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Title: Estimating Single-Insult Failure Probabilities from Multiple-Insult Accelerated Tests

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Estimating Single-Insult Failure Probabilities from Multiple-Insult Accelerated Tests

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Overview

- Motivation and Motivating Example
- Probability Model
- Application
- Software

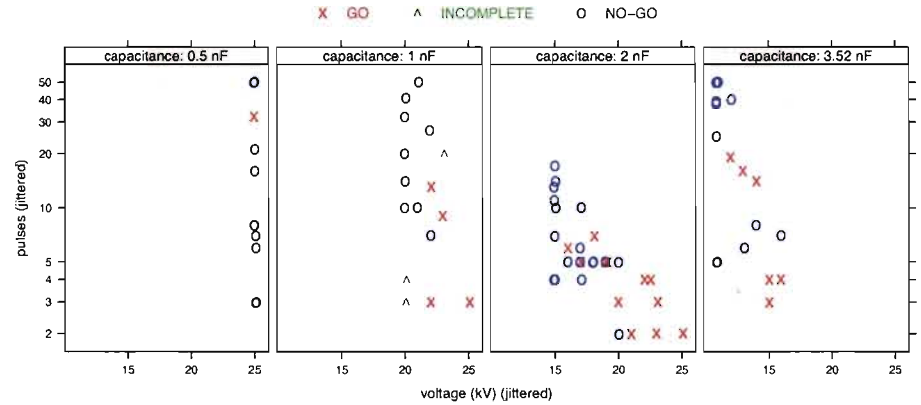
Motivation

- If a worker or an environment "insults" a detonator (det), how will det respond?
- An insult could be a physical shock (drop a tool on a det), electrostatic discharge (worker shuffles their feet while walking), etc.
- The det's response could be either "Go" or "No-Go"
- Engineers/Researchers want to know Pr(Go) for a single insult

Example from LANL

- ESD is controlled using voltage (kV) and capacitance (nF)
- Model Pr(Go) as a function of voltage and energy (mJ) where $E = \frac{1}{2}cv^2$
- It is believed that a det will Go if both energy and voltage exceed some unknown threshold value
- Interested in estimating:
  - Pr(Go) for a single insult
  - Voltage, Energy combinations that give 1×10^{-6} probability on first insult
- Dets are expensive therefore multiple insults are applied to each det
- Model needs to account for potential degradation with each additional insult

Example from LANL



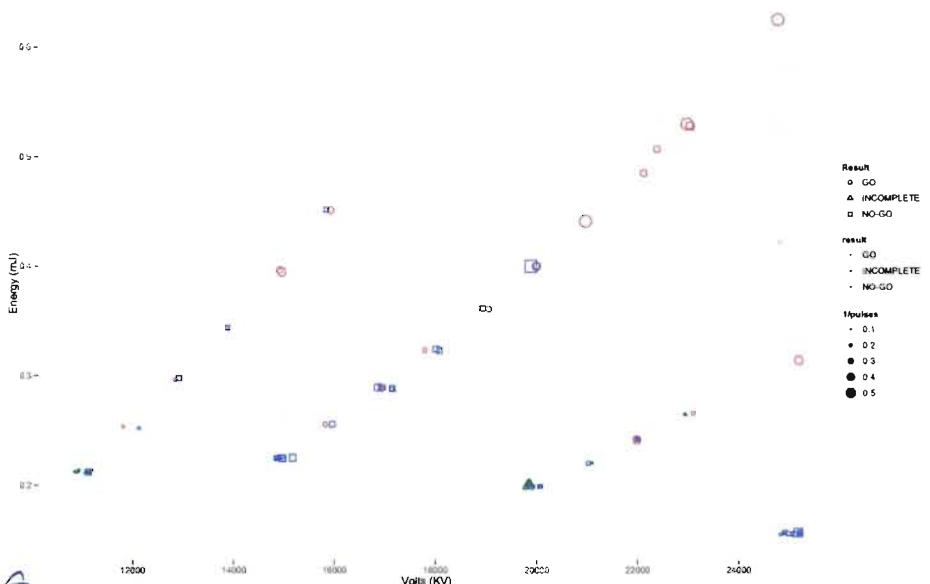
Example from LANL

- 75 dets were exposed to different levels of electric static discharge (ESD)
- Data contains:
  - Identifier for det
  - Capacitance and voltage for which a det was tested
  - Number of insults till either a Go or operator deterimines that det will never Go (No-Go)
  - Response, either Go or No-Go
  - 3 censored observations
- Censored observations occurred because operator determined det was being tested at too low of energy (E) and voltage (V) combination and moved to higher conditions (to conserve resources)

Model Motivation

- The fraction of units that will go with repeated insults varies with E and V
  - Model assumes limited failure population (LFP)
- Det experts claim that both E and V must exceed thresholds for a Go to occur
  - Model incorporates soft thresholds with E and V

Example from LANL



Probability Model

- Notation:
  - N is the number of insults till an event (Go/No-Go)
  - x_i is accelerating variable i which needs to exceed a soft threshold for $i = 1, \ldots, k$
  - p is the proportion of the population that will Go (equivalently the probability of a Go for an infinite number of hits)
  - $F_i(N; \boldsymbol{\theta}_i, x_i)$ is the cumulative probability for a det that will Go when x_i is the limiting factor
  - $\boldsymbol{\theta}_i$ is the parameter vector for distribution function F_i

Probability Model

- For an integer n

$$\Pr(N \leq n) = pF(n) = pF_1(n; \boldsymbol{\theta}_1, x_1) \cdots F_k(n; \boldsymbol{\theta}_k, x_k)$$

- Notice that as $n \to \infty$, $\Pr(N \leq n) \to p$, the failure proportion
- We model F_i using the Extended Generalized Gamma (EGG) Family of distributions $\forall i$
- $\boldsymbol{\theta}_i = (\mu_i, \sigma_i, \lambda_i)$
- We assume that:
  - $\mu = \mathbf{X}\boldsymbol{\alpha}$
  - $\log(\sigma) = \mathbf{X}\boldsymbol{\beta}$
  - $\lambda = \mathbf{X}\boldsymbol{\gamma}$
  - $\text{logit}(p) = \mathbf{X}\boldsymbol{\delta}$

EGG Family

- For fixed λ, EGG are log-location-scale distributions
- $\lambda = 0$ yields lognormal
- $\lambda = 1$ yields Weibull
- Vander Wiel (2009) reparameterizes this family so that if $Y \sim \mathrm{EGG}(m, s, \lambda)$, $\log(Y)$ has median m and density at m equal to $1/\sqrt{2\pi}s$ regardless of λ (Scott's EGG distribution)

Likelihood Contribution

- If a Go is observed at insult n, we treat as a discrete event $pF(n) - pF(n-1)$
- If a No-Go is observed, then we use $1 - p$ (survival function with $n = \infty$)
- If unit is censored at insult n, we treat as right censored $1 - pF(n)$

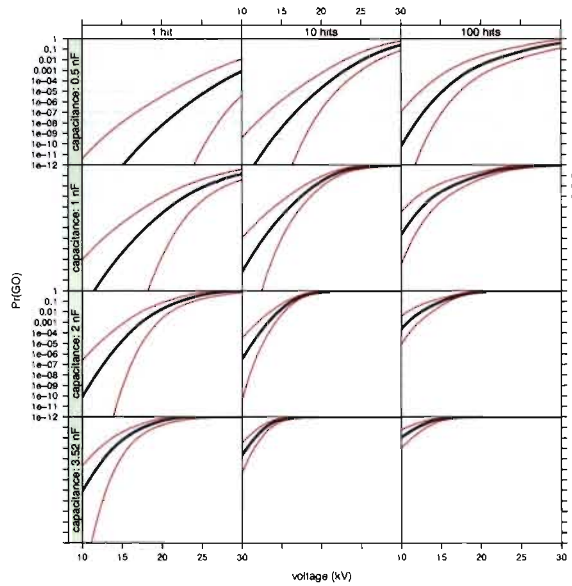
Prior Distributions

- $\alpha \sim$ wide normal
- $\beta \sim$ wide normal
- γ is not assigned a distribution (i.e., we assume we know this value)
- $\delta \sim$ wide normal

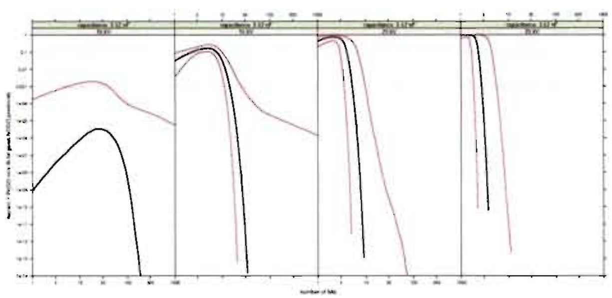
Data Analysis

- Scott and I are finishing up some general software that will utilize the product of CDFs
- The data analysis already performed only has one CDF where the location parameters is a function of log voltage and log energy
- We set $\lambda = 1$ (Weibull) as this is the most conservative for estimating lower tail quantiles

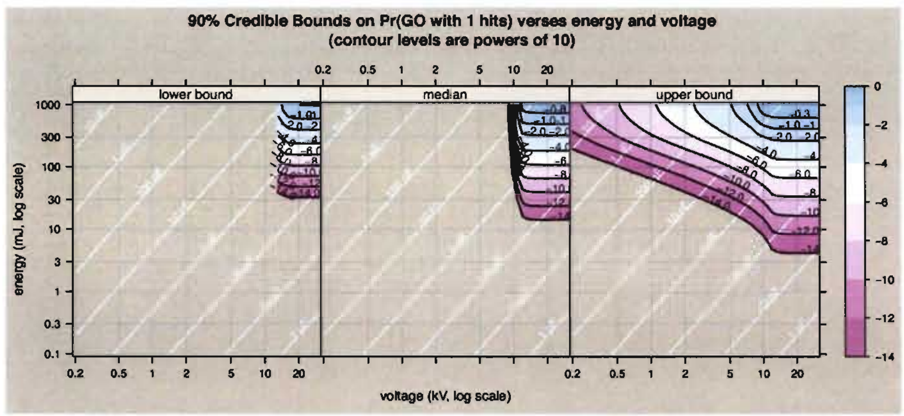
Posteriors: Pr(Go)



Posteriors: Hazard Function



Posteriors: Pr(Go) for $n = 1$



Where Do We Go From Here

- Model Assessment
- Develop frequentist based estimation to satisfy all philosophies

Informative Priors

Define

$$V50 = \text{voltage for dielectric breakdown on half of units on 1st insult,}$$
$$P = \text{relative witdh of the soft threshold for voltage; namely}$$
$$V10 = V50/(1+P) \qquad V90 = V50 \times (1+P).$$

An expert who has tested other similar families of detonators provided plausible conservative ranges:

$$V50 : 10 \text{ kV to } 30 \text{ kV}$$
$$P : 10\% \text{ to } 50\%$$

We translate the "plausible" ranges into independent priors:

$$\log(V50) \sim \text{N with 0.1 below and above given range}$$
$$\log(1+P) \sim \text{N with 0.1 below and above given range}$$

This implies priors on *functions of the parameters for the voltage CDF:*

$$\alpha_1/\alpha_2 \sim \textit{informative Normal}$$

Details on Prior to Honor range for V50

$$\Pr\{\text{dielectric breakdown on 1st insult at voltage } V\} = \Phi\left(\frac{\log 1 - \alpha_1 - \alpha_2 \log(V50/V_0)}{e^{\beta}}\right)$$

where Φ is the SEV distribution CDF with median zero and density at zero.
Let $z_p = \Phi^{-1}(p)$ and note that $z_{0.5} = 0$.
Then, V50 satifies

$$\frac{\log 1 - \alpha_1 - \alpha_2 \log(V50/V_0)}{e^{\beta}} = z_{0.5} = 0$$

and therefore

$$\log(V50) - \log(V_0) = -\alpha_1/\alpha_2.$$

Details on Prior to Honor range for V50

Take the nominal voltage (kV) to be the center of the elicited range on the log scale

$$\log(V_0) = (\log(10) + \log(30))/2$$

Therefore we obtain $\Pr(V50 < 10) = \Pr(V50 > 30) = 0.1$ by taking

$$\alpha_1/\alpha 2 \sim N\left[0, \left(\frac{\log(30/10)}{2 \times 1.28}\right)^2\right].$$

Details on Prior to Honor range for P

V10 satisfies

$$\frac{\log 1 - \alpha_1 - \alpha_2 \log(V10/V_0)}{e^{\beta}} = z_{0.1}$$

and similarly for V90. Therefore

$$\log(V10/V_0) = \frac{\alpha_1 + z_{0.1}e^{\beta}}{-\alpha_2} \quad \textit{and} \quad \log(V90/V_0) = \frac{\alpha_1 + z_{0.9}e^{\beta}}{-\alpha_2}.$$

But $V90/V10 = (1+P)^2$ and therefore

$$\log(1+P) = \frac{1}{2}\log(V90/V10) = \frac{1}{2\alpha_2}\left[\left(\alpha_1 + z_{0.1}e^{\beta}\right) - \left(\alpha_1 + z_{0.9}e^{\beta}\right)\right]$$
$$= \frac{e^{\beta}}{2}\frac{z_{0.1} - z_{0.9}}{\alpha_2}$$

Therefore we obtain $\Pr(P < 0.1) = \Pr(P > 0.5) = 0.1$ by taking

$$\text{RHS above} \sim N\left[\frac{\log(1.1)+\log(1.5)}{2}, \left(\frac{\log(1.5)-\log(1.1)}{2\times 1.28}\right)^2\right].$$