

A scaling rule for the collision energy dependence of a rotationally inelastic differential cross-section: a case study of $NO(X) + He$

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Method:

Exact Quantum Treatment (QM): Solve large sets (>1000) of coupled differential equations that contain all the relevant scattering channels $j, l, \Omega, \varepsilon$ to result the T matrix $T_{j',l',\Omega',\varepsilon',j,l,\Omega,\varepsilon}^J$ scattering process at each J and parity. In our case one has to do this for $J=0.5$ up to $J=120.5$. This all requires a state of the art computational effort!

Quasi-Quantum Treatment (QQT): employs a Feynman path alike integral over the angular variables in the kinematic apse frame. *QQT* assigns a phase factor to each scattering (ray) trajectory according its path length and its De Broglie wavelength. As a result the *QQT* provides a valuable physical insight while requiring very little computational effort^[1,2]. The *QQT* scattering amplitude follows from: $g(\beta, p; j', m'_a \leftarrow j=0, m_a) = C(\beta) \langle j', m'_a | g_{geom}(\gamma_a; \beta) \cdot \exp[i\eta_{j' \leftarrow j}(\gamma_a; \beta)] | j=0, m_a \rangle$ (1)

where the phase shift is: $\eta_{j' \leftarrow j}(\gamma_a; \beta, p) = -\mathbf{a}(\beta, p) \cdot \mathbf{R}_S(\gamma_a) = -[k_{\perp} - (-1)^p k'_{\perp}] \cdot \mathbf{R}_S(\gamma_R) \cdot \cos(\gamma_R - \gamma_a)$ (2)

and the geometric scattering amplitude is: $g_{geom}(\gamma_a; \beta) \equiv k \sqrt{\frac{d\sigma_{geom}(\gamma_a; \beta)}{d\omega_a}} = k \sqrt{\cos \beta} |\rho_1(\gamma_a) \cdot \rho_2(\gamma_a)|$ (3)

An important difference between the *QM* and *QQT* treatment is that the former is addressed in the collision frame, the state-to-state DCS is described by the spherical angles θ , while the latter is addressed in the kinematic apse frame, where the DCS is given by the spherical angles β .

Extension of *QQT* into the classically forbidden region:

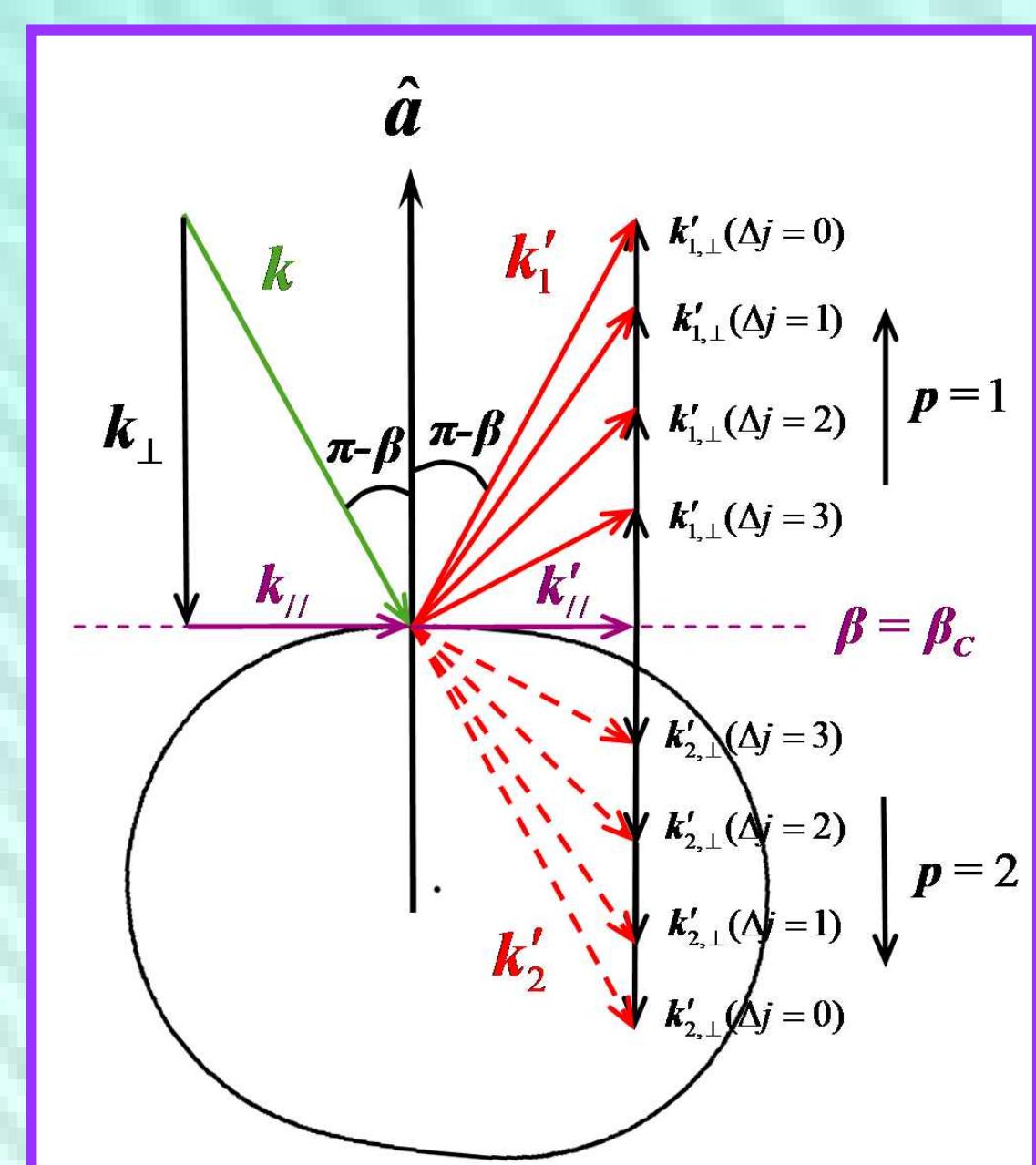


Fig. 1. Representation of the classically allowed (solid lines) and classically forbidden (dashed lines) Feynman paths that contribute to the scattering amplitude within the *QQT* formalisms.

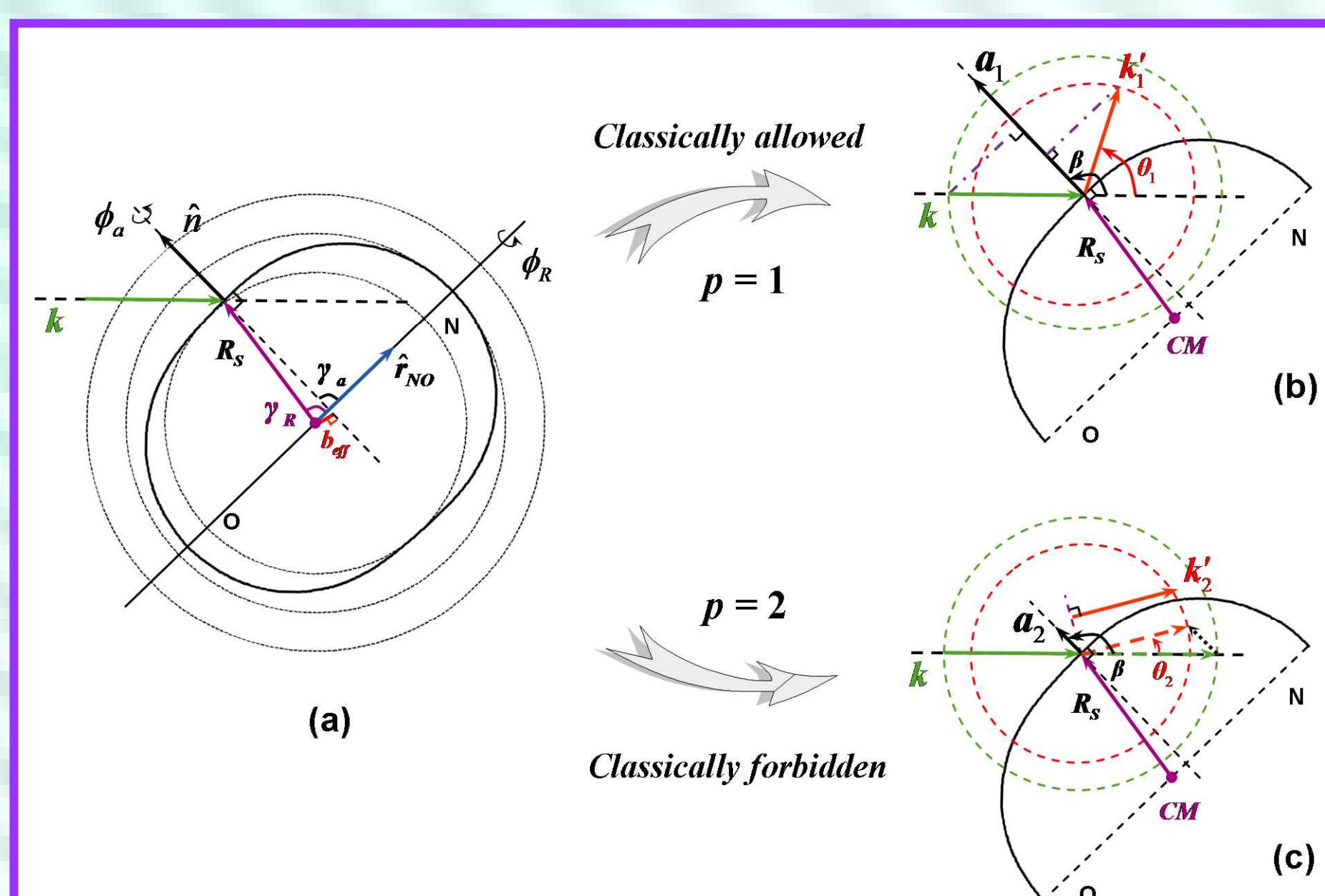


Fig. 2 Representation of the angles defining the *QQT* collision geometry (left hand panel), and the classically allowed (upper right hand panel) and classically forbidden (lower right hand panel) Feynman paths that contribute to the scattering amplitude within the *QQT* formalism.

Scaling process:

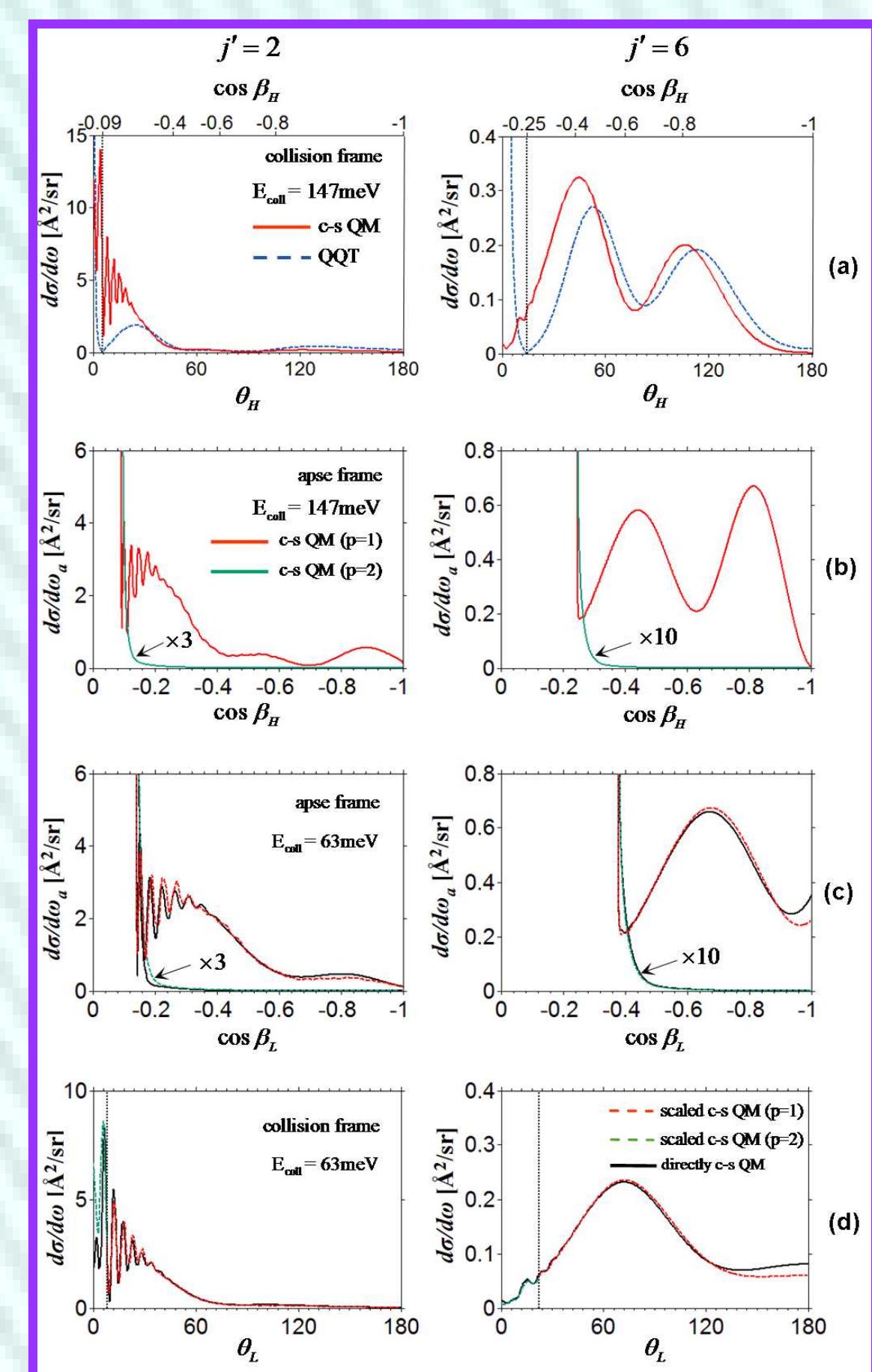


Fig. 3. Illustration of the step-by-step process used to scale the closed-shell QM DCSs from a high collision energy E_H to a lower collision energy E_L . The scattering of $NO(X) + He$ from $j=0$ to $j'=2$ and to $j'=6$ are considered as examples here.

The relation between the scattering angle θ and apse angle β is:

$$\cos \beta = \hat{a} \cdot \hat{Z} = \frac{k' \cos \theta - k}{\sqrt{k'^2 - 2kk' \cos \theta + k^2}} \quad (4)$$

$$\theta = \arccos \left[\left(\frac{k}{k'} \right) \sin^2 \beta + (-1)^p |\cos \beta| \left(1 - \left(\frac{k}{k'} \right)^2 [\sin^2 \beta] \right)^{0.5} \right] \quad (5)$$

$$\text{the cutoff angle is: } -1 \leq \cos \beta \leq \cos \beta_c = -\sqrt{1 - (k'/k)^2} \quad (6)$$

The relation between the apse and collision frame is:

$$\frac{d\sigma_{f \leftarrow i}}{d\omega}(\theta, \phi) = \frac{d\sigma_{f \leftarrow i}}{d\omega_a}(\beta, \alpha) \cdot \left| \frac{d\cos \beta(\theta)}{d\cos \theta} \right| \quad (7)$$

$$\text{where: } \frac{d\cos \beta}{d\cos \theta} = \frac{(k')^2 \cdot [k' - k \cos \theta]}{\{(k')^2 - 2kk' \cos \theta + k^2\}^{1.5}} \quad (8)$$

The phase shift, geometry scattering amplitude and the differential cross section calculated at collision energy of E_H and E_L relates to each other by:

$$g_{geom}^L(\gamma_a; \cos \beta_L) = \sqrt{\frac{k_L}{k_H}} g_{geom}^H(\gamma_a; \cos \beta_H = \frac{k_L}{k_H} \cdot \cos \beta_L) \quad (9)$$

$$\eta_{f \leftarrow i}^L(\gamma_a; \cos \beta_L, p) = \eta_{f \leftarrow i}^H(\gamma_a; \cos \beta_H = \frac{k_L}{k_H} \cdot \cos \beta_L, p) \quad (10)$$

$$\frac{d\sigma_{f \leftarrow i}^{QQT;L}(\cos \beta_L)}{d\omega_a} = \frac{k_H}{k_L} \frac{d\sigma_{f \leftarrow i}^{QQT;H}(\cos \beta_H = \frac{k_L}{k_H} \cos \beta_L)}{d\omega_a} \quad (11)$$

Results:

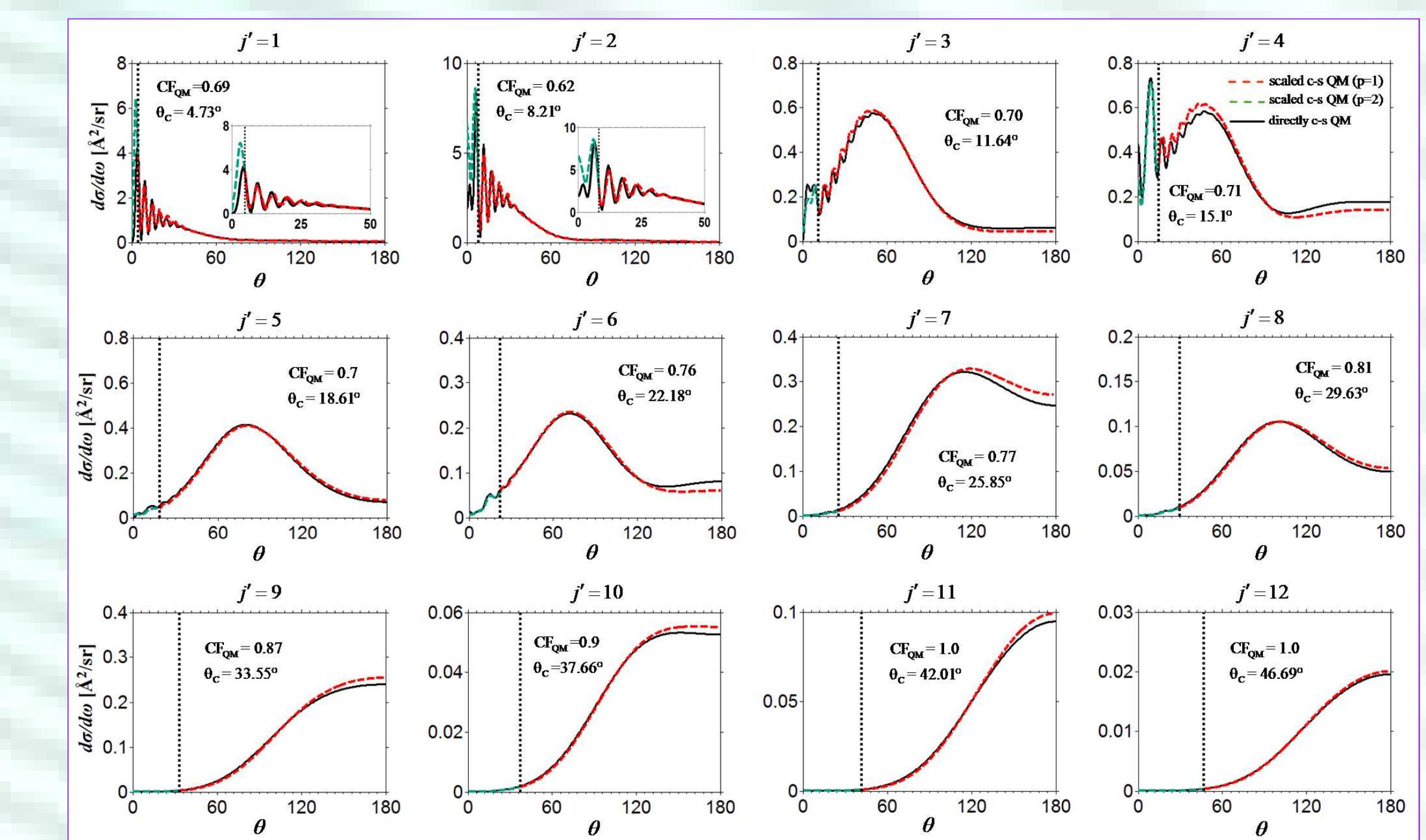


Fig. 4. Complete set of results for the comparison of the *closed-shell* QM $NO(X) + He$ DCSs from initial state $j=0$ to final states $j'=1-12$ scaled from a collision energy of 147meV to 63meV (dashed lines) with those DCSs calculated directly at 63meV (solid lines). Insets in the first two panels show a detailed comparison at low scattering angles. The point at which each transition becomes classically forbidden is shown as a dashed vertical line.

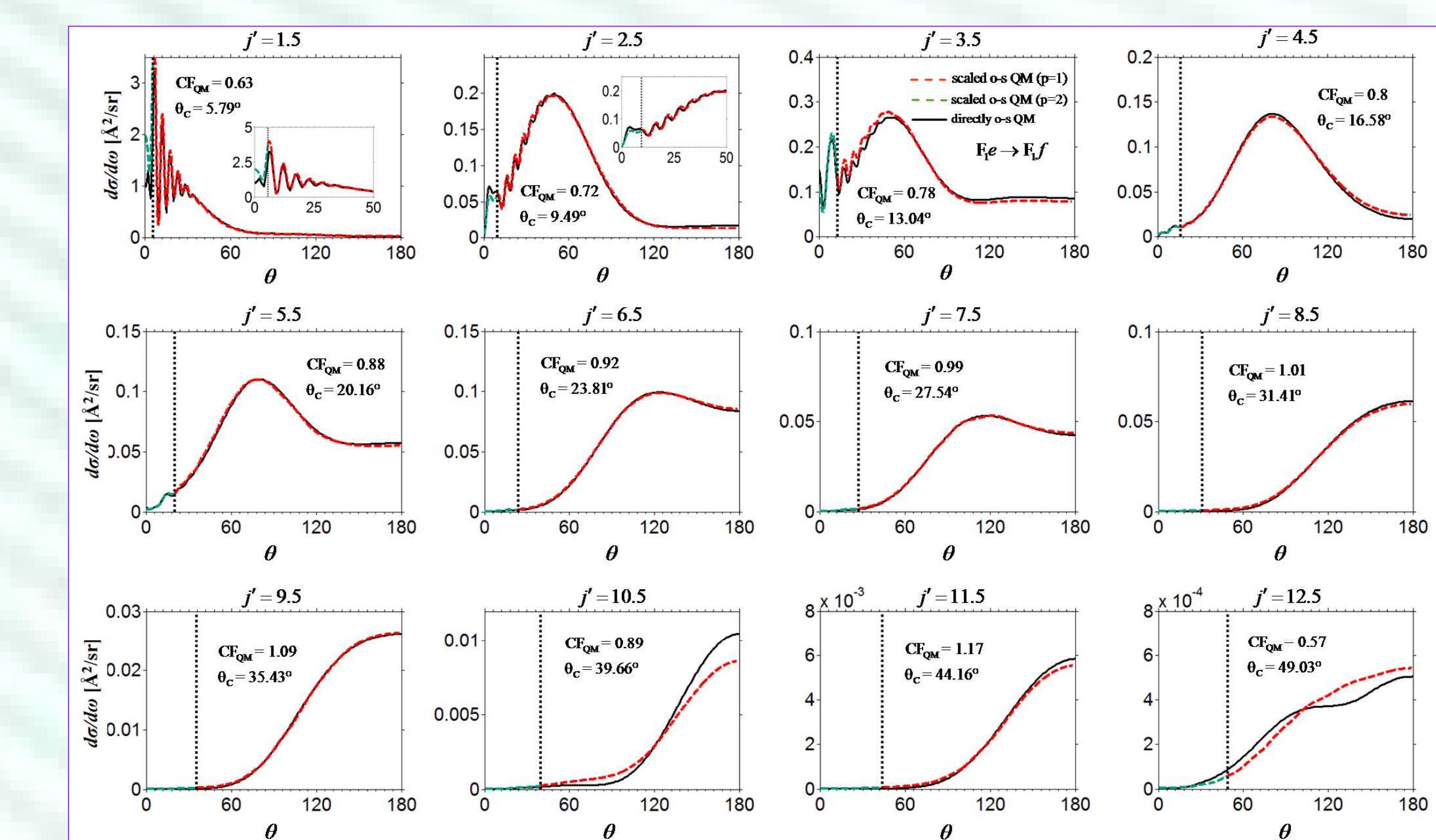


Fig. 5. Complete set of results for the comparison of the *open-shell* spin-orbit conserving QM $NO(X) + He$ DCSs from initial state $j=0.5$, e to final states $j'=1.5-12.5$, f scaled from a collision energy of 147meV to 63meV (solid lines) with those DCSs calculated directly at 63meV (dashed lines). Insets in the first two panels show a detailed comparison at low scattering angles.

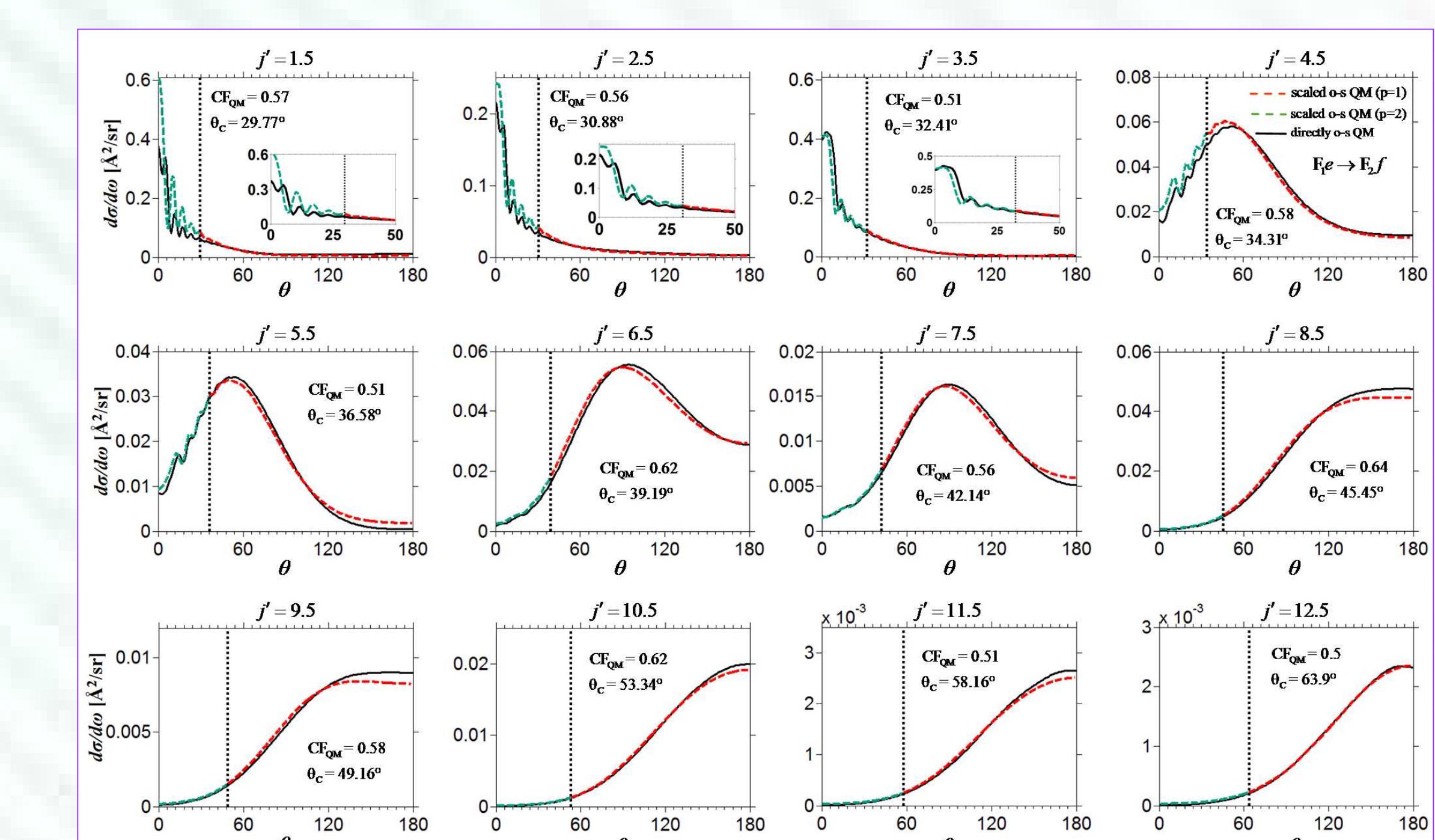


Fig. 6. same as fig.5 but for the *open-shell* spin-orbit changing QM $NO(X) + He$ DCSs transition from $e \rightarrow f$

References:

- [1] A. Gijsbertsen, H. Linnartz, C.A. Taatjes, S. Stolte, *J. Am. Chem. Soc.* 128(2005)72.
- [2] A. Ballast, A. Gijsbertsen, S. Stolte, *Mol. Phys.* 106(2008)315.
- [3] J. Kłos, G. Chalasinsky, M.T. Berry, R. Bukowski, S.M. Cylbulski, *J. Chem. Phys.* 112(2000)2195.