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# A Parsimonious Approach to QMU and Validation

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# Motivation

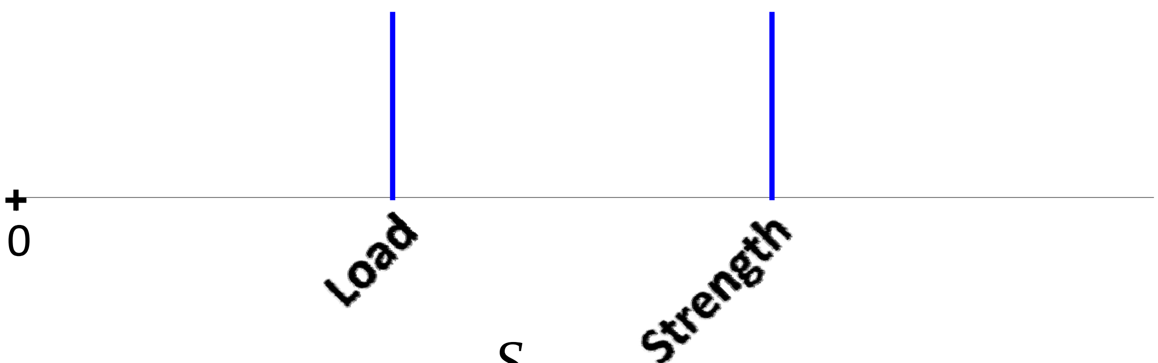
- **Validation is essential to the assurance of model adequacy (perhaps accuracy) and**
- **Validation is required more frequently prior to use of mathematical models for prediction**
- **Yet, practitioners of traditional finite element modeling are unaccustomed to specification of quantities required in validation, specifically**
  - **Measures of response appropriate for validation**
  - **Validation metrics**

# Introduction

- We seek to specify a validation procedure
  - Written in terms of quantities with which most designers will be comfortable, and
  - Using data typically available to the designer
- We develop the procedure for structural dynamic systems
- We then assess the potential for adequacy of the procedure

# Introduction

- **Design is often performed in a deterministic, factor of safety or margin of safety framework**
- **Example**




$$Factor\ of\ safety = \frac{S}{L}$$

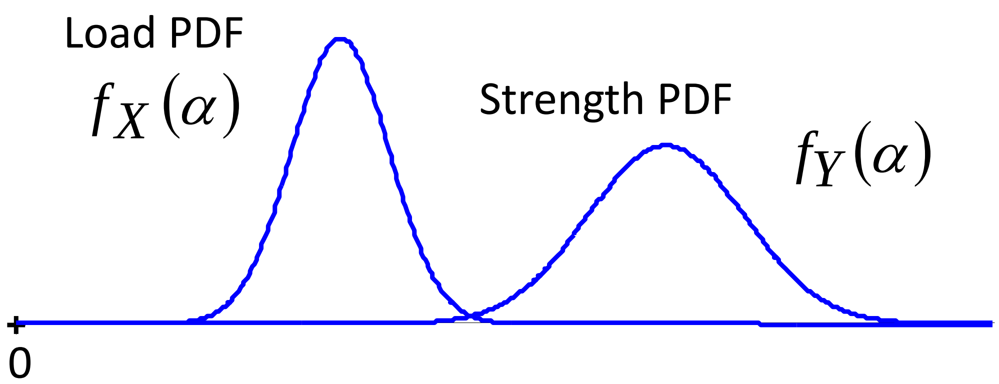
$$Margine\ of\ safety = \frac{S}{L} - 1$$

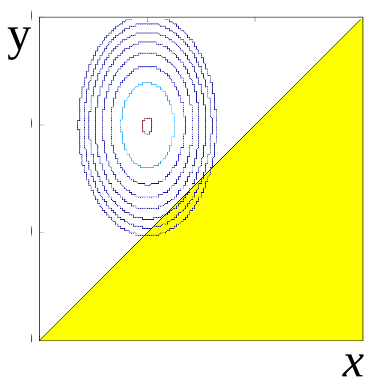
Advantages:
- Easy to apply
- Easy to interpret

Disadvantages:
- Because randomness is not accommodated, probability of failure may be very high or very low
- Because strength is truly random, it may be difficult to specify
- Because load may be random, it may be difficult to specify

# Introduction

- **Design can also be performed in a probabilistic framework**
- **Example**




Probability of failure

$$p_f = \iint_{y-x<0} f_X(x) f_Y(y) dxdy$$

Advantage:

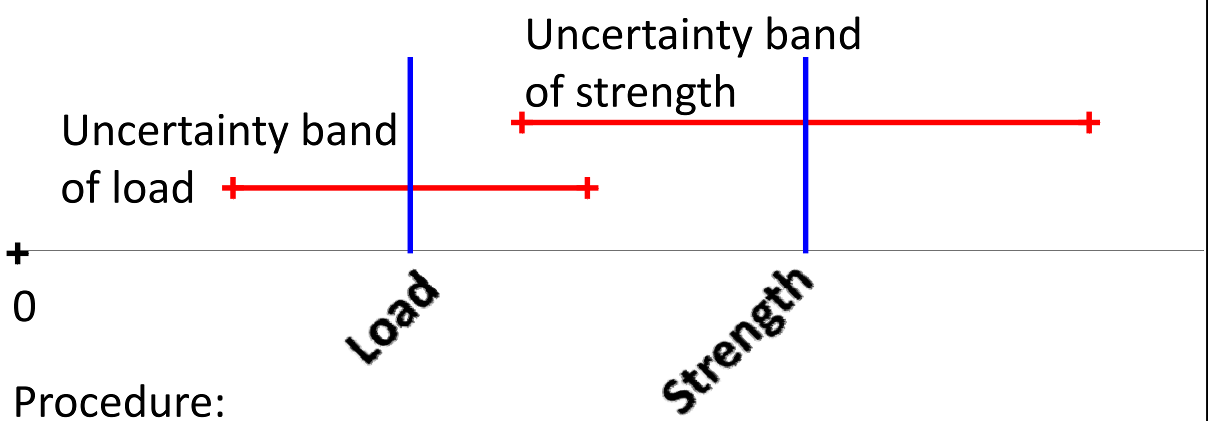
- Randomness in load and strength accommodated

Disadvantages:

- Distributions of load and strength must be established – usually based on limited data
- Probability of failure depends on tail behaviors of load and strength distributions

# Introduction

- **More recently, designs are performed in a quantification of margins and uncertainties framework**
- **Example**




Procedure:
- Load and strength are "best estimates"
- When max of load uncertainty band substantially surpasses min of strength uncertainty band – re-design
- When uncertainty bands do not overlap – design all right
- When overlap of uncertainty bands is small …

Advantages:
- Technique does not require specific knowledge of load and strength distributions
- Yields qualitative design conclusion

Disadvantages:
- Requires some method to establish uncertainty bands, or
- Requires expert "guess" at uncertainty bands

# QMU and Validation

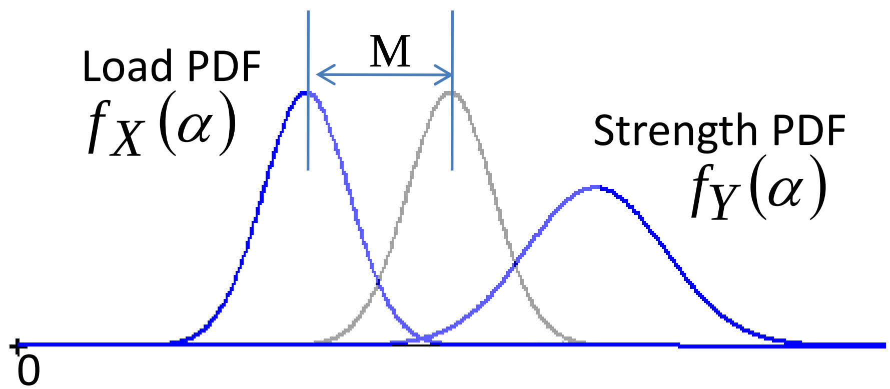
**We propose a UQ technique that combines the best aspects of the three approaches specified, above**

- **Identify "central" load probability distribution**
  - **Internal load in structure involves externally applied load, and**
  - **Structural model**
- **Identify structural strength probability distribution**
- **Estimate probability, $p_u$, of "undesirable" event**
  - **Undesirable event: Multiple of load, $kX$, surpasses strength, $Y$**
- **Limit $p_u$ to "reasonable levels (say, [0.05,0.95]; depends on data available for probabilistic estimates)**

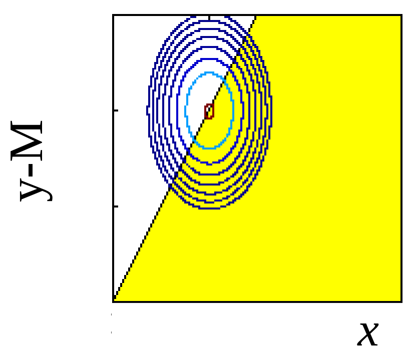
**The approach can be used for *design to avoid undesirable events***

# QMU and Validation

- **Probability of undesirable event** $p_u$
- **Example**




Probability of undesirable event



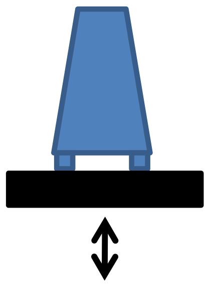

$$p_u = P(kX + M > Y)$$
$$= \iint_{y-kx<M} f_X(x)\, f_Y(y)\, dx dy$$

Advantages:
- Uses familiar frameworks of factor of safety/reliability/QMU
- Randomness in load and strength accommodated but characterized by “central” values

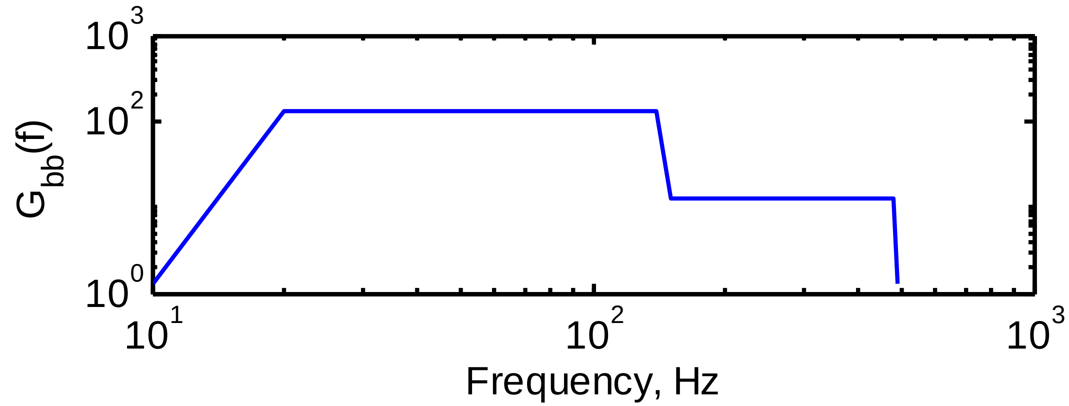
Disadvantages:
- Differs from existing techniques

# UQ – Example

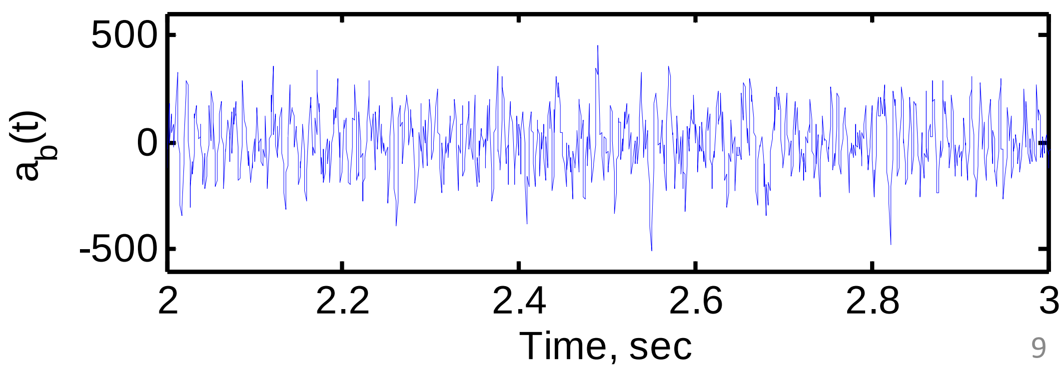
- **Aerospace component (nonlinear base connection) excited at base with zero-mean, stationary random vibration input**
- **Critical response in structure near tip**



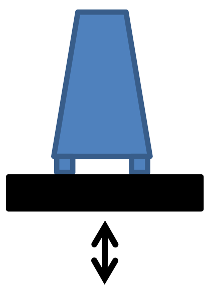
**Input acceleration spectral density**



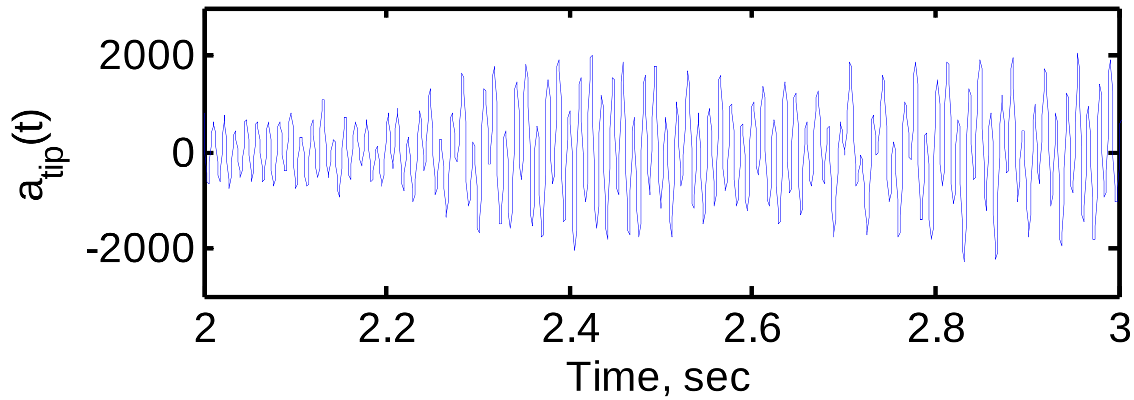

**Realization of input acceleration**



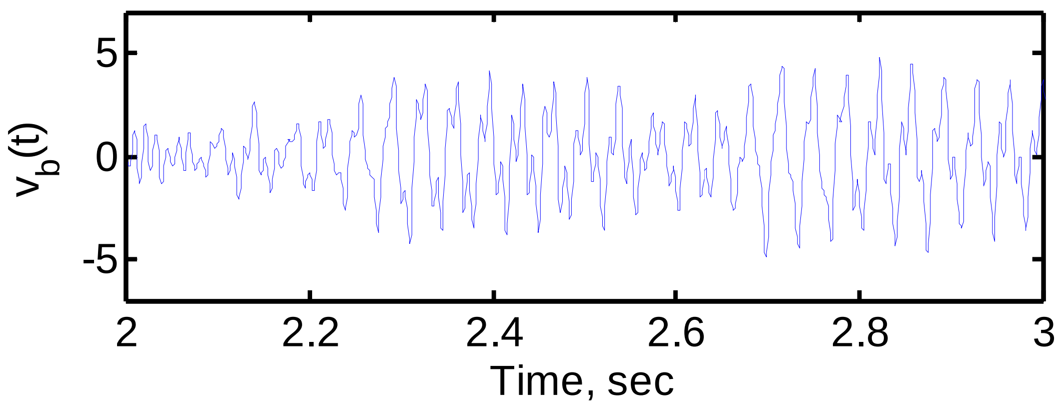
# UQ – Example



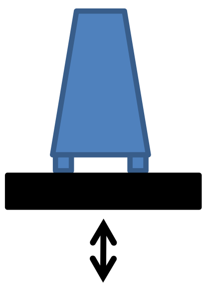
**Response acceleration near tip**



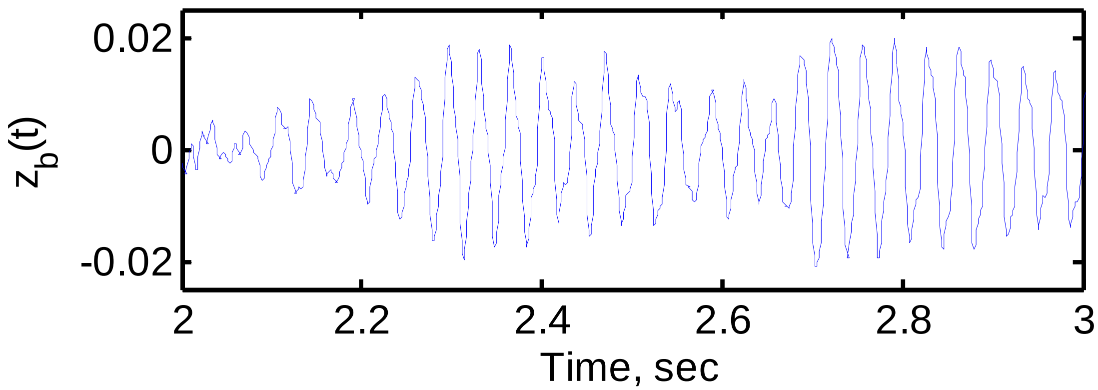

**Response relative velocity across base**



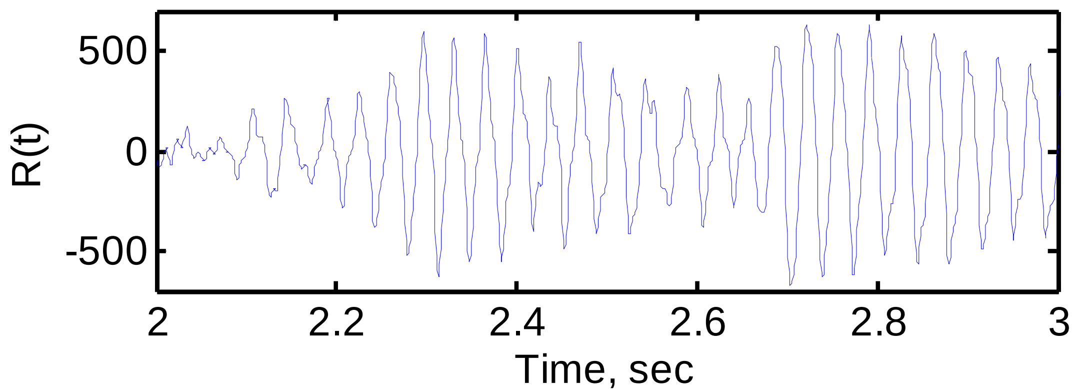
# UQ – Example



**Response relative displacement across base**



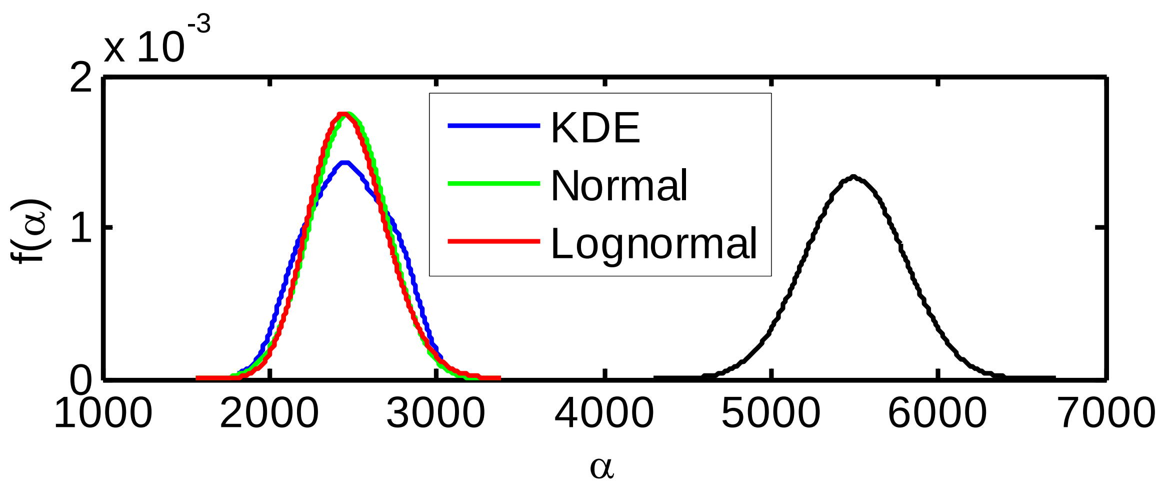

**Force across base**



# UQ – Example

- **Ten experiments performed**
  - **Peak acceleration response measured during each experiment**
  - **PDF of peaks approximated three ways (blue, green, red)**
- **Failure level PDF known from experiment (black)**
- **Probability of undesirable event is** $p_u = P(2X > Y)$

| **Estimates of** $p_u$ | |
|---|---|
| **KDE** | **0.19** |
| **Normal** | **0.16** |
| **Lognormal** | **0.16** |

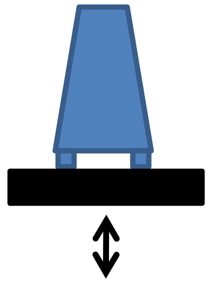


# Validation

- To use this UQ technique in validation
  - Identify (calibrate) structure model parameters with calibration experiments
  - Perform validation experiments with physical system – Estimate $p_u$
  - Use model to predict validation data – Estimate $p_u$
- If model-estimated $p_u$ "satisfactorily" matches physical $p_u$ then model is validated

**Technique is particularly effective when analysis objective is probabilistic assurance of system sufficiency.**

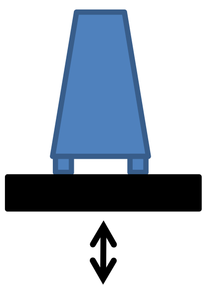
# Validation - Example



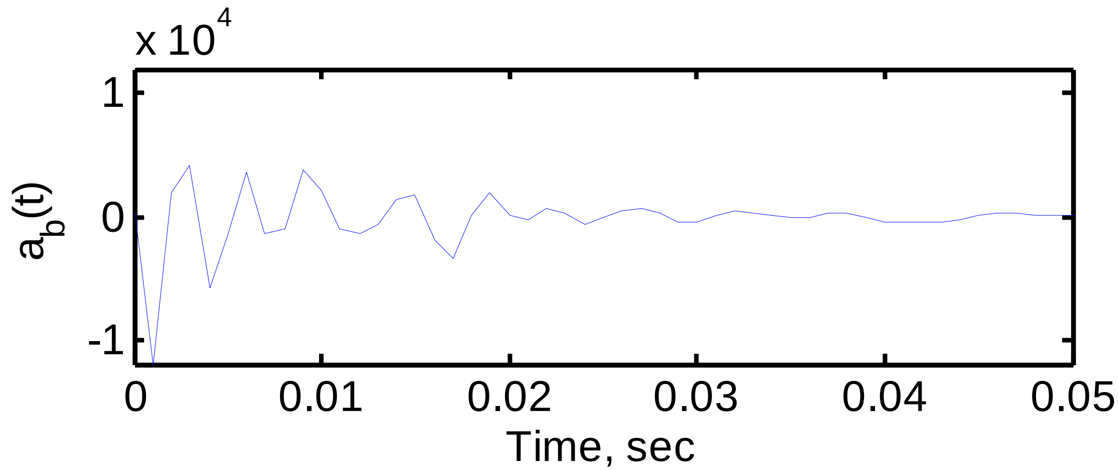
- Same system as that considered above
- Load is oscillatory shock
- Ten experiments performed – Peak response near tip is critical response measure – Estimate $p_u$ from experiments
- Model:
  - Calibrate linear model with random vibration experiments
  - Model base spring as linear and stochastic
  - Generate twenty realizations of stochastic model
  - Compute response of each to each of twenty experimental inputs
  - Compute $p_u$ for each realization of stochastic model
  - Compare model-predicted results to experimental result

# Validation – Example

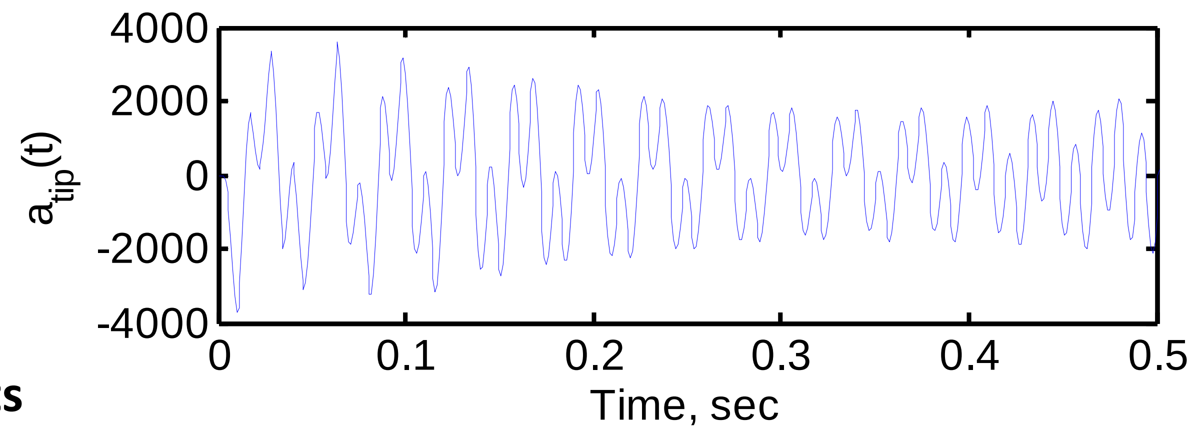
**Experimental structure**



**Base acceleration shock excitation**



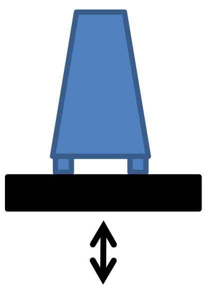

**Response near tip**



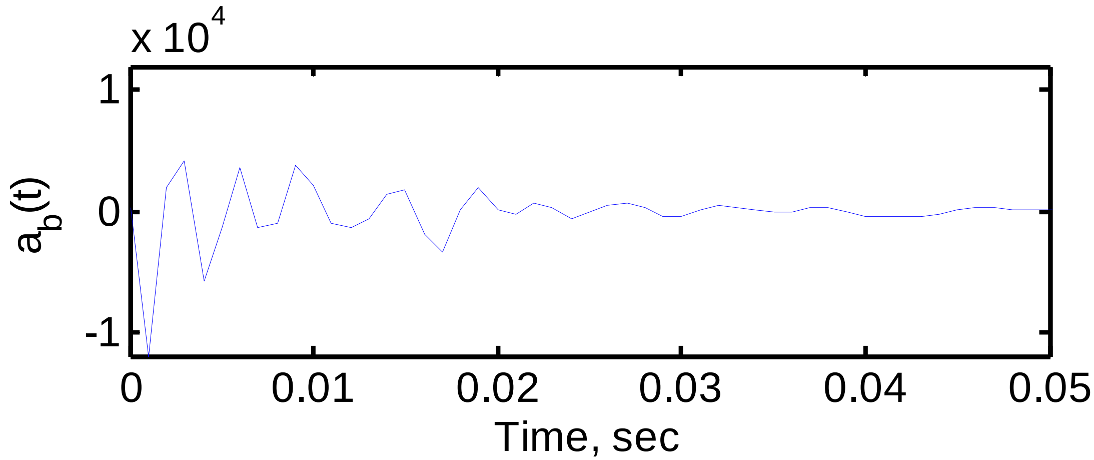

**10 Experimental inputs**
**10 Experimental responses**

# Validation – Example

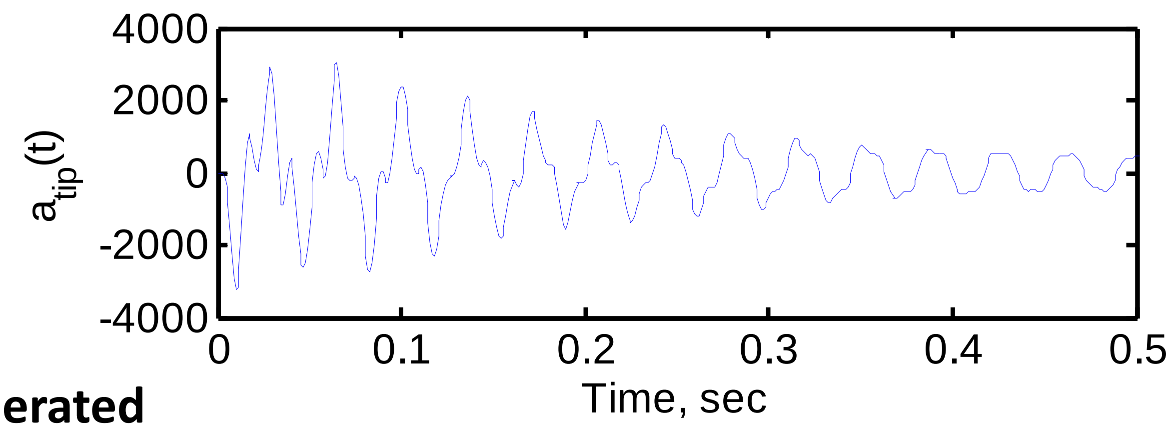
**Model**



**Base acceleration shock excitation**



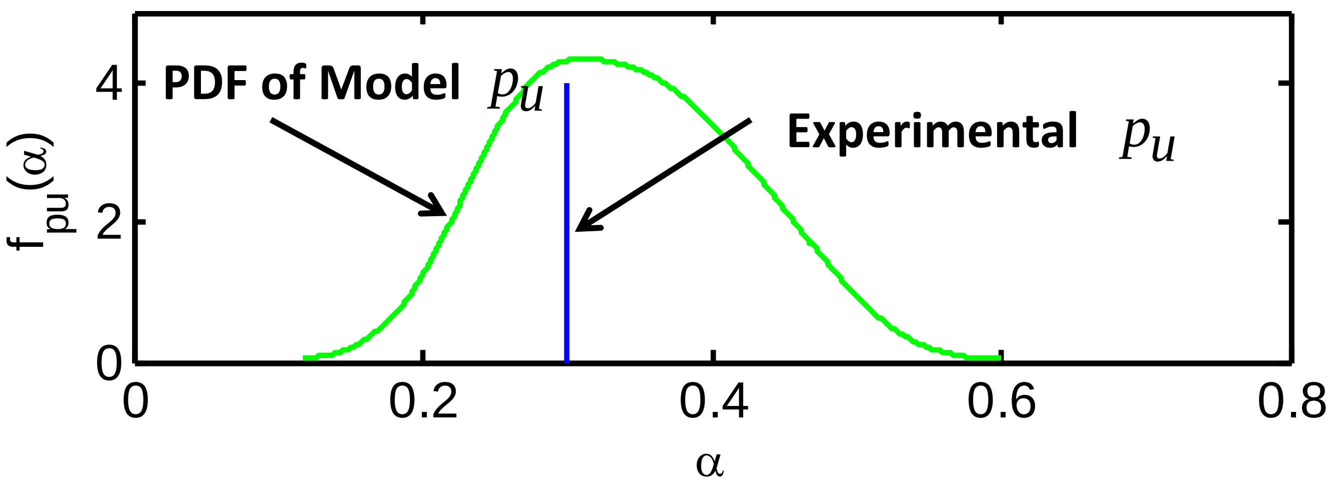

**Response near tip**




**20 model structures generated**
**10 experimental inputs**
**200 model responses**

# Validation - Example

- **Use computed $p_u$ for each shock model to estimate PDF of model $p_u$**
- **Perform test of hypothesis "Experimental $p_u$ is realization of model-generated $p_u$."**
- **If not rejected, then model is validated with respect to $p_u$ …**



# Summary

- Framework for probabilistic analysis/QMU/design developed
- Defines and uses probability of undesirable event

  $p_u = P(kX > Y)$
- Advantages:
  - Looks like factor of safety design
  - Accommodates randomness
  - Based on "heart" of data, not tails
- Can be applied to model validation when
  - Multiple experiments available
  - Probabilistic assurance of system sufficiency desired