SAND2011-7092C

Modeling of General 1-D Periodic Leaky-Wave Antennas in Layered Media using EIGER™

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Introduction

LWA Geometry: Printed circuit lines with 1D periodic modulation or waveguides with a 1D array of periodic slots both with fully 3D currents over a grounded layered medium.

Goals:

- Evaluate the dispersion diagrams for all the propagating modes: bound (non-radiating) and leaky modes to aid in leaky-wave antenna design.
- Direct calculation of corresponding radiation patterns.

Michalski formulation for layered media:

$$E(\mathbf{r}) = -j\omega\int_s \underline{\mathbf{G}}_{\mathrm{A}}^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')\,ds' - \nabla\int_s K_{\Phi}^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\nabla'\cdot\mathbf{J}(\mathbf{r}')\,ds'$$
$$-\nabla\int_s P_z^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\,\mathbf{z}_0\cdot\mathbf{J}(\mathbf{r}')\,ds' - \frac{1}{\varepsilon}PV\int_s \nabla\times\underline{\mathbf{G}}_{\mathrm{F}}^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{M}(\mathbf{r}')\,ds' \pm \frac{\mathbf{M}(\mathbf{r}')}{2}\delta_{\mathbf{r},\mathbf{r}'}$$

$$H(\mathbf{r}) = \pm\frac{\mathbf{J}(\mathbf{r}')}{2}\delta_{\mathbf{r},\mathbf{r}'} + \frac{1}{\mu}PV\int_s \nabla\times\underline{\mathbf{G}}_{\mathrm{A}}^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')\,ds' - j\omega\int_s \underline{\mathbf{G}}_{\mathrm{F}}^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{M}(\mathbf{r}')\,ds'$$
$$-\nabla\int_s Q_z^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\,\mathbf{z}_0\cdot\mathrm{M}(\mathbf{r}')\,ds' - \nabla\int_s K_{\Psi}^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\nabla'\cdot\mathbf{M}(\mathbf{r}')\,ds'$$

K. A. Michalski and D. Zheng, "Electromagnetic scattering by sources of arbitrary shape in layered media, Part I: Theory," *IEEE Trans. Antennas Propag.*, vol. 38, no. 3, pp.335-344, Mar. 1990.

The Complex Wave Numbers are Found from the Eigenvalues of the Integral Equation.

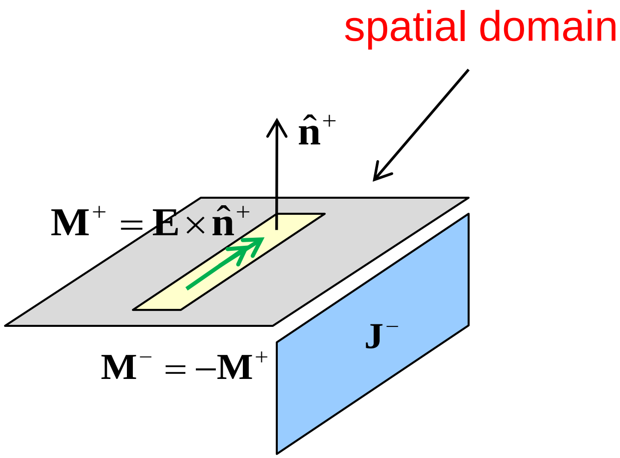
$$\mathbf{J}\mathbf{A} = \sum_n I_n \quad {}_n \qquad \mathbf{M}\mathbf{A} = \sum_n I_n^M \quad {}_n$$

Metallic conductors (EFIE):

$$\mathbf{E}_{\tan}^{\alpha}(\mathbf{J}^{\alpha}, \mathbf{M}^{\alpha}) = 0, \; \alpha = \pm$$

Slots in pec ground planes:

$$\left[\hat{\mathbf{n}} \times \mathbf{H}^{\alpha}(\mathbf{J}^{\alpha}, \mathbf{M}^{\alpha})\right]_{\alpha=-}^{\alpha=+} = 0$$



These equations can be effectively solved by means of the MoM in the spatial domain:

$$[\mathbf{Z}(k_{x0})][I_n] = [0] \quad \Rightarrow \quad \det[\mathbf{Z}(k_{x0})] = 0$$

$$k_{x0} = \beta - j\alpha \quad \Rightarrow \quad k_{xn} = k_{x0} + \frac{2\pi n}{p}$$

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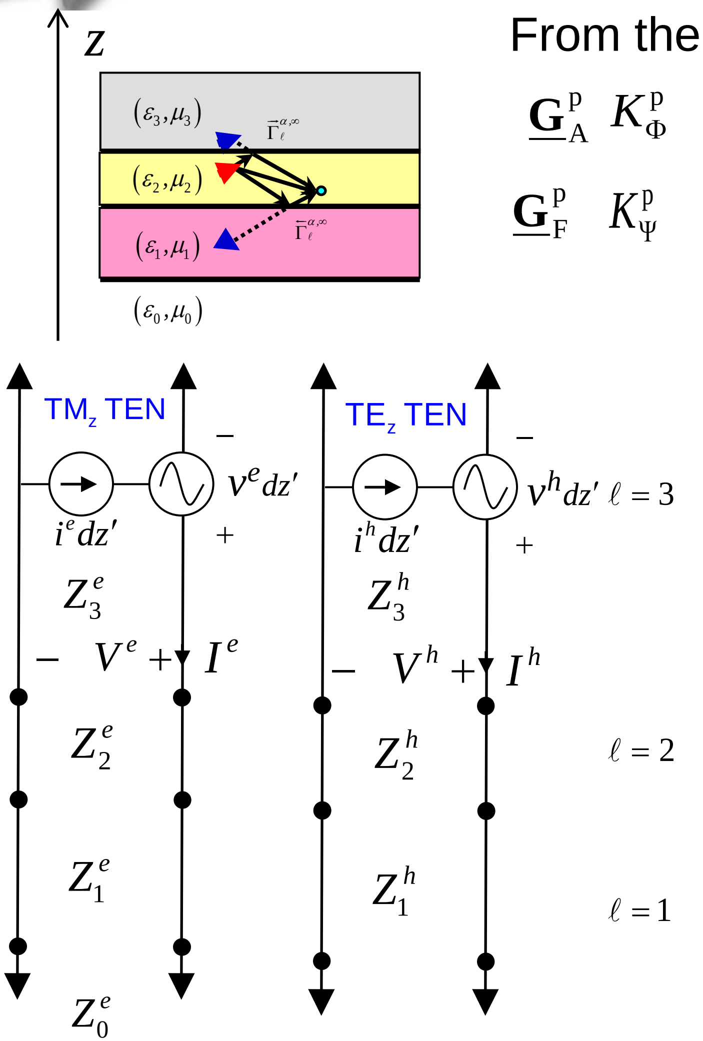
Spectral Representation of Green's Functions



From the periodicity, the Green's functions

$\underline{\mathbf{G}}_A^p \quad K_\Phi^p \quad P_z^p$

$\underline{\mathbf{G}}_F^p \quad K_\Psi^p \quad Q_z^p$

$$G^p = \frac{1}{2\pi p}\sum_{n=-\infty}^{+\infty} e^{-jk_{x_n}\Delta x}\int_{-\infty}^{+\infty} \tilde{G}\,(k_{t_n}, z, z')\; e^{-jk_y\Delta y}\mathrm{d}k_y$$

$$k_{t_n}^2 = k_{x_n}^2 + k_{y_n}^2$$

$$\tilde{K}_\Phi = \frac{V_I^h - V_I^e}{k_t^2}, \qquad \tilde{P}_z = \frac{j\omega\mu_0\mu_{r'}}{k_t}\frac{V_V^h - V_V^e}{k_t}$$

$$\mathbf{G}_A = \begin{pmatrix} \dfrac{V_I^h}{j\omega} & 0 & 0 \\ 0 & \dfrac{V_I^h}{j\omega} & 0 \\ \dfrac{j\omega\mu_0\mu_r(I_I^e - I_I^h)k_x}{k_t^2} & \dfrac{j\omega\mu_0\mu_r(I_I^e - I_I^h)k_y}{k_t^2} & \dfrac{\mu_0\mu_r I_V^e}{j\omega\varepsilon_0\varepsilon_{r'}} \end{pmatrix}$$

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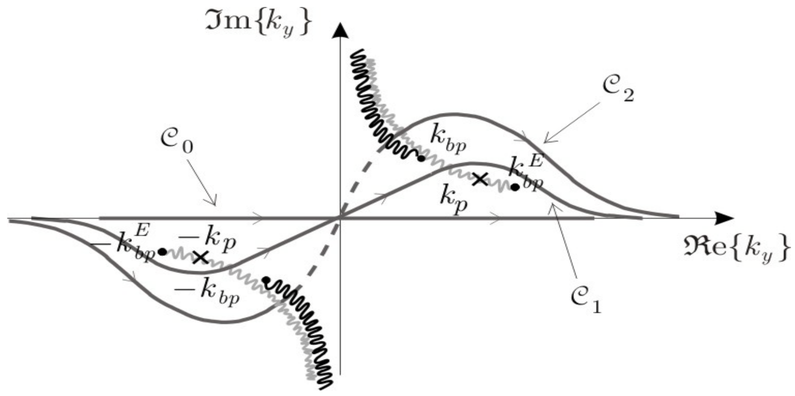
Integration Paths

$$G^{\mathrm{p}} = \frac{1}{2\pi p} \sum_{n=-\infty}^{+\infty} e^{-jk_{x_n}\Delta x} \int_{-\infty}^{+\infty} \tilde{G}\,(k_{t_n}, z, z')\, e^{-jk_y \Delta y} \mathrm{d}k_y$$

The singularities of the integrand are
the singularities of the multilayered Green's functions
plus the singularities of the extracted terms (homogeneous-medium problem).



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Asymptotic Extractions

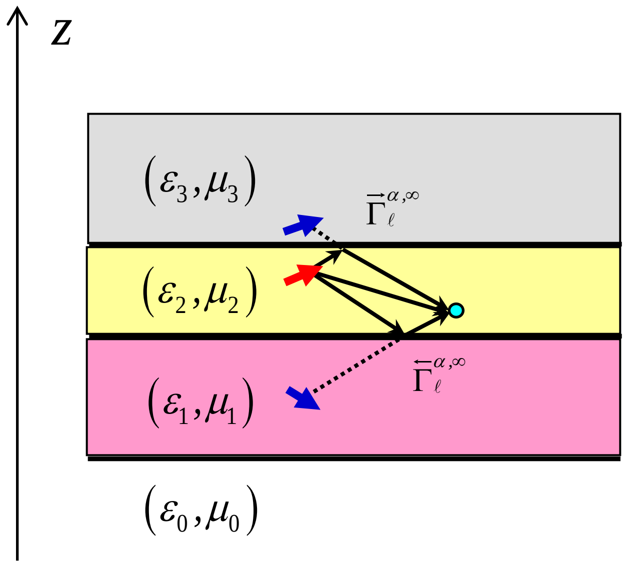
Depending on the component, different asymptotic behaviors are extracted.

$G^{\mathrm{p}}_{A,xx}$ $G^{\mathrm{p}}_{A,yy}$ K^{p}_{Φ} $G^{\mathrm{p}}_{A,zz}$

Kummer extraction

$$G^{\mathrm{p}} = \frac{1}{2\pi p}\sum_{n=-\infty}^{+\infty}\left\{e^{-jk_{x_n}\Delta x}\int_{-\infty}^{+\infty}\left[\tilde{G}(k_{t_n},z,z') - \sum_{i=-1}^{+1} C_i\,\tilde{g}(k_{t_n},\Delta z_i)\right]e^{-jk_y\Delta y}\,\mathrm{d}k_y\right\} + \sum_{i=-1}^{+1} C_i\, g^{\mathrm{p}}(\Delta\mathbf{r}_i)$$

Accelerated through Ewald



$$\tilde{g}(k_{t_n},\Delta z) = \frac{e^{-jk_{z_n}|\Delta z|}}{2jk_{z_n}} \qquad k_{z_n} = (k_s^2 - k_{t_n}^2)^{1/2}$$

$k_s = k$ of source layer

$$g^{\mathrm{p}}(\Delta\mathbf{r}) = \sum_{n=-\infty}^{\infty}\frac{e^{-jk_sR_n}}{4\pi R_n}e^{-jnk_{x0}p} \qquad R_n = |\Delta\mathbf{r} - np\hat{\mathbf{x}}|$$

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Vertical Currents (1)

Different terms need to be extracted from the series for vertical currents

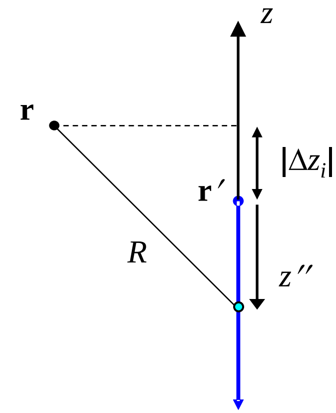
$$P_z^{\mathrm{p}} = \frac{1}{2\pi p}\sum_{n=-\infty}^{+\infty}\left\{e^{-jk_{x_n}\Delta x}\int_{-\infty}^{+\infty}\left[\tilde{P}_z(k_{t_n}, z, z') - \sum_{i=-1}^{+1}\frac{C_i^z}{jk_z}\tilde{g}(k_{t_n}, \Delta z_i)\right]e^{-jk_y \Delta y}\mathrm{d}k_y\right\} + \sum_{i=-1}^{+1} C_i^z g^{z,\mathrm{p}}(\Delta \mathbf{r}_i)$$

The extracted terms contain the extra factor $1/k_z$

$$g^{z,\mathrm{p}}(\Delta \mathbf{r}_i) = \int_{|\Delta z_i|}^{+\infty} g^{\mathrm{p}}(\Delta x\,\hat{\mathbf{x}} + \Delta y\,\hat{\mathbf{y}} + z''\,\hat{\mathbf{z}})\,\mathrm{d}z''$$

Accordingly, in the space domain a different homogeneous-medium Green's function must be used, that is of the form

This is the potential produced by a periodic "half-line source" that starts at a vertical distance $|\Delta z_i|$ from the observation point and extends vertically to infinity.



Vertical Currents (2)

Similar expressions can be obtained for the nondiagonal dyadic elements

$G^{\mathrm{p}}_{A,zx}$ and $G^{\mathrm{p}}_{A,zy}$

$$G^{\mathrm{p}}_{A,z\upsilon} = \frac{1}{2\pi p}\sum_{n=-\infty}^{+\infty}\left\{e^{-jk_{x_n}\Delta x}\int_{-\infty}^{+\infty}\left[\tilde{G}_{A,z\upsilon}(k_{\mathrm{t}_n}, z, z') - \sum_{i=-1}^{+1} C_i^{z\upsilon}\frac{k_\upsilon}{jk_z}\tilde{g}(k_{\mathrm{t}_n}, \Delta z_i)\right]e^{-jk_y\Delta y}\mathrm{d}k_y\right\}$$

$$+\, j\,\hat{\boldsymbol{\upsilon}}\cdot\sum_{i=-1}^{+1} C_i^{z\upsilon}\nabla g^{z,\mathrm{p}}(\Delta\mathbf{r}_i)$$

where $\upsilon = x, y, \quad \hat{\boldsymbol{\upsilon}} = \hat{\mathbf{x}}, \hat{\mathbf{y}}$

The factors k_x and k_y appearing above the term k_z correspond to a differentiation with respect to x and to y, respectively, in the spatial domain, leading to the

gradient of the half-line source potential.

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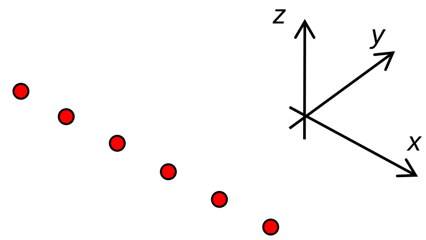
Free-space Acceleration with Ewald

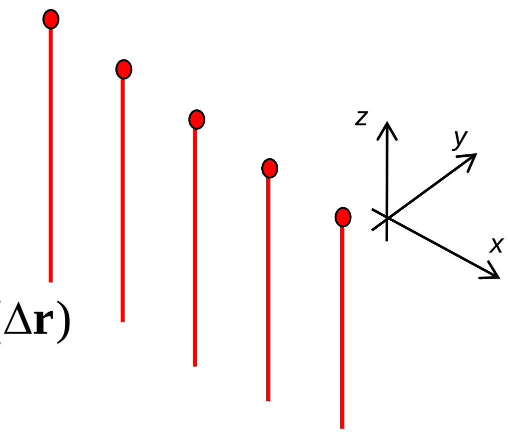
The terms extracted from the "**planar**" components are Green's functions for a **1-D array of point sources in free-space**. They are summed back with the Ewald approach:



$$g^{\mathrm{p}}(\Delta\mathbf{r}) = \sum_{n=-\infty}^{\infty} \frac{e^{-jk_s R_n}}{4\pi R_n} e^{-jnk_{x0}p} = g_{\mathrm{spectral}}^{\mathrm{E}}(\Delta\mathbf{r}) + g_{\mathrm{spatial}}^{\mathrm{E}}(\Delta\mathbf{r})$$

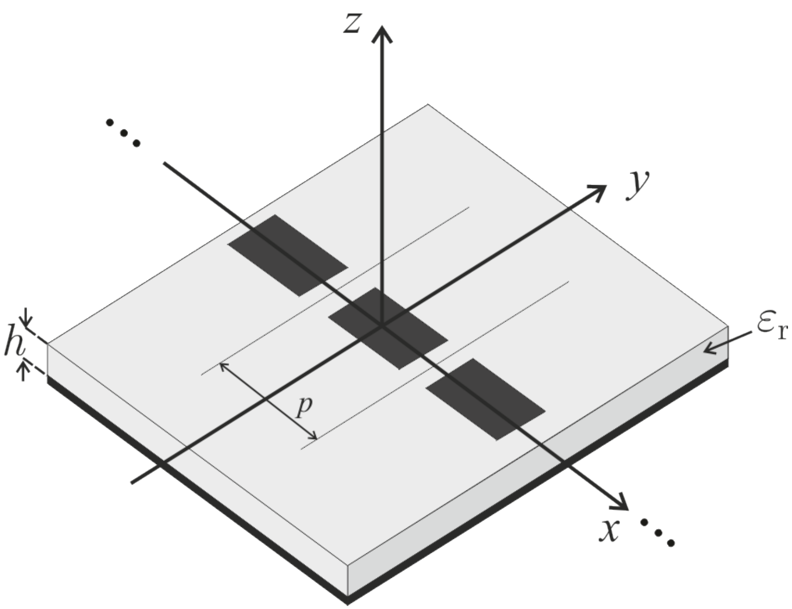
Algebraic convergence (series) — Gaussian convergence ($g_{\mathrm{spectral}}^{\mathrm{E}}$, $g_{\mathrm{spatial}}^{\mathrm{E}}$)

The terms extracted from the **vertical** components are Green's functions for a **1-D array of half-line sources in free-space**. A modified Ewald approach has been developed to accelerate these series:



$$g^{z,\mathrm{p}}(\Delta\mathbf{r}) = \int_{|\Delta z_i|}^{+\infty} g_{\mathrm{spectral}}^{\mathrm{E}}(\Delta\mathbf{r})dz'' + \int_{|\Delta z_i|}^{+\infty} g_{\mathrm{spatial}}^{\mathrm{E}}(\Delta\mathbf{r})dz'' = g_{\mathrm{spectral}}^{z,\mathrm{E}}(\Delta\mathbf{r}) + g_{\mathrm{spatial}}^{z,\mathrm{E}}(\Delta\mathbf{r})$$

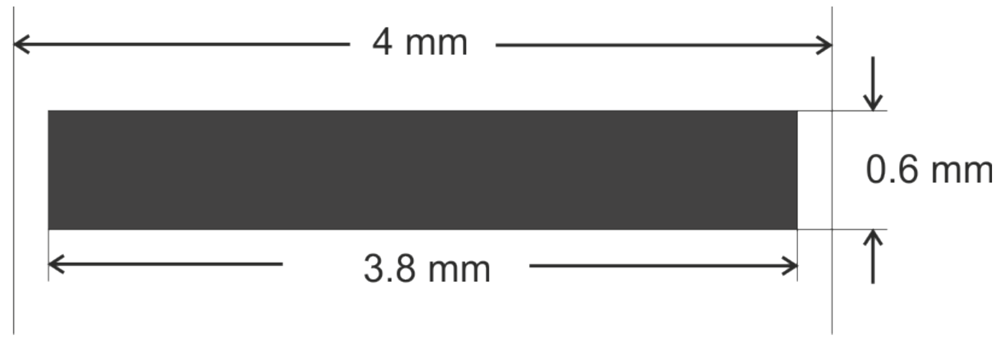
Gap-Coupled Periodic Microstrip line



$$\varepsilon_r = 10.2$$

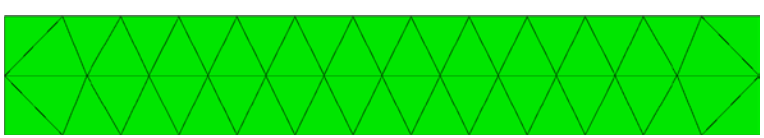
$$h = 0.762\,\text{mm (30 mil)}$$

$$p = 4\,\text{mm}$$

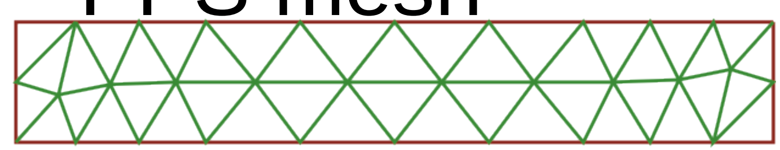


$$L = 3.8\,\text{mm} \qquad w = 0.6\,\text{mm}$$

Eiger Mesh



PPS mesh



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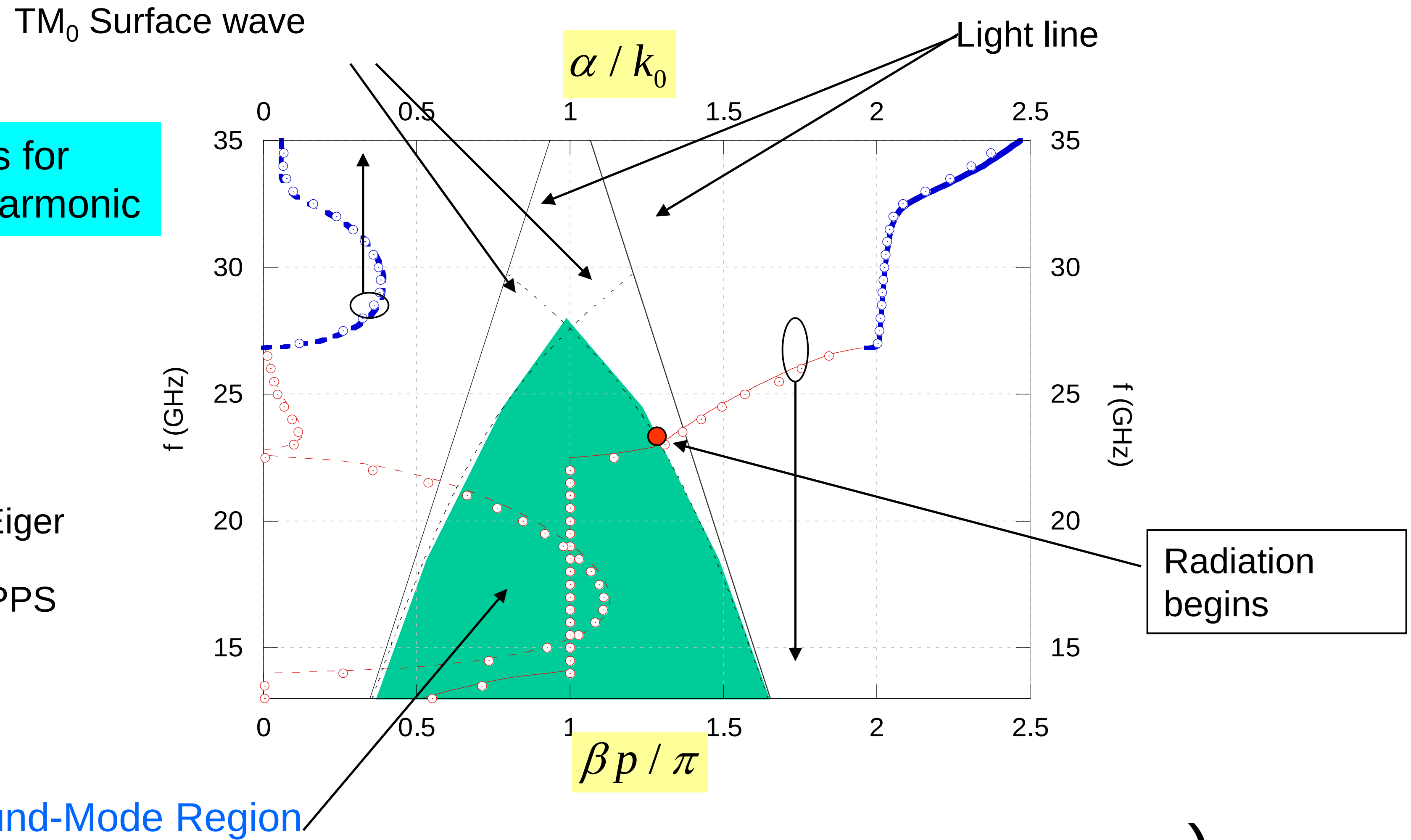
Validation for Gap-Coupled Periodic Microstrip line

Results for n = 0 harmonic

Dots: Eiger

Lines: PPS



Blue lines: forward radiation from n = -1 (improper)

Red lines: backward radiation from n = -1 (proper)

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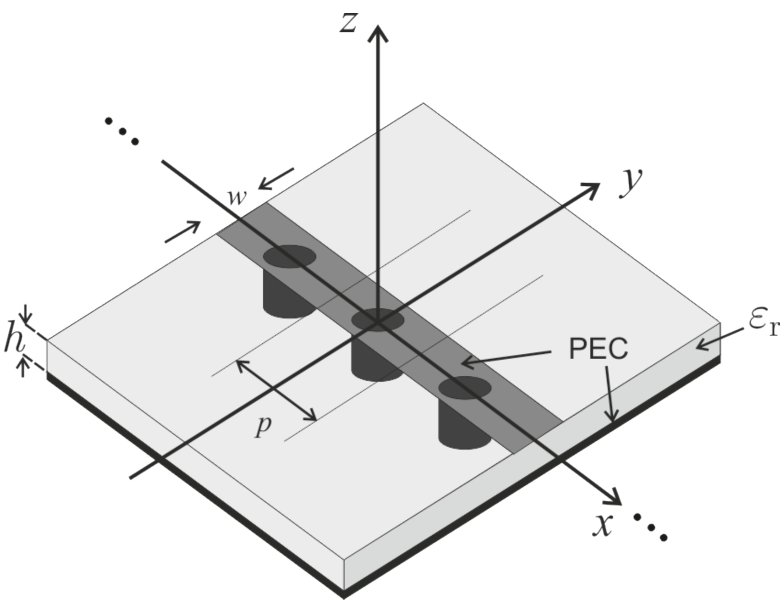
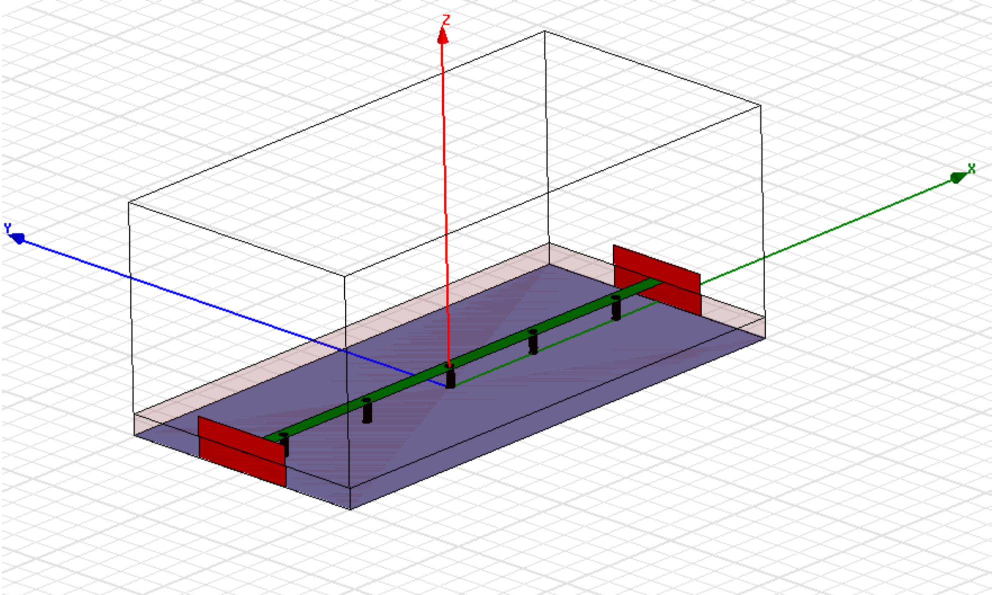
Microstrip Periodically Loaded with Vertical Strips



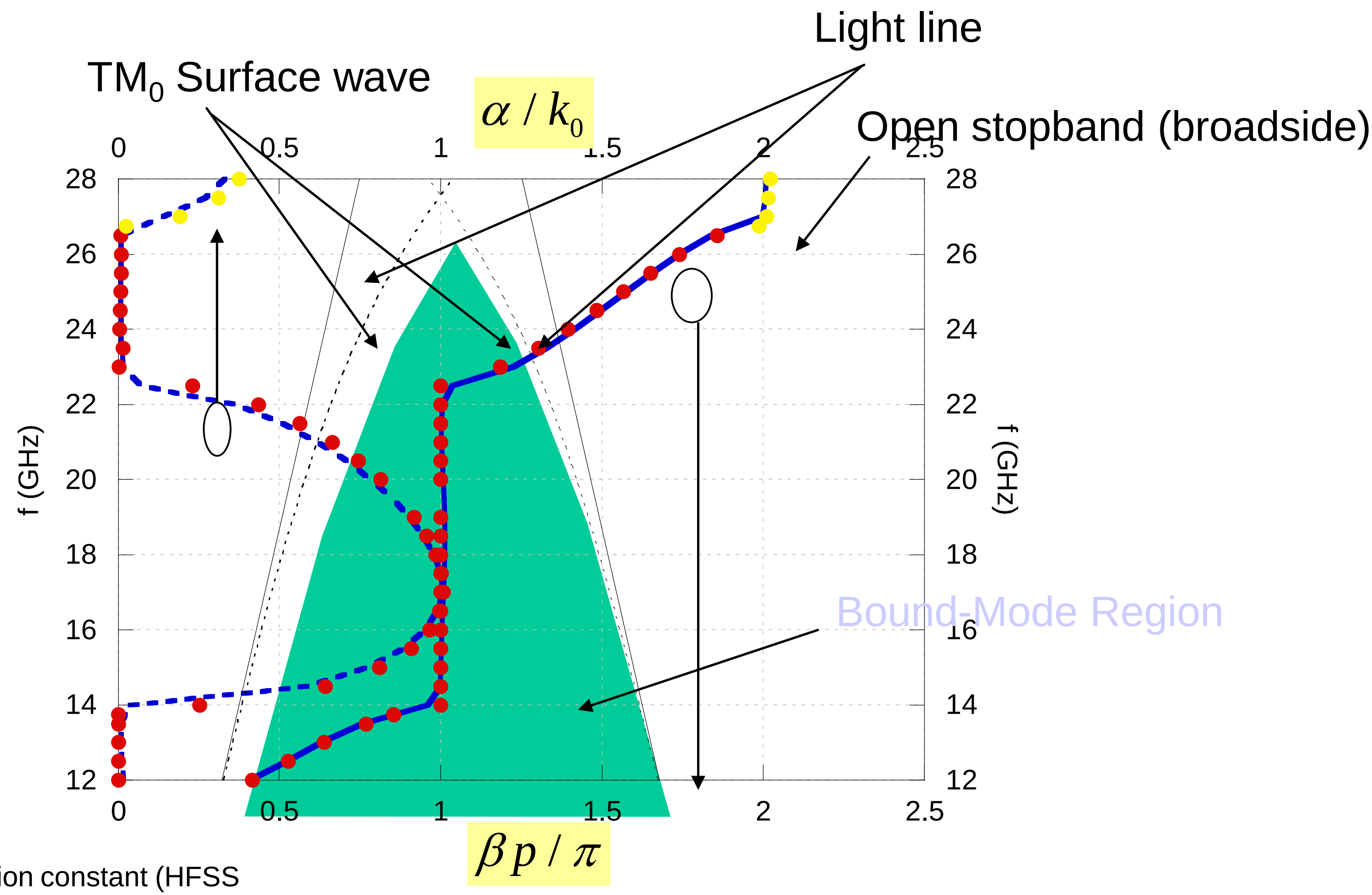


Approximate method:

G. Valerio, S. Paulotto, P. Baccarelli, P. Burghignoli, A. Galli, *'Sapienza' University of Rome, Italy, "***Improving Modal Analysis of 1D-Periodic Lines Based on the Simulation of Finite Structures,"** in Proceedings of IEEE AP-S/URSI 2010/

The 5-cell structure analyzed with HFSS, with vertical PEC posts of radius $a = w/4$, where w is the width of the strip.

Microstrip Periodically Loaded with Vertical Strips



Attenuation constant (HFSS + Approximate Method)

Phase constant (HFSS + Approximate Method)

EIGER improper solution (the approximate method is not able to distinguish the spectral character)

EIGER proper solution

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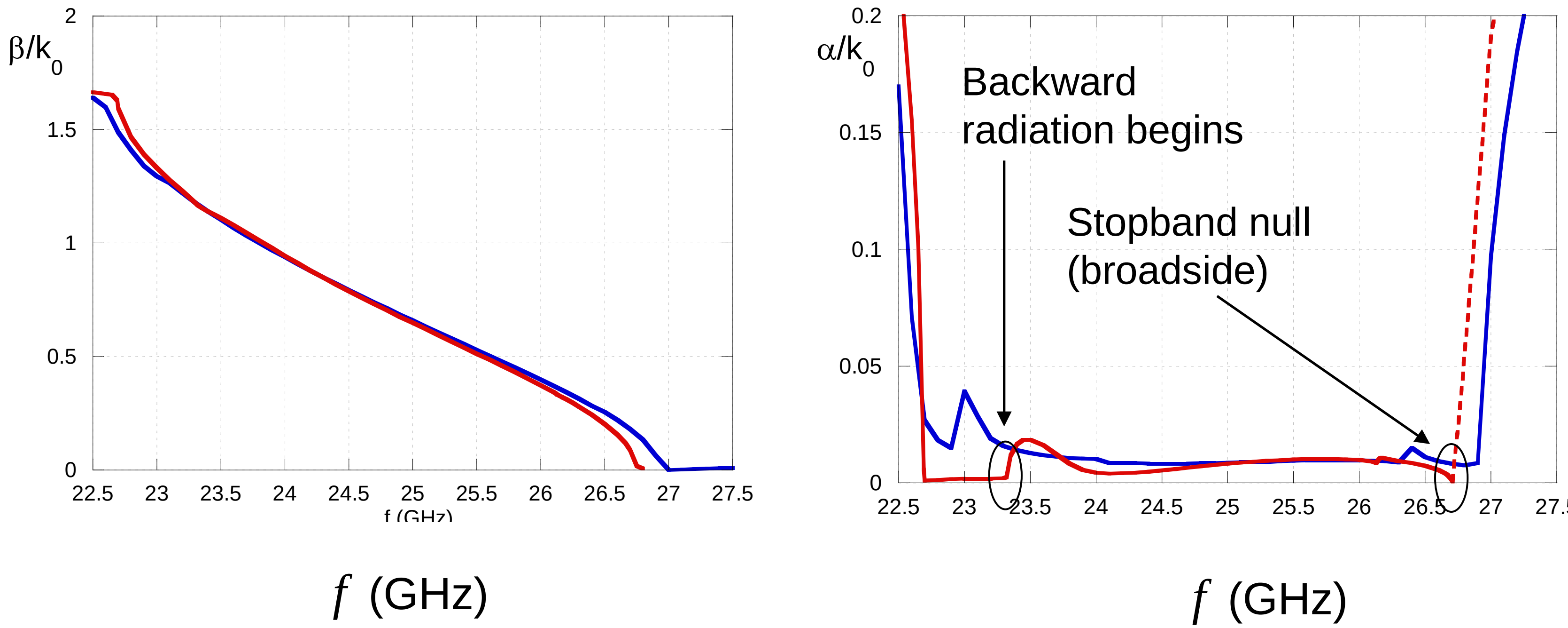
Microstrip Periodically Loaded with Vertical Strips

Zoom into the radiation region



HFSS + Approximate Method

EIGER proper solution

EIGER improper solution

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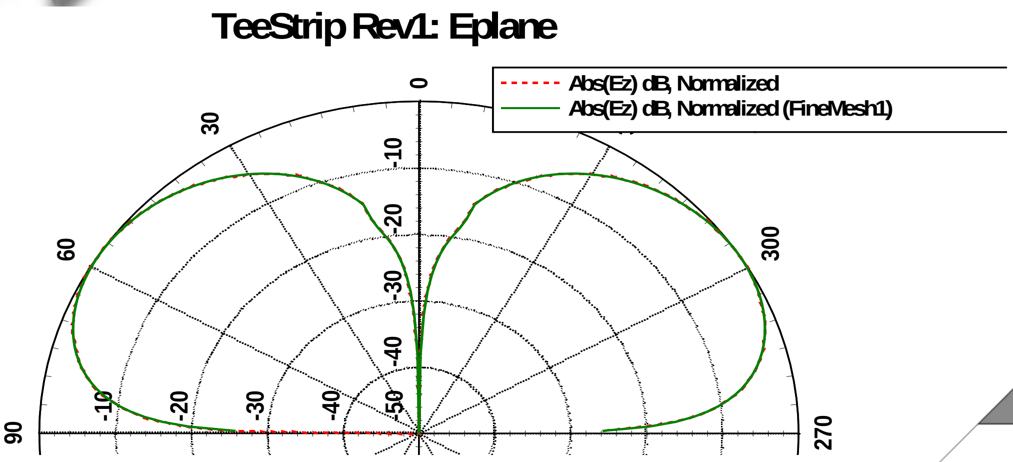
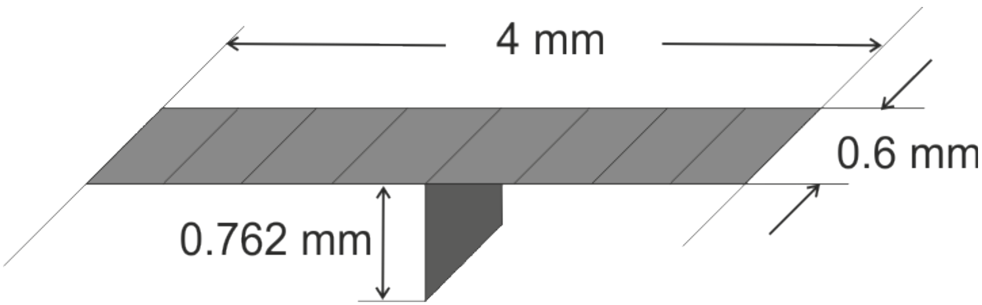
Microstrip Periodically Loaded with Vertical Strips: Radiation Pattern





Picture of source needed

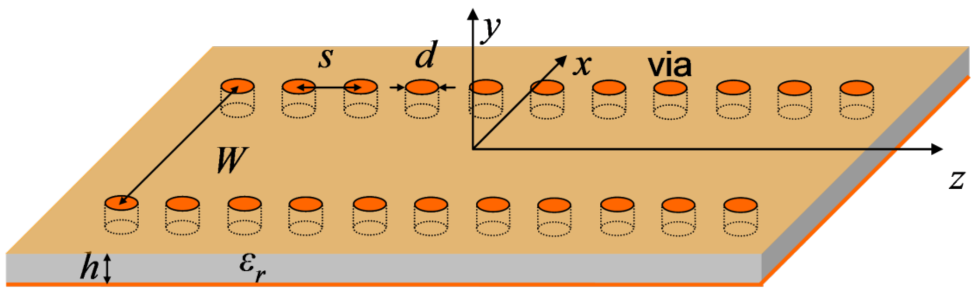
Dipole source failed to excite the leaky wave mode

Goal: Model Substrate Integrate Waveguide

- Substrate integrated waveguide (SIW) has been recently investigated for its significant advantages such as low cost, low loss, and easy integration with planar circuits.
- The SIW consists of a wide microstrip line that is shorted at the edges with conductive vias, acting as a rectangular waveguide † .

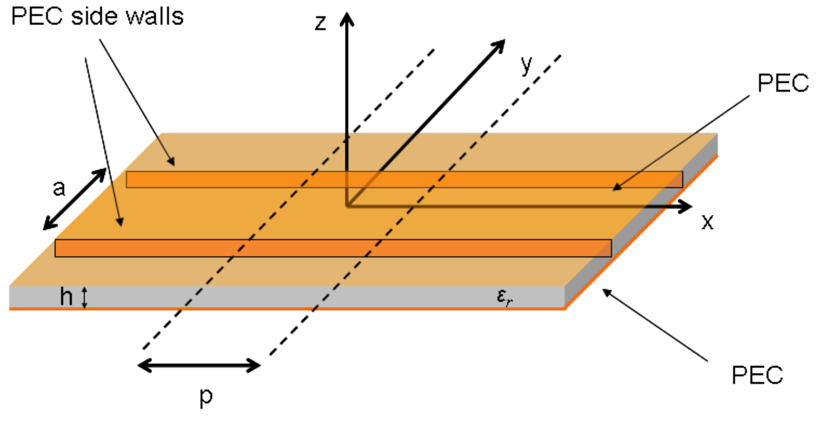
$$k_z = \sqrt{k_0^2 \varepsilon_r \mu_r - \left(\frac{m\pi}{W_e}\right)^2}$$

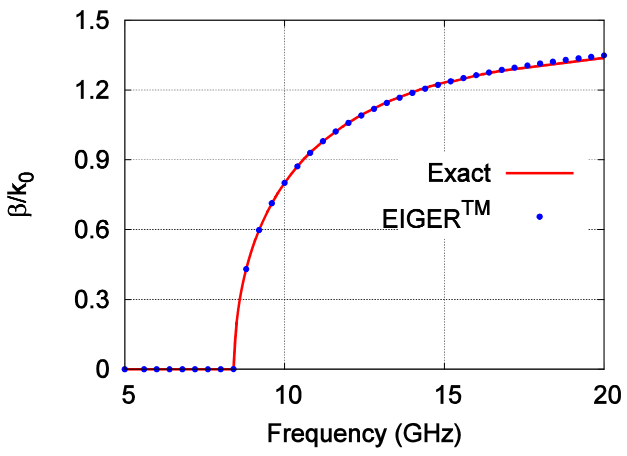
$$W_e = W - 1.08\left(\frac{d^2}{s}\right) + 0.1\left(\frac{d^2}{W}\right)$$



† F. Xu and K. Wu, IEEE T-MTT, vol. 53, no. 1, pp. 66-73, Jan. 2005

A First Step is to Model a Closed Waveguide





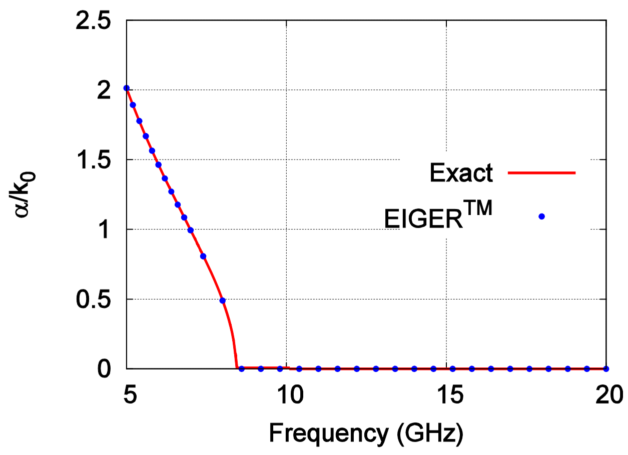


Figure 3: A dielectric-filled rectangular waveguide test case with dimensions *a* and *h* equal to 12 mm and 1.524 mm, respectively. The waveguide is filled with a dielectric having ε_r = 2.2. An artificial 1-D periodic spacing of 3 mm has been used for the periodic Green's function.

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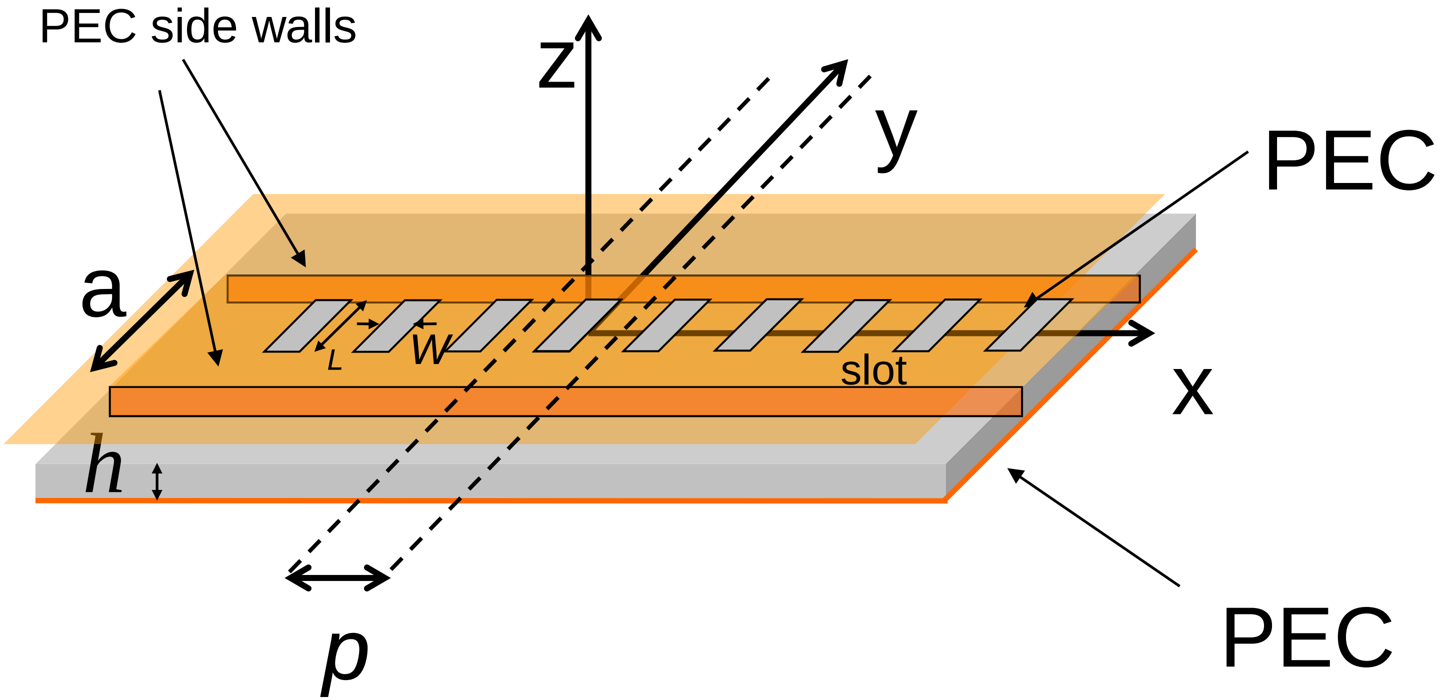
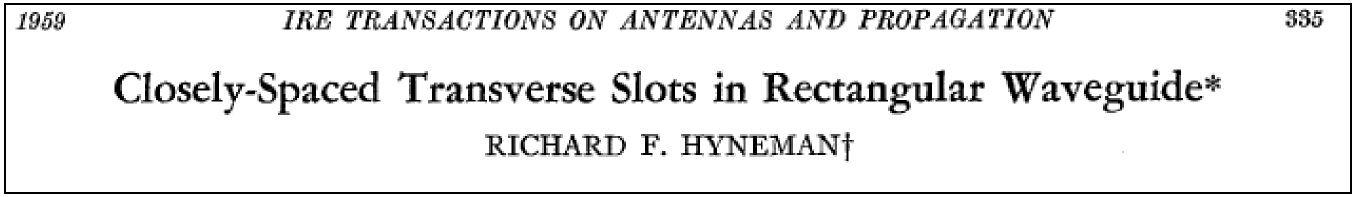
A Next Step is to Model a Periodically Slotted Waveguide

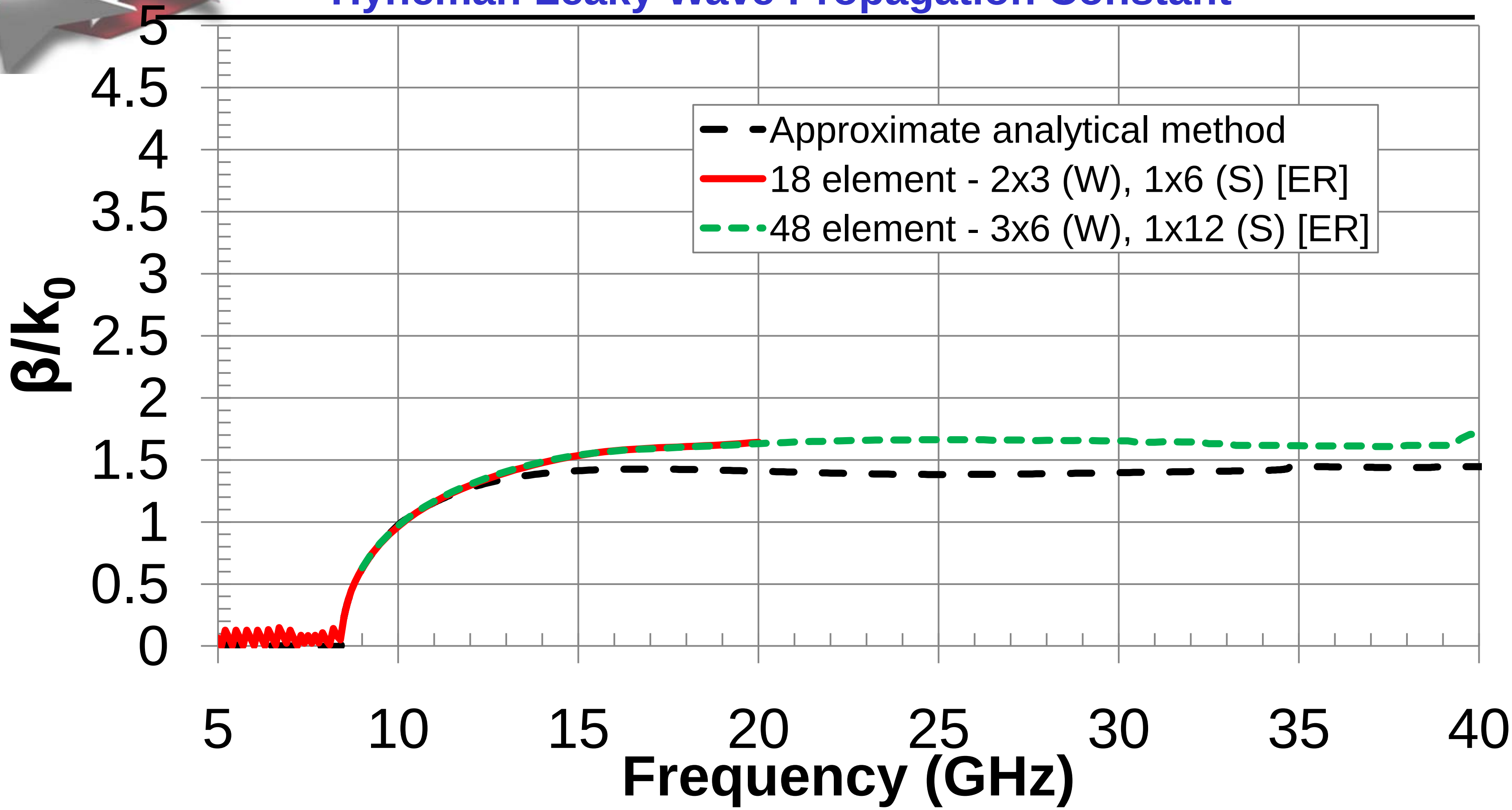



1959 IRE TRANSACTIONS ON ANTENNAS AND PROPAGATION 335

Closely-Spaced Transverse Slots in Rectangular Waveguide*

RICHARD F. HYNEMAN†

Hyneman Leaky Wave Propagation Constant



Approximate analytical method

Investigation of Leaky-Wave Antenna Based on Dielectric-Filled Rectangular Waveguide with Transverse Slots," J. Liu, D. R. Jackson, and Y. Long, IEEE AP-S Intl. Symp., July 11-17, 2010, Toronto, Ontario, Canada (Symp. Digest)

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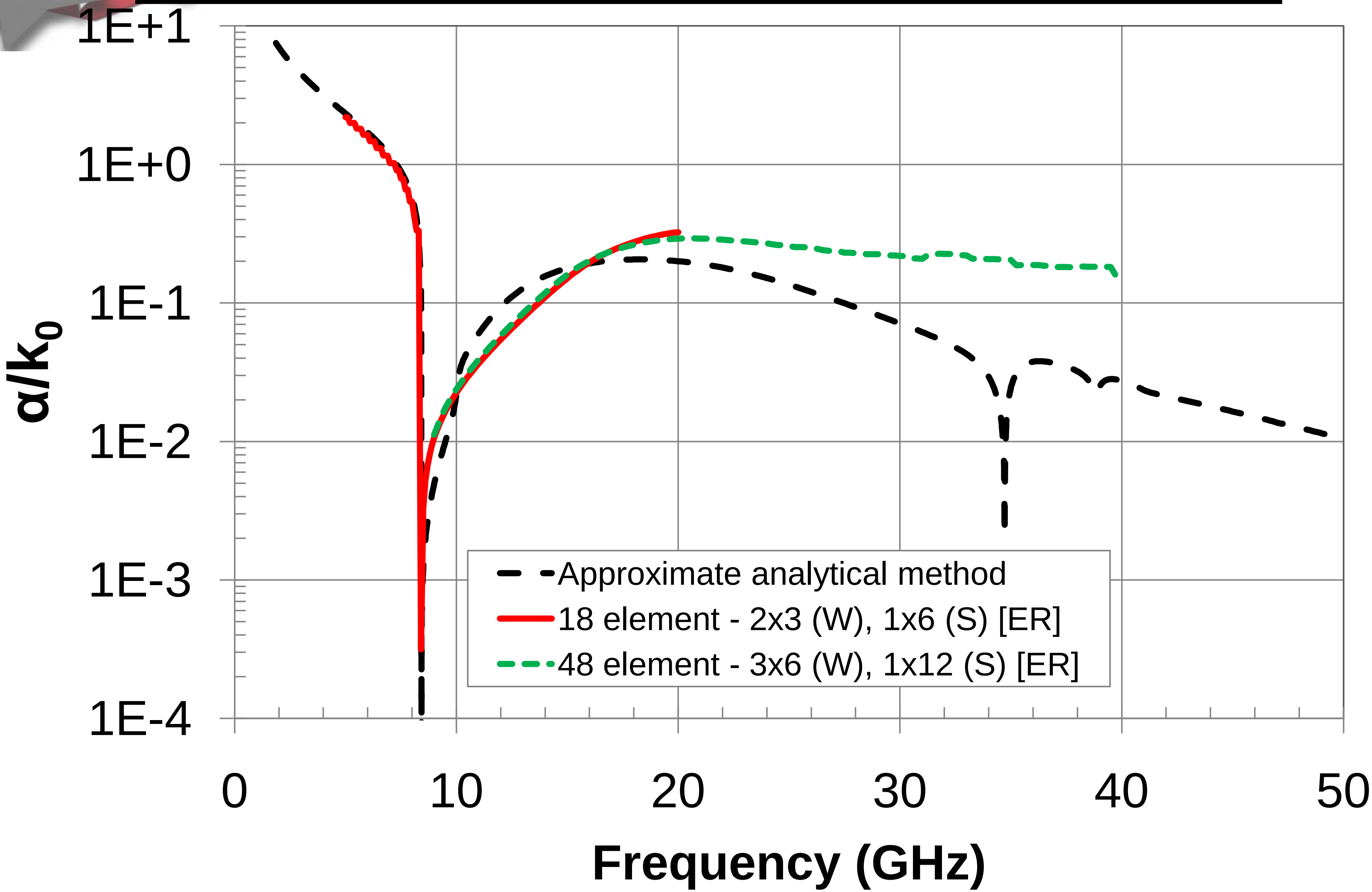
Hyneman Leaky Wave Attenuation Constant



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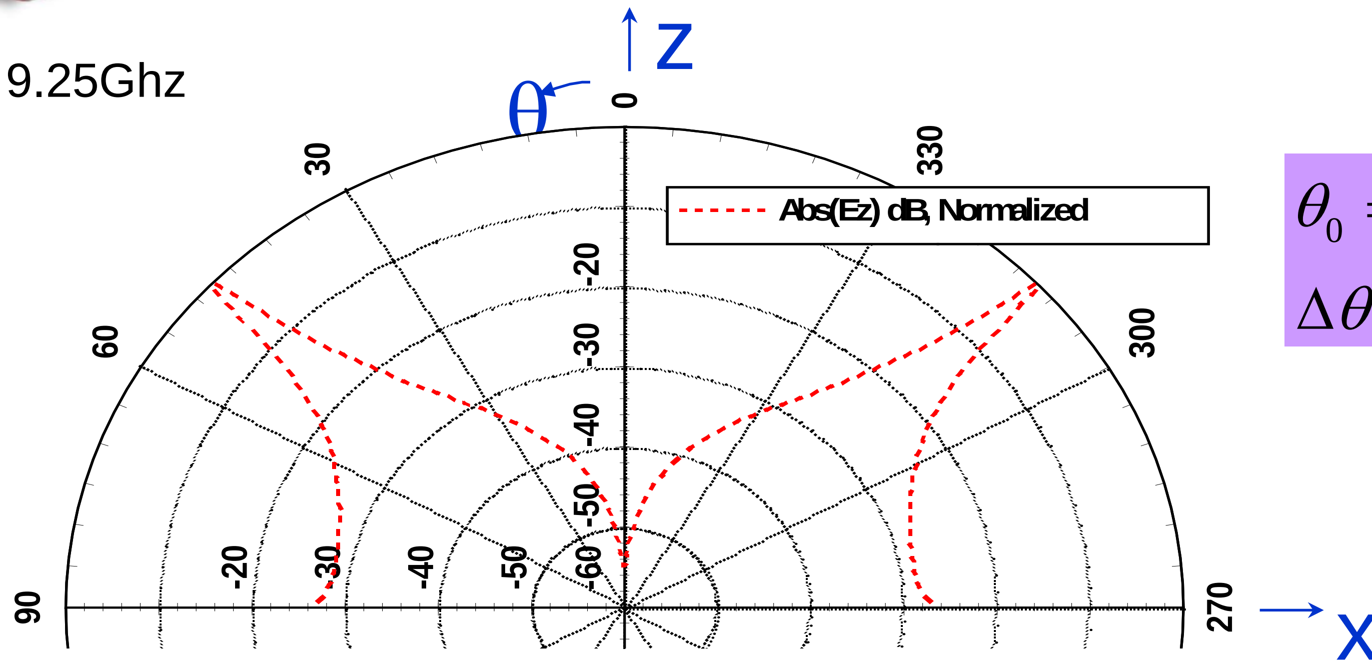
Slotted Waveguide: Phi=0 Plane (□ to Slot)

F = 9.25Ghz



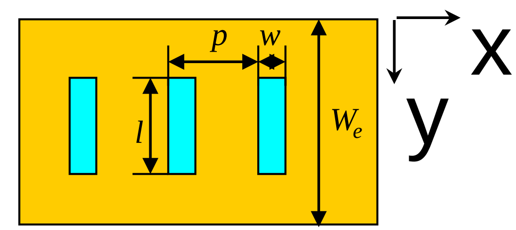
$$\theta_0 = 47.75^\circ$$

$$\Delta\theta = 1.4^\circ$$

$$\cos\theta_0 = \beta_x / k_0 = 0.672$$

$$\Delta\theta = 2\frac{\alpha_x / k_0}{\cos\theta_0} \Rightarrow \alpha_x / k_0 = 0.0082$$

RWG



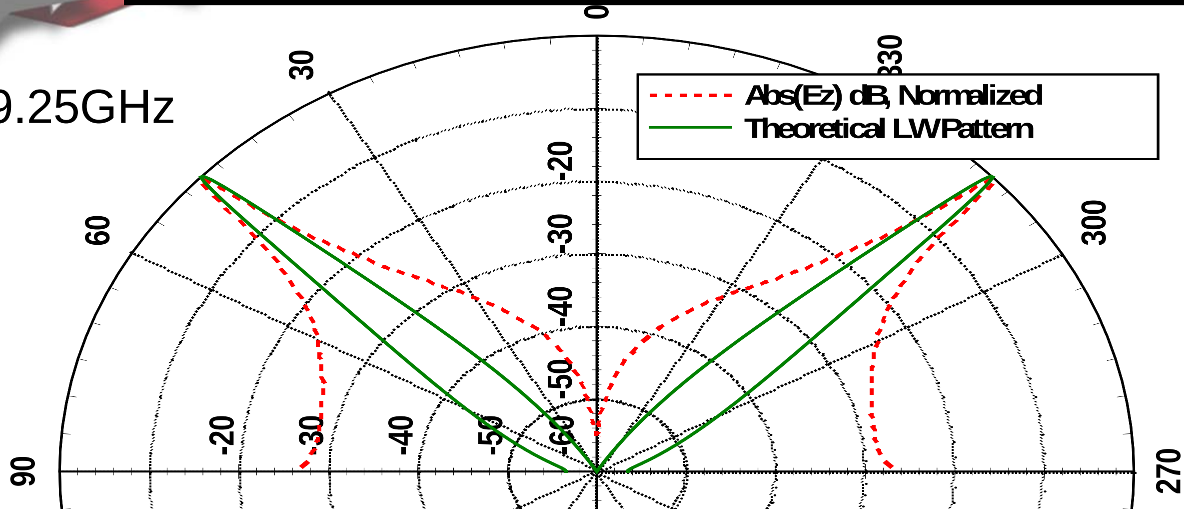
$\varepsilon_r = 2.2,\ h = 1.524$ mm

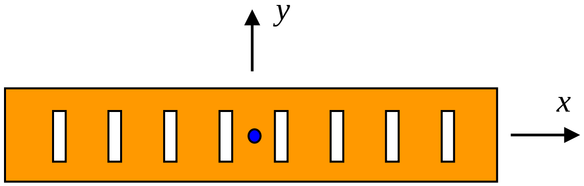
$l = 6$ mm, $w = 0.56$ mm, $p = 3$ mm

$W_e = 12$ mm

Comparing Simulated Pattern to Theoretical LW Pattern

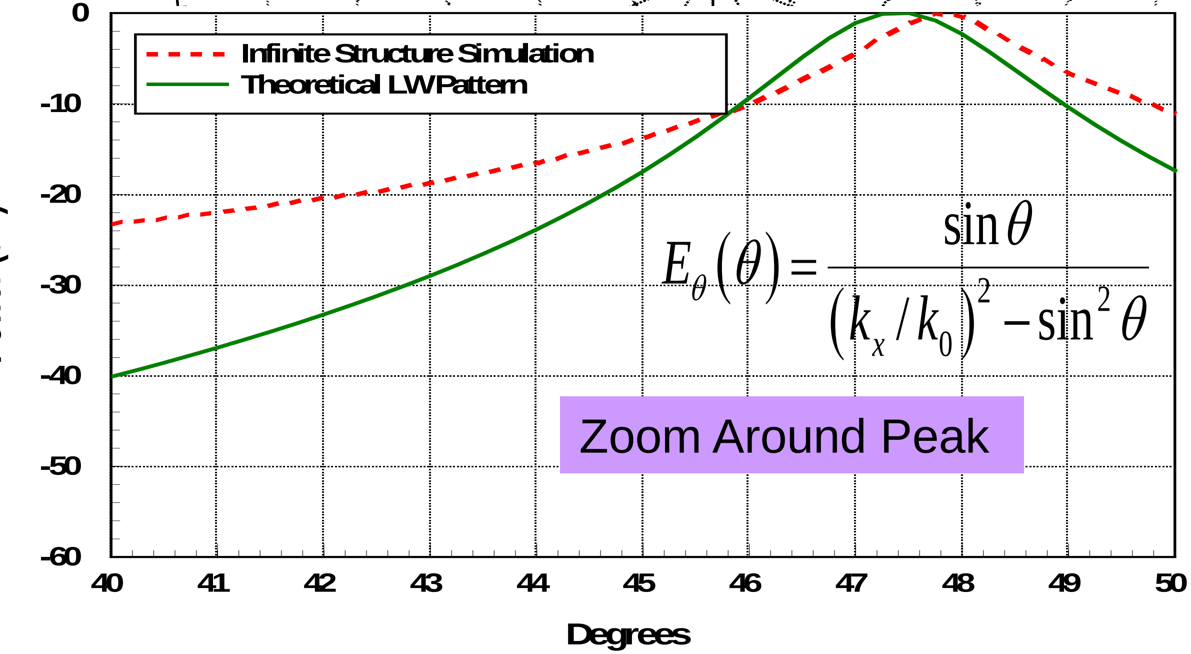
F = 9.25GHz





Vertical dipole

Theoretical LW Pattern

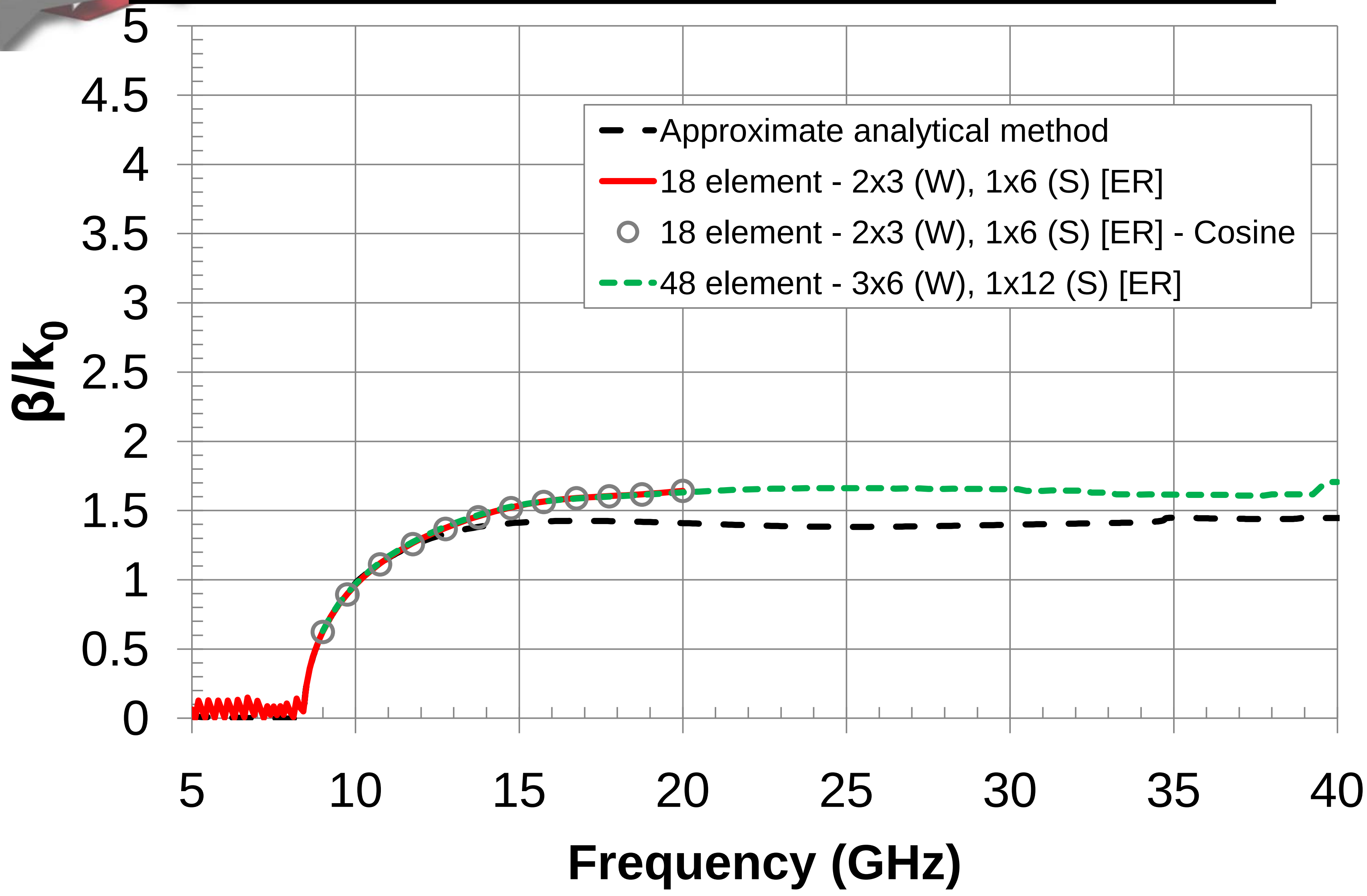


$$E_\theta(\theta) = \frac{\sin\theta}{(k_x/k_0)^2 - \sin^2\theta}$$

$$k_x = \beta - j\alpha$$

Zoom Around Peak

Hyneman Leaky Wave Propagation Constant



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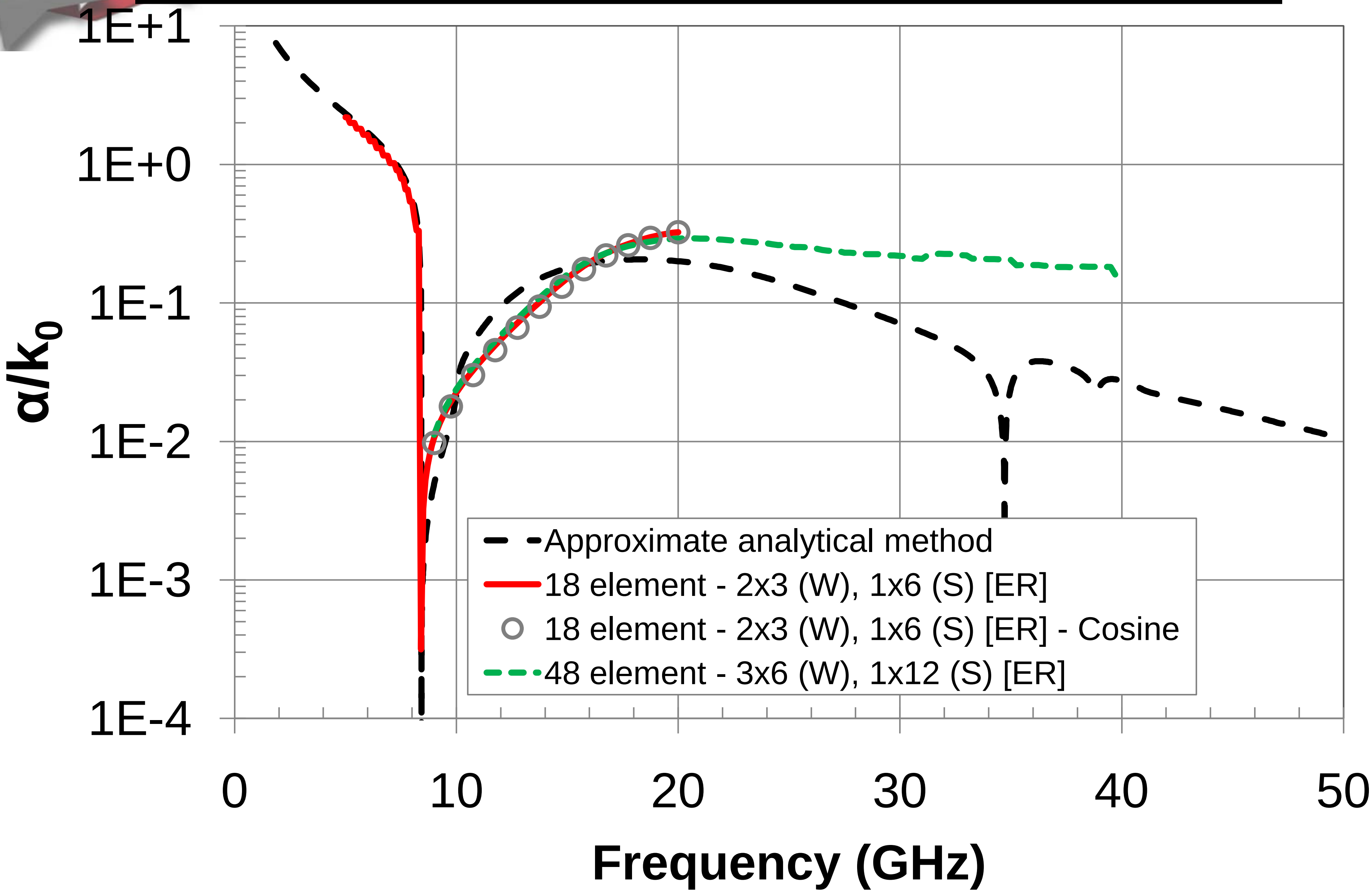
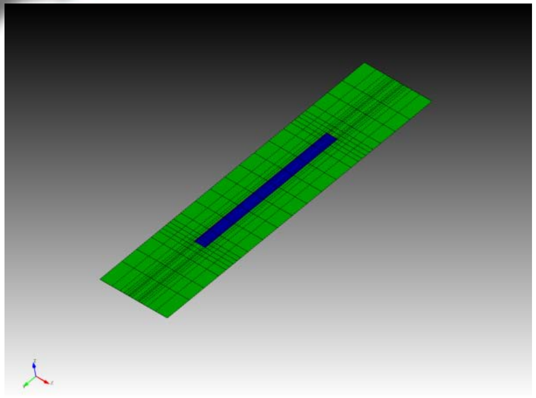
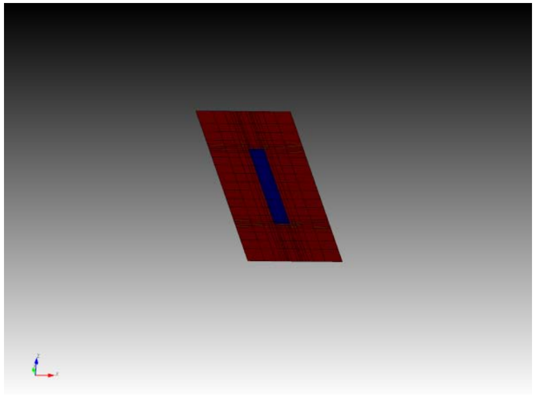
Hyneman Leaky Wave Attenuation Constant



A two dimensionally periodic test case





Field below .5 mm below center of slot

Ex = (0.13875E-01,-0.72604E+00) slot only

Ex = (-0.11165E+00,-0.51954E+00) metal and aperture

Note : We are runing this now and hope to get a better answer

LABORATORY DIRECTED RESEARCH & DEVELOPMENT

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Summary

- **Validated EIGER™ for modeling dispersion diagrams for planar, 1D periodic, leaky wave antennas**
  - PPS (Periodic Planar Simulator) code from Sapienza Universita di Roma
- **Added the capability to model dispersion diagrams for fully 3D leaky wave antennas with**
- **Resolve the discrepancy between the approximate analytical method and EIGER™**

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Thank you!