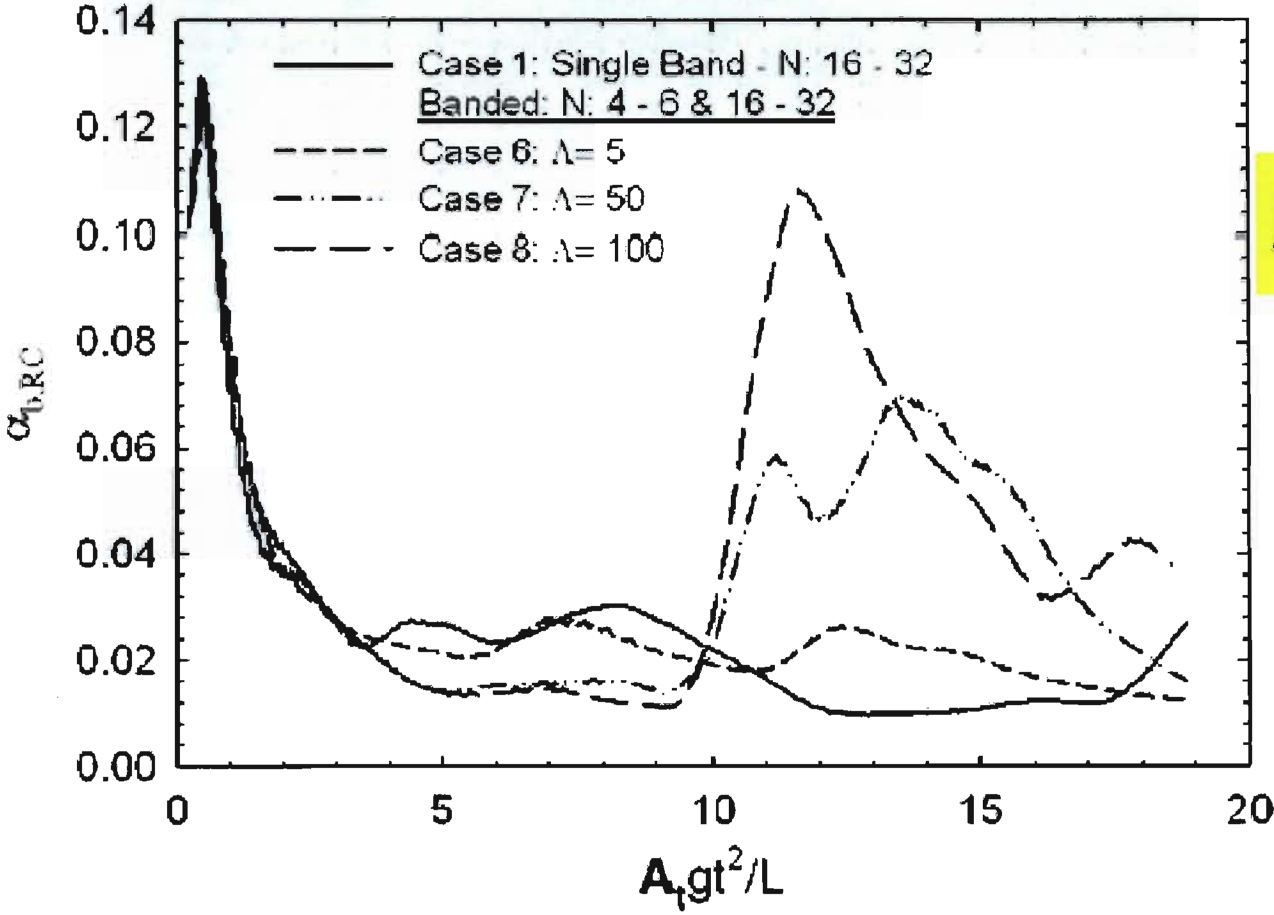


$A_t g t^2/L = 5.31$

$A_t g t^2/L = 19.62$

Why a Modal Model?

3-D ILES Simulations of Banded Spectra and Late-Time Appearance of Long Wavelengths (Banerjee & Andrews 2009)



$$h_b = \alpha A_t g t^2$$

$$\alpha_b = \frac{\dot{h}_b^{\,2}}{4 A_t g h_b}$$

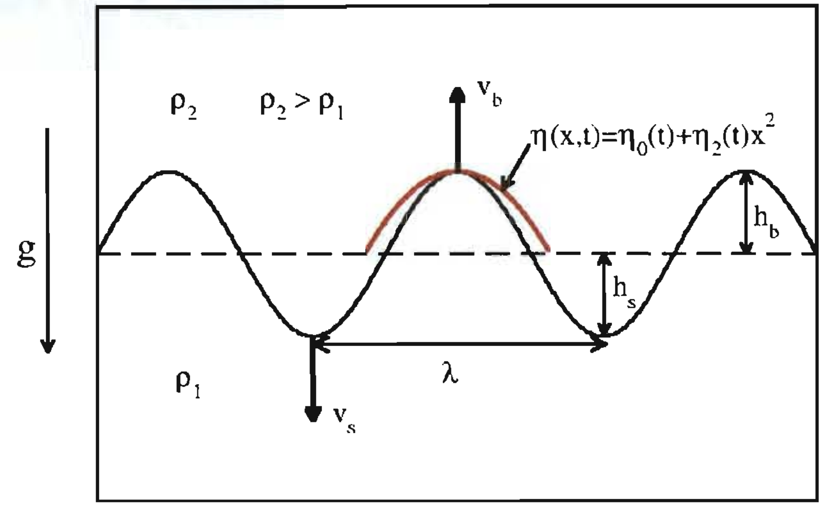
A Potential Flow Model for Single Mode Perturbation

Goncharov model:

$$\Delta\phi^{h/l} = 0$$

$$\phi^h = a(t)J_0(kr)e^{-k(z-\eta_0)}$$

$$\phi^l = b_1(t)J_0(kr)e^{k(z-\eta_0)} + b_2(t)z$$

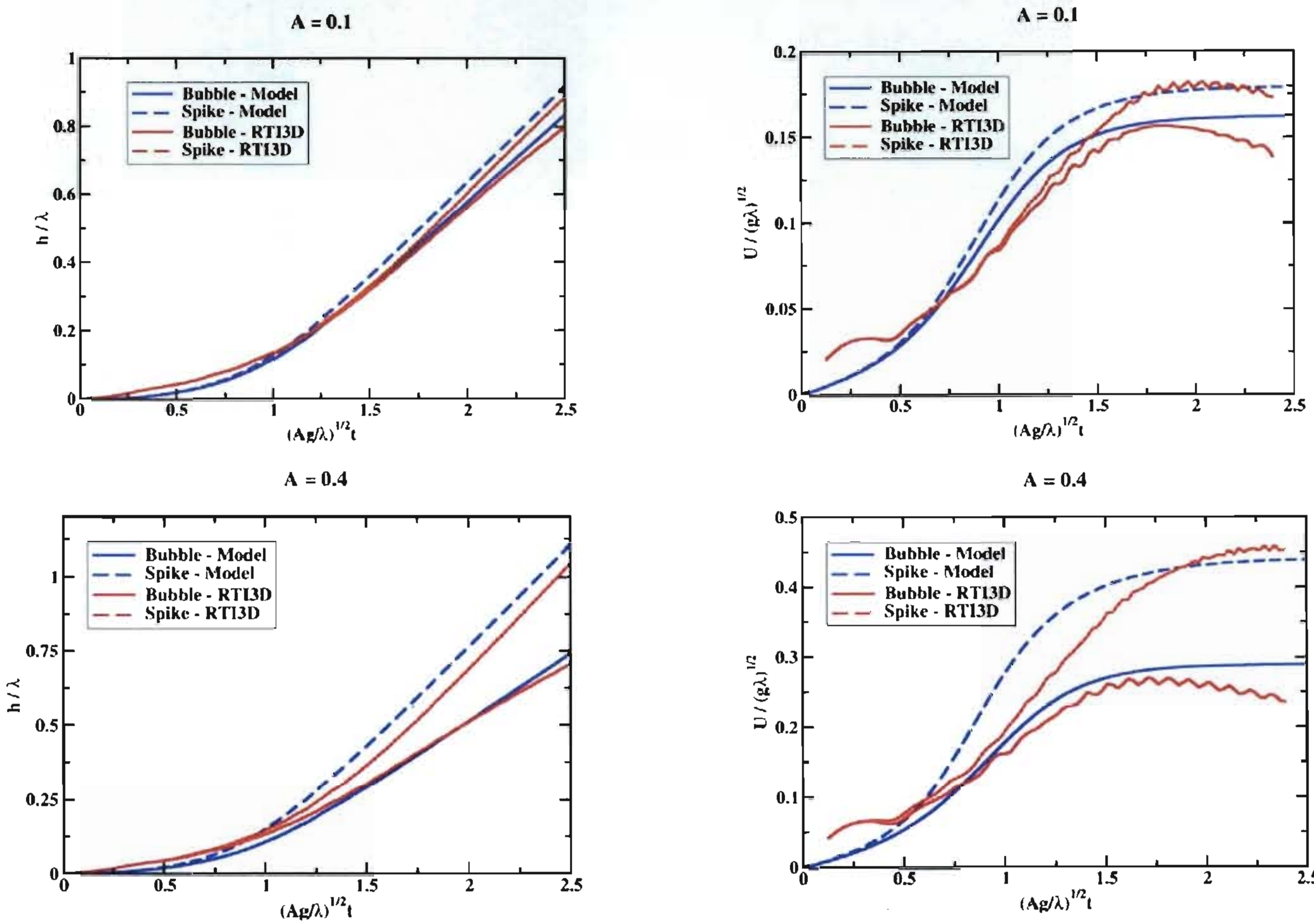


$$\partial_t\eta + v_r^{h/l}\partial_r\eta = v_z^{h/l} \qquad [v_z - v_r\partial_r\eta] = 0 \qquad [Q] = Q^h - Q^l$$

$$\left[\rho\left(\partial_t\phi + \frac{1}{2}v^2 + g\eta\right)\right] = P$$

The velocity potential are expended to 2nd order and plugged in the interfacial conditions

Single Mode Model Results

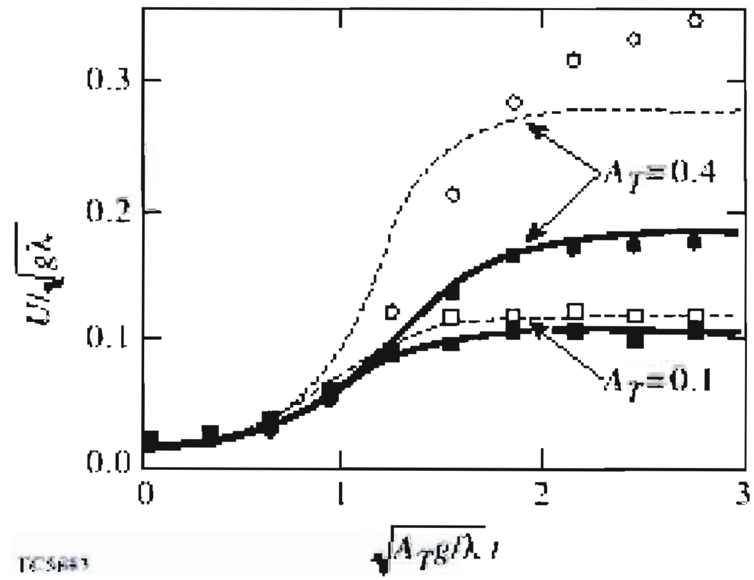


The Goncharov model performs well for low Atwood numbers

Single Mode Model Summary

- Nonlinear model
- Valid on a large range of A_T ($0 \le A_T \le 0.4$)
- Good prediction for bubble

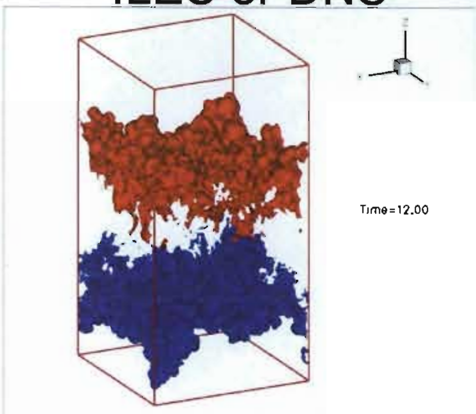
- Spike inaccurate for high A_T

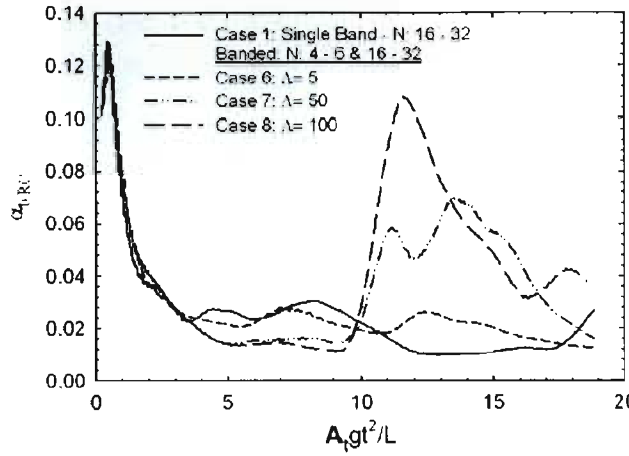


Goncharov, PRL, **88**, 2002

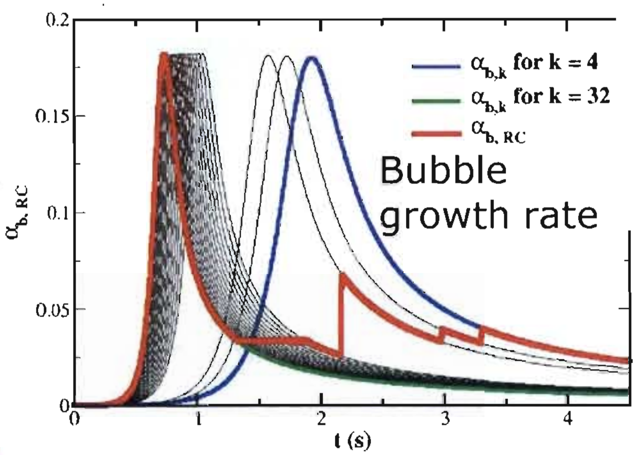
Multi-mode Study Using ODE's to Predict the Envelope of the Bubble Mix Region (Andrews+Rollin)

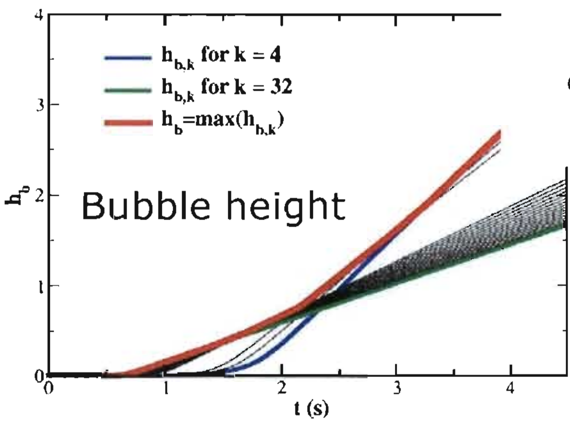
ILES or DNS



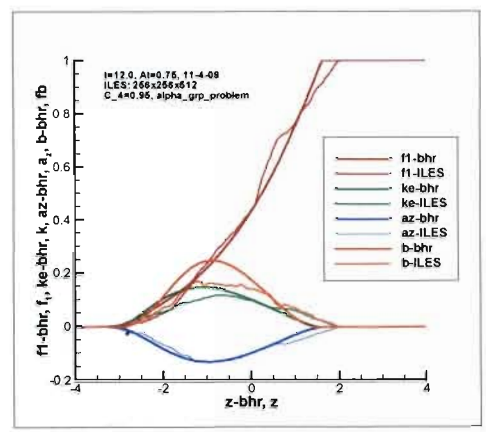


ODE Model





- At late time, h_b is governed by the longest wavelength evolution
- Each band of the initial spectrum is 'seen' in the growth rate



Weakly Nonlinear Model Summary

- Nonlinear model
- Valid for all Atwood number
- Multimode model, i.e., handle mode coupling

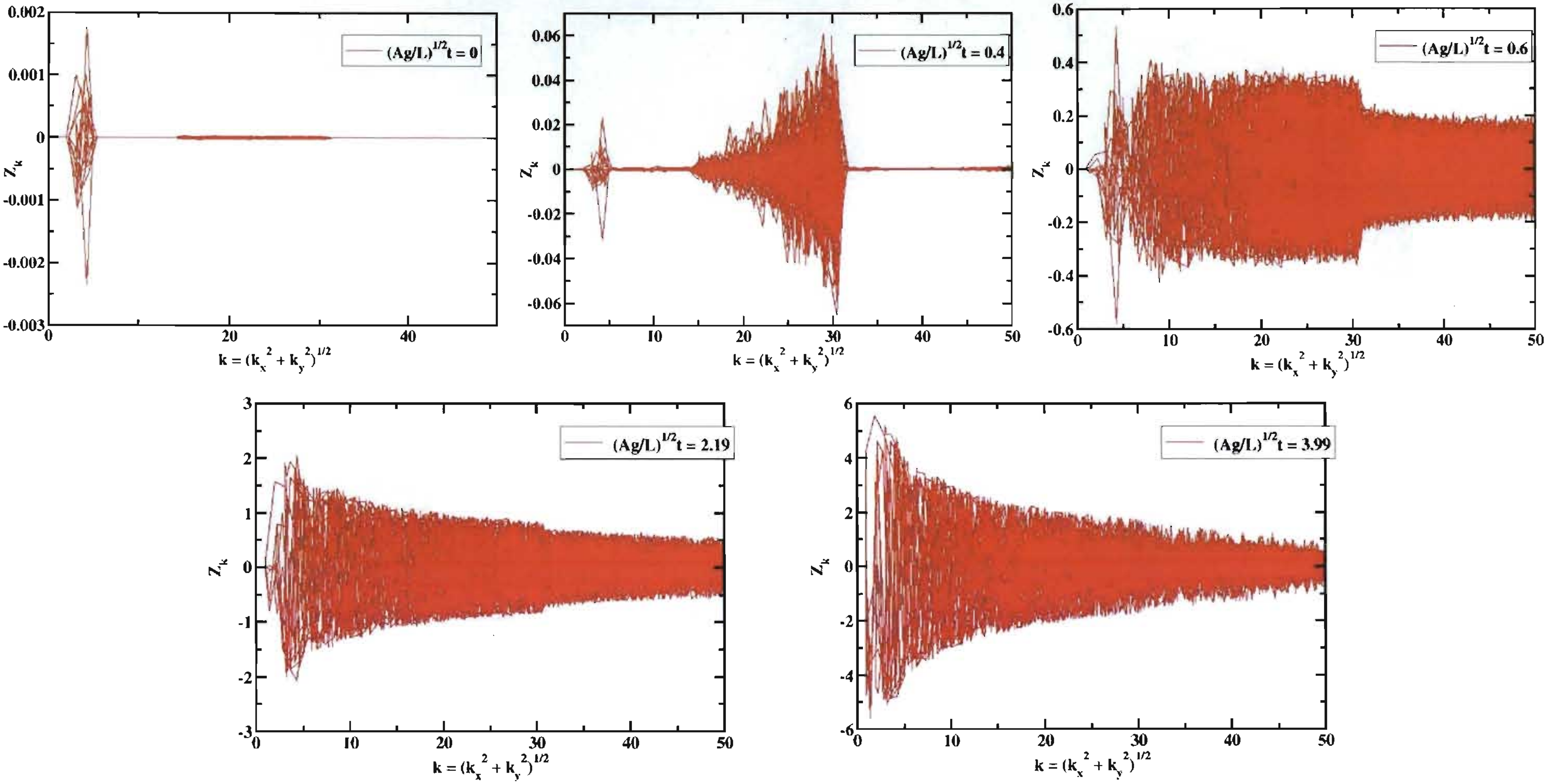
- Valid until early transition to nonlinear behavior, so here is what we do:

For all k,

$$\begin{cases} \ddot{Z}_k = G(k) + A_T k \sum_{\vec{m}} \left\{ \ddot{Z}_m Z_n \left(1 - \hat{m}\cdot\hat{k}\right) + \dot{Z}_m \dot{Z}_n \left(\frac{1}{2} - \hat{m}\cdot\hat{k} - \frac{1}{2}\hat{m}\cdot\hat{n}\right)\right\} & \text{Before k saturates} \\ \ddot{Z}_k = A_T k \sum_{\vec{m}} \left\{ \ddot{Z}_m Z_n \left(1 - \hat{m}\cdot\hat{k}\right) + \dot{Z}_m \dot{Z}_n \left(\frac{1}{2} - \hat{m}\cdot\hat{k} - \frac{1}{2}\hat{m}\cdot\hat{n}\right)\right\} & \text{After k has saturated} \end{cases}$$

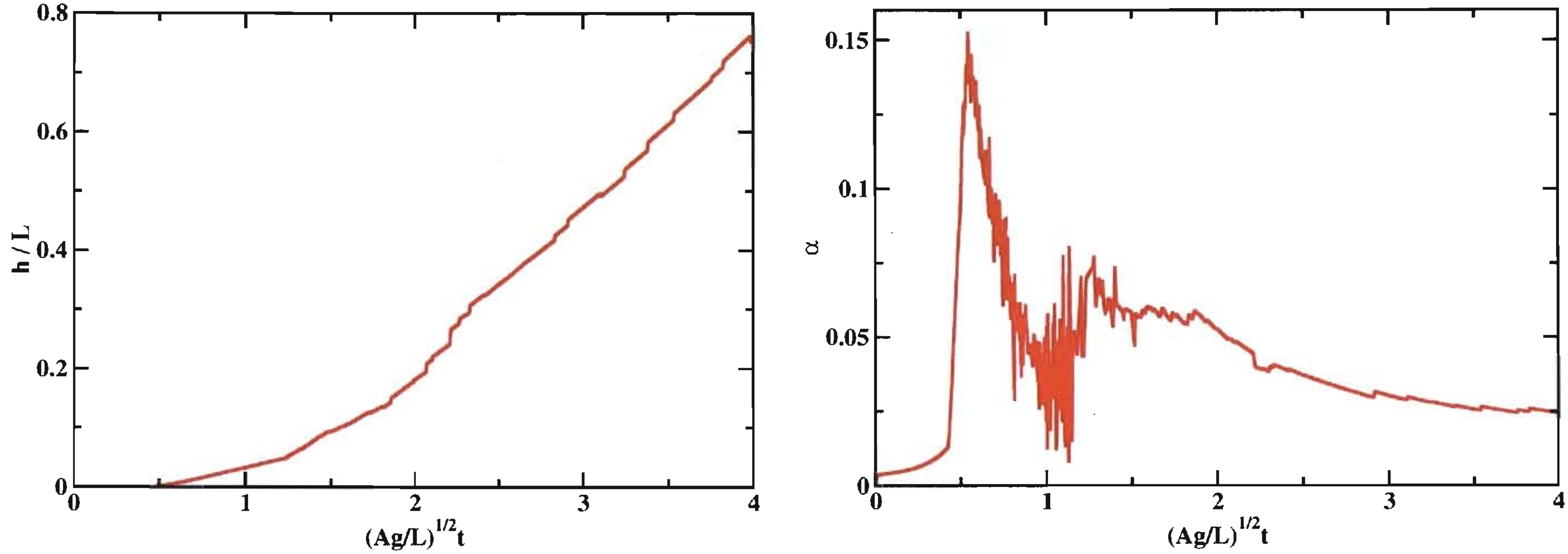
$|m|, |n| < |k|$

Banded Spectrum Case



Existing long wavelength in the initial spectrum is not washed out by mode coupling

Banded Spectrum Case



The mixing layer expansion experience an "extra kick" when the long wavelengths of the initial pertubation become the dominant modes

BHR Turbulence Model for RT Instability

LDRD

Selected Besnard-Harlow-Rauenzhan (BHR) turbulence model, but could use others just as well:

- **Single-point turbulent transport model**
- **Designed for variable density turbulence**

D. Besnard, F. H. Harlow, R. Rauenzhan, LA-10911-MS (1987)

Model Variables:

$$k = \frac{1}{2}\overline{u_i' u_i'} \qquad a_i = \frac{\overline{\rho' u_i'}}{\overline{\rho}} \qquad b = -\overline{\rho' v'} \qquad S = \frac{k^{3/2}}{\varepsilon} \qquad \nu_t = C_\mu k^{1/2} S$$

Governing equation for the variable S:

$$\partial_t S = \left(\frac{3}{2} - C_4\right) a_z g \frac{S}{k} + \frac{1}{\rho}\partial_z\left(\rho \frac{\nu_t}{\sigma_S}\partial_z S\right) - \left(\frac{3}{2} - C_2\right) k^{1/2}$$

BHR initiated with:

- **Profiles for:** $k \quad a_i \quad b \quad S$
- **Values for:** $C_4 \quad C_2 \quad C_\mu \quad \sigma_S \quad \ldots$

C_4 .. Controls RT mix width

Self-Similar Solution for "Dynamic" C_4 Derivation

Self-similar soln. into BHR ➡ $k = \alpha_k A_T^2 g^2 t^2 \quad a_z = \alpha_a A_T g t \quad S = \alpha_S A_T g t^2$

$$\partial_t S = \left(\frac{3}{2} - C_4\right) a_z g \frac{S}{k} + \frac{1}{\rho}\partial_z\left(\rho \frac{\nu_t}{\sigma_S}\partial_z S\right) - \left(\frac{3}{2} - C_2\right) k^{1/2}$$

Obtain algebraic eqn. for α's ➡

$$2\alpha_S = \left(\frac{3}{2} - C_4\right)\frac{\alpha_a \alpha_S}{\alpha_k A} - \left(\frac{3}{2} - C_2\right)\alpha_k^{1/2}$$

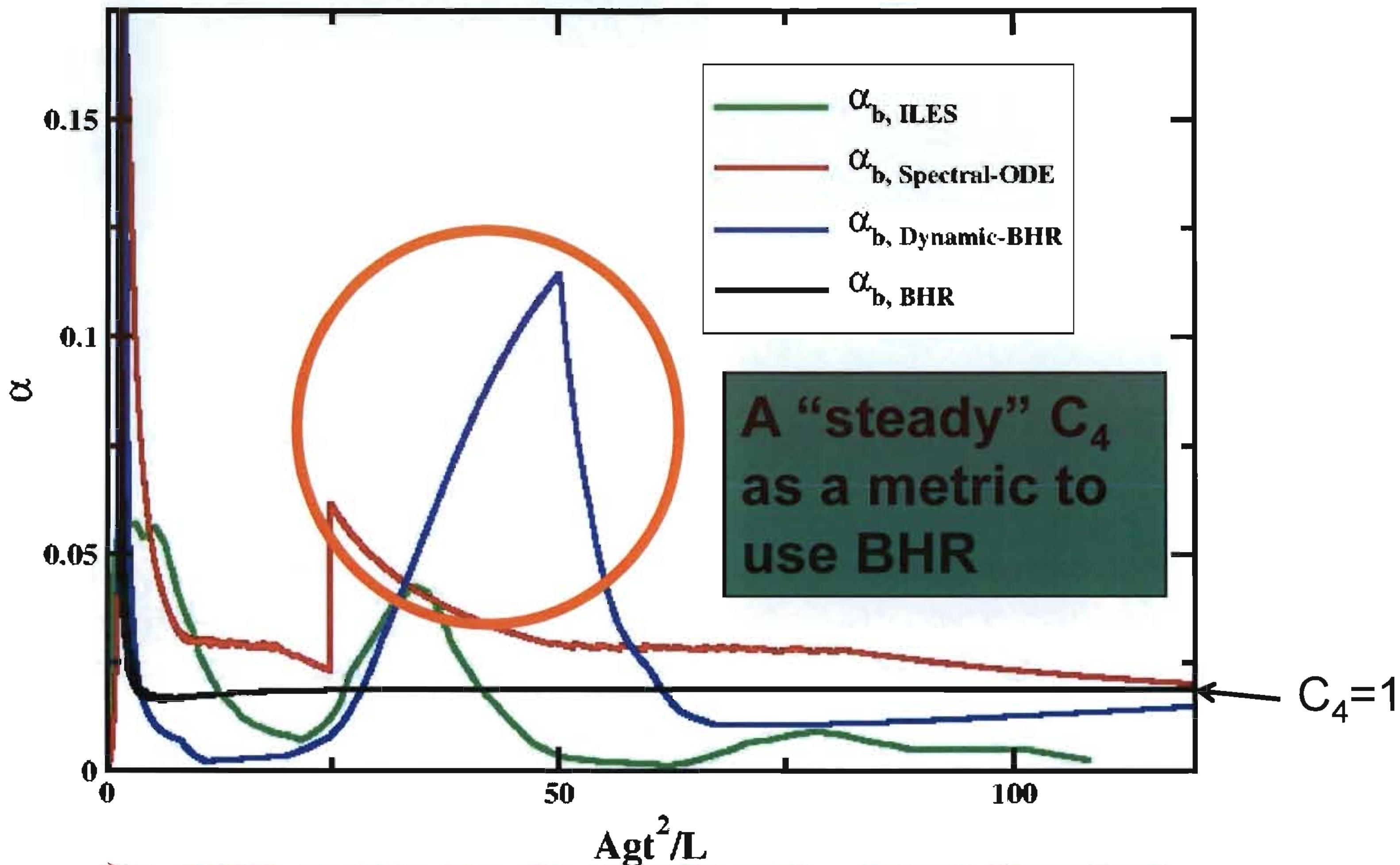
Solve for BHR coefficient ➡

$$C_4 = f(\alpha_k, \alpha_S, \alpha_{a_z}, C_2)$$

Ballistic mode metrics ➡

$$\alpha_k = \frac{\frac{d^2 k}{dt^2}}{2 A_T^2 g^2}, \alpha_a = \frac{\frac{da}{dt}}{A_T g}, \alpha_s = \frac{\frac{d^2 S}{dt^2}}{2 A_T g}$$

Bubble Growth Rate Prediction with a Dynamic C_4



➢ **BHR captures dynamics of α for a banded spectrum with "dynamically" prescribed C_4**

Two-Fluid Initial Profiles for BHR Variables

$$\overline{\rho} = f_l \rho_l + f_h \rho_h \qquad \overline{\mathbf{u}} = f_l \mathbf{u}_l + f_h \mathbf{u}_h$$

$$k = C_k \frac{3}{2}\left(\vec{v_b} - \vec{v_s}\right)^2 \frac{f_h f_l \rho_h \rho_l}{\left(f_h \rho_h + f_l \rho_l\right)^2}$$

Isotropy hypothesis

$$a_z = C_{a_z} \frac{f_h f_l}{f_h \rho_h + f_l \rho_l}\left(\rho_h - \rho_l\right)\left(\vec{v_s} - \vec{v_b}\right)$$

$$b = C_b \frac{f_h f_l \left(\rho_h - \rho_l\right)^2}{\rho_h \rho_l}$$

$$S = C_S \left(h_b + h_s\right)\left(4 f_h f_l\right)^{1/2}$$

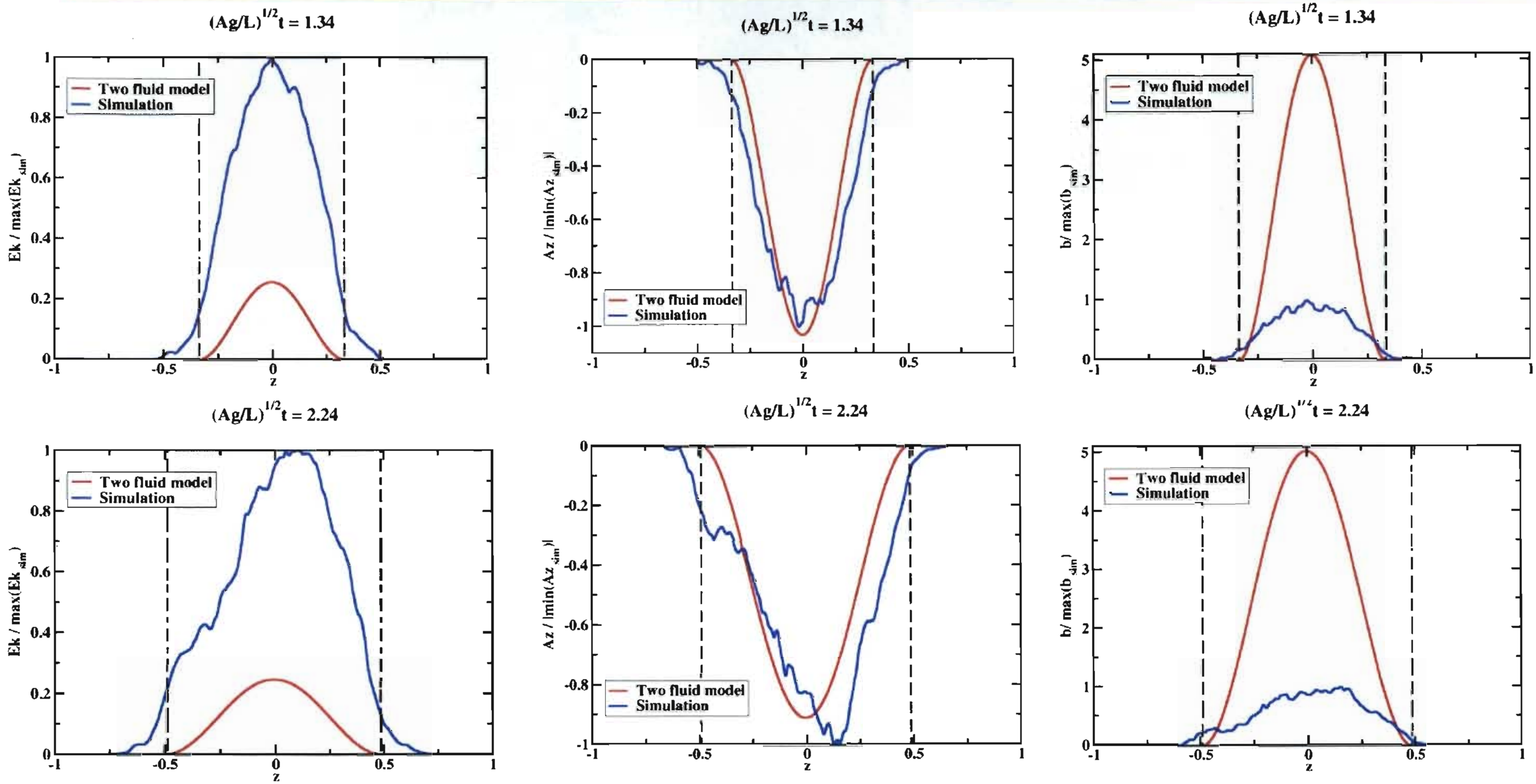
Self-similarity hypothesis

Derived for low Atwood number

$$C_k = C_S = C_b = C_{a_z} = 1$$

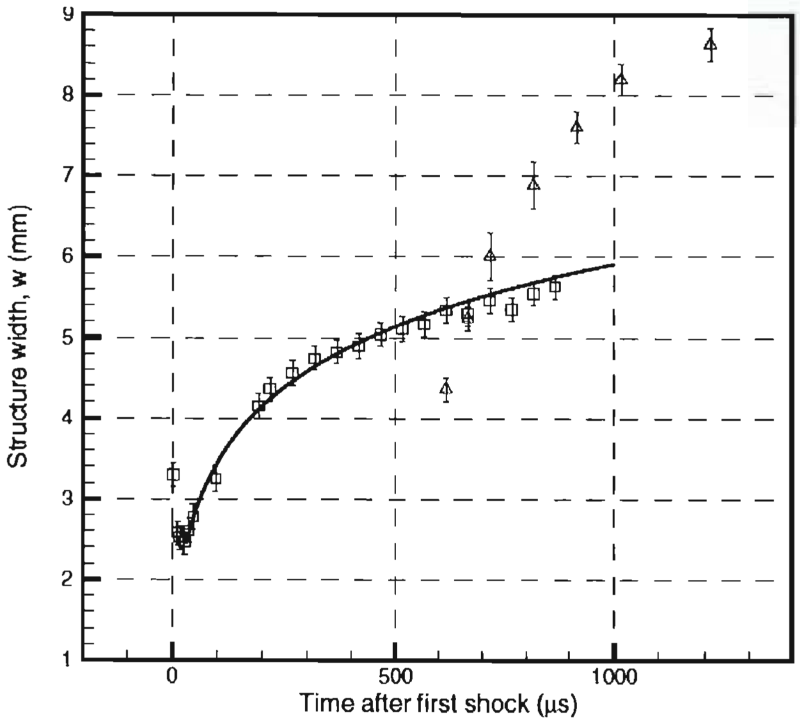
Profiles of f, and values for v_b, v_s, h_b and h_s come from the ballistic model

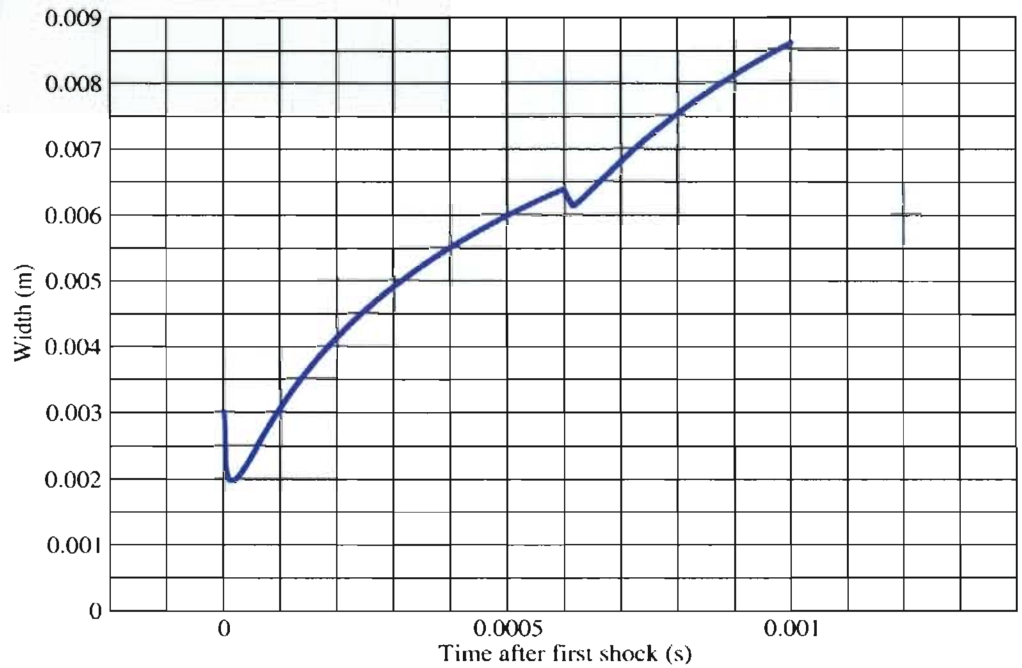
Two fluid model predictions



Two-fluid formulation produces reasonable profiles that need to be adjusted correction coefficients

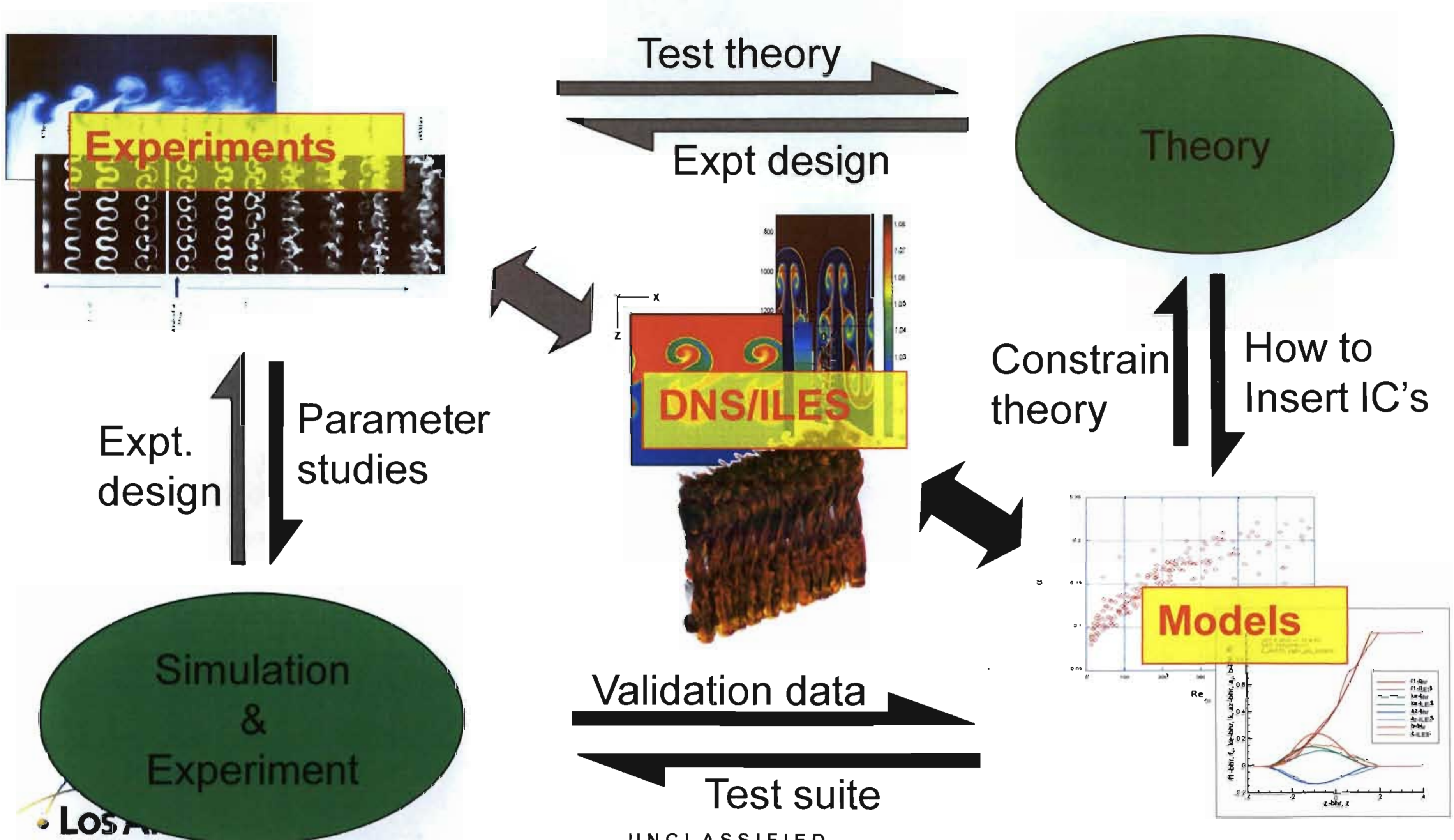
Application to RM instability





The Goncharov model applied to the gas curtain experiment produces a very close result

A Useful Integrated DR?



Did we validate the Hypothesis?

Initially seeded small amplitude, long wavelength, perturbations can develop at late-time and be used to control turbulent transport and mixing effectiveness.

- **Prestridge et al. RM experiments clearly show $\eta=\kappa\delta$ effect of IC's, and in the bi-polar discovery of Grinstein**
- **Grinstein/Gowardhan identified the $\eta\sim1$ transition, a new design criteria**
- **Livescu/Wei quantified a variety of initial condition phenomena, including strong asymmetry associated with bi-modal IC's and phase shifts**
- **Livescu/Wei demonstrated RT 3-D DNS simulations with different initial spectra that have different late-time(?) characteristics**
- **Andrews/Rollin constructed a ballistic model that facilitates evaluation of the development of complex initial spectra for RT, and shown how it connects to turbulence model initialization**

SPARES

LDRD
innovation for our nation

Progress Toward an ODE model with Mode Coupling for Non-Linear Evolution of Multi-Mode IC's

Driving idea:

- **A source term to the Goncharov nonlinear ODE for single mode evolution that expresses the contribution by coupling of the wavenumber of interest with a direct neighbor**

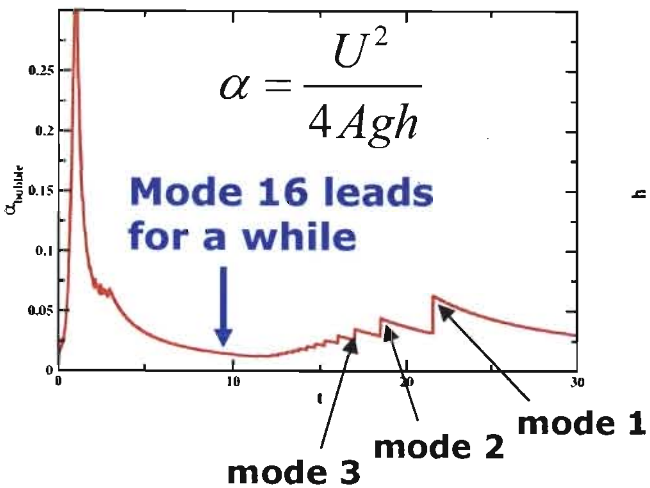
$$\ddot{h}_{b,k} = G(k, A, g, etc...) + A \times k \times F(k, k+1) \times \left(\ddot{h}_{b,k+1} h_{b,k+1} + \dot{h}^2_{b,k+1}\right)$$

- $G(k, A, g, etc...)$ **is given by Goncharov's model for a single mode perturbation**
- $F(k, k+1)$ **is a coupling factor of order 1**
- **The ODE are solved for all modes and the dominant mode gives the height of the mixing layer**

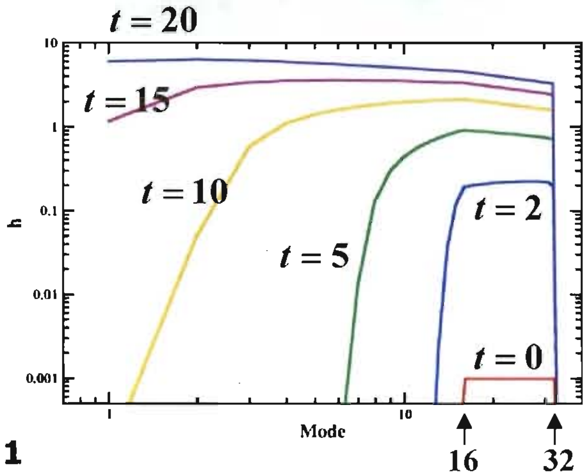
$$h_b(t) = \max_k (h_{b,k}(t))$$

ODE model with Mode Coupling for Non-Linear Evolution of Multi-Mode IC's

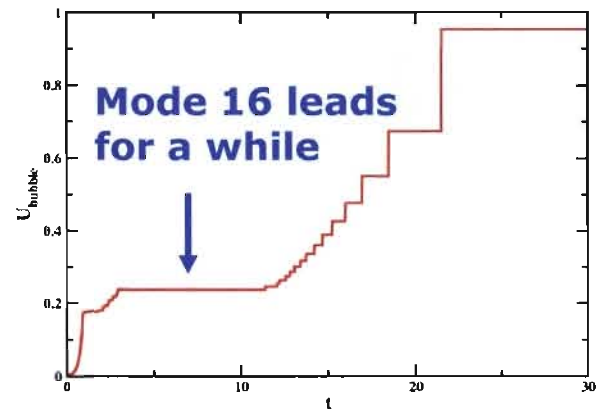
Bubble growth rate



Height evolution for each mode

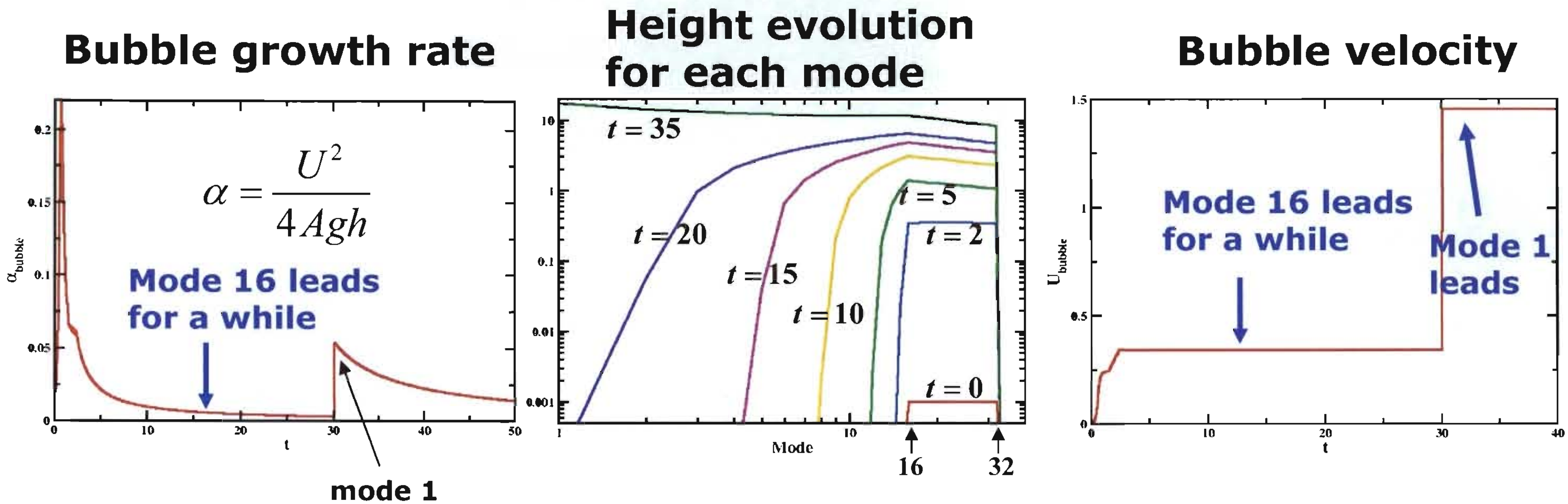


Bubble velocity



- The model generates modes toward lower wavenumbers
- Work is needed for the coupling factor as the seeded modes have a much too low generated amplitude.

ODE model with Mode Coupling for Non-Linear Evolution of Multi-Mode IC's



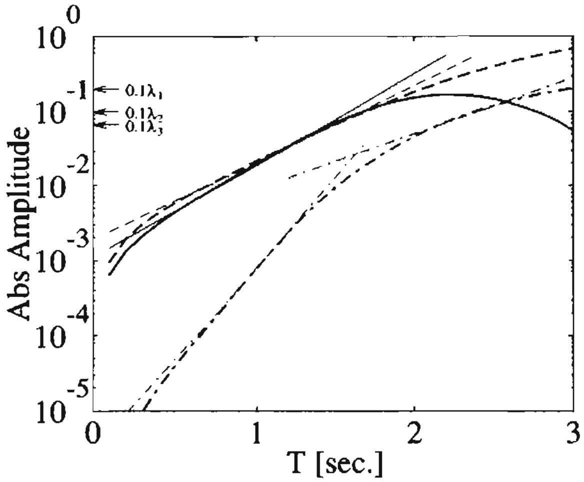
- The model generates modes toward lower wavenumbers

- Improvement is needed for the coupling factor as the seeded modes have a much too low generated amplitude.

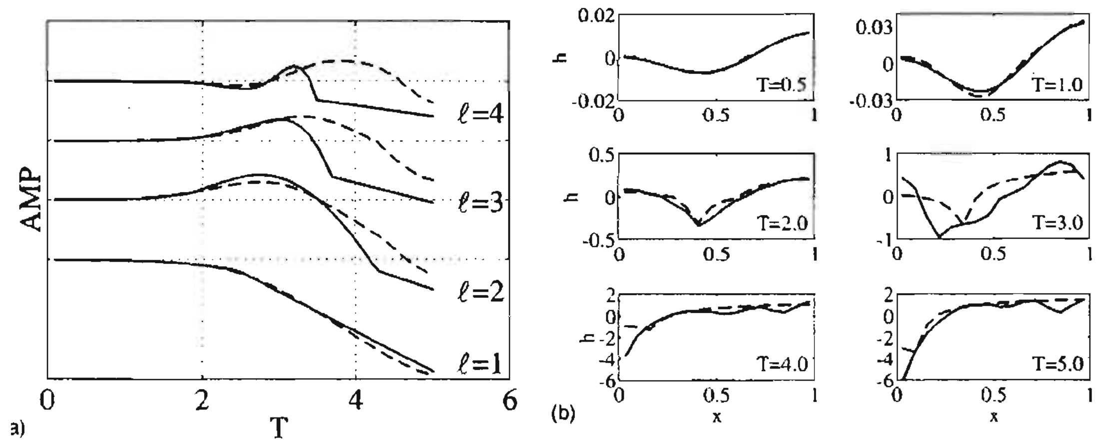
Post-saturation treatment

Ofer *et al.*, Phys. Plasmas, **3** (1996)

Evolution of a two mode initial perturbation, modes 2 & 3



Evolution of a two mode initial perturbation, modes 1 & 2

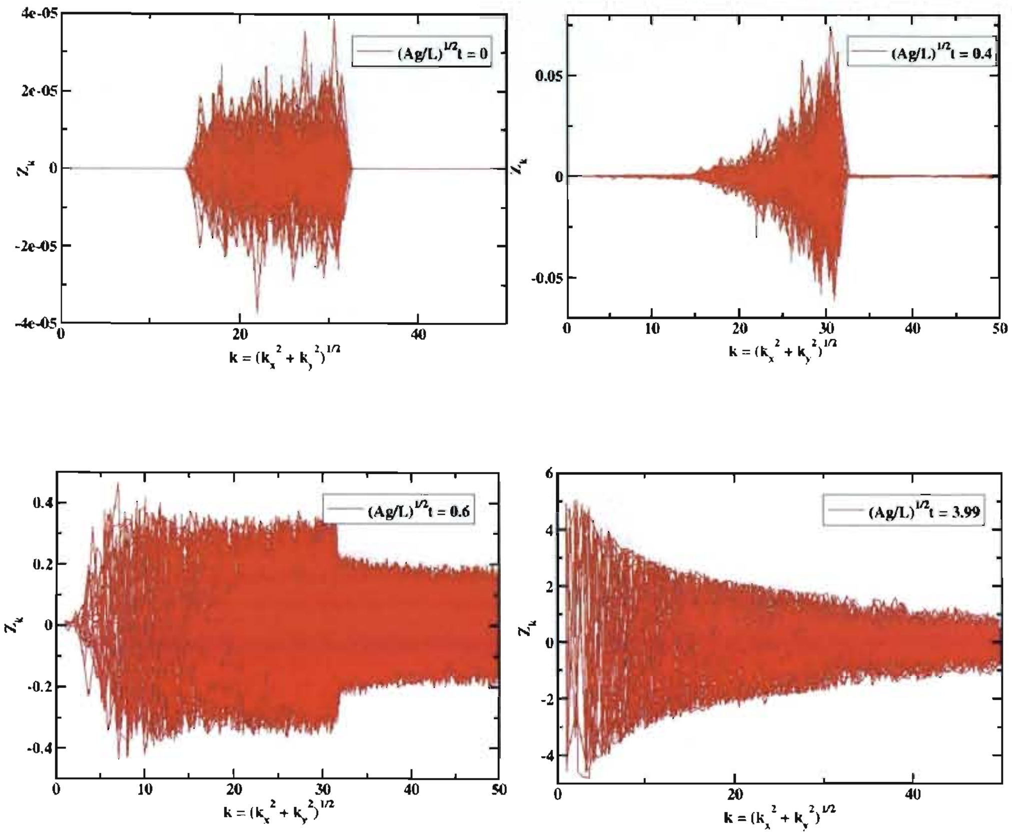


A saturated mode cease to contribute to mode coupling

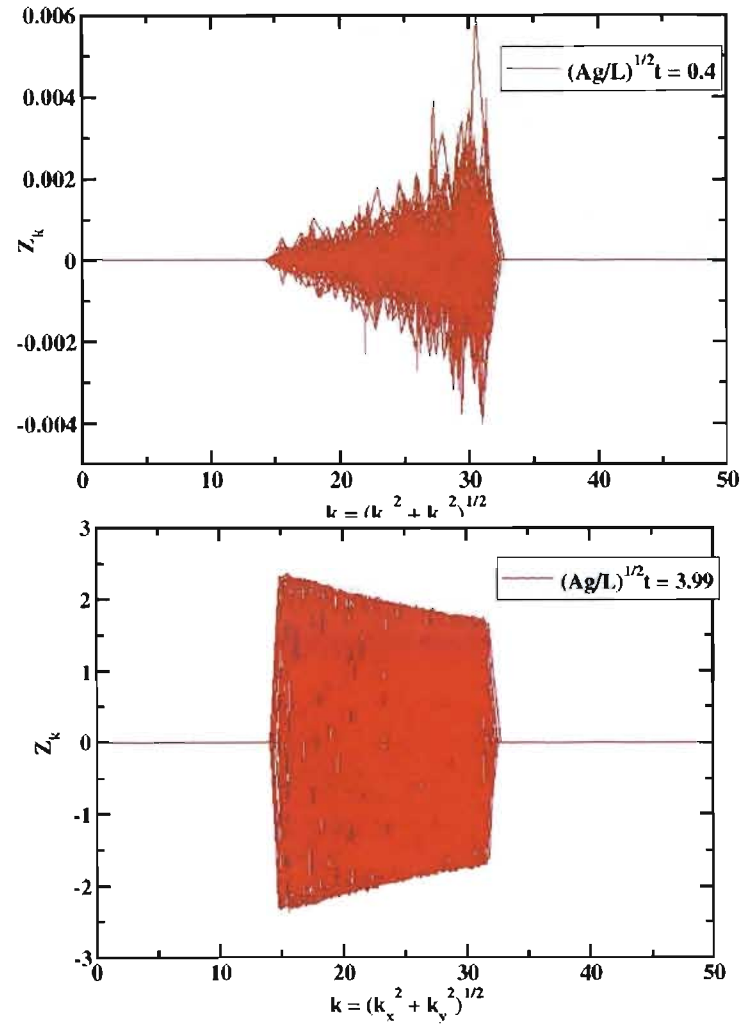
A saturated mode k can only be affected by two lower-k modes. Its velocity can never exceed its saturation velocity.

Modal Model Behavior

Mode Coupling

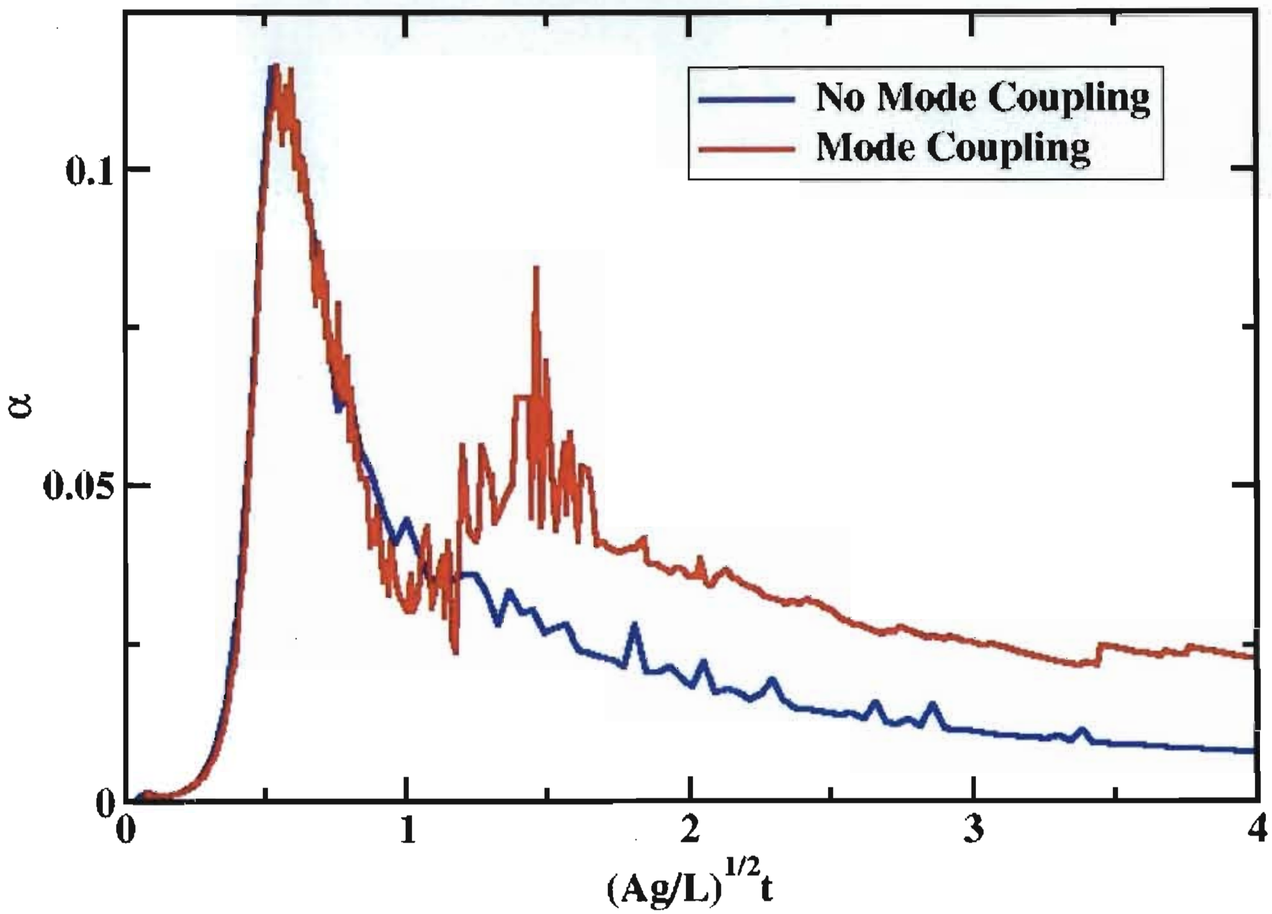


No Mode Coupling



The mode coupling function will "populate" the entire spectrum

Modal Model Behavior



Mode coupling is at the origin of self similarity

A Modal Model for Multimode RT: Linear regime

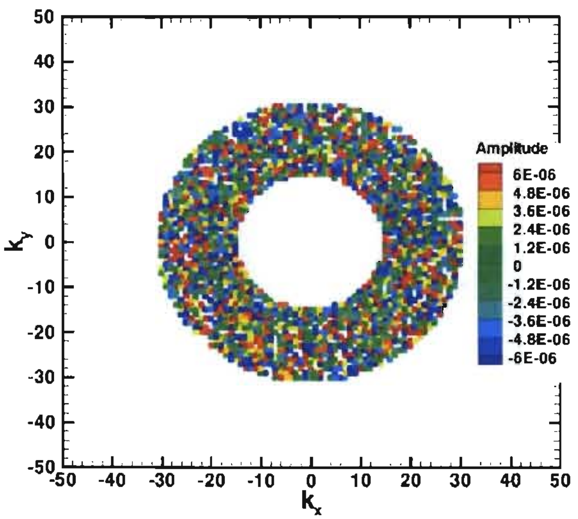
An modal model for multimode RT built from the "fusion" between a potential flow model for single mode and a weakly nonlinear model:

For all k,

$$\ddot{Z}_k = \frac{4(k-8\eta_2)}{k^2 - 4A_T k\eta_2 - 32A_T\eta_2^2}\left(-\dot{Z}_k^2 k^2 \frac{(5A_T-4)k^2 + 16(2A_T-3)k\eta_2 + 64A_T\eta_2^2}{8(k-8\eta_2)^2} - A_T g\eta_2\right.$$
$$\left. + A_T k \sum_{\vec{m}} \left\{ \ddot{Z}_m Z_n\left(1-\hat{m}\cdot\hat{k}\right) + \dot{Z}_m \dot{Z}_n \left(\frac{1}{2} - \hat{m}\cdot\hat{k} - \frac{1}{2}\hat{m}\cdot\hat{n}\right)\right\}\right)$$

$$k = \sqrt{k_x^2 + k_y^2}$$

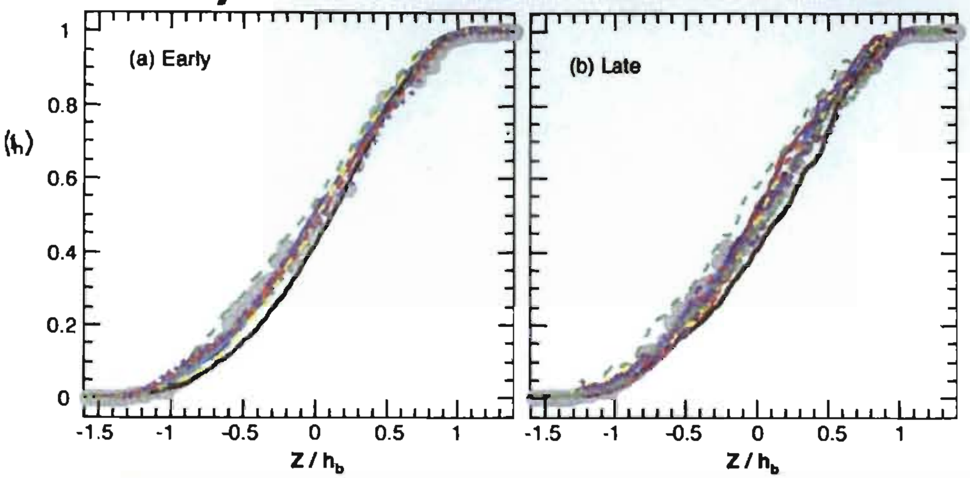
Initial perturbation in wave space



Using a two dimensional initial perturbation spectrum for the model allow a one-to-one match with ICs for 3D simulations

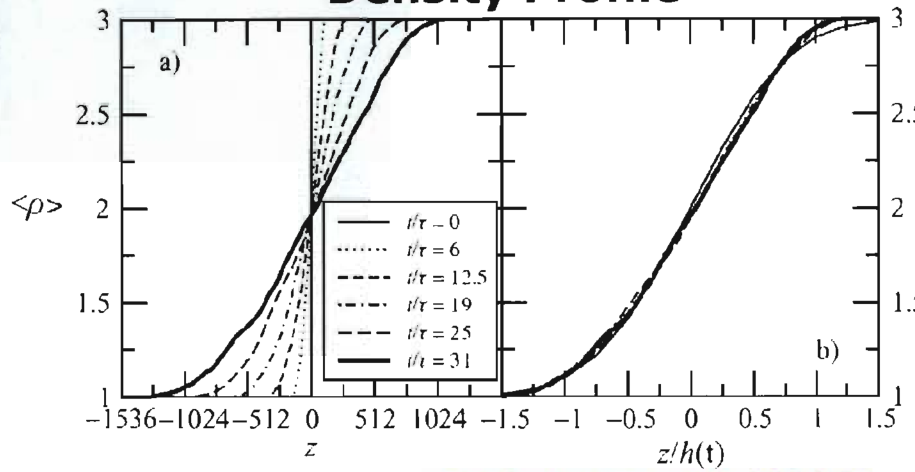
Approximation for Density Profile

Heavy Fluid Volume Fraction Profile



Dimonte *et al.*, Phys. of Fluids, **16** (2004)

Density Profile



Livescu *et al.*, J. Turbulence, **10** (2009)

$$\rho = f_l \rho_l + f_h \rho_h$$

$$\begin{cases} f_l = \dfrac{\rho - \rho_h}{\rho_l - \rho_h} \\ f_h = 1 - f_l \end{cases}$$

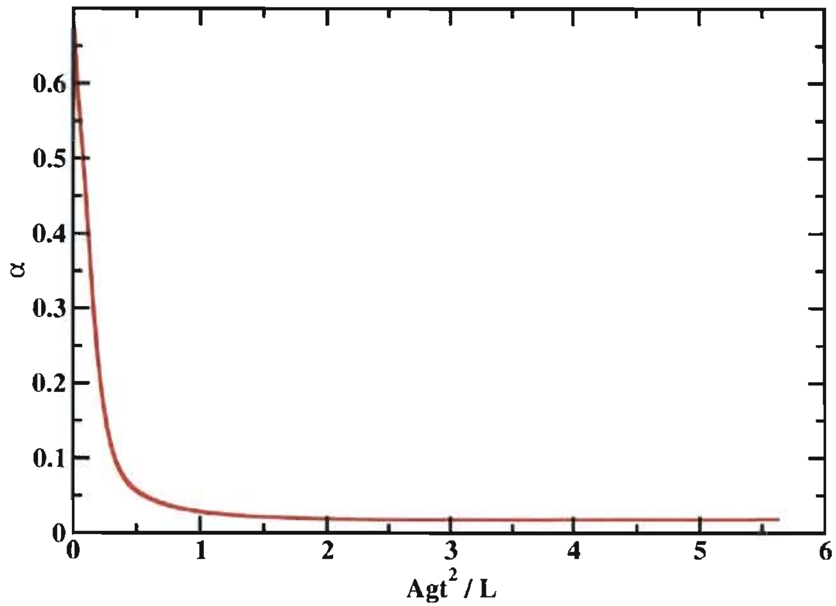
$$\begin{cases} f_h = 0 & \text{if} \quad z < -h_s \\ f_h = 0.5\dfrac{z + h_s}{h_s} & \text{if} \quad -h_s \le z < 0 \\ f_h = 0.5\dfrac{z}{h_b} + 0.5 & \text{if} \quad 0 \le z \le h_b \\ f_h = 1 & \text{if} \quad z > h_b \end{cases}$$

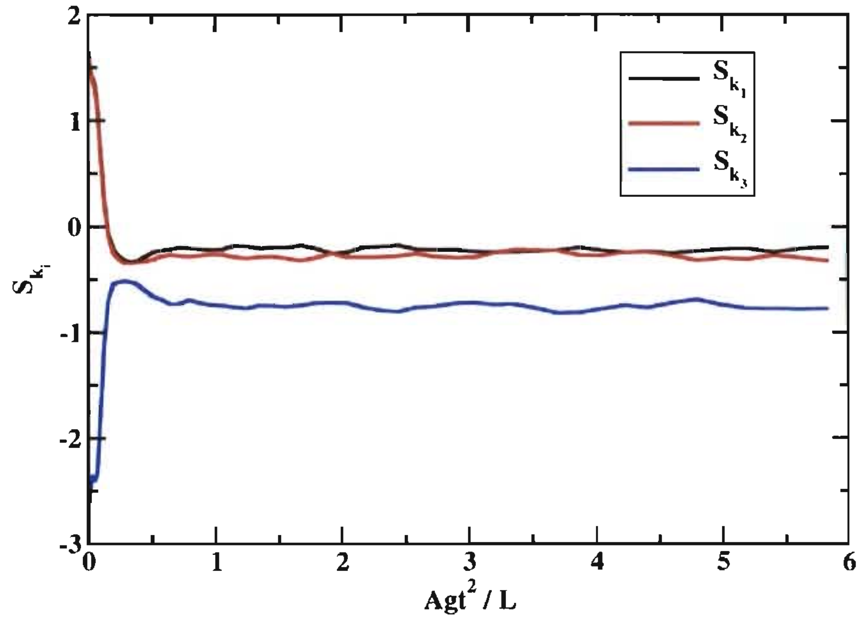
For a smooth mixture fraction description

$$\tilde{f}_h(z) = \int_{h_s}^{h_b} (z - h_s)^{a-1} (h_b - z)^{b-1} dz$$

$$f_h(z) = \frac{\tilde{f}_h(z)}{\tilde{f}_h(h_b)}$$

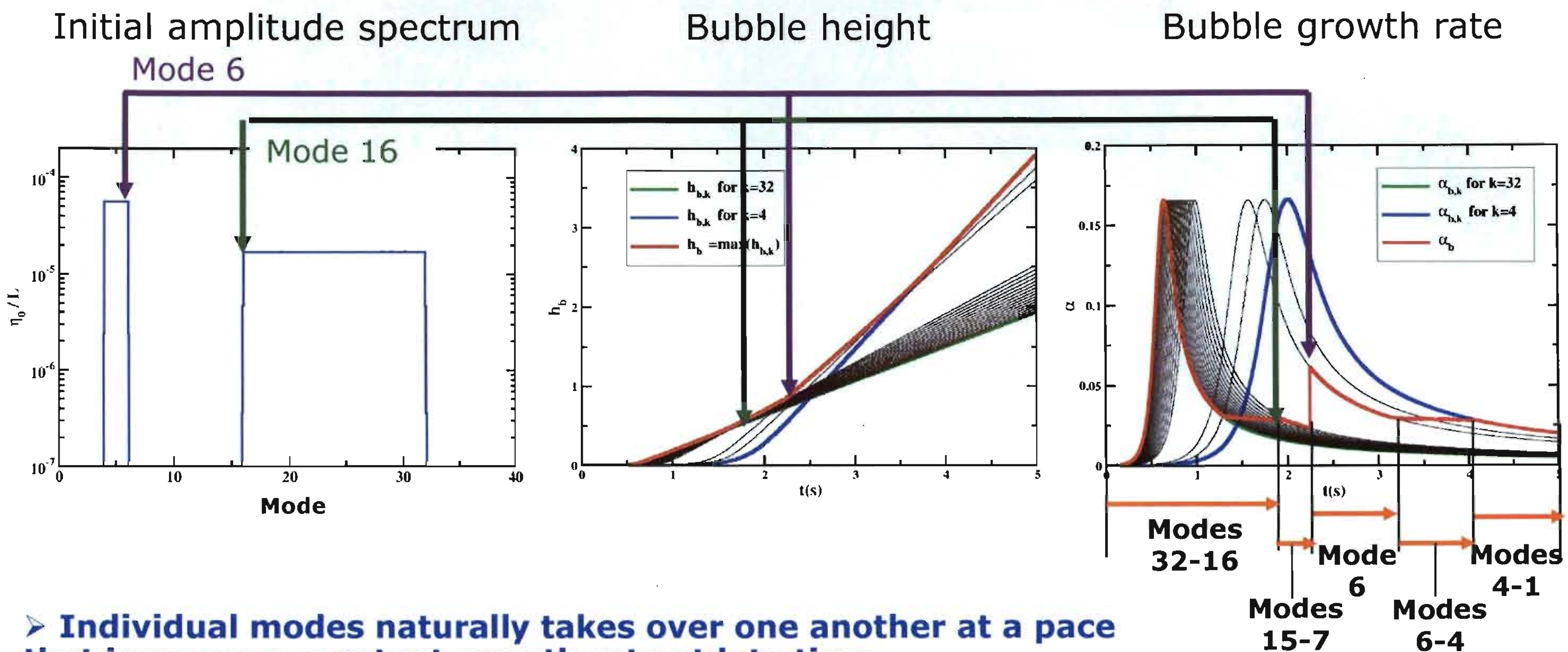
Establishement of a nonlinear cascade process





It appears that the establishment of a nonlinear cascade process occurs at about the same time as the mixing layer growth becomes self-similar

Multi-mode Study Using Ballistic ODE's to Predict the Envelope of the RT Bubble Mix Region



- **Individual modes naturally takes over one another at a pace that imposes a constant growth rate at late time**
- **Each band of the initial spectrum is 'seen' in the growth rate**
- **At late time, h_b is governed by the longest wavelength evolution**
- **Our ballistic model cannot be used after the longest wavelength of the initial perturbation spectrum has become dominant**

$$\alpha = \frac{U^2}{4Agh}$$

A Weakly Nonlinear Model for Multimode Perturbation

Haan's model:

$$\Delta\phi^{h/l} = 0$$

$$Z(\vec{x},t) = \sum_{\vec{k}} Z_k(t) e^{i\vec{k}\cdot\vec{x}}$$

$$\partial_t Z + \partial_x Z \cdot \partial_x \phi\big|_Z + \partial_y Z \cdot \partial_y \phi\big|_Z = \partial_z \phi\big|_Z$$

$$\phi^h(\vec{x},z,t) = \sum_{\vec{k}} \phi_k^h(t) e^{-kz} e^{i\vec{k}\cdot\vec{x}}$$

$$\left[\rho\left(\partial_t\phi + \frac{1}{2}v^2 + gZ\right)\right] = P$$

$$\phi^l(\vec{x},z,t) = \sum_{\vec{k}} \phi_k^l(t) e^{kz} e^{i\vec{k}\cdot\vec{x}}$$

$$\ddot{Z}_k = \gamma(k)^2 Z_k + A_T k \sum_{\vec{m}} \left\{ \ddot{Z}_m Z_n \left(1 - \hat{m}\cdot\hat{k}\right) + \dot{Z}_m \dot{Z}_n \left(\frac{1}{2} - \hat{m}\cdot\hat{k} - \frac{1}{2}\hat{m}\cdot\hat{n}\right)\right\}$$

Mode coupling term

$$\vec{n} = \vec{k} - \vec{m} \qquad \gamma(k) = \sqrt{A_T g k}$$

Haan's model allow mode generation