

Coupled Flow and Mechanics in Porous and Fractured Media

Mario J Martinez, Joseph E Bishop, Patrick K Notz and Daniel Z Turner

**Engineering Sciences Center
Sandia National Laboratories
Albuquerque, NM, USA**

Presented at the 2011 US National Congress on Computational Mechanics

This material is based upon work supported by the Sandia National Laboratories LDRD program and as part of the Center for Frontiers of Subsurface Energy Security, an Energy Frontier Research Center funded by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences under Award Number DE-SC0001114.

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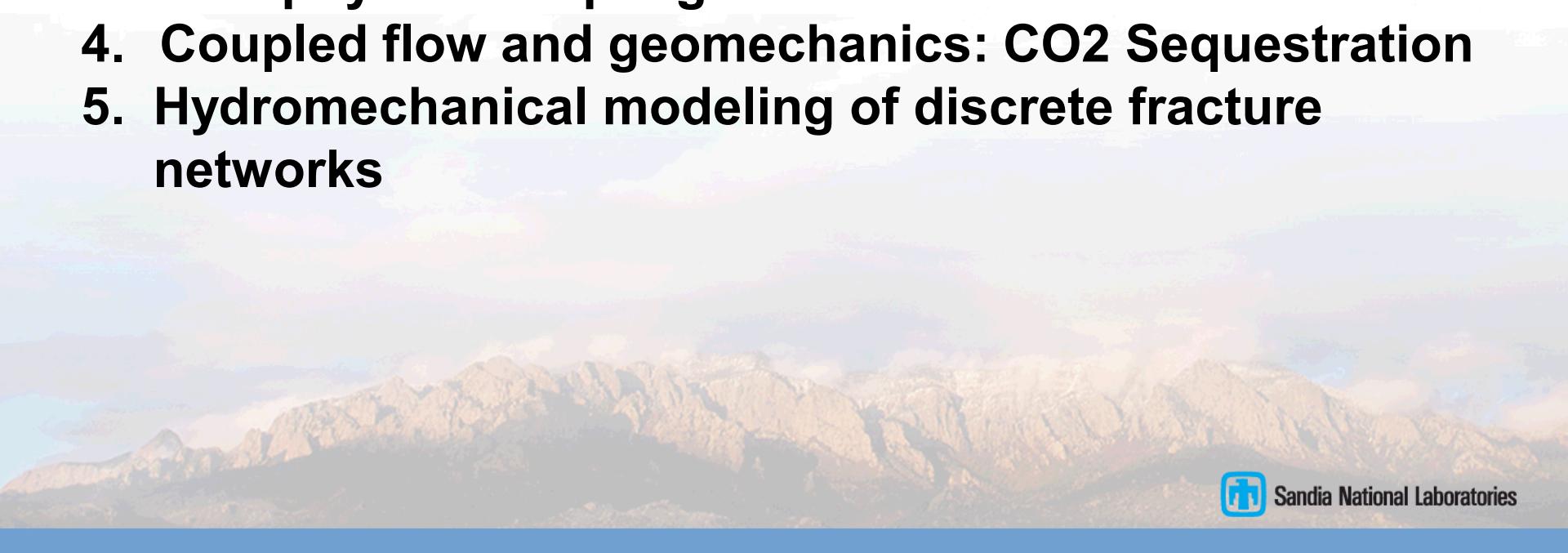


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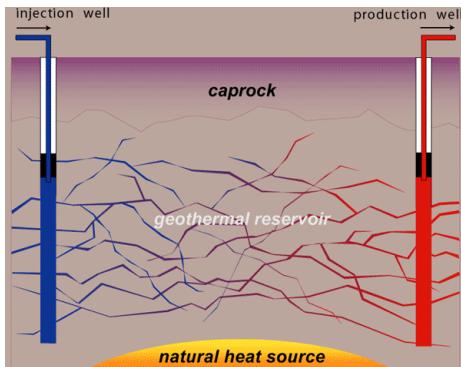
Outline

- 1. Motivation: Geosciences at Sandia National Laboratories**
- 2. Sierra software system**
 - i. Geoscience capabilities**
 - ii. Examples**
- 3. Multiphysics coupling in Sierra**
- 4. Coupled flow and geomechanics: CO2 Sequestration**
- 5. Hydromechanical modeling of discrete fracture networks**

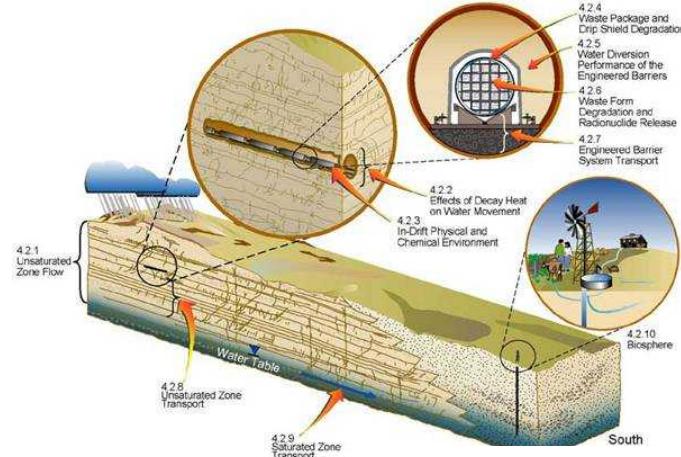


Geoscience Applications at SNL

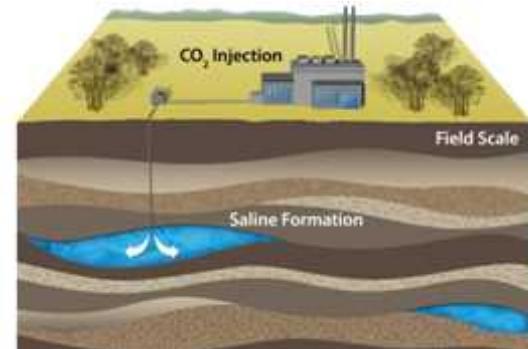
Engineered Geothermal



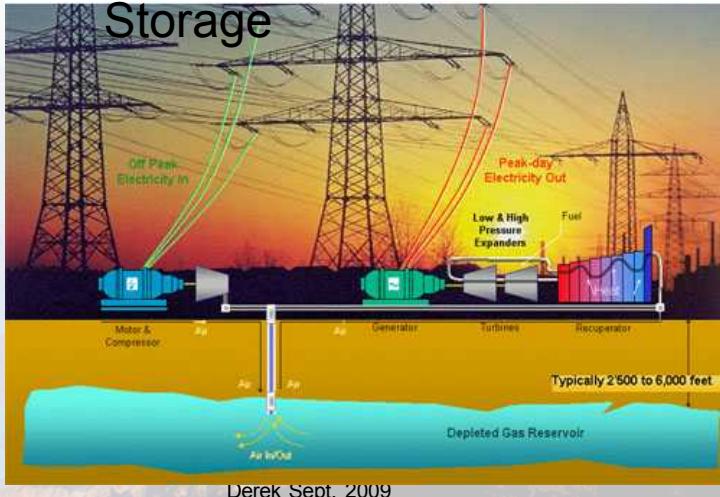
Nuclear Waste Isolation



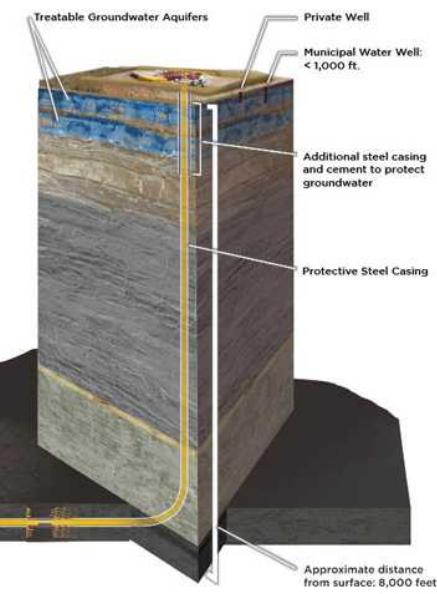
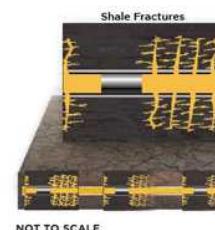
CO₂ Sequestration



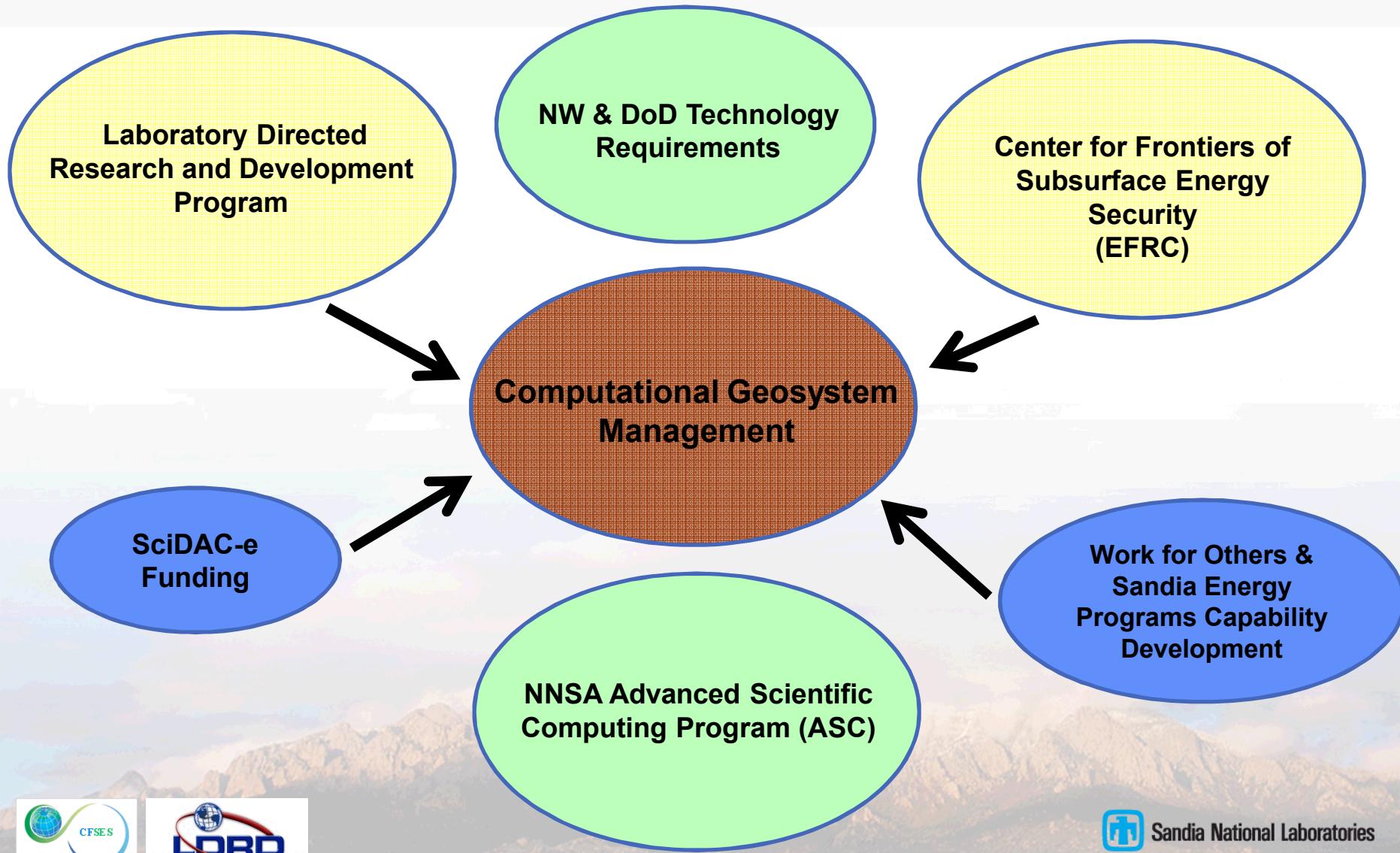
Compressed Air Energy Storage



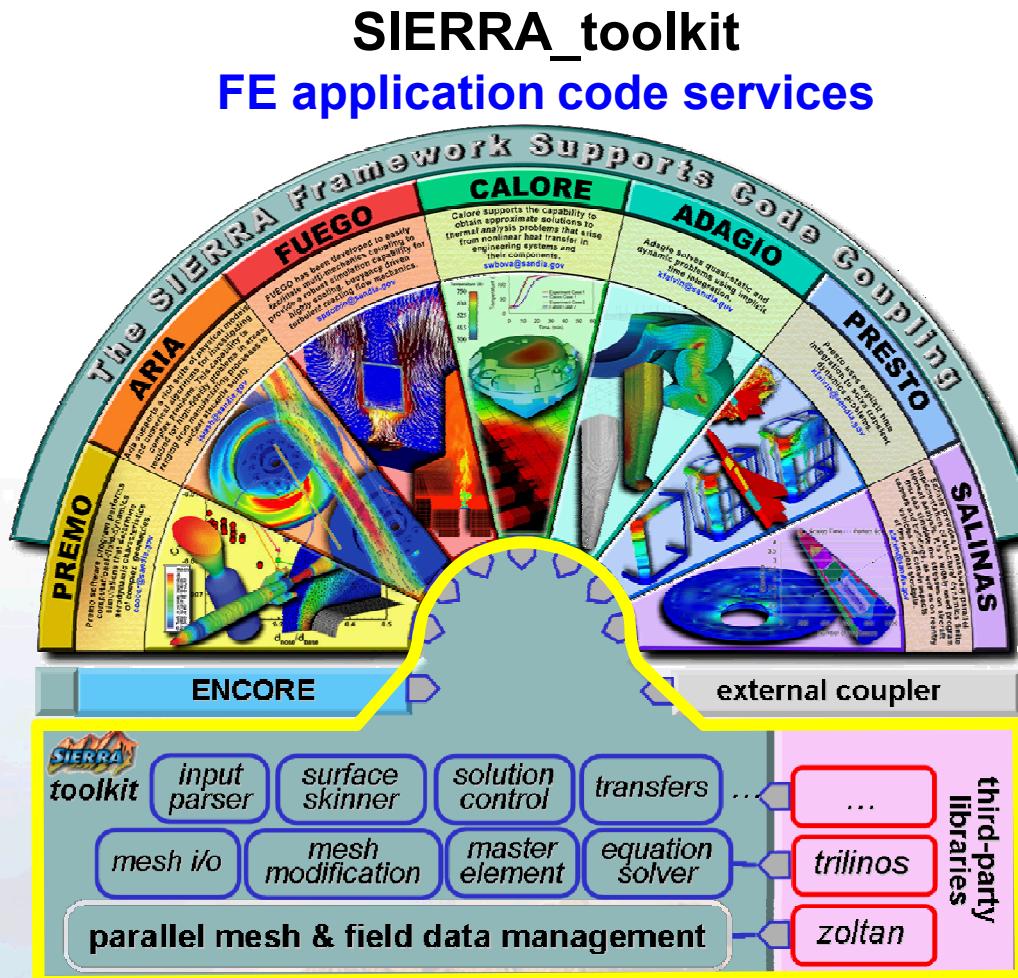
Hydraulic Fracturing



Sandia Computational Geoscience Research and Subsurface Management Program



Subsurface simulation software leverages SIERRA Mechanics Foundation



Services provided to mechanics applications:

- Mesh & field data management (parallel, distributed)
- Transfer operators for mapping field variables from one mechanics to another
- Solution controller for code coupling: Arpeggio
- Includes third party libraries (e.g. solver libraries, MPI communications package)
- Accommodates heterogeneity



SIERRA Mechanics Represents Enabling Capability for Coupled Geoscience Multiphysics Simulation

- Large deformation, large strain mechanics
- Nonlinear constitutive behavior

Coupling: temperatures, pore pressures, saturations, capillary pressure

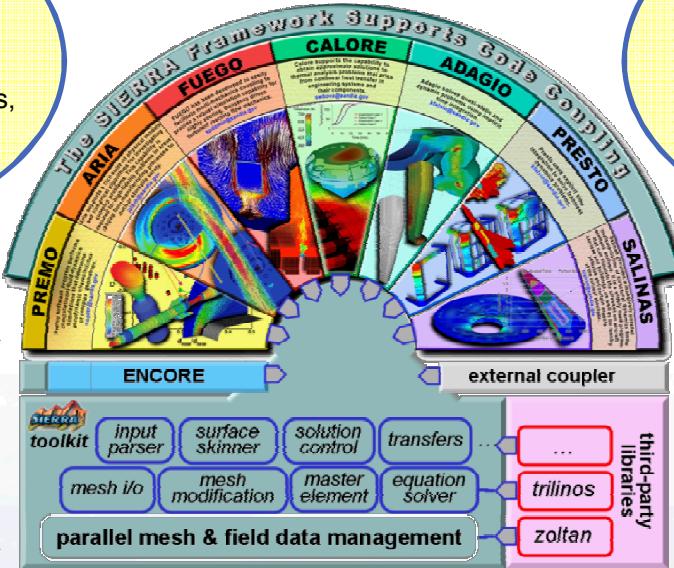
- Multiphase, multicomponent, noisothermal porous flow

Coupling: displacements, stresses, permeability, porosity

- Heterogeneity
- Material
- Initial state

- Sorption
- Reaction-transport modeling
- Water/rock interaction kinetics
- Brine/CO₂ reactions

Coupling: temperature, porosity, Darcy velocity, permeability



Coupled Algorithms



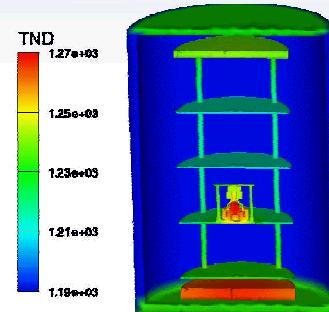
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SNL Thermal/Fluid Modeling Capabilities in Sierra Mechanics

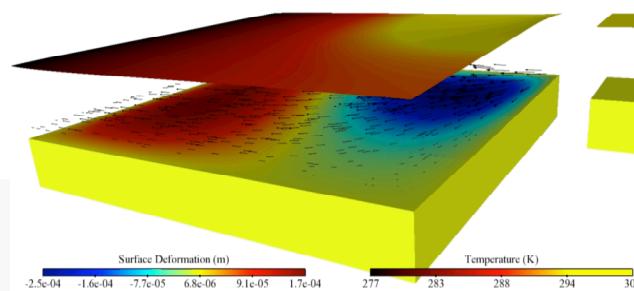
- **Aria** – Non-Newtonian, Multi-physics, and Free Surface Flows

- Fully-Coupled, Galerkin FEM (GFEM)
- Complex material response
- Level sets for surface tracking
- Flexible coupling schemes



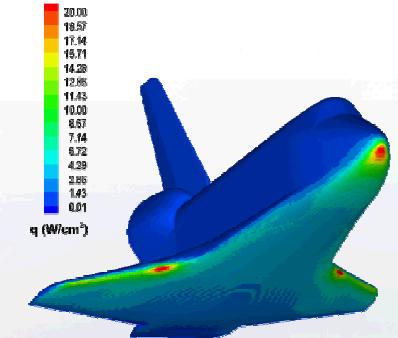
- **Calore** – Heat Transfer, Enclosure Radiation and Chemistry

- Galerkin FEM (GFEM)
- Dynamic enclosures
- Element birth death
- Contact



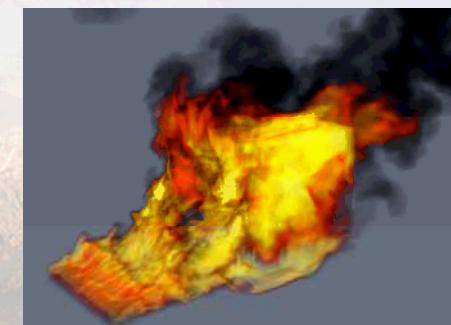
- **Premo** – Compressible Fluid Mechanics

- Fully-Coupled, Edge-Based Finite Volume (node centered)
- Subsonic through hypersonic
- Laminar and turbulent



- **Fuego** – Low Speed, Variable Density, Chemically Reacting Flows (Fire)

- Loosely-Coupled, Control Volume FEM (CVFEM)
- Eddy dissipation and mixture fraction reaction models
- RANS and LES based turbulence models
- Pressurization models





CDFEM Capability Development Status

Objective:

Implement CDFEM capability in Sierra Mechanics to track moving interfaces and capture interfacial physics between multiple materials or phases

Approach:

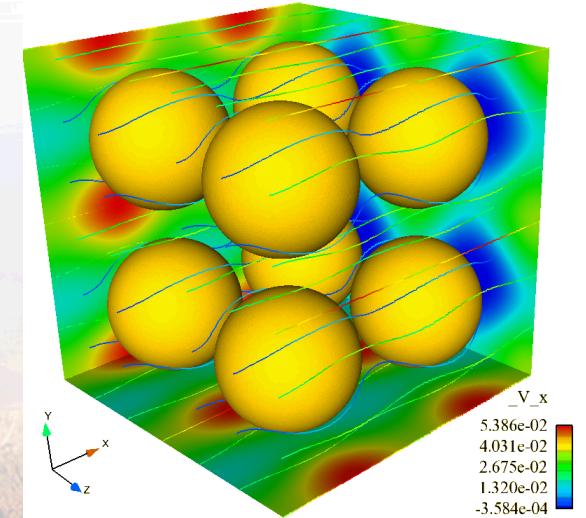
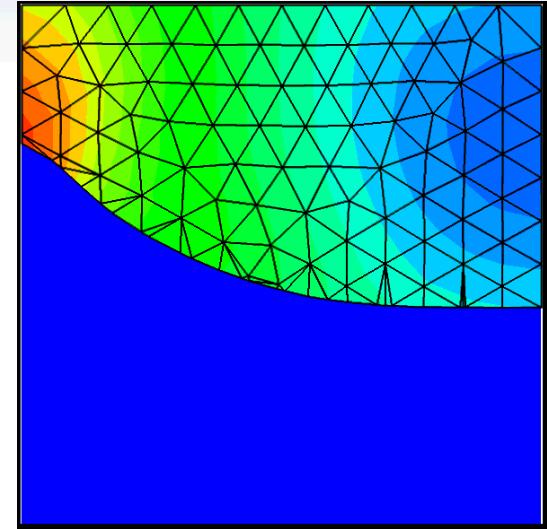
- Use level set technology to capture implicit interfaces
- Decompose non-conformal elements into conformal ones and obtain solution on conformal elements
- Use ALE technology to handle nodes that change phase

Accomplishments:

- Implemented CDFEM for steady state and transient interface problems
- Capability verified to scale well in parallel
- Tested on air-water capillary flows, solid suspension flows, and thermal phase change problems
- Verified accuracy of CDFEM to be 2nd order for potential flow, conduction, two fluid viscous flow, and blunt body flow problems

Status:

- Extending scheme to track interface based on various interfacial physics including reactions, phase change, and thermal degradation
- Extending capability to multiple phases (more than two)

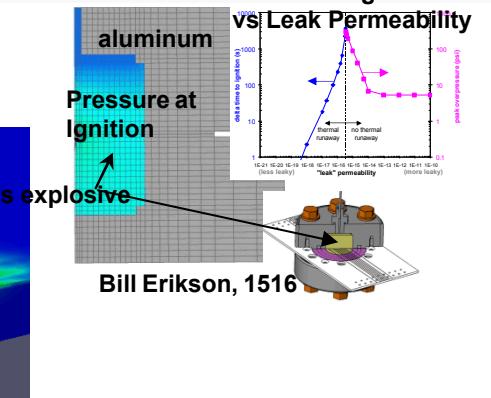


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Overview of Porous Flow in Aria

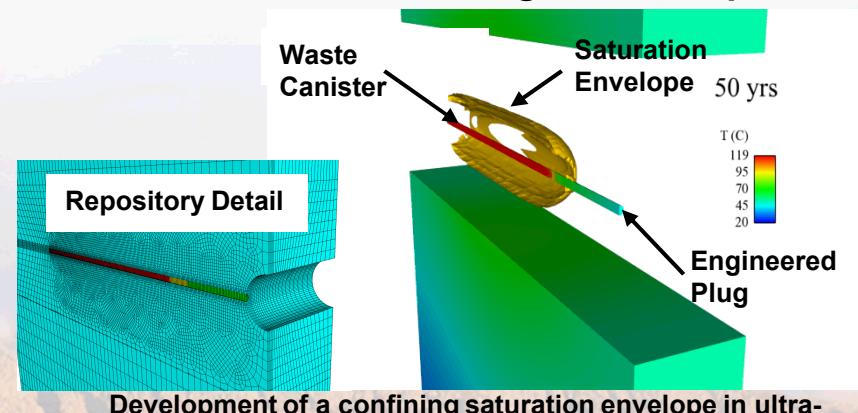
- Leveraged development under LDRD & EFRC
 - Targets SNL activities in energy security, conventional munitions, thermal batteries, heat pipes, ...
- Current capabilities
 - Single phase heat and reactive mass flow
 - Immiscible two-phase flow
 - Two-phase, two-component (air & water) evaporating/condensing thermal model
 - Chemically reactive flows (e.g. calcite mineralization)
 - Spatially heterogeneous material and transport properties
 - Couples with mechanics and other Sierra physics modules
- Capability under development
 - Nonisothermal two-phase CO₂-H₂O-NACL EOS with general phase behavior
 - Advanced discretization schemes (UT technology)

Modeling Cook-Off in Granular Explosives



CO₂ saturation levels in a brine-filled reservoir represented with uncorrelated heterogeneous permeability

Heat-Generating Waste Disposal



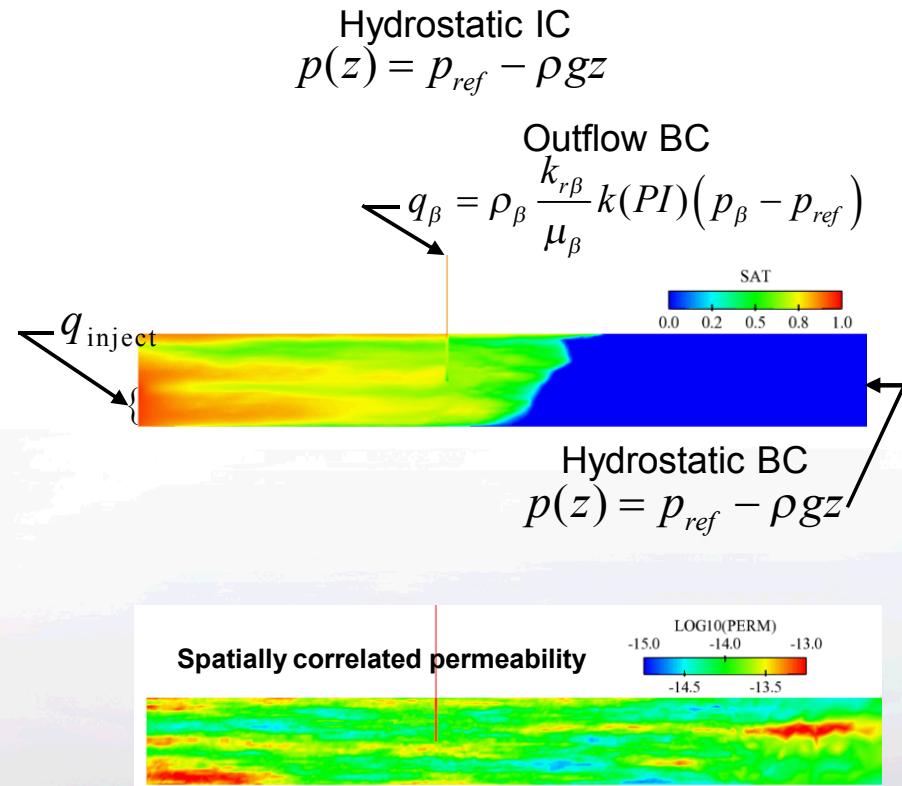
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Boundary and Initial Conditions

- Initial Conditions
 - Element block dependent
 - Function of coordinates
 - Linear, quadratic built-in
 - Output file from a previous solution
 - User Plug-in
- Dirichlet BCs (nodeset, sideset)
 - `f(t,x,soln_vector,expression)`
 - Many built-in forms
 - Plug-in
- Flux BCs (sidesets)
 - `f(t,x,soln_vector,expression)`
 - Many built-in forms
 - Third-type BCs
 - Outflow (e.g. wells)
 - Periodic
 - XFER – values set from a transfer
 - Encore function
 - Plug-in
- Distinguishing Condition (constraint)
 - Replaces original equation
 - Implemented in weak form
 - E.g. Kinematic BC with deforming mesh (ALE):

$$R^i = \int_S (\mathbf{n} \bullet (\mathbf{v} - \dot{\mathbf{x}}) - v_{leak}) N^i \, dS = 0$$

Fingering of CO₂ Injected into a Heterogeneous Aquifer with Leaky Well



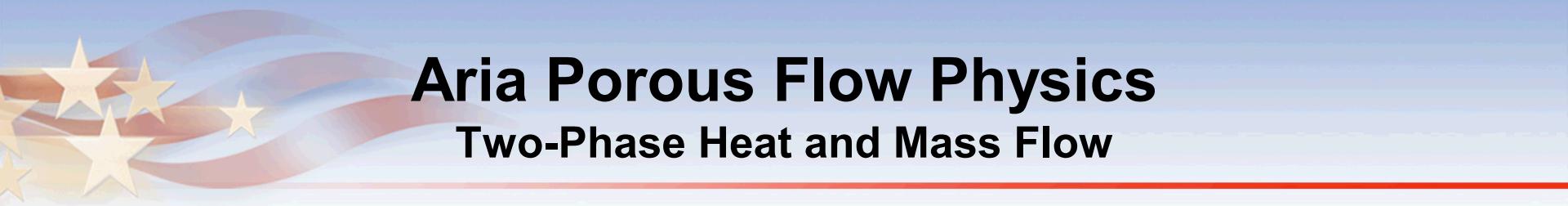


Aria Multiphase Porous Flow Physics

Two-Phase Two-Component Non-Isothermal Model (water, air, energy)

- Water - steam tables, equilibrium thermodynamic phase partitioning
- Air - ideal gas, Henry's Law partitioning
- Energy - conduction, latent and sensible energy transfer, including binary diffusion of heat
- Evaporating/condensing flows
- Phase appearance/disappearance via persistent variables approach
- General specification of transport property dependence on solution vector (e.g., densities, viscosities, diffusion coeffs. depend on T, P, phase volume fraction)
- Capillary pressure, relative permeability models
- Specification of heterogeneous property fields (e.g. permeability, porosity, GSLIB linkage)
- Other EOS under development





Aria Porous Flow Physics

Two-Phase Heat and Mass Flow

Mathematical Model

- **Two-Phase Component Mass and Energy Balances:**

$$\frac{\partial}{\partial t} \begin{bmatrix} \phi(S_l \rho_l^w + S_g \rho_g^w) \\ \phi(S_l \rho_l^a + S_g \rho_g^a) \\ (1-\phi) \rho_s e_s + \phi(\rho_l S_l e_l + \rho_g S_g e_g) \end{bmatrix} + \nabla \bullet \begin{bmatrix} \mathbf{F}_l^w + \mathbf{F}_g^w \\ \mathbf{F}_l^a + \mathbf{F}_g^a \\ \mathbf{q}_e \end{bmatrix} + \begin{bmatrix} Q_w \\ Q_a \\ Q_e \end{bmatrix} \begin{array}{l} \text{water} \\ \text{air} \\ \text{energy} \end{array}$$

- **Net Mass Flux:**

$$\mathbf{F}_\beta^\alpha = Y_\beta^\alpha \rho_\beta \mathbf{v}_\beta + \mathbf{J}_\beta^\alpha$$

α = component

β = phase

- **Darcy Velocity:**

$$\mathbf{v}_\beta = -\frac{k_{r\beta}}{\mu_\beta} \mathbf{k} \bullet (\nabla P_\beta - \rho_\beta \mathbf{g})$$

- **Binary Diffusion (gas phase):**

$$\mathbf{J}_g^\alpha = -\rho_g D_g^\alpha \nabla Y_g^\alpha$$



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Aria Porous Flow Physics

Two-Phase Heat and Mass Flow

Mathematical Model (cont.)

- **Total Energy Flux (heat conduction, convection, binary diffusion):**

$$\mathbf{q}_e = -\lambda_T \nabla T + \sum_{\beta} \rho_{\beta} \mathbf{v}_{\beta} h_{\beta} + \sum_{\alpha} h_g^{\alpha} \mathbf{J}_g^{\alpha}$$

- **Saturation Constraint:** $S_l + S_g = 1$
- **Mixing Rules:** $\sum_{\beta=l,g} Y_{\beta}^{\alpha} = 1, \quad \alpha = w \text{ (water), } a \text{ (air)}$
- **Capillary Pressure:** $P_g - P_l = P_c(S_l)$
- **Relative Permeability:** $k_{r,\beta} = f(S_l, T, \dots)$





Spatial Discretization

- **Unstructured Grid Finite Element-Based Discretization**
- **Finite-dimensional basis representation:** $f(x, u) = \sum_J N_J(x) f_J(u)$
- **Residual-based implicit weak formulation:**

$$\begin{bmatrix} R_{w,I} \\ R_{a,I} \\ R_{e,I} \end{bmatrix} = \int_{\Omega} N_I \sum_J N_J \begin{bmatrix} \dot{d}_{w,J} \\ \dot{d}_{w,J} \\ \dot{e}_J \end{bmatrix} d\Omega - \int_{\Omega} \nabla N_I \bullet \begin{bmatrix} \mathbf{F}_w \\ \mathbf{F}_a \\ \mathbf{q}_e \end{bmatrix} d\Omega - \int_{\Omega} N_I \begin{bmatrix} Q_w \\ Q_a \\ Q_e \end{bmatrix} d\Omega + \int_{\Gamma} N_I \begin{bmatrix} \mathbf{F}_w \bullet \mathbf{n} \\ \mathbf{F}_a \bullet \mathbf{n} \\ \mathbf{q}_e \bullet \mathbf{n} \end{bmatrix} d\Gamma$$

Hybrid Features:

- **Mass lumping for multiphase systems**
- **Control-volume finite element upwind scheme for convective terms**
 - **Supports tensor permeability**





Nonlinear Solution Procedures

Discretization leads to nonlinear system:

$$\mathbf{R}(\mathbf{u}) = (\mathbf{R}_w, \mathbf{R}_a, \mathbf{R}_e)^T = \mathbf{0}$$

Nonlinear Solve:

- Newton Iteration

$$\mathbf{J}(\mathbf{u}^q) \delta \mathbf{u}^{q+1} = -\mathbf{R}(\mathbf{u}^q)$$

$$\mathbf{u}^{q+1} = \mathbf{u}^q + \delta \mathbf{u}^{q+1}$$

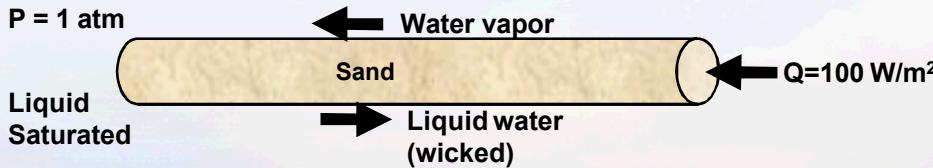
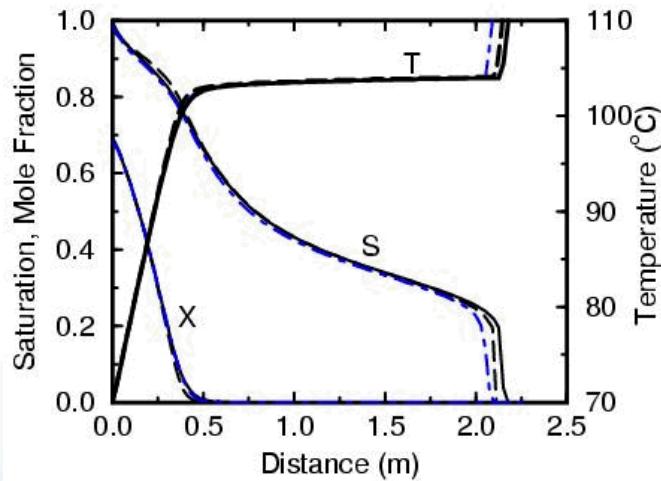


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Coupled Mass and Heat Flow Examples

Porous Heat Pipe with Non-Condensible Gas

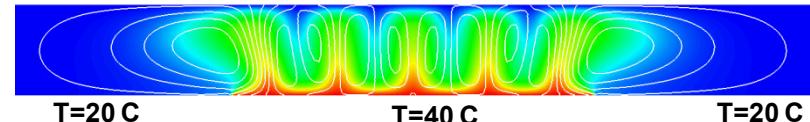
(solid – PorSalsa/upwind, dashed – TOUGH2/upwind, dash-dot – Aria/gfem)



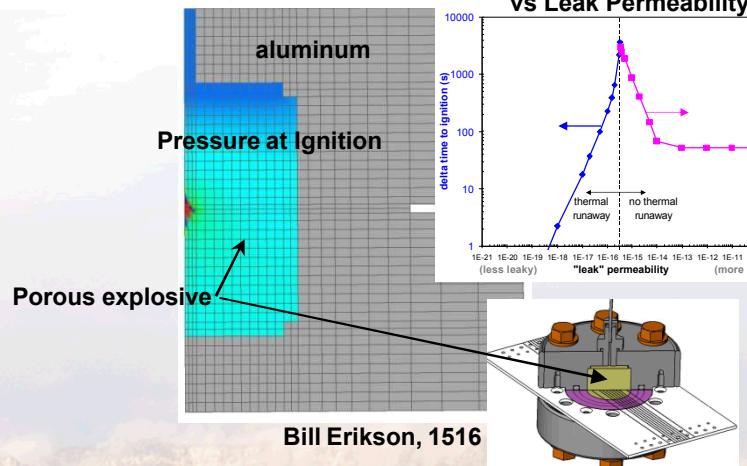
Elder Problem

Buoyant Convection in a Porous Layer

$Ra = 150$



Modeling Cook-Off in Granular Explosives



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CO₂ Leakage Through an Abandoned Well

Reference Problem Description:

- 3D model of leakage during supercritical CO₂ injection into a brine aquifer
- Single CO₂ injection well
- Two aquifers separated by an aquitard
- One leaky well, 100 m from injection well
- 500 k elements, 1200 day injection

Assumptions:

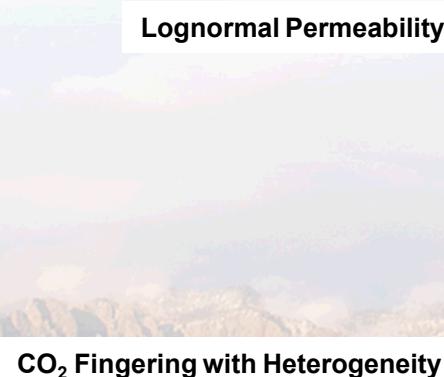
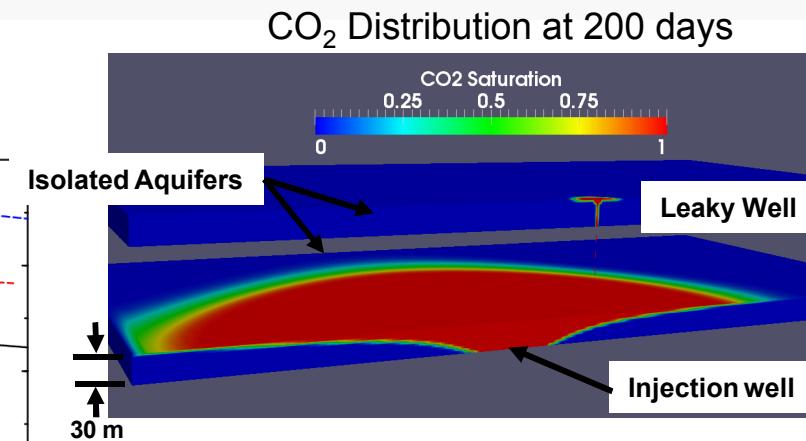
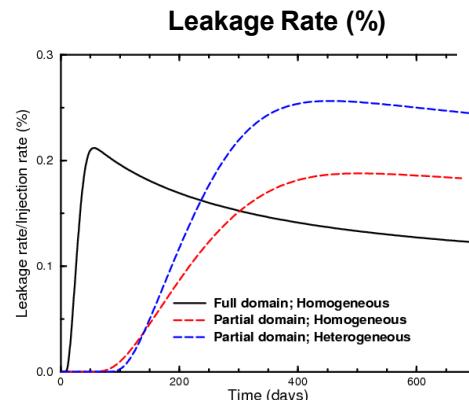
- Isothermal injection process
- CO₂ and brine immiscible phases
- Isotropic formation
- Neglect capillary pressure

Results:

- Computed leakage rate and arrival times compare well with benchmark study

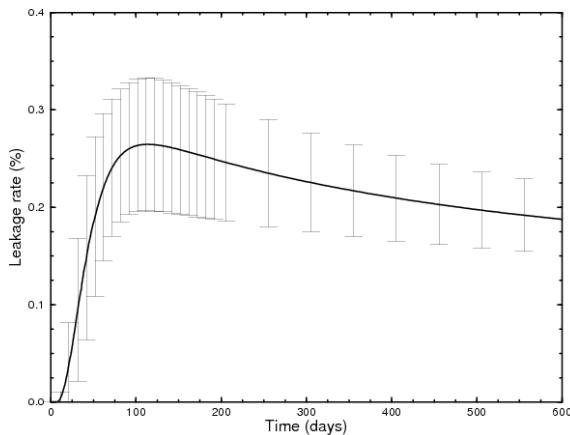
Effects of Heterogeneous Permeability:

- Geologic aquifers are highly heterogeneous
- Truncated domain allows multiple realizations
- Lognormal distribution of permeability, normal distribution of porosity
- Highly non-uniform (fingering) injection in the presence of heterogeneity
- Lognormal permeability increases the leakage rate



CO₂ Leakage Through an Abandoned Well Effects of Heterogeneity

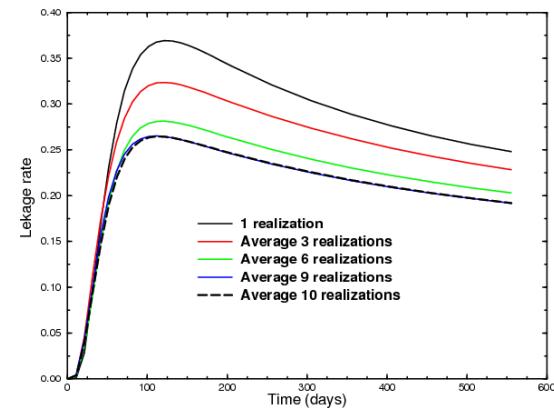
Average Leakage Rate and Std. Dev.



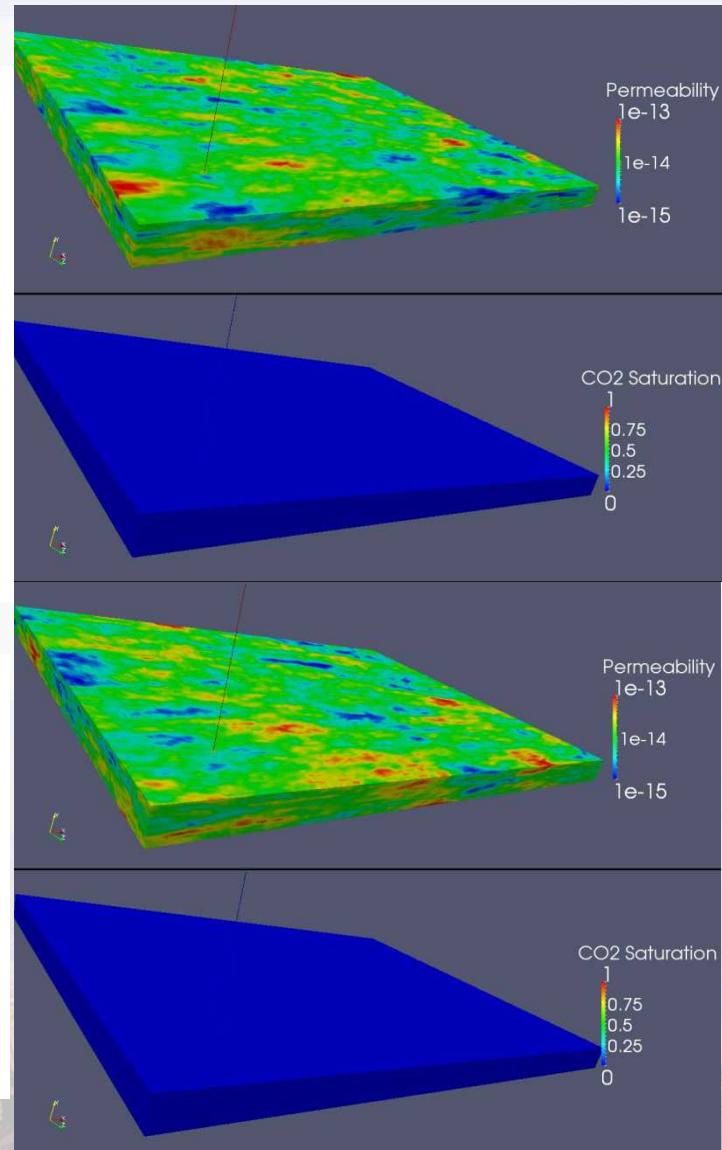
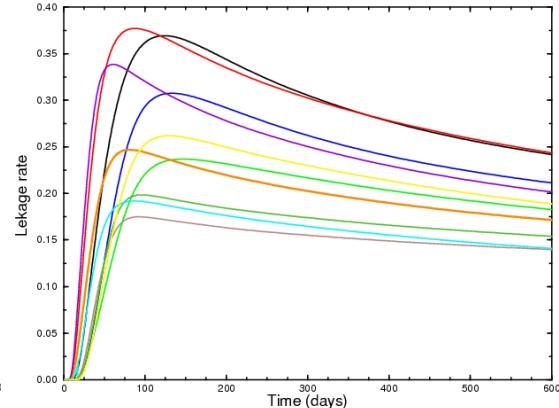
Some Results (10 realizations)

- Correlation between fast paths and permeability distribution is evident
- Leakage, arrival time are heavily dependent on permeability distribution
- Standard deviations are substantial
- Appears useful results can be obtained from a few realizations

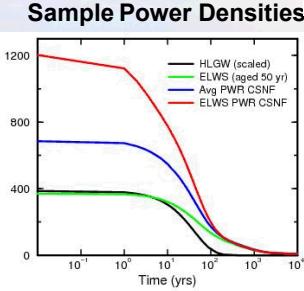
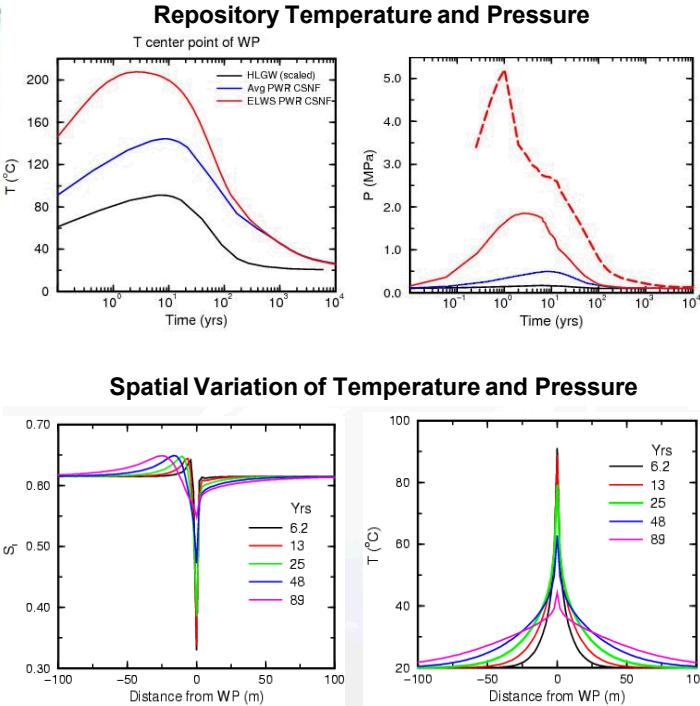
Leakage Rate: Running Average



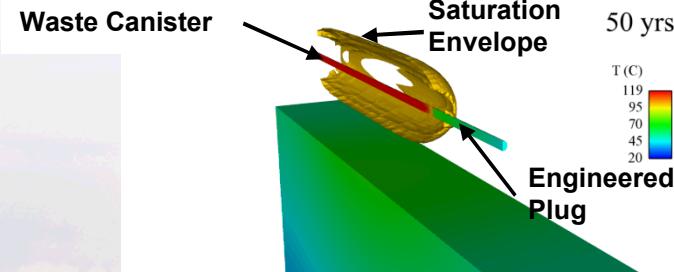
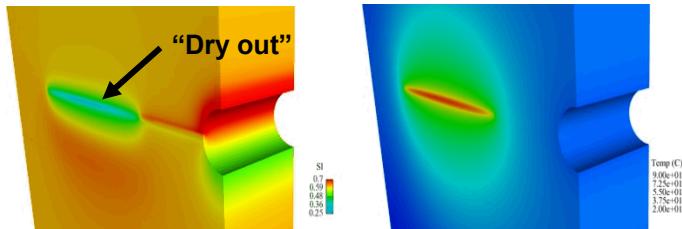
Leakage Curves



High Level Waste Disposal in Clay Thermo-Hydrologic Features



High decay powers in ultra-low permeability clays can result in dry out regions and saturation envelopes.



Development of a confining saturation envelope in ultra-low permeability clays, trapping gases within.



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Overview of Geomechanics in Adagio

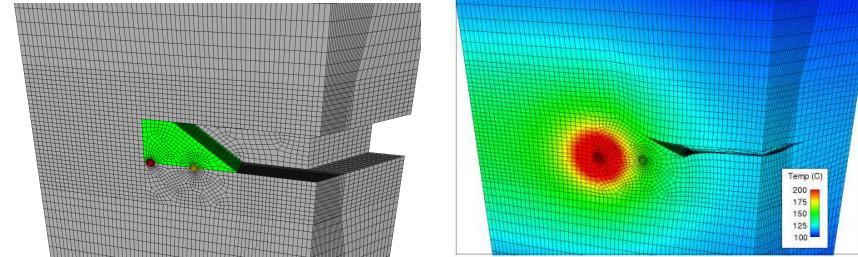
Features:

- Large deformation, large strain kinematics
- Robust contact algorithms (both detection and application)
- Based on iterative (matrix-free) solvers with low order hourglass stabilized 8-node hexahedron element
- Efficient constitutive model implementations

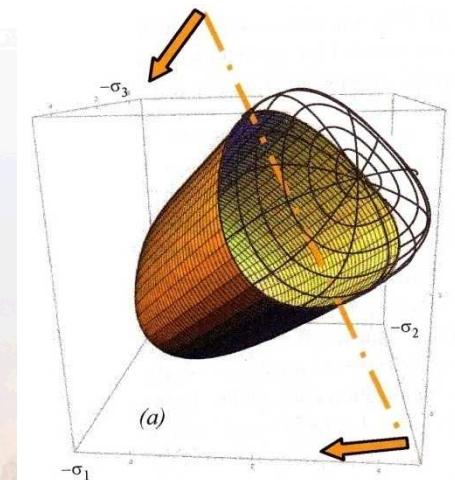
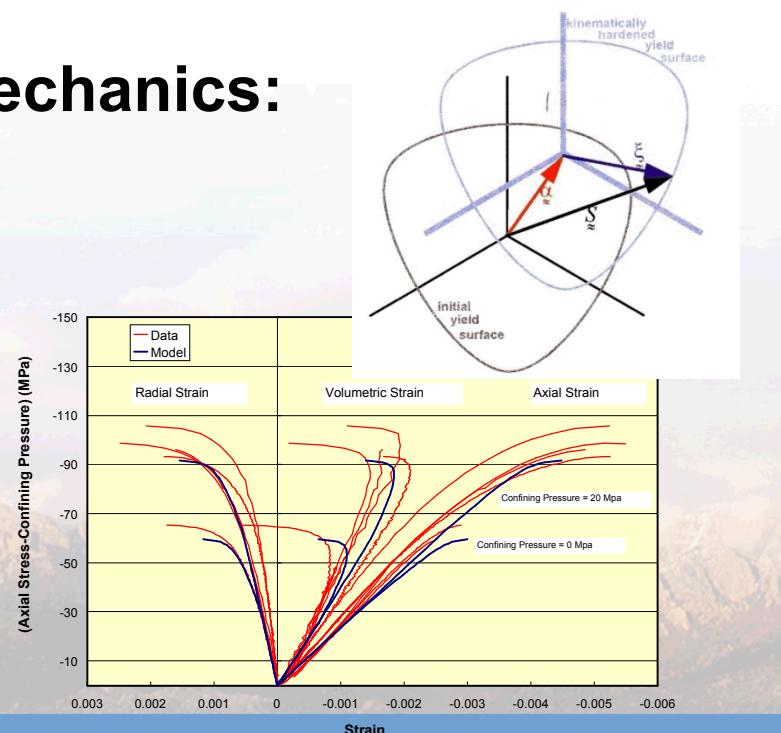
Models for Geomechanics:

- Elastic
- Elastic/Plastic
- Soil Foam
- Power Law Creep
- MD Creep Model
- Crushed Salt Creep Model
- Clay
- GeoModel

Thermally Enhanced Creep Closure

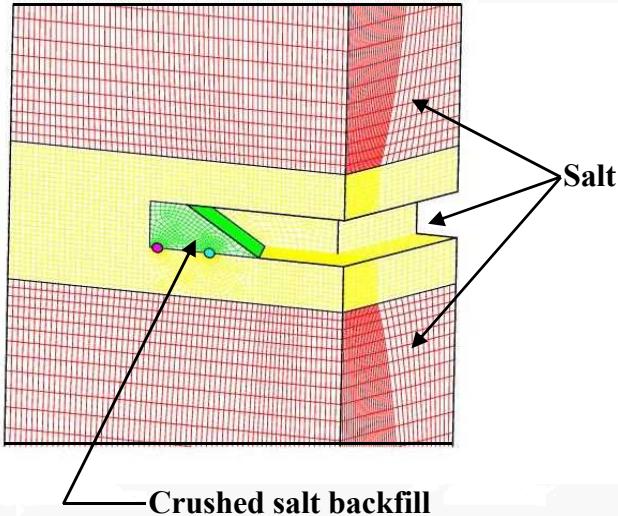


Undeformed and Deformed Storage Tunnels in a Heated Salt Repository



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Sierra Mechanics Simulation of Salt Repository



- Three-dimensional fully coupled thermal/mechanical analysis
- Massively Parallel Calculation - 96 processors
- Dissimilar meshes and domains for thermal and structural mechanics
- Contact surfaces used for both thermal and structural problems

Thermal Analysis Features:

- 904736 nodes / 864927 elements
- Contact surfaces used to accommodate heat conduction between contacting surfaces (alcove and haulage way)
- Re-computation of radiation view factors for deforming heated room surfaces

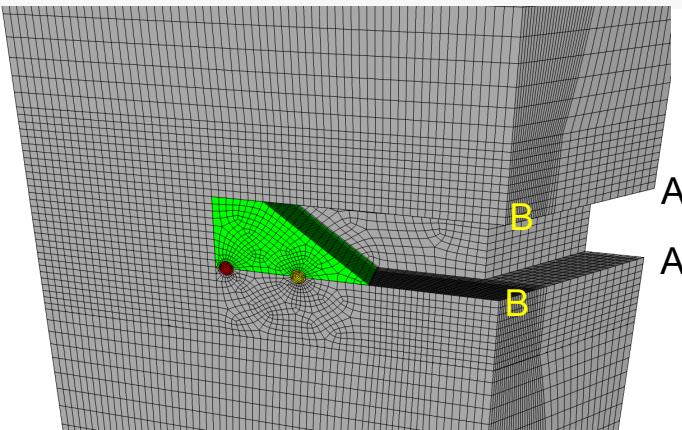
Structural Analysis Features:

- Quasistatic analysis with 294698 nodes / 279537 elements
- Large deformation, large strain formulation
- Nonlinear power law secondary creep model for salt
- Volumetric compaction model for the crushed salt
- Contact surfaces defined to allow arbitrary roof, rib, and floor contact
- Temperature dependent material properties

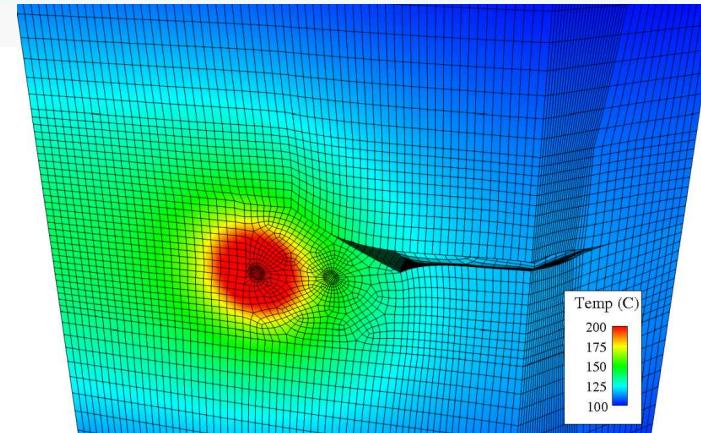


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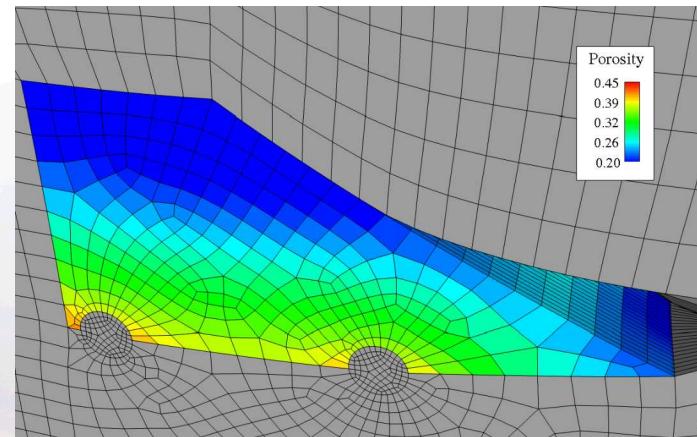
Generic Salt Repository Closure Details



Cross-section A-A



Generic salt repository final deformed state

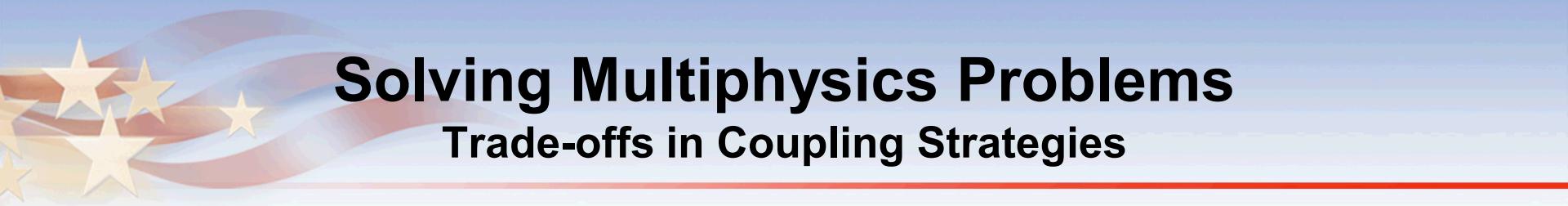


Corresponding porosity in crushed salt backfill

Cross-section B-B



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Solving Multiphysics Problems

Trade-offs in Coupling Strategies

Full Coupling

- Provides a consistent solution
- Known convergence properties
- Expensive to solve
- Requires compatible algorithms among physics

Tight / Loose Coupling

- Can be efficient to solve (smaller, nicer matrices)
- Allows separate meshes, time steps, discretizations
- Best de-coupling may be unknown
- May be more expensive in the long run





Building Blocks for Coupled Equations

Region: a mesh and set of fields

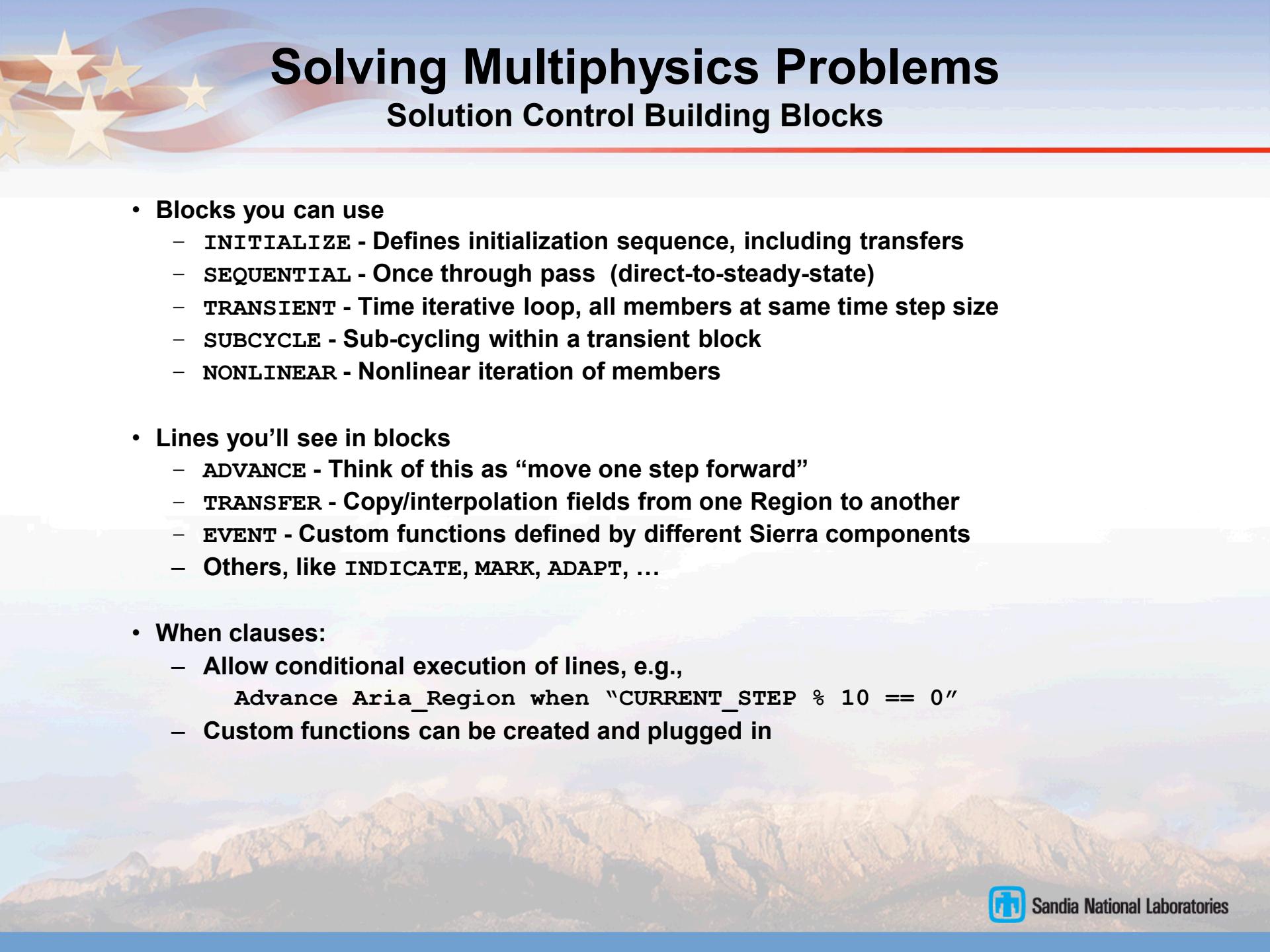
- One or more equations, constraints, etc.
- Aria: implicitly solved equations in a single matrix
 - Newton (Analytic + AD + FD)
 - Newton Finite Difference
 - Trilinos::NOX

Multiple Regions can be coupled with “**transfers**”

- Fields are copied/interpolated between Regions
- Entire mesh or select blocks, sidesets, etc.
- Can have different discretizations ($Q1 \leftrightarrow P0$)
- Sequencing is user defined



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Solving Multiphysics Problems

Solution Control Building Blocks

- **Blocks you can use**
 - **INITIALIZE** - Defines initialization sequence, including transfers
 - **SEQUENTIAL** - Once through pass (direct-to-steady-state)
 - **TRANSIENT** - Time iterative loop, all members at same time step size
 - **SUBCYCLE** - Sub-cycling within a transient block
 - **NONLINEAR** - Nonlinear iteration of members
- **Lines you'll see in blocks**
 - **ADVANCE** - Think of this as “move one step forward”
 - **TRANSFER** - Copy/interpolation fields from one Region to another
 - **EVENT** - Custom functions defined by different Sierra components
 - **Others**, like **INDICATE**, **MARK**, **ADAPT**, ...
- **When clauses:**
 - Allow conditional execution of lines, e.g.,
`Advance Aria_Region when "CURRENT_STEP % 10 == 0"`
 - Custom functions can be created and plugged in



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Input File Structure Reveals Execution Hierarchy

Domain

```
Begin Sierra job_name
  Define Materials
  Define "Finite Element Model":
    Mesh & Materials
  Define Linear Solvers
```

Procedure

```
Begin Procedure procedure_name
  Define Solution Control
    (e.g., time stepping)
```

Region

```
Begin Aria Region region_name
  Nonlinear Solver Settings
  Equation Definitions
  Boundary Conditions
  Sources
  Post-processors
  Output File Definition
```

```
End
```

```
End
```

Procedure: solution control instructions

- multiple procedures allowed within a domain

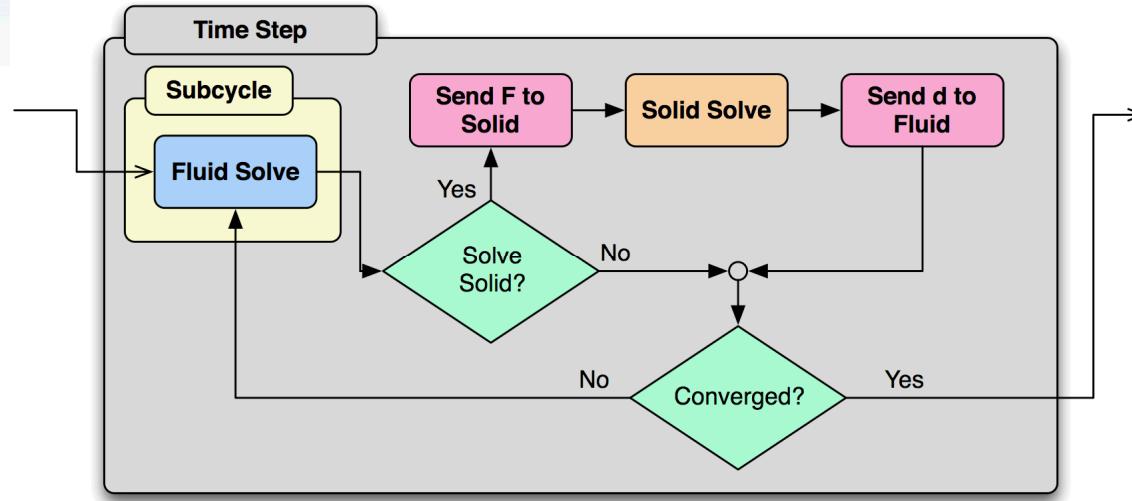
Region: a mesh and set of fields

- multiple regions allowed within a procedure



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Conditional Loose Coupling with Nonlinear Iteration and Subcycling



```
Begin System Main
  Use Initialize MyInit
  Begin Transient MyTransient
    Begin Nonlinear MyNonlinearLoop
      Begin Subcycle MySubcycle
        Advance AriaRegion
      End
      Transfer ForceAriaForceAdagio when "Solve_Solid()"
      Advance AdagioRegion when "Solve_Solid()"
      Transfer DispAdagioDispAria when "Solve_Solid()"
    End
  End
End
```



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Coupling Multiphase Flow and Geomechanics

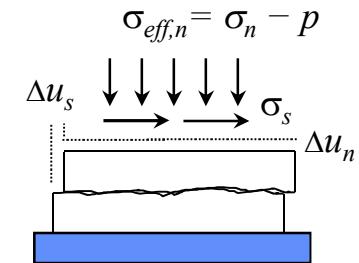
Effective stress principle: $\bar{\sigma}_{eff} = \sigma^T + \alpha p_{eff}$ e.g., $p_{eff} = p_w S_w + p_n S_n$

- effective stress is used the constitutive models
- equilibrium is based on total stress – balance of external forces

Porosity maps from deformation gradient:

- deforming grids in flow and mechanics
- permeability models can depend on porosity or other damage criteria

$$\frac{1 - \phi_0}{1 - \phi} = \det \mathbf{F}(\mathbf{u})$$



Coupled Flow and Geomechanics

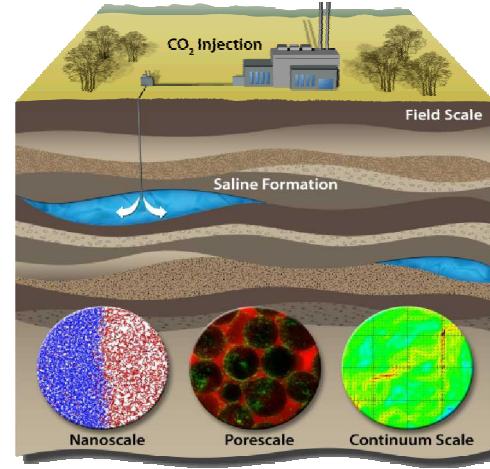
Injection into an Reservoir/Caprock System

Motivation

Global consumption of fossil fuels has significantly increased levels of atmospheric CO₂, a greenhouse gas. Carbon capture and storage (CCS) is a promising mitigation strategy. CCS consists of capturing and sequestering CO₂ emissions from large “point sources”, for example, coal-fired power plants.

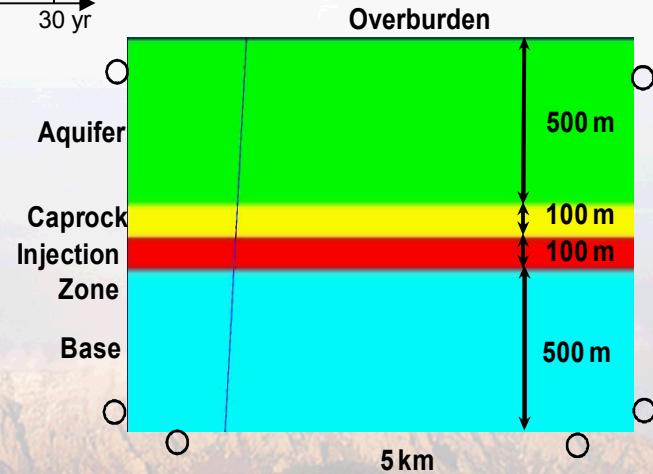
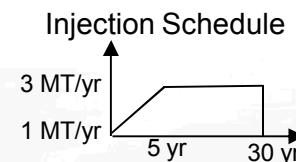
We are developing multiscale multiphysics simulation tools for investigating both the short and long term coupled chemical and mechanical processes encountered during subsurface CO₂ sequestration.

Problem Schematic



Problem Features and Goals

- Injection of large volumes of supercritical CO₂ into a layered brine reservoir-caprock system at greater than 1km depth.
- High pressures can damage either the injection reservoir (e.g. plastic deformation), or the confining caprock (fracture generation or activation)
- We use a coupled multiphase flow and geomechanics numerical models to determine conditions for failure of the caprock due to injection pressures, flow rates, permeabilities, thus determining operation limits for design of safe CO₂ injection.



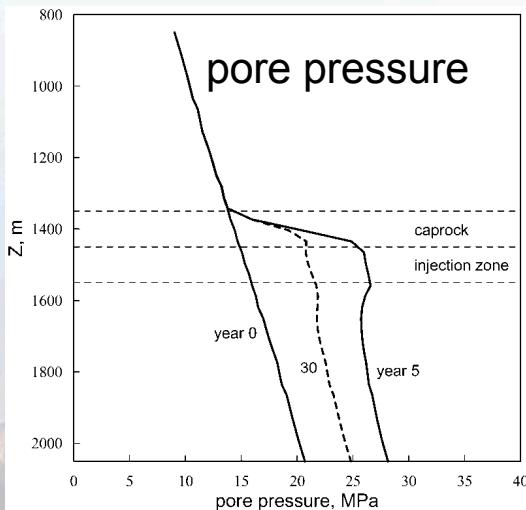
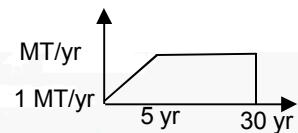
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Coupled Flow and Geomechanics

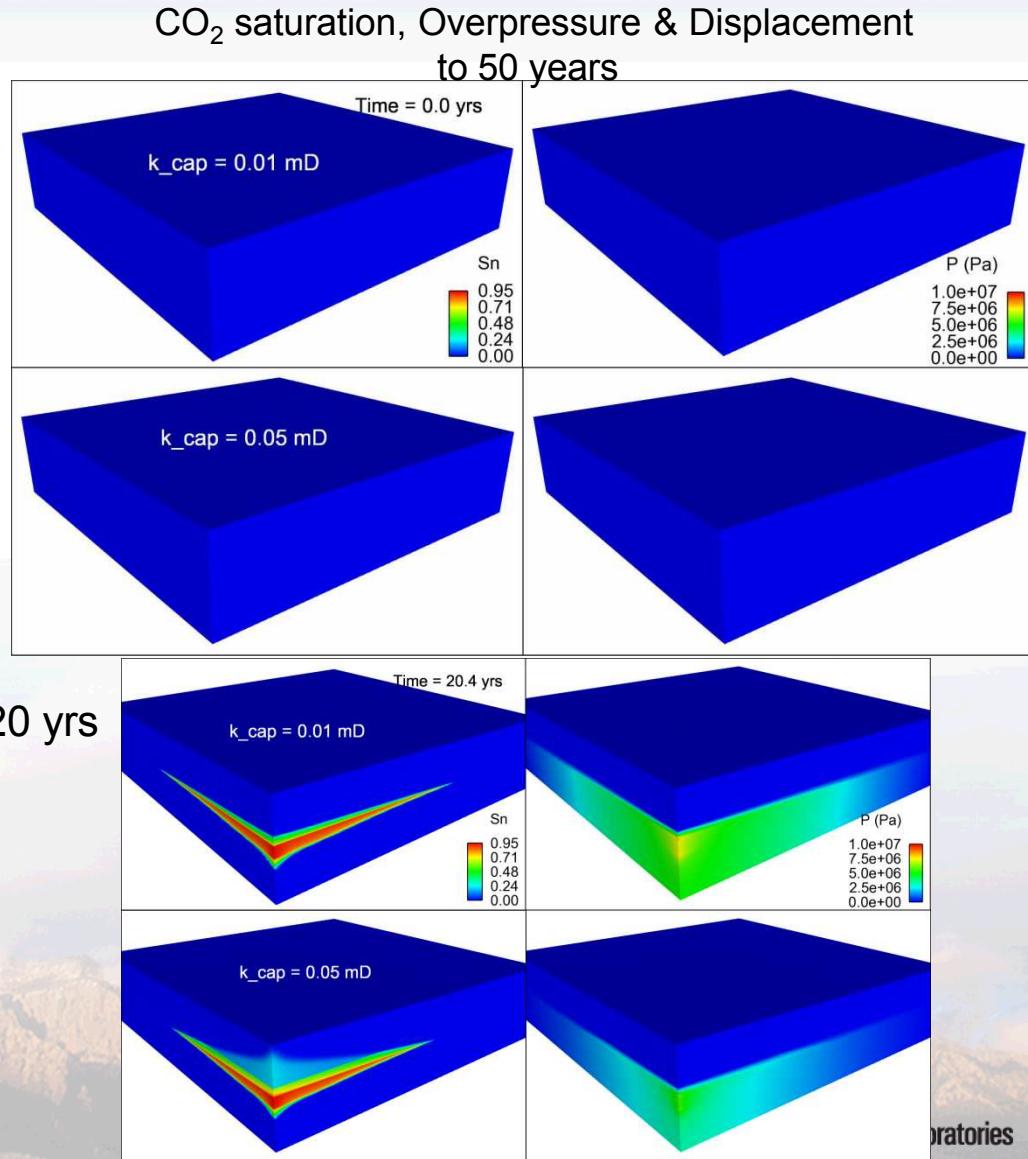
Flow, CO₂ transport and Deformation

Results

- “gravity-override” for injected lighter, less viscous CO₂
- higher overpressure, further penetration with low permeability caprock
- 50 μ D caprock leaks during injection. Buoyant leaked CO₂ rises and pools at the upper impermeable surface



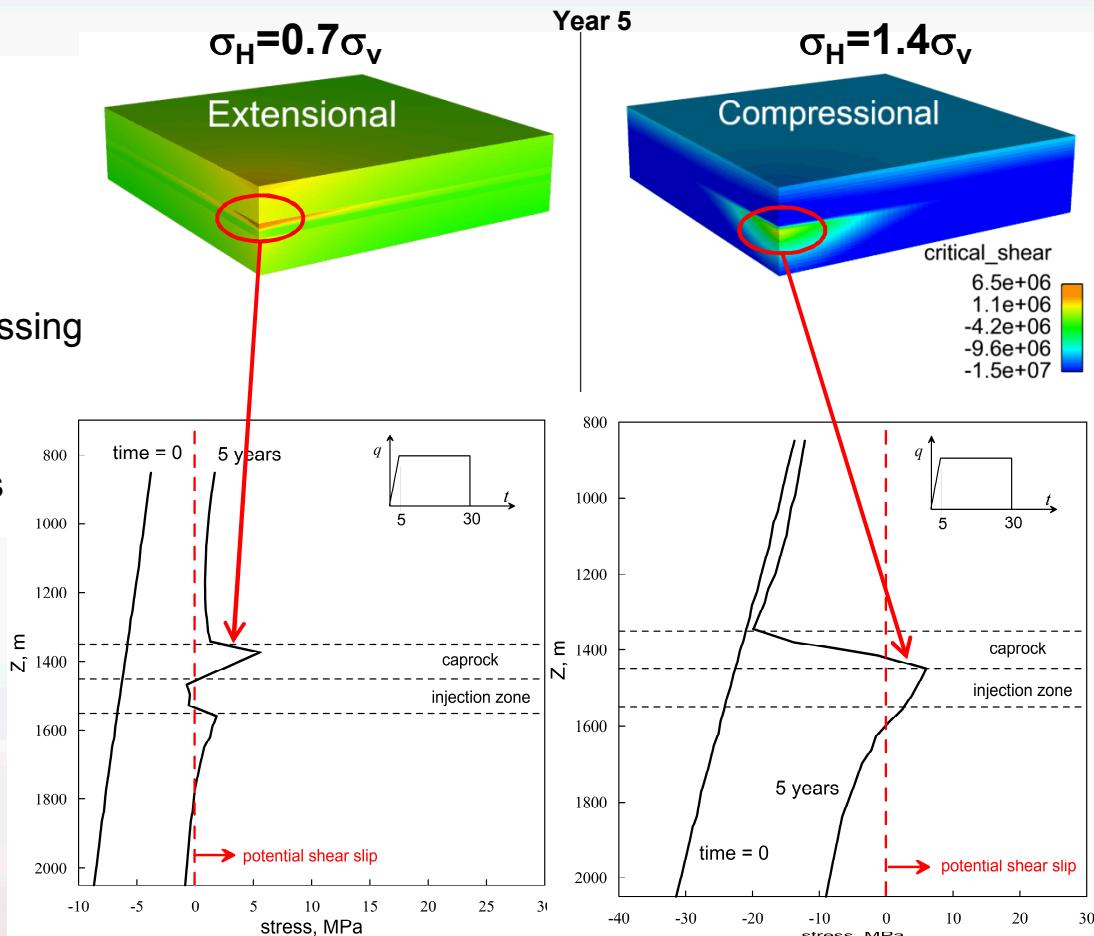
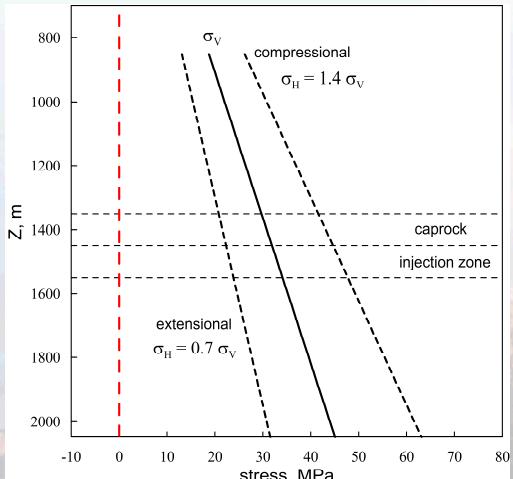
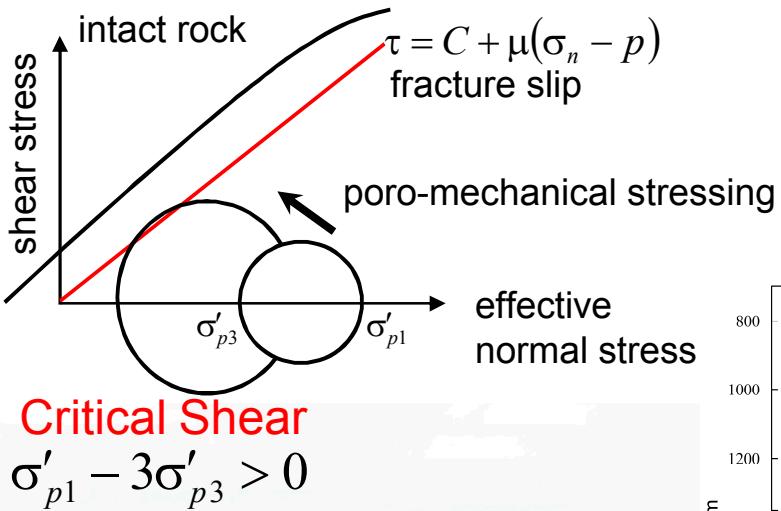
20 yrs



Coupled Flow and Geomechanics

Effect of Regional Stress State

Linear Mohr-Coulomb



Extensional regional stresses are more dangerous to caprock integrity



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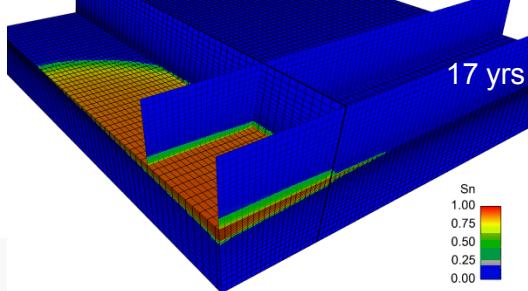
Coupled Flow and Geomechanics

Hydromechanical Effects of Faults

Some faults could go undetected and may pose a risk to sequestration of CO₂ by reactivation due to injection pressures. This study considers possible hydromechanical effects due to a low and high permeability fault.

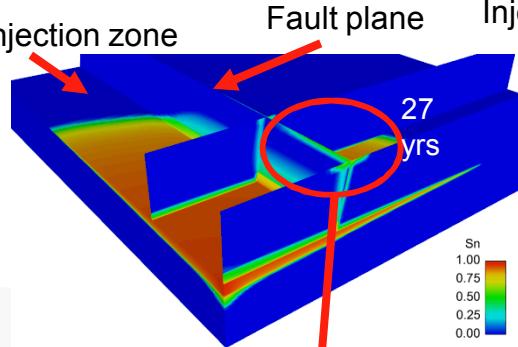
Low Permeability Fault

Interior view of CO₂ Saturation

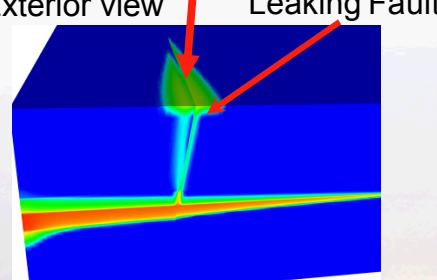


High Permeability Fault

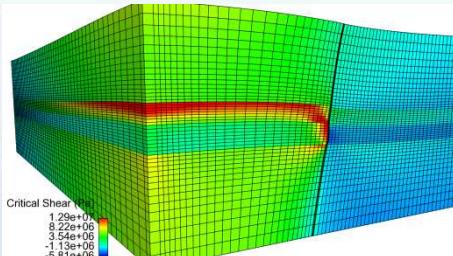
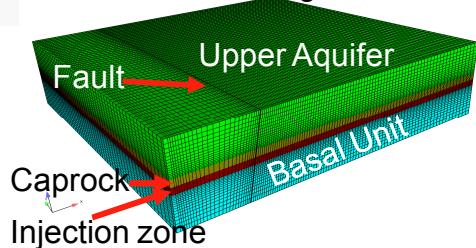
Top of injection zone



Exterior view

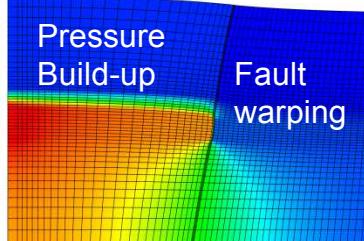


Discrete Geologic Model



Pressure Build-up

Fault warping



Low permeability fault impedes CO₂ injection, diverts flow along fault and builds pressure behind the fault, thereby shearing/warping the fault and inducing critical shear failure in both the caprock and fault.

High permeability fault creates a pathway for leakage of CO₂ through the caprock, ultimately pooling at the top of the upper aquifer, which is capped by an impermeable boundary.

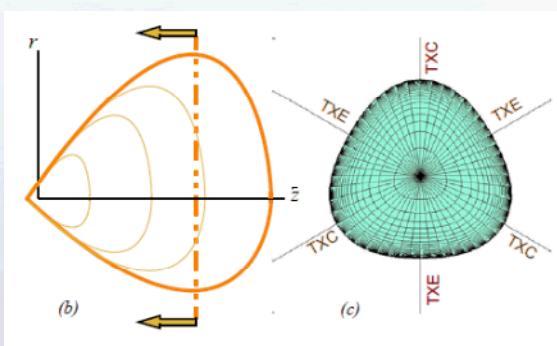
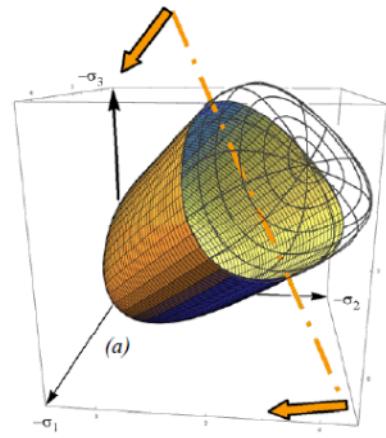


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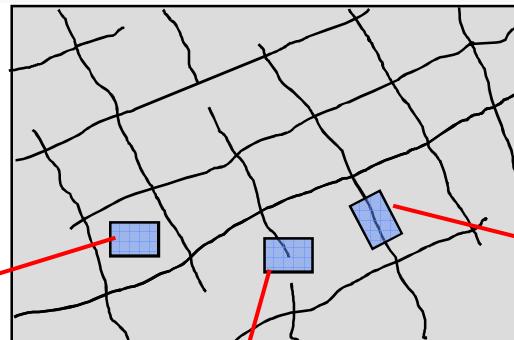
Hydromechanical Coupling in Fractured Rock

bulk constitutive properties

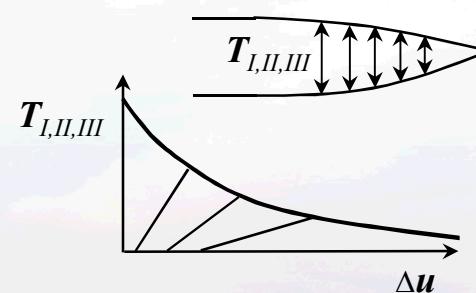
(Sandia GeoModel
Fossum & Brannon, 2004)



Fractured Porous Rock



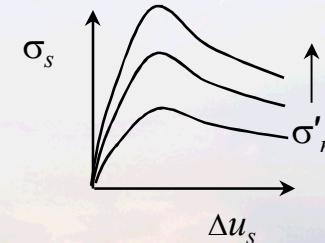
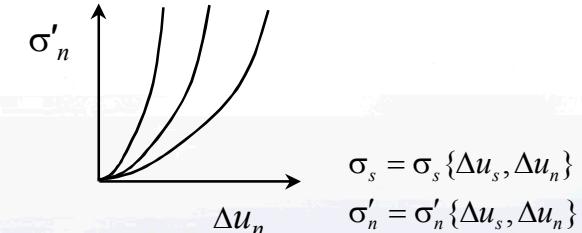
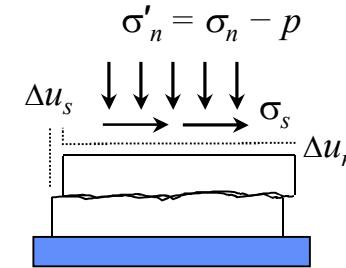
crack-tip cohesive properties



additional challenges

- scale dependence
- history dependence
- precipitation
- dissolution

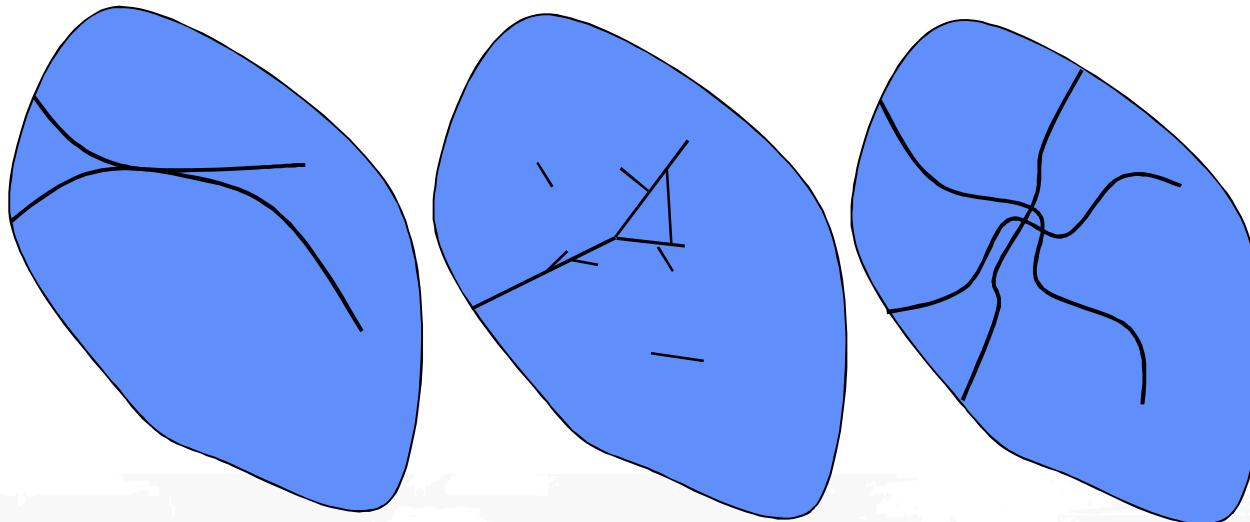
fracture contact properties



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Computational Challenges to Allowing Cracks to Grow Arbitrarily



- Do we restrict branching?
- Do we restrict initiation?
 - from surface only?
 - from crack tips only?
 - from existing cracks only?
- Constraints on turning angles?
- Constraints on crossing angles?
- Constraints on minimum fragment size?

What about 3D?

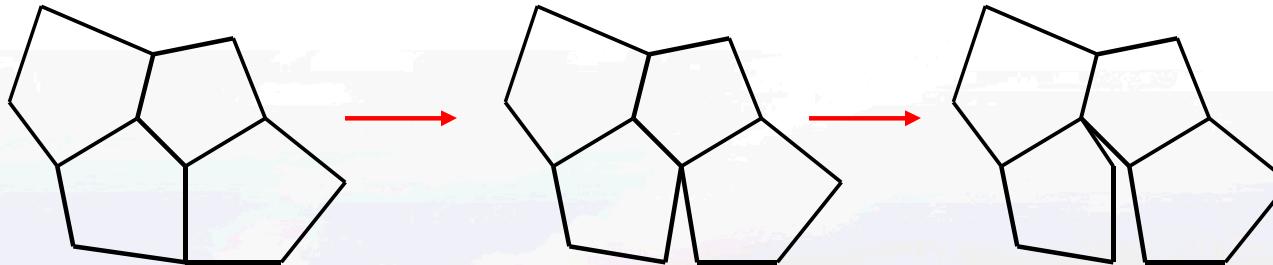


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Computational Approach

- Random Voronoi tessellation (mesh)
- Polyhedral finite-elements
- Fracture only allowed at element edges.
- *Dynamic* mesh connectivity
- Insert cohesive tractions on new fracture surfaces.



dynamic insertion of cohesive tractions based on . . .

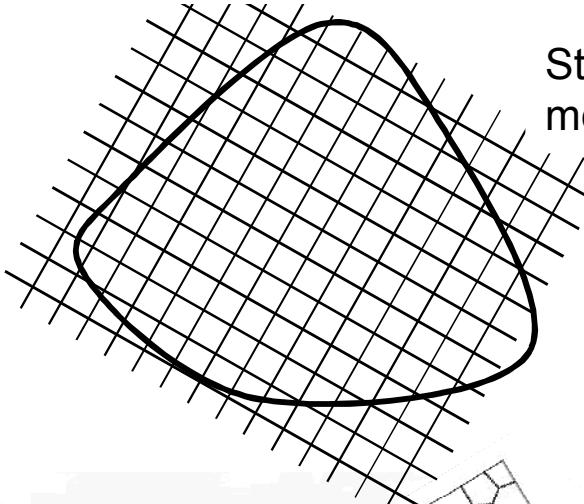
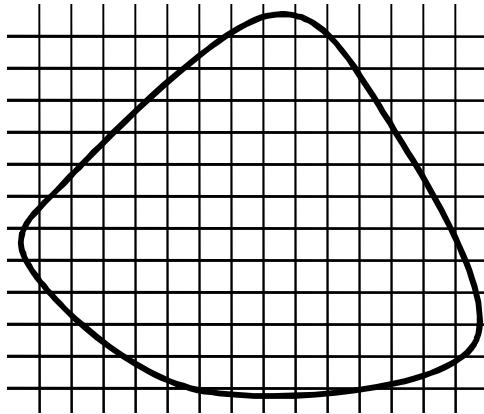
Pandolfi, A. and Ortiz, M. (2002) 'An efficient adaptive procedure for three-dimensional fragmentation simulations,' *Engineering with computers*, 18, 148-159.



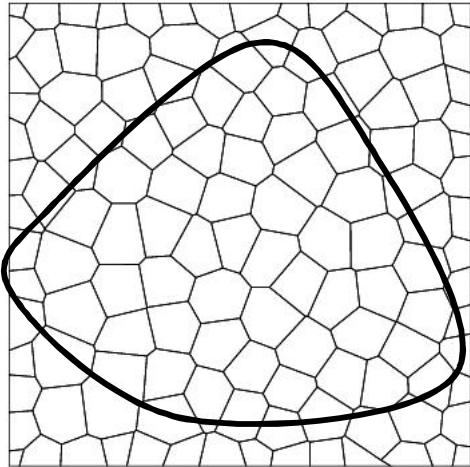
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Eliminating Mesh Induced Crack Bias

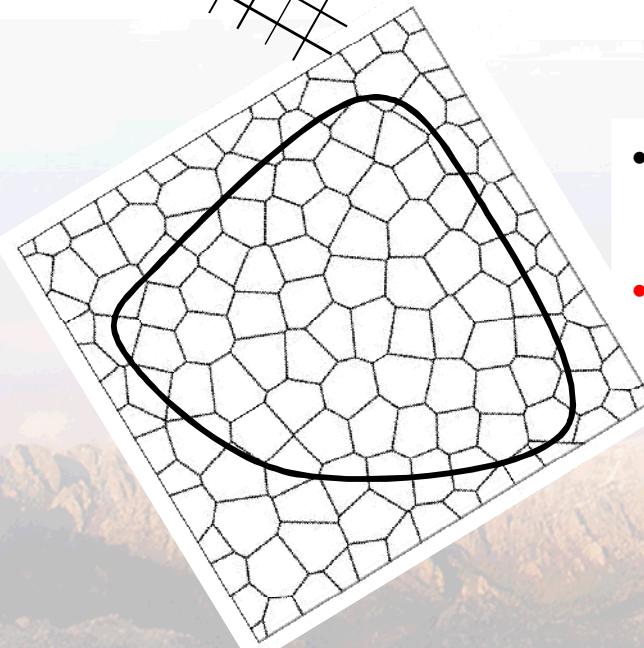
If cracks can grow only at element edges, then need to eliminate any directional bias in crack growth.



Structured grids can result in strong mesh induced bias (nonobjective).



Voronoi tessellation of
with random seeding

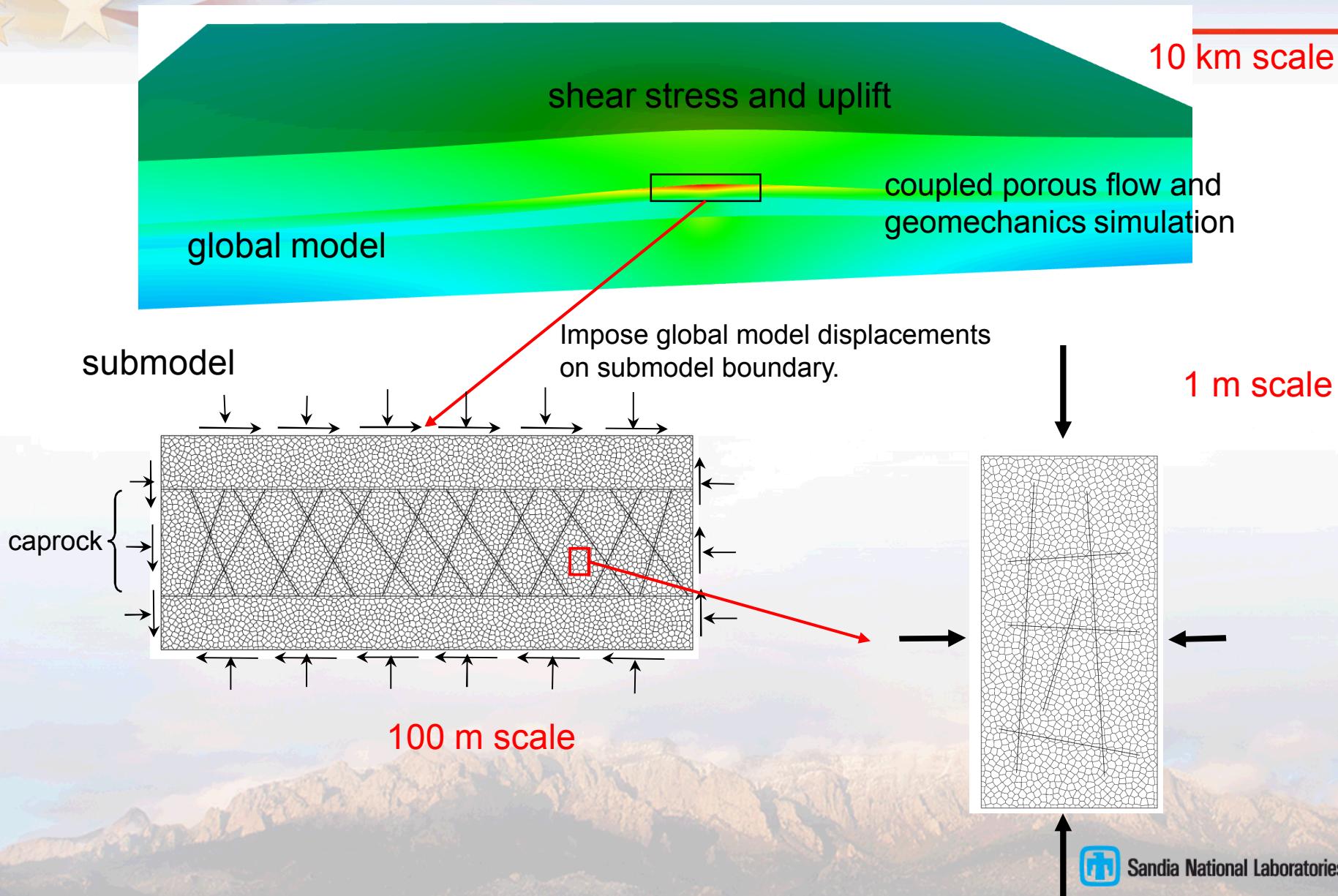


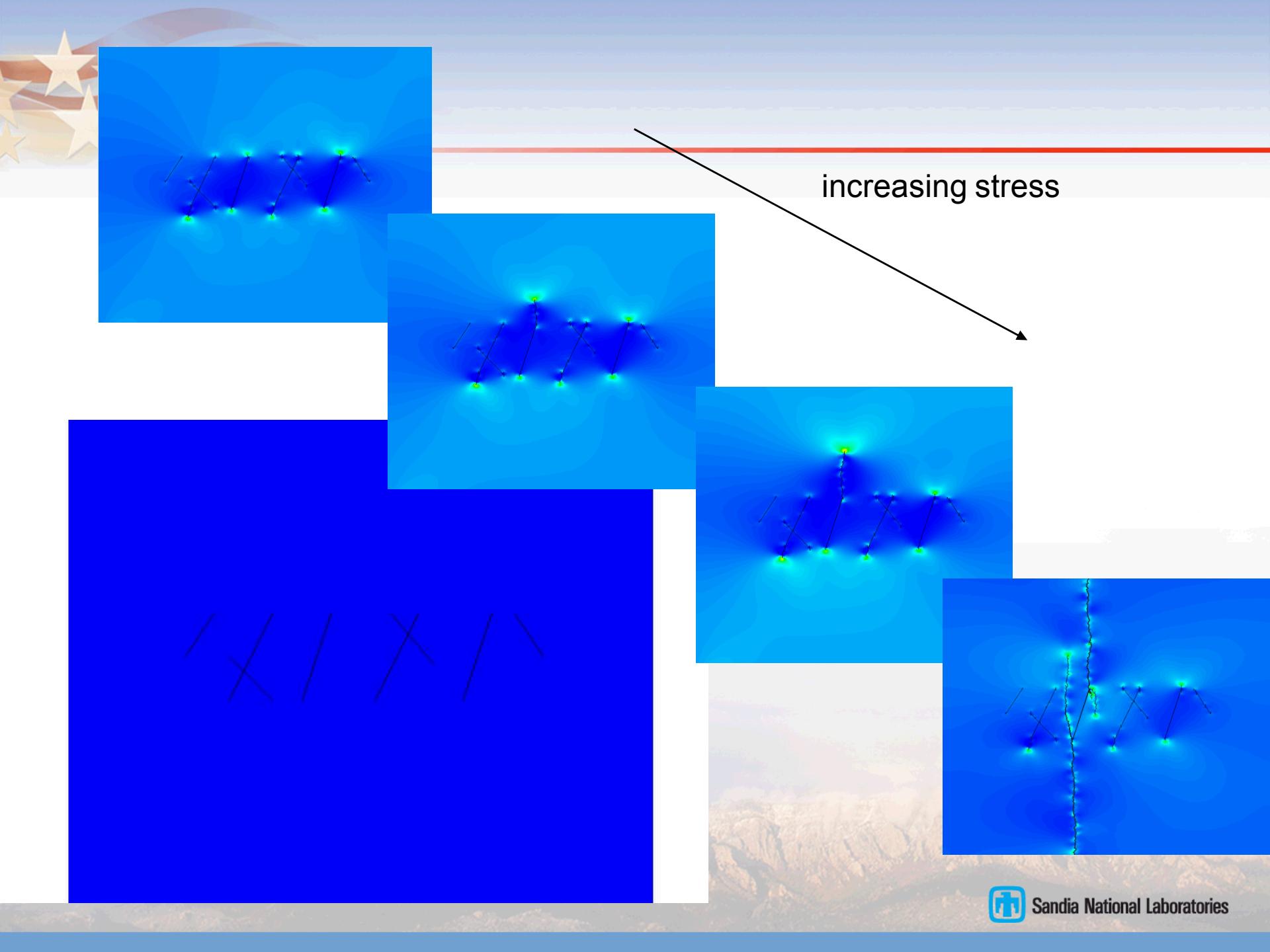
- need to use 'random' discretizations
- **statistically isotropic**



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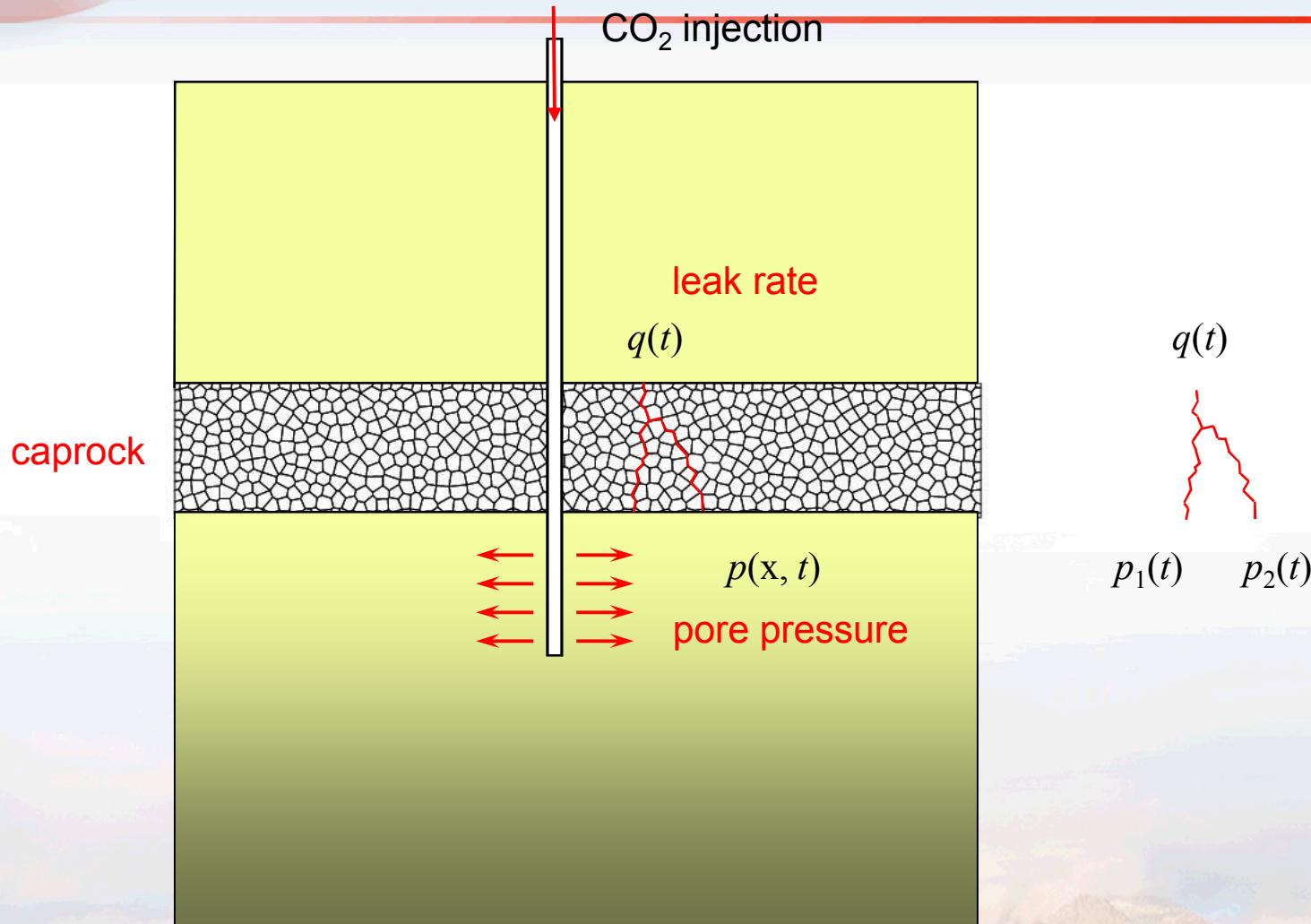
Multiscale analysis of caprock integrity during CO₂ injection





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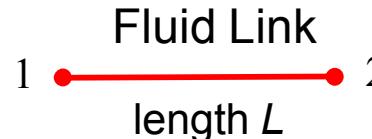
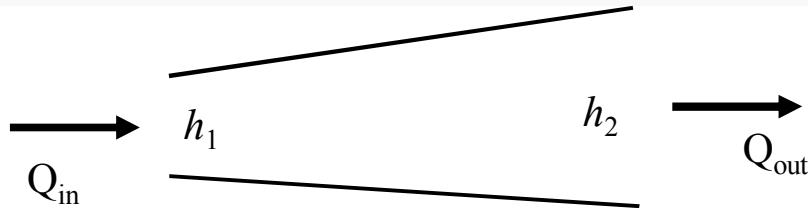
Fluid Flow in 2D Discrete Fracture Networks



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Fluid Flow in 2D Discrete Fracture Networks

Solve fluid network to get nodal pressures and flow rates.



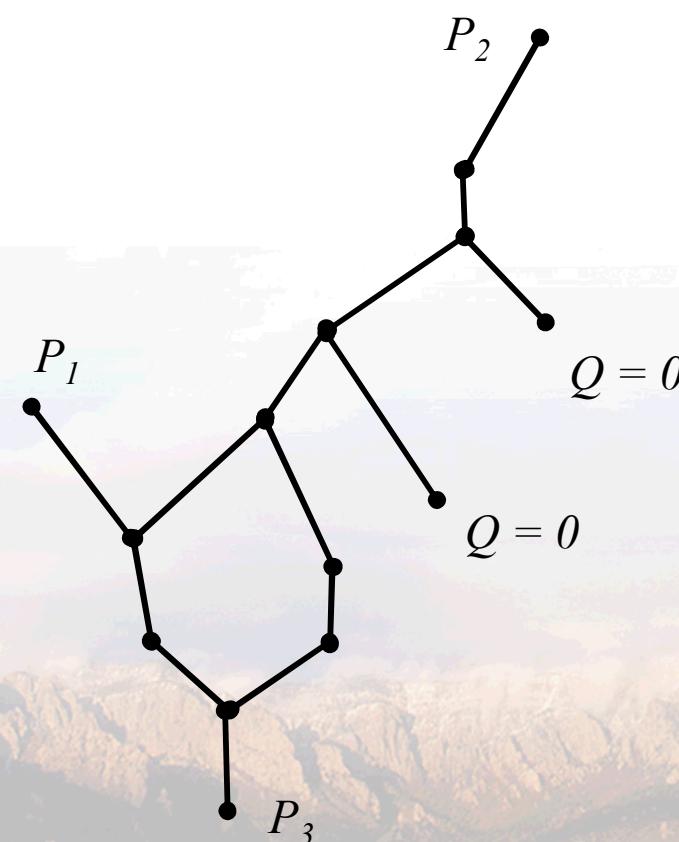
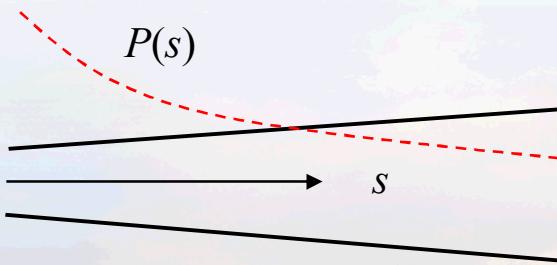
$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \frac{T}{\mu} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

$$T = \frac{h_1^2 h_2^2}{6L} \frac{1}{h_1 + h_2}$$

Reynold's lubrication equation

$$\nabla(\rho \mathbf{Q}) = 0$$

$$\mathbf{Q} = -\frac{h^3}{12\mu} (\nabla p - \rho g h)$$

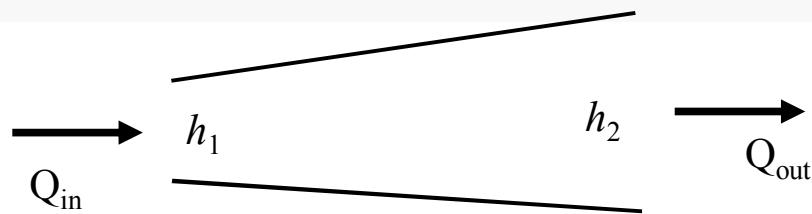


Q = flow rate
 P = pressure
 μ = viscosity
 T = transmissibility



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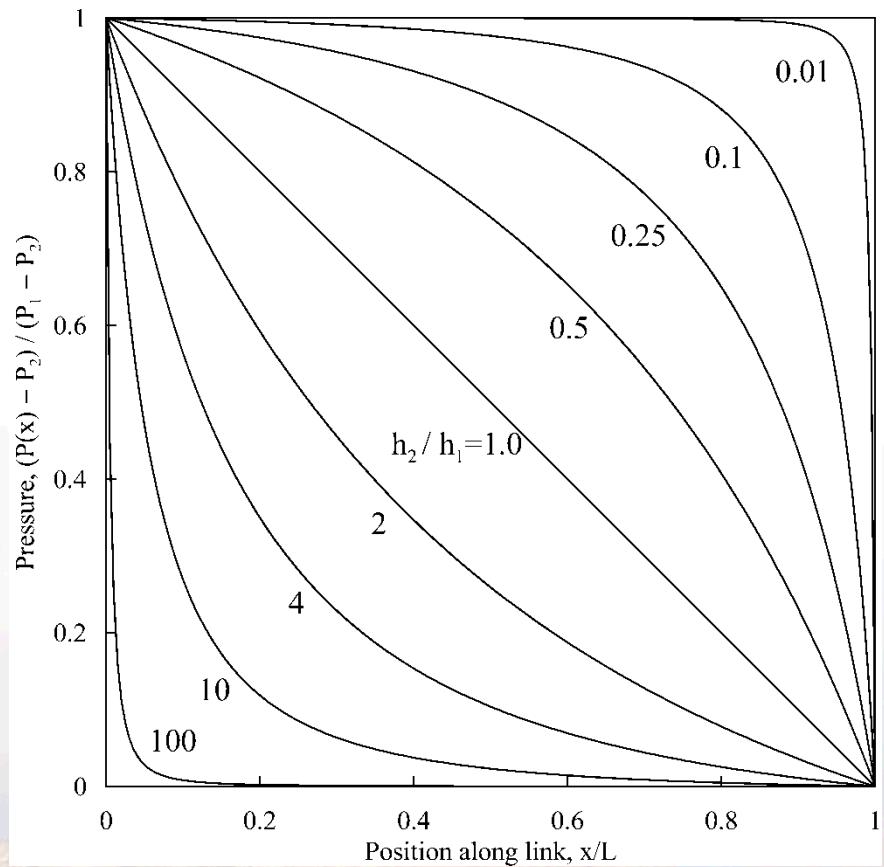
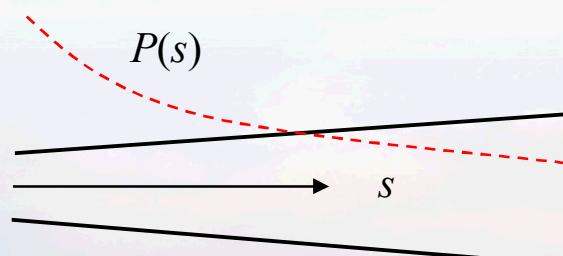
Fluid Flow in Discrete Fracture Networks



Reybold's lubrication equation

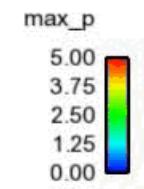
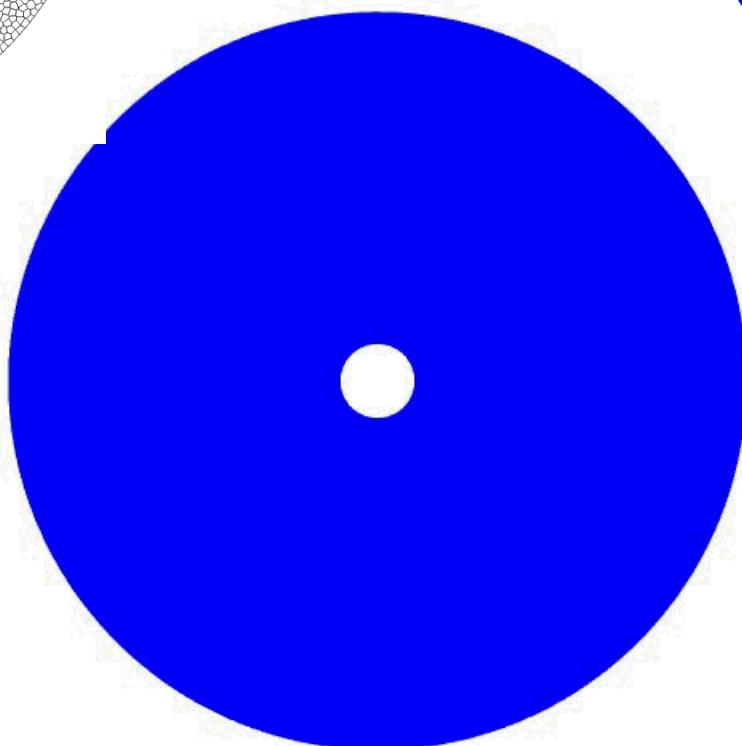
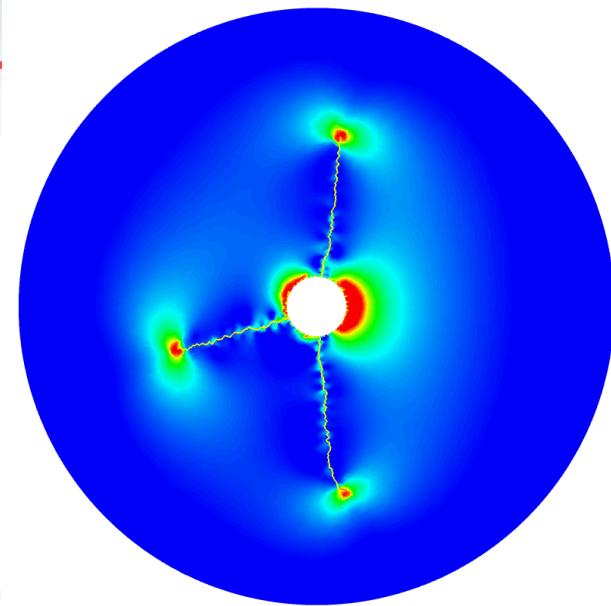
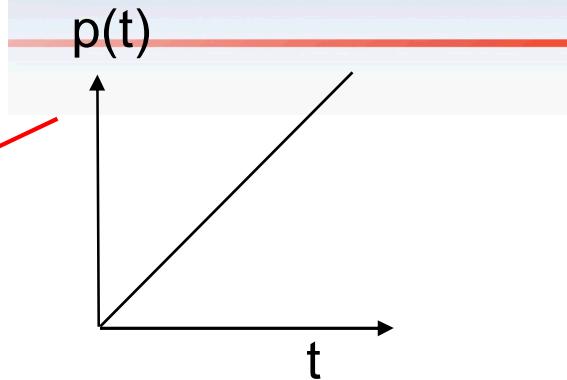
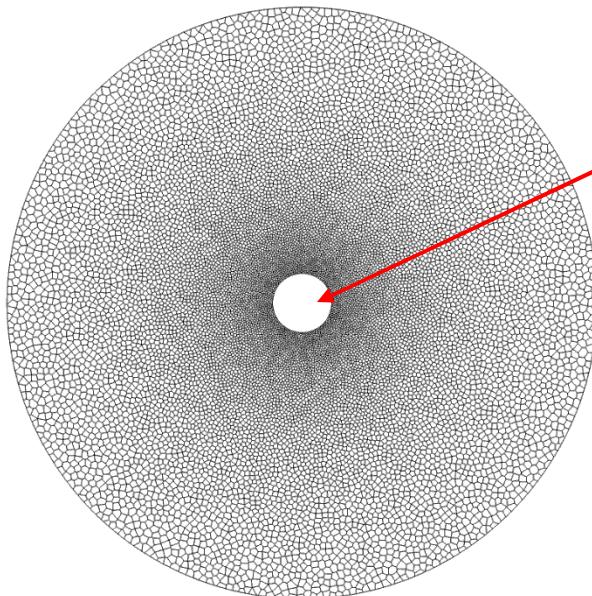
$$\nabla(\rho\mathbf{Q}) = 0$$

$$\mathbf{Q} = -\frac{h^3}{12\mu}(\nabla p - \rho gh)$$



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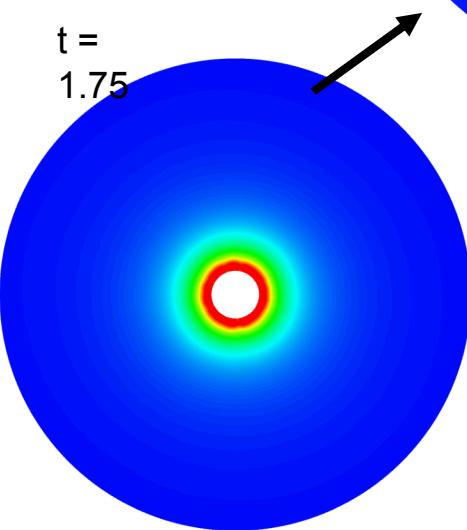
Hydraulic Fracture Simulation



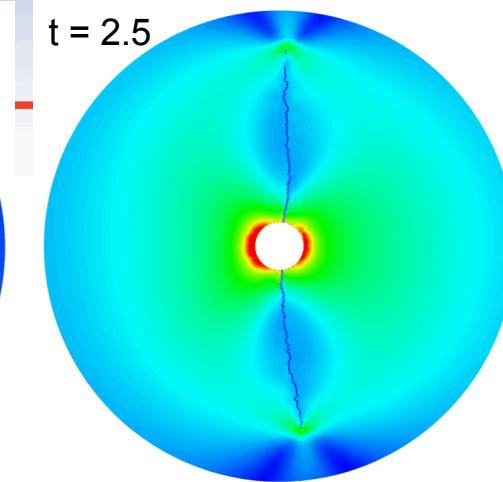
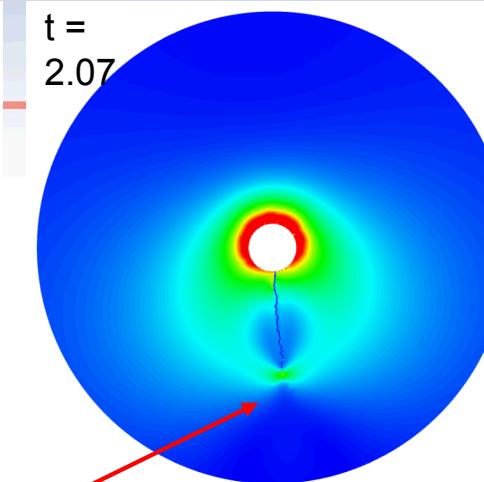
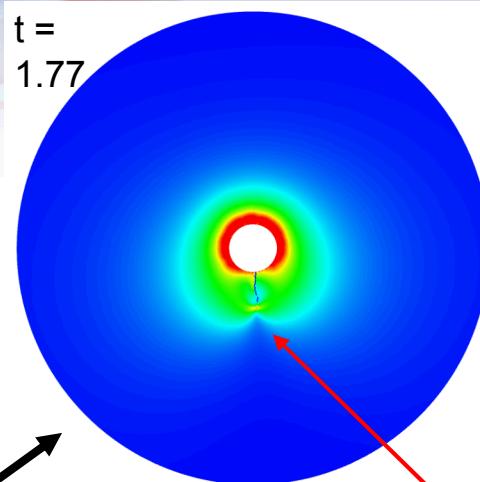
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Hydraulic Fracture Simulation

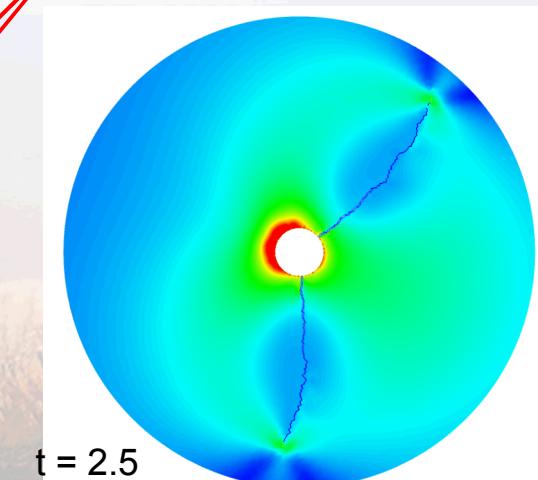
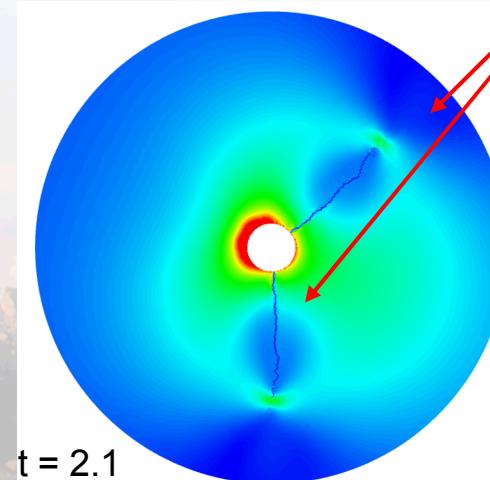
fluid pressure
up to crack tip



fluid pressure up
to cohesive tip



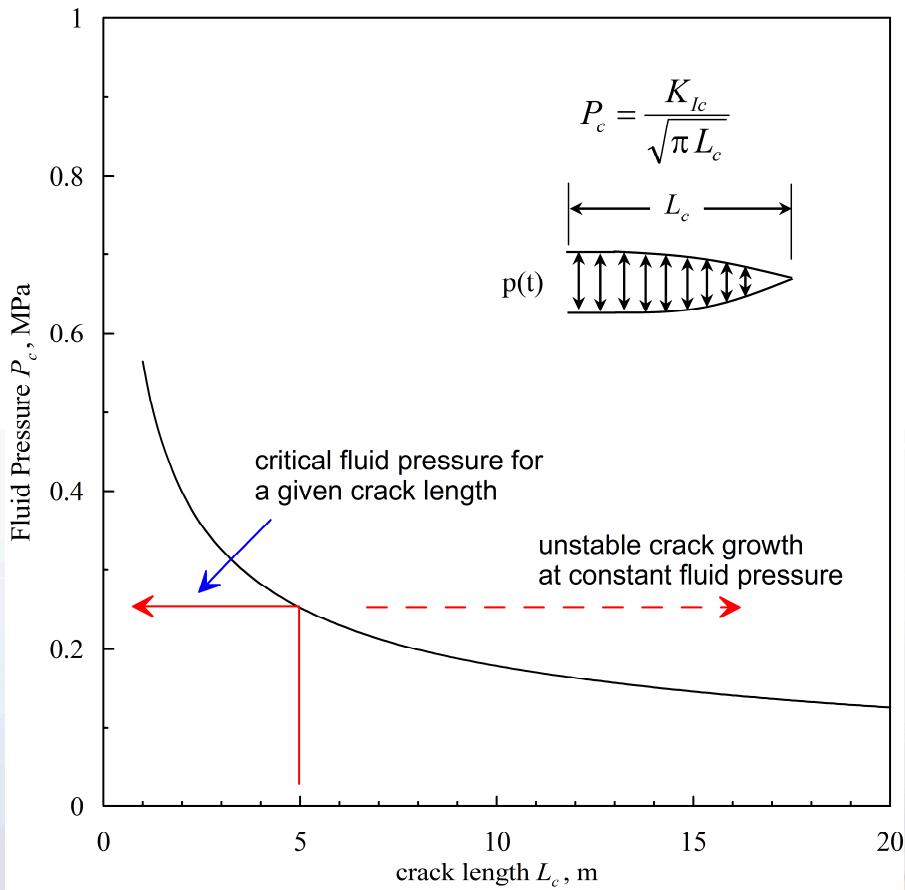
Both cracks
propagate
simultaneously.



atories

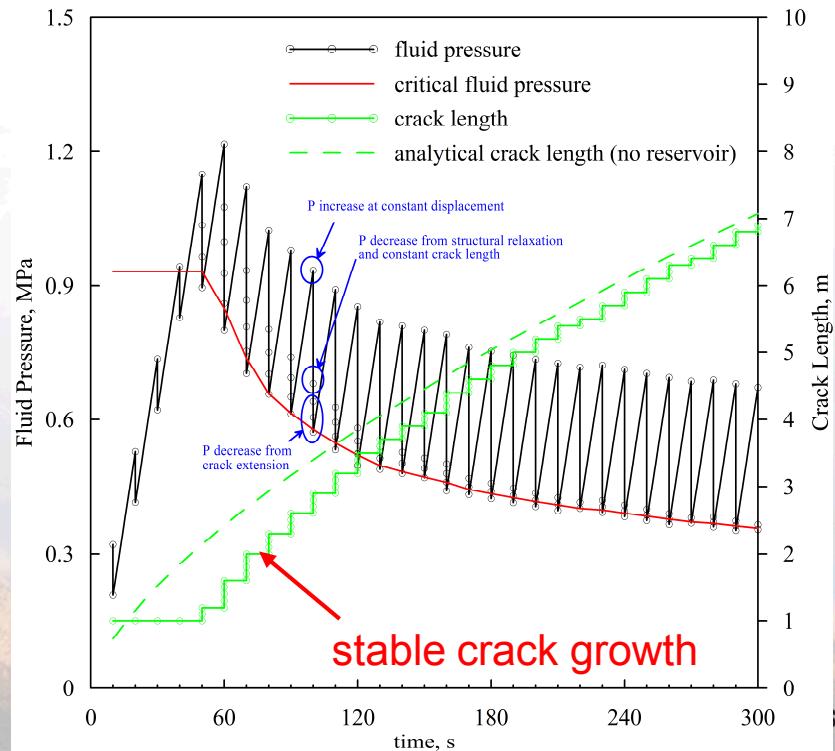
Hydraulic Fracture Simulation

- Constant fluid pressure causes unstable crack growth.
- Use fluid-mass control.



```

do
  increment fluid mass,  $\Delta m$ 
  equilibrate at constant crack length,  $a$ .
  while ( $K_I > K_{Ic}$ )
    increment crack length,  $\Delta a$ .
    equilibrate at constant crack length,  $a + \Delta a$ .
  end while
end do
  
```





Future Directions

- Models of perm(stress, porosity, damage)
- Multiphysics coupling of dynamic discrete fracture network generation
 - One-way coupled (continuum → discrete) dynamic discrete fracture
 - Flow physics on shells
 - 3D Voronoi mesh in Sierra
- Thermal CO₂-NACL-H₂O phase behavior
- Geochemistry





Backup Slides



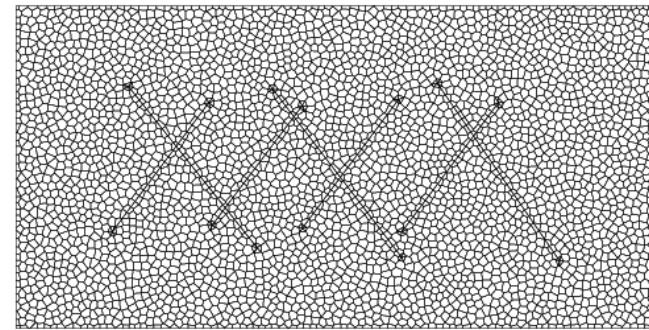
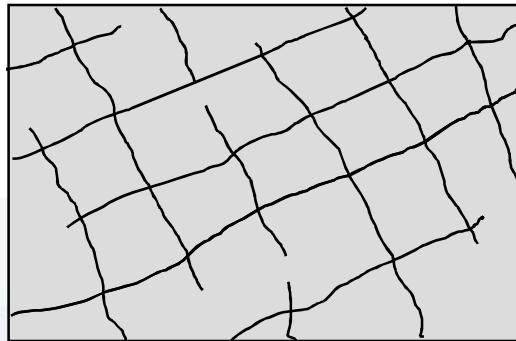
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MeshingGenie (Trilinos)

(Ebeida, M., Knupp, P., Vitus Leung, Sandia National Laboratories)

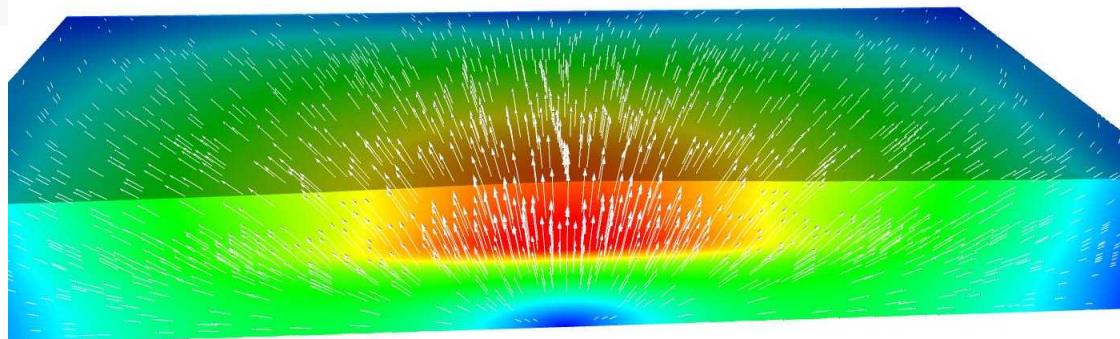
Fractured Rock



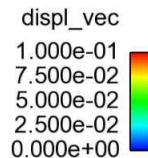
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Injection Induced Uplift

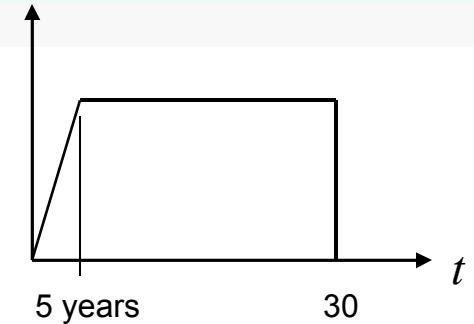
5.32 years



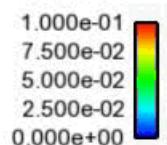
displacement field $\times 1000$ at year 5



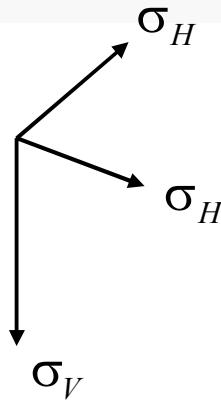
injection rate



displ_vec

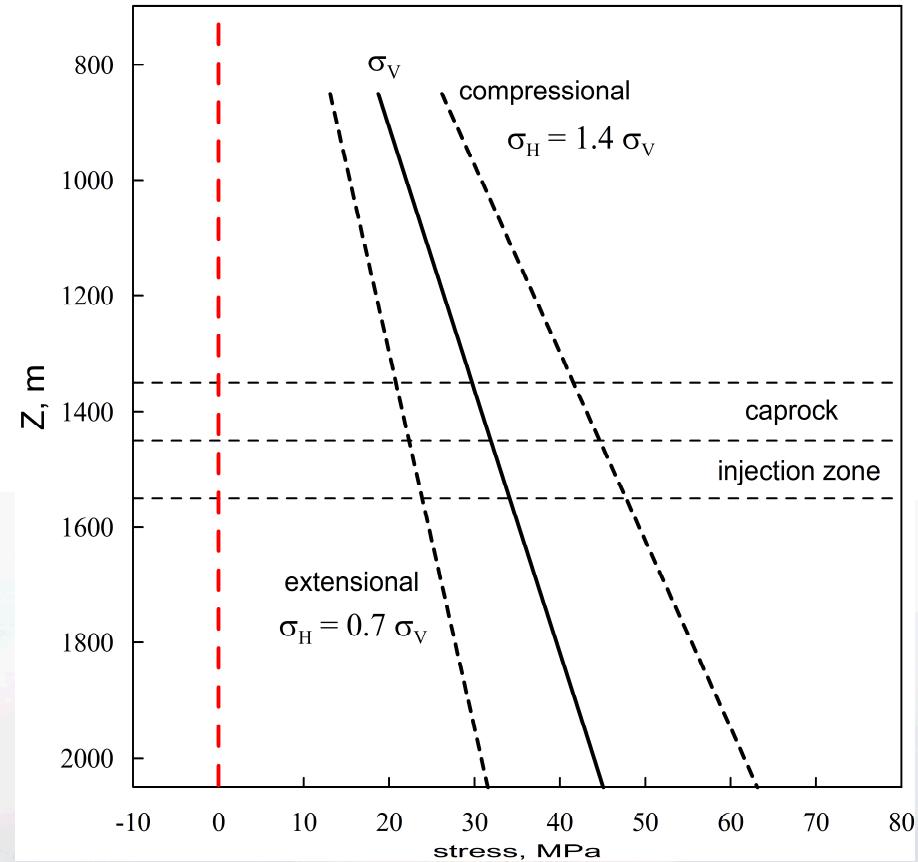


Initial Stress State



Look at two initial stress regimes

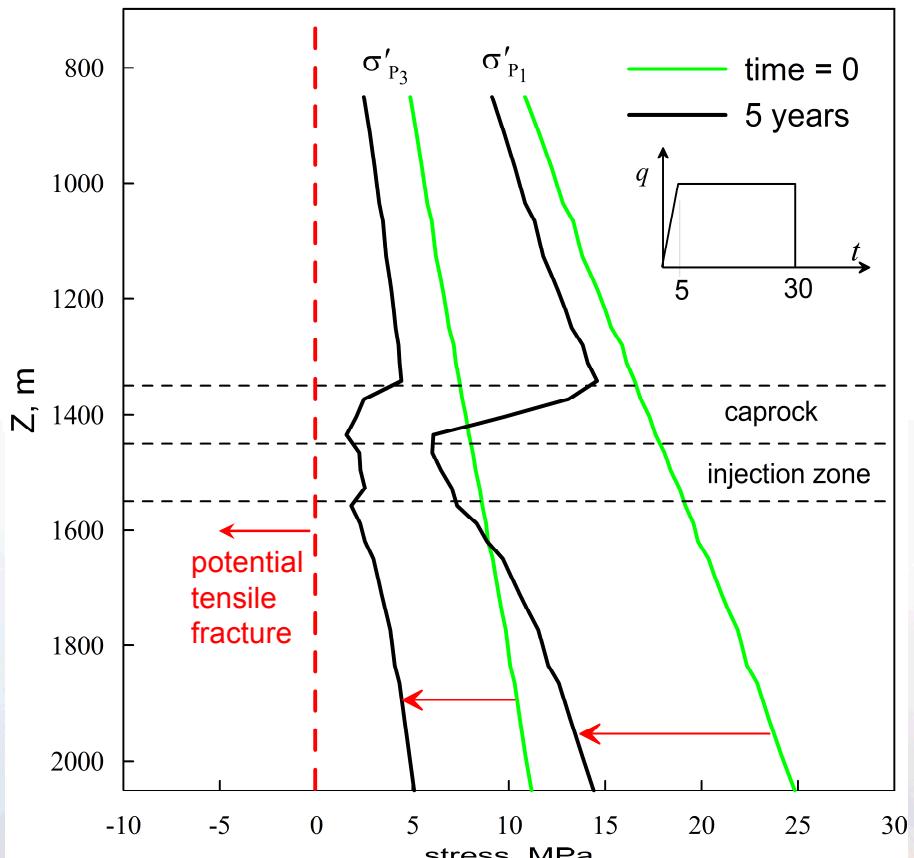
1. extensional $\sigma_H < \sigma_V$
2. compressional $\sigma_H > \sigma_V$



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Extensional Initial Stress

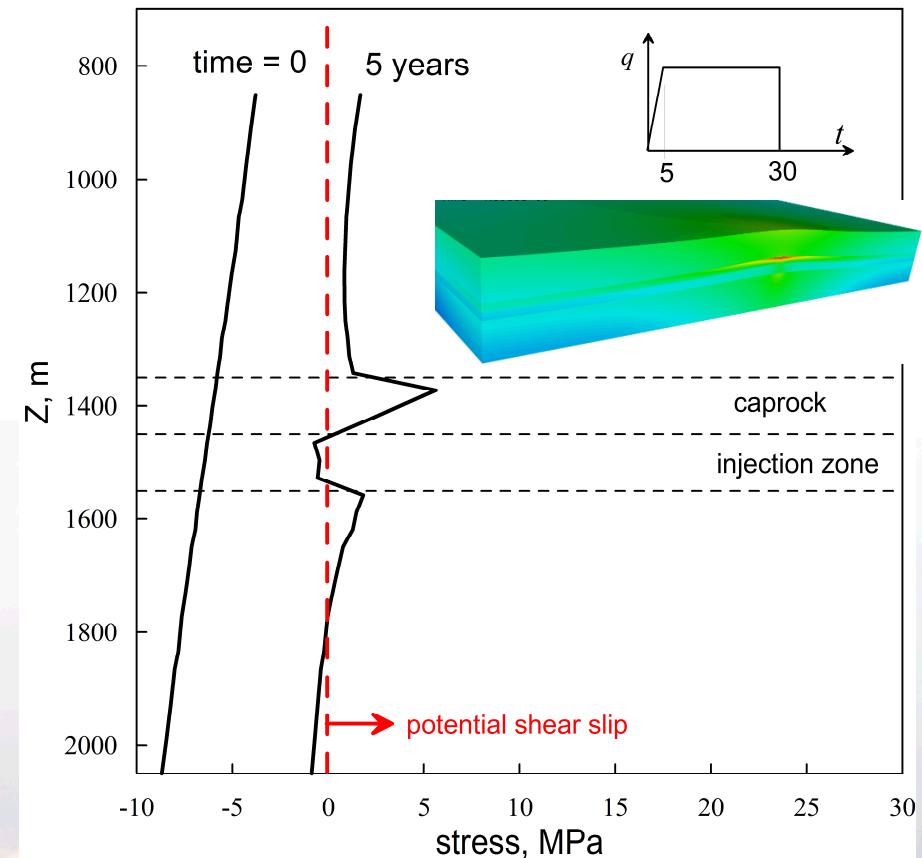
effective principal stresses



effective principal stress

critical shear

$$\sigma'_{p1} - 3\sigma'_{p3} > 0$$



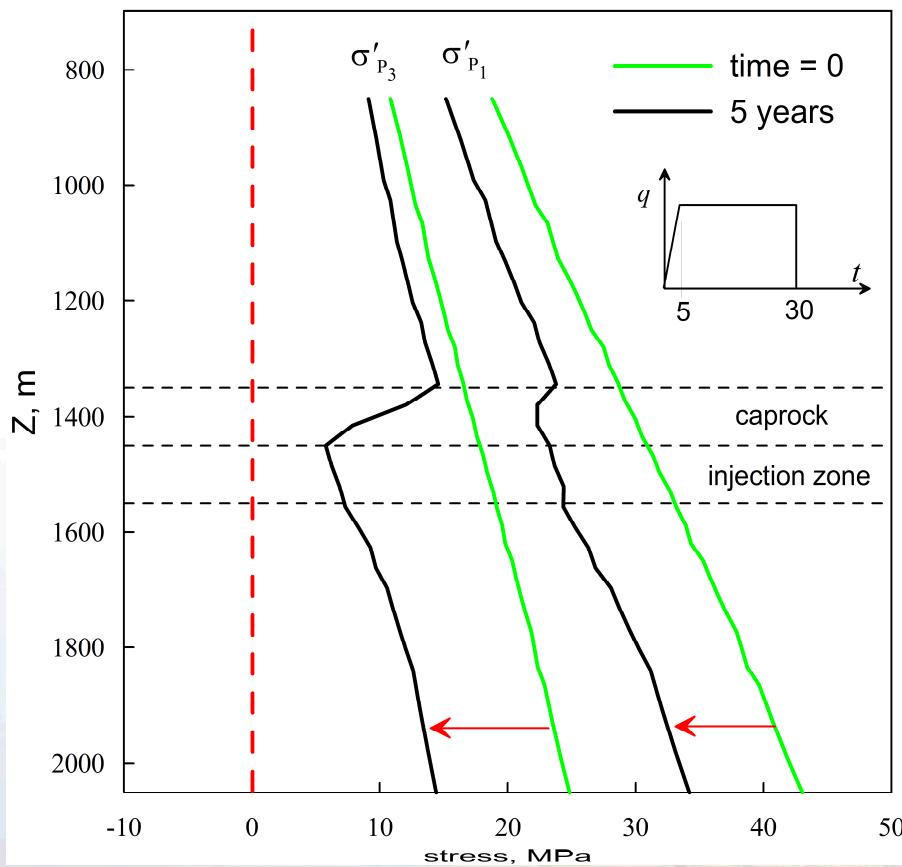
critical shear stress



Sandia National Laboratories

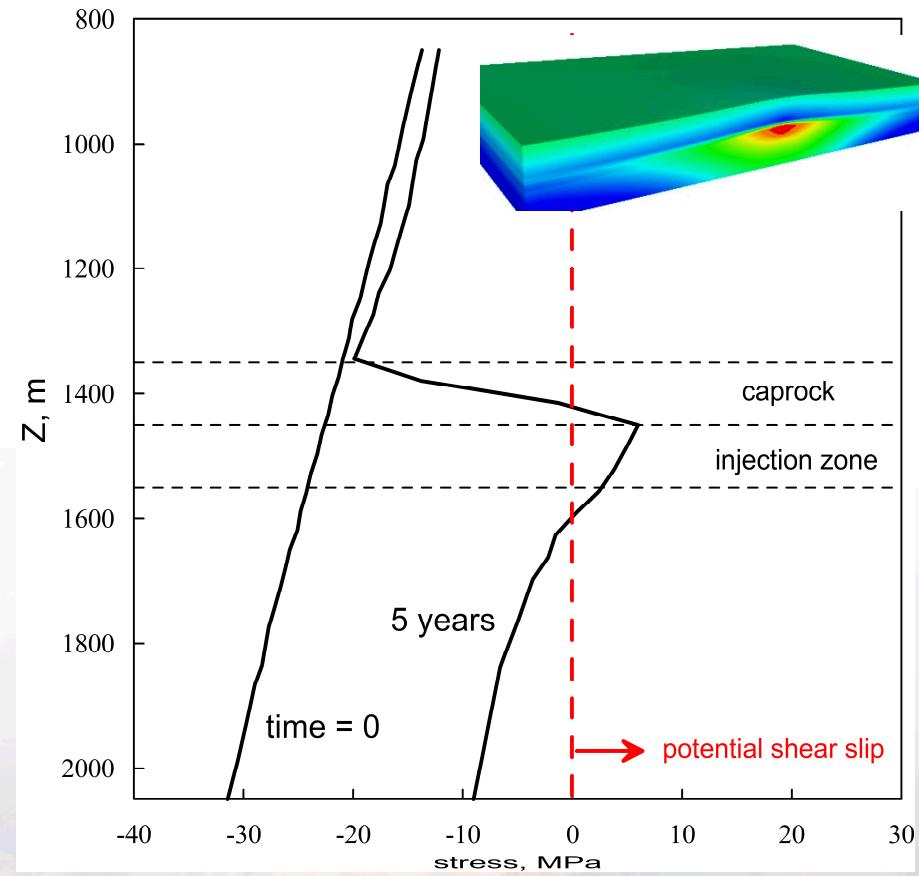
Compressional Initial Stress

effective principal stresses



critical shear

$$\sigma'_{p1} - 3\sigma'_{p3} > 0$$



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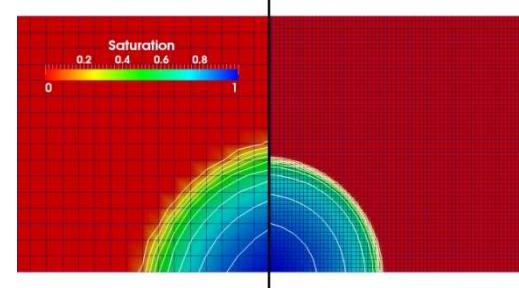
Aria Multiphase Porous Flow Physics

Two-Phase Immiscible Flow

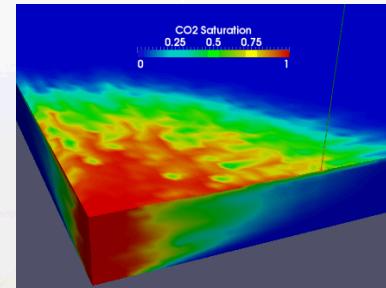
- Compressible (fluids and/or formation), buoyancy effects
- General dependence of thermophysical and transport properties on solution vector
- Capillary pressure (optional)
- Relative permeability
- Specification of heterogeneous transport property fields (e.g. permeability, porosity)
- Can be coupled to energy equation

Benchmark Problem

Displacement of oil by water flood without capillary pressure or gravitational effects.



Grid effects using upwind CVFEM scheme



Injected CO₂ saturation levels in a brine filled reservoir represented with heterogeneous permeability



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Aria Porous Flow Physics

Immiscible Flow

Mathematical Model

- **Two-Phase Immiscible Mass Balances:**

$$\frac{\partial(\rho_w \phi S_w)}{\partial t} = \nabla \bullet \left(\rho_w \frac{k_{rw}}{\mu_w} \mathbf{k} \bullet (\nabla p - \rho_w \mathbf{g}) \right) + Q_w$$

$$\frac{\partial(\rho_n \phi S_n)}{\partial t} = \nabla \bullet \left(\rho_n \frac{k_{rn}}{\mu_n} \mathbf{k} \bullet (\nabla p + \nabla p_c - \rho_n \mathbf{g}) \right) + Q_n$$

- **Thermophysical property models (new models are easily incorporated):**

$$\rho_w = \rho_{w,0} (1 + \kappa_{Tw} (p - p_o))$$

$$\rho_n = \rho_{n,0} (1 + \kappa_{Tn} (p_n - p_{n,o}))$$

$$p_n = p + p_c (S_w)$$

$$S_w = 1 - S_n$$

$$\phi = \phi_0 (1 + C_r (p - p_0))$$

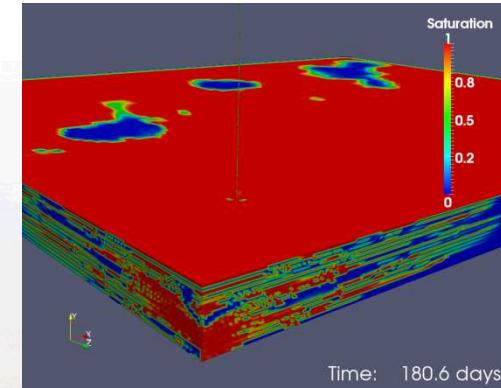
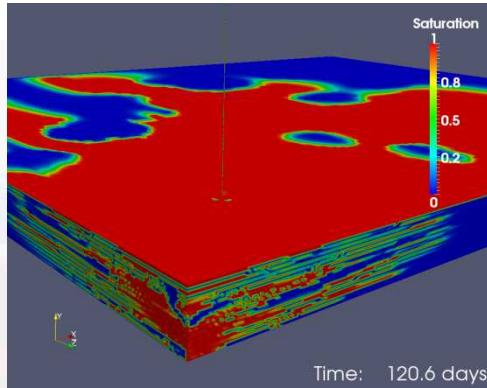
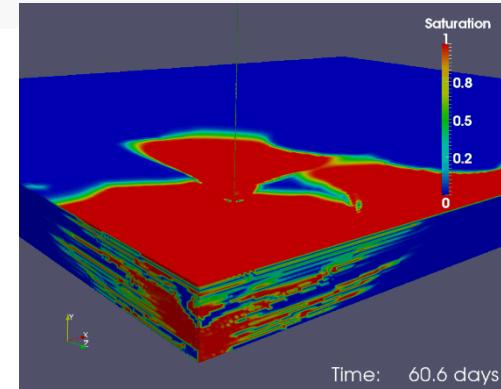
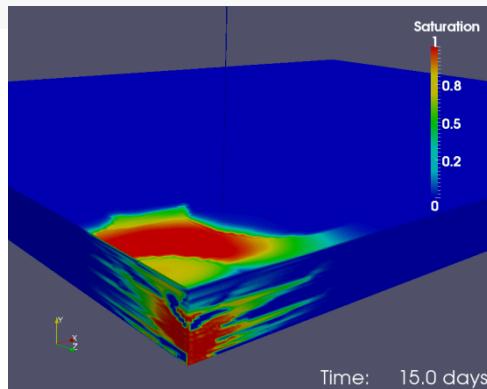


CO₂ Leakage Through an Abandoned Well Heterogeneous Layered Media

Some Results

(10 realizations)

- Correlation between fast paths and permeability distribution is evident
- Leakage, arrival time are heavily dependent on permeability distribution
- Heterogeneous layer media results in similar leakage rates.



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Contact Algorithm

 A node-face contact capability is implemented with a two-part strategy:

1. A search algorithm to define contact constraints which
 - operates on the current configuration
 - produces a list of interactions
2. An enforcement algorithm to enforce the contact constraints
 - Includes the physics, i.e. friction model

The enforcement algorithm satisfies equilibrium across the contact interface and also enforces frictional sliding based on the following slip condition:

$$\Phi := \|t_T\| - \mu t_N \leq 0 \quad \text{no slip due to shear}$$

$$\Phi := \|t_T\| - \mu t_N > 0 \quad \text{slip will occur due to shear}$$

3D disposal room closure behavior in salt at different locations along drift



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