Inverse Structural Acoustic Source and Material Identification in a Massively Parallel Finite Element Framework

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Inverse Problems- Motivation

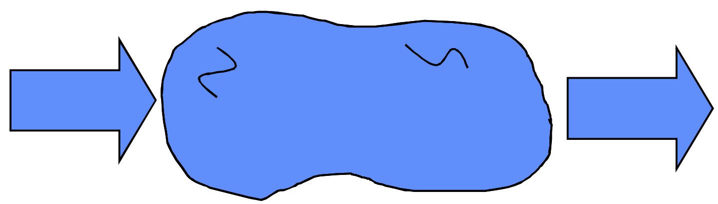
- Characterizing energy sources from experimental measurements is a common need in structural acoustics
  - acoustic testing of aerospace structures, damage or defect identification from acoustic emission, earthquake modeling, nonproliferation
- Determining unknown material properties from measurements is a common need in engineering analysis
  - Model calibration, Subsurface modeling, medical ultrasonics
- For applications that involve complex geometries and/or sources, finite element modeling is needed for an accurate solution of the forward problem.
- Goal: leverage existing massively parallel finite element technology developed for forward problems to solve the inverse problem.

Inverse Problems: The physical View

The direct or forward problem

External inputs (known)

e.g. forces, fluxes, etc.



System response (unknown)

e.g. displacements, temperature, concentrations, etc.

The System (known)

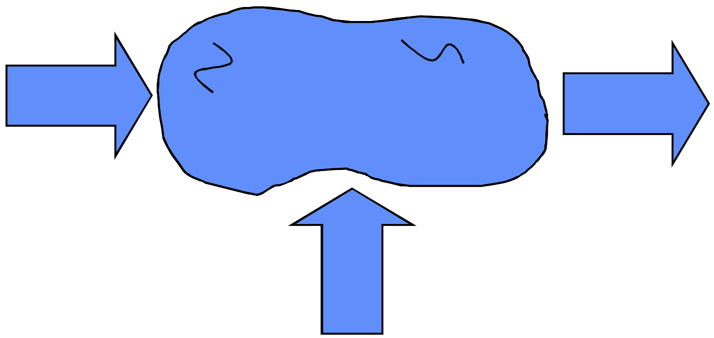
e.g. geometry, material properties, etc.

Inverse Problems: The physical View (2)

One type of inverse problem

External inputs (unknown)

e.g. forces, fluxes, etc.



System response (partially known)

e.g. displacements, temperature, concentrations, etc.

The System (unknown)

e.g. geometry, material properties, etc.

Inverse Problems (3)
Challenges

- Usually, inverse problems are ill-posed.
  - Solution may not exist.
  - Solution may not be unique.
  - Solution may be unstable. That is, it may be sensitive to small changes in the input data.
- Can be very computationally demanding.

Inverse Problems of Interest

- Source inversion for acoustics, structures, and coupled structural acoustics
  - Determine amplitudes of acoustic sources, given microphone response measurements
  - Determine amplitudes of structural tractions or pressures, given accelerometer measurements
  - Determine both acoustic and structural sources, given both microphone and accelerometer data
- Material inversion for elastic materials in frequency domain and nonlinear joints in time domain
  - Determine structural damping parameters to calibrate finite element models
  - Modified Error in Constitutive Equations (MECE)
  - L2 (Least Squares) minimization.

Source Inversion Methodology

- PDE-constrained optimization approach
  - Offers flexibility and extensibility
  - Applicable to time-domain, frequency-domain, and nonlinear problems. Can be tailored to each application.
  - Applicable to large numbers of design variables.
  - Allows significant code sharing with material inversion capability (backward time integrators for adjoint problems, experimental data manager, objective function, etc)
- Massively parallel finite element code Sierra-SD is used for solving the forward and adjoint problems.
- Optimization code ROL is used for solving the optimization problem.

Structural Acoustic Equations of Motion

acoustics

$$\nabla^2 \phi = \frac{1}{c^2}\ddot{\phi}, \ \text{ in } \Omega_f \times (0,T)$$

$$\nabla \phi \cdot \boldsymbol{n}_f = -\rho_f \ddot{u}_n, \ \text{ on } \partial\Omega_f^N \times [0,T]$$

$$\phi = 0, \ \text{ on } \ \partial\Omega_f^D \times [0,T]$$

$$\phi(0,T) = 0, \ \text{ in } \Omega_f$$

$$\dot{\phi}(0,T) = 0, \ \text{ in } \Omega_f$$

solid mechanics

$$\nabla \cdot \boldsymbol{\sigma} = \rho \ddot{\boldsymbol{u}}, \ \text{ in } \Omega \times (0,T)$$

$$\boldsymbol{\sigma n} = \boldsymbol{h}, \text{ on } \partial\Omega^N \times [0,T]$$

$$\boldsymbol{\sigma} = \boldsymbol{D} : \nabla \boldsymbol{u}, \ \text{ in } \Omega \times [0,T]$$

$$\boldsymbol{u} = \boldsymbol{0}, \ \text{ on } \ \partial\Omega^D \times [0,T]$$

$$\boldsymbol{u}(0,T) = \boldsymbol{0}, \ \text{ in } \Omega$$

$$\dot{\boldsymbol{u}}(0,T) = \boldsymbol{0}, \ \text{ in } \Omega$$

Time domain

$$[M]\boldsymbol{a}(t) + [C]\boldsymbol{v}(t) + [K]\boldsymbol{u}(t) = \boldsymbol{f}(t)$$

Frequency domain (Helmholtz)

$$[H(\omega)]\boldsymbol{z}(\omega) = \boldsymbol{F}(\omega)$$

$$[H(\omega)] = -\omega^2[M] + i\omega[C] + [K]$$

Structural Acoustic Equations of Motion

Fully coupled formulation

$$\begin{bmatrix} M_s & 0 \\ 0 & M_a \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} C_s & L^T \\ -L & C_a \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{bmatrix} f_s \\ f_a \end{bmatrix}$$

Condensed notation

$$[M]\boldsymbol{a}(t) + [C]\boldsymbol{v}(t) + [K]\boldsymbol{u}(t) = \boldsymbol{f}(t)$$

We will use the condensed notation in following slides

Statement of Inverse Problem

Minimize objective function

$$J(\{\boldsymbol{u}\},\{\boldsymbol{p}\}) = \frac{\kappa}{2}\Big(\{\boldsymbol{u}\}-\{\boldsymbol{u}_m\}\Big)^{\mathrm{T}}[Q]\Big(\{\boldsymbol{u}\}-\{\boldsymbol{u}_m\}\Big)+\mathscr{R}(\{\boldsymbol{p}\}),$$

- $\{\boldsymbol{u}\}$ State variables (displacement, pressure)
- $\{\boldsymbol{u}_m\}$ Measured data (displacement, pressure)
- $\{\boldsymbol{p}\}$ Unknown parameters (loads, material parameters)
- $[Q]$ Weight matrix

Subject to equations of motion

$$[M]\boldsymbol{a}(t)+[C]\boldsymbol{v}(t)+[K]\boldsymbol{u}(t)=\boldsymbol{f}(t)$$

Time domain

$$[H(\omega)]\boldsymbol{z}(\omega)=\boldsymbol{F}(\omega)$$

$$[H(\omega)]=-\omega^2[M]+i\omega[C]+[K]$$

Frequency domain

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Statement of Inverse Problem (2)

The Lagrangian

$$\mathscr{L}(\{\boldsymbol{d}\},\{\hat{\boldsymbol{d}}\},\{\boldsymbol{p}\}) = \tilde{J}(\{\boldsymbol{p}\}) + \hat{\boldsymbol{u}}_0^{\mathrm{T}}\Big([M]\boldsymbol{a}_0 + [C]\boldsymbol{v}_0 + [K]\boldsymbol{u}_0 - \boldsymbol{f}_0(\{\boldsymbol{p}\})\Big)$$
$$+ \sum_{k=1}^{N}\Big\{\hat{\boldsymbol{u}}_k^{\mathrm{T}}\Big([M]\boldsymbol{a}_k + [C]\boldsymbol{v}_k + [K]\boldsymbol{u}_k - \boldsymbol{f}_k(\{\boldsymbol{p}\})\Big)$$
$$+ \hat{\boldsymbol{v}}_k^{\mathrm{T}}[M]\Big(\boldsymbol{v}_k - \boldsymbol{v}_{k-1} - \Delta t\,[(1-\gamma)\boldsymbol{a}_{k-1} + \gamma\boldsymbol{a}_k]\Big)$$
$$+ \hat{\boldsymbol{a}}_k^{\mathrm{T}}[M]\Big(\boldsymbol{u}_k - \boldsymbol{u}_{k-1} - \Delta t\,\boldsymbol{v}_{k-1} - \frac{\Delta t^2}{2}[(1-2\beta)\boldsymbol{a}_{k-1} + 2\beta\boldsymbol{a}_k]\Big)\Big\}$$

where

$$\{\boldsymbol{d}(\{\boldsymbol{p}\})\} = \Big\{\{\boldsymbol{u}\},\{\boldsymbol{v}\},\{\boldsymbol{a}\}\Big\}$$

Optimality conditions

- Optimality is obtained by setting derivatives of Lagrangian to zero
- We will adopt a reduced space approach where we derive reduced gradients from full space approach
- Reduced space approach can be derived from full space

Formulation of Source Inversion Problem – Frequency Domain

$$J(\{u\}_1, .., \{u\}_{N_f}) = \frac{\kappa}{2} \sum_{i=1}^{N_f} \left(\overline{\{u\}_i - \{u_m\}_i}\right)^T [Q]\left(\{u\}_i - \{u_m\}_i\right) + \mathcal{R}(\{p\})$$

Objective Function

$$\mathcal{L} = J + \sum_{i=1}^{N_f} \mathrm{Re}\left(\overline{\{w\}}_i^T \left([H(\omega_i)]\{u\}_i - \{F(\{p\})\}_i\right)\right)$$

Lagrangian

<u>KKT conditions:</u>

$$D_{\{w\}_j}\mathcal{L} \cdot \{\delta w\} = 0 \implies [H(\omega_j)]\{u\}_j = \{F(\{p\})\}_j$$

Forward problem

$$D_{\{u\}_j}\mathcal{L} \cdot \{\delta u\} = 0 \implies [H(\omega_j)]\{w\}_j = \kappa[Q]\left(\{u_m\}_j - \{u\}_j\right)$$

Adjoint problem

$$D_{\{p\}}\mathcal{L} \cdot \{\delta p\} = -\sum_{i=1}^{N_f} \mathrm{Re}\left((\overline{\{w\}}_j^T \left[\frac{\partial \{F(\{p\})\}_j}{\partial \{p\}}\right] \{\delta p\} \right) + D_{\{p\}}\mathcal{R}(\{p\}) \cdot \{\delta p\}$$

Gradient

Formulation of Source Inverse Problem – Time Domain

$$J(\{\boldsymbol{u}\},\{\boldsymbol{p}\}) = \frac{\kappa}{2}\Big(\{\boldsymbol{u}\}-\{\boldsymbol{u}_m\}\Big)^{\mathrm{T}}[Q]\Big(\{\boldsymbol{u}\}-\{\boldsymbol{u}_m\}\Big)+\mathscr{R}(\{\boldsymbol{p}\}),$$

Objective Function

$$\begin{aligned}\mathscr{L}(\{\boldsymbol{d}\},\{\hat{\boldsymbol{d}}\},\{\boldsymbol{p}\}) = {} & \tilde{J}(\{\boldsymbol{p}\}) + \hat{\boldsymbol{u}}_0^{\mathrm{T}}\Big([M]\boldsymbol{a}_0+[C]\boldsymbol{v}_0+[K]\boldsymbol{u}_0-\boldsymbol{f}_0(\{\boldsymbol{p}\})\Big)\\ & + \sum_{k=1}^{N}\Big\{\hat{\boldsymbol{u}}_k^{\mathrm{T}}\Big([M]\boldsymbol{a}_k+[C]\boldsymbol{v}_k+[K]\boldsymbol{u}_k-\boldsymbol{f}_k(\{\boldsymbol{p}\})\Big)\\ & \quad + \hat{\boldsymbol{v}}_k^{\mathrm{T}}[M]\Big(\boldsymbol{v}_k-\boldsymbol{v}_{k-1}-\Delta t[(1-\gamma)\boldsymbol{a}_{k-1}+\gamma\boldsymbol{a}_k]\Big)\\ & \quad + \hat{\boldsymbol{a}}_k^{\mathrm{T}}[M]\Big(\boldsymbol{u}_k-\boldsymbol{u}_{k-1}-\Delta t\,\boldsymbol{v}_{k-1}-\frac{\Delta t^2}{2}[(1-2\beta)\boldsymbol{a}_{k-1}+2\beta\boldsymbol{a}_k]\Big)\Big\}\end{aligned}$$

Lagrangian

$$\nabla_{\{\boldsymbol{p}\}}\tilde{J} = \nabla_{\{\boldsymbol{p}\}}\mathscr{L} = -\sum_{k=1}^{N}\hat{\boldsymbol{u}}_k^{\mathrm{T}}\Big(\nabla_{\{\boldsymbol{p}\}}\boldsymbol{f}_k(\{\boldsymbol{p}\})\Big)+\nabla_{\{\boldsymbol{p}\}}\mathscr{R}.$$

Gradient Equation

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Statement of Inverse Problem (3)

Gateaux derivatives of the Lagrangian with respect to adjoint variables

$$
\begin{aligned}
\nabla_{\boldsymbol{a}_0}\mathscr{L}\cdot\delta\boldsymbol{a}_0 &= \delta\boldsymbol{a}_0^{\mathrm{T}}\Big([M]\hat{\boldsymbol{u}}_0 - \frac{\Delta t^2}{2}(1-2\beta)[M]\hat{\boldsymbol{a}}_1 - \Delta t(1-\gamma)[M]\hat{\boldsymbol{v}}_1\Big),\\
\nabla_{\boldsymbol{u}_k}\mathscr{L}\cdot\delta\boldsymbol{u}_k &= \delta\boldsymbol{u}_k^{\mathrm{T}}\Big([M](\hat{\boldsymbol{a}}_k - \hat{\boldsymbol{a}}_{k+1}) + [K]\hat{\boldsymbol{u}}_k + \kappa[Q](\boldsymbol{u}_k - \boldsymbol{u}_{m_k}),\Big),\\
\nabla_{\boldsymbol{v}_k}\mathscr{L}\cdot\delta\boldsymbol{v}_k &= \delta\boldsymbol{v}_k^{\mathrm{T}}\Big([C]\hat{\boldsymbol{u}}_k - \Delta t[M]\hat{\boldsymbol{a}}_{k+1} + [M]\hat{\boldsymbol{v}}_k - [M]\hat{\boldsymbol{v}}_{k+1}\Big),\\
\nabla_{\boldsymbol{a}_k}\mathscr{L}\cdot\delta\boldsymbol{a}_k &= \delta\boldsymbol{a}_k^{\mathrm{T}}\Big([M]\hat{\boldsymbol{u}}_k - \beta\Delta t^2[M]\hat{\boldsymbol{a}}_k - \frac{\Delta t^2}{2}[M](1-2\beta)\hat{\boldsymbol{a}}_{k+1},\\
&\quad -\Delta t[M](\gamma\hat{\boldsymbol{v}}_k + (1-\gamma)\hat{\boldsymbol{v}}_{k+1})\Big),\\
\nabla_{\boldsymbol{u}_N}\mathscr{L}\cdot\delta\boldsymbol{u}_N &= \delta\boldsymbol{u}_N^{\mathrm{T}}\Big([M]\hat{\boldsymbol{a}}_N + [K]\hat{\boldsymbol{u}}_N + \kappa[Q](\boldsymbol{u}_N - \boldsymbol{u}_{m_N})\Big),\\
\nabla_{\boldsymbol{v}_N}\mathscr{L}\cdot\delta\boldsymbol{v}_N &= \delta\boldsymbol{v}_N^{\mathrm{T}}\Big([C]\hat{\boldsymbol{u}}_N + [M]\hat{\boldsymbol{v}}_N\Big),\\
\nabla_{\boldsymbol{a}_N}\mathscr{L}\cdot\delta\boldsymbol{a}_N &= \delta\boldsymbol{a}_N^{\mathrm{T}}\Big([M]\hat{\boldsymbol{u}}_N - \Delta t^2\beta[M]\hat{\boldsymbol{a}}_N - \Delta t\gamma[M]\hat{\boldsymbol{v}}_N\Big).
\end{aligned}
$$

Statement of Inverse Problem (4)

(i) Final conditions

$$
\begin{aligned}
[C]\,\hat{\boldsymbol{u}}_N + [M]\hat{\boldsymbol{v}}_N &= \mathbf{0} \\
\hat{\boldsymbol{u}}_N &= \Delta t^2 \beta\, \hat{\boldsymbol{a}}_N + \Delta t\, \gamma \hat{\boldsymbol{v}}_N \\
[M]\,\hat{\boldsymbol{a}}_N + [K]\hat{\boldsymbol{u}}_N &= \kappa [Q](\boldsymbol{u}_{m_N} - \boldsymbol{u}_N)
\end{aligned}
$$

(ii) Backward transition equations

$$
\begin{aligned}
\hat{\boldsymbol{u}}_k - \beta \Delta t^2 \hat{\boldsymbol{a}}_k - \Delta t\, \gamma \hat{\boldsymbol{v}}_k &= \frac{\Delta t^2}{2}(1-2\beta)\hat{\boldsymbol{a}}_{k+1} + \Delta t (1-\gamma)\hat{\boldsymbol{v}}_{k+1} \\
[C]\,\hat{\boldsymbol{u}}_k + [M](\hat{\boldsymbol{v}}_k - \Delta t\, \hat{\boldsymbol{a}}_{k+1} - \hat{\boldsymbol{v}}_{k+1}) &= \mathbf{0} \\
[M]\,\hat{\boldsymbol{a}}_k + [K]\hat{\boldsymbol{u}}_k &= [M]\hat{\boldsymbol{a}}_{k+1} + \kappa [Q](\boldsymbol{u}_{m_k} - \boldsymbol{u}_k)
\end{aligned}
$$

(iii) Last transition equation

$$
\hat{\boldsymbol{u}}_0 = \frac{\Delta t^2}{2}(1-2\beta)\hat{\boldsymbol{a}}_1 + \Delta t (1-\gamma)\hat{\boldsymbol{v}}_1
$$

Statement of Inverse Problem (5)

Gateaux derivatives of the Lagrangian with respect to design variables

$$\nabla_{\{\boldsymbol{p}\}}\mathscr{L}(\{\boldsymbol{d}\},\{\hat{\boldsymbol{d}}\},\{\boldsymbol{p}\})\cdot\{\delta\boldsymbol{p}\} = \nabla_{\{\boldsymbol{d}\}}\mathscr{L}\cdot\{\delta\boldsymbol{d}\} + \nabla_{\{\boldsymbol{p}\}}\mathscr{L}\cdot\{\delta\boldsymbol{p}\}$$

$\nabla_{\{\boldsymbol{d}\}}\mathscr{L} = \boldsymbol{0}$ (from adjoint solution)

Gradient Equation

$$\nabla_{\{\boldsymbol{p}\}}\tilde{J} = \nabla_{\{\boldsymbol{p}\}}\mathscr{L} = -\sum_{k=1}^{N}\hat{\boldsymbol{u}}_k^{\mathrm{T}}\Big(\nabla_{\{\boldsymbol{p}\}}\boldsymbol{f}_k(\{\boldsymbol{p}\})\Big) + \nabla_{\{\boldsymbol{p}\}}\mathscr{R}.$$

Solution of Inverse Problem

Do until tolerance < eps

Solve forward problem

↓

Solve adjoint problem

↓

Compute gradients, Hessians

↓

Optimization step

↓

Receive design variable updates from optimization solver

end

Sierra-SD: A Brief History

- Sierra-SD was created in the 1990's at Sandia National Laboratories for large-scale structural analysis
- Intended for extremely complex structural and structural acoustics models
  - Commonly used to solve models with 100's of millions of degrees of freedom
- Scalability is the key
  - Sierra-SD can solve n-times larger problem using n-times many more compute processors, in nearly constant CPU time

Rapid Optimization Library (ROL)

- **Challenge: Optimization software typically lacks support for large-scale computing.**
  - optimization variables cannot be distributed across processors
    → limited to very small inverse and optimal design problems
  - no support for iterative linear system solvers
    → slow (or no) convergence due to solver inexactness
- **Requirements:**
  - **Matrix-free:** The application developer, *not the optimization software*, defines how matrices and vectors are stored/used, and chooses the linear system solver.
  - **Robust:** The optimization software manages the linear solver accuracy.
    - **ROL/PEOpt is based on vector-space abstractions, through polymorphism mechanisms of C++.**
    - **User defines: copy, axpy, scal, zero, innr; and objective function evaluation, gradient, Hessian.**
    - **ROL/PEOpt supports a variety of algorithms: nonlinear CG, Gauss-Newton, full-space SQP, etc., all with trust regions and line search.**

Available in Trilinos in 2013! Denis Ridzal, Joseph Young (Sandia)

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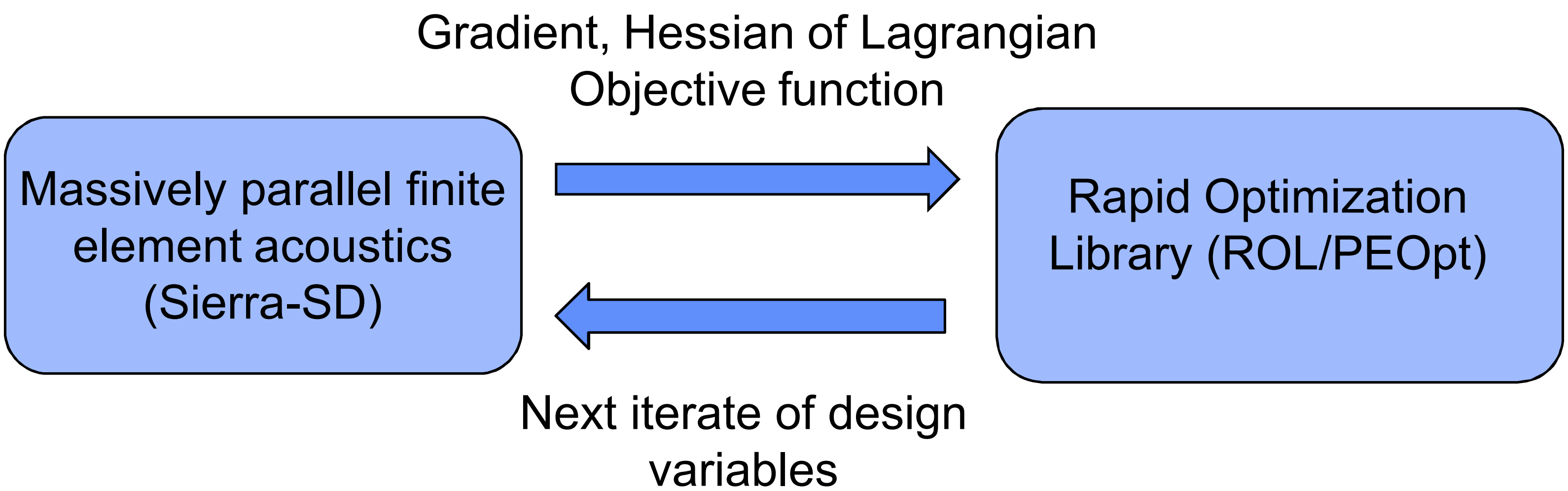
Interaction of Finite Element and Optimization Codes

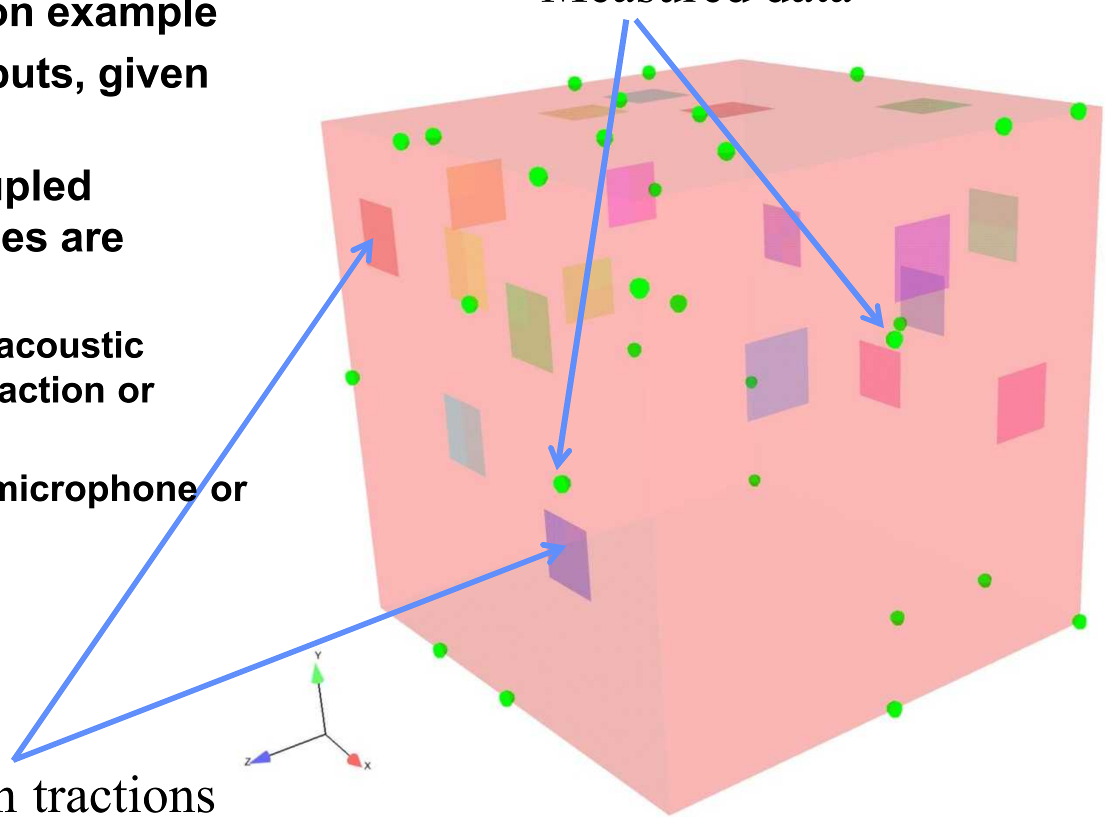
Finite Element and Optimization Codes operate as independent entities



- The adjoint method is used to compute the gradients and Hessians

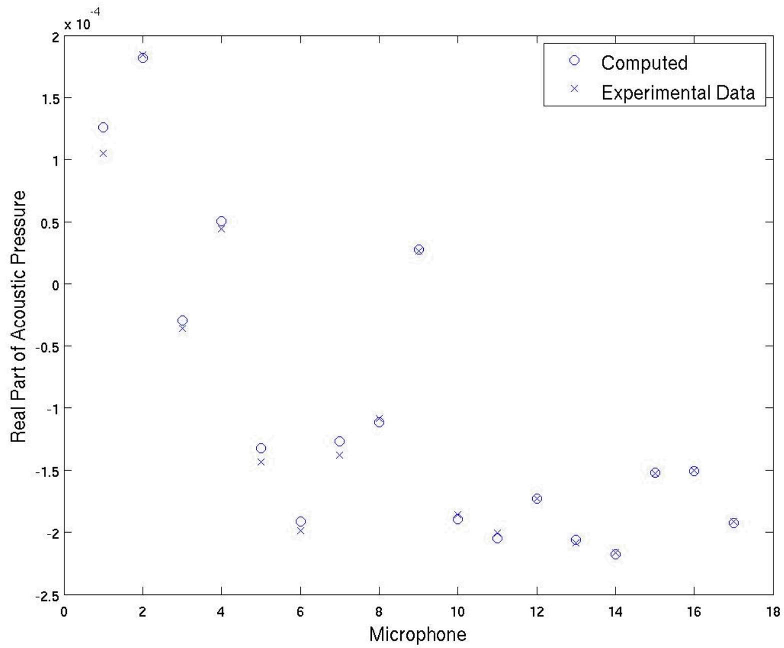
3D Source Inversion Test

- Schematic of source inversion example
- Goal is to predict traction inputs, given measurement data
- Acoustic, structural, and coupled structural acoustic approaches are identical
  - Unknown tractions could be acoustic particle velocity, structural traction or structural pressure
  - Measurement data could be microphone or accelerometer
- Two approaches:
  1. Time domain
  2. Frequency domain

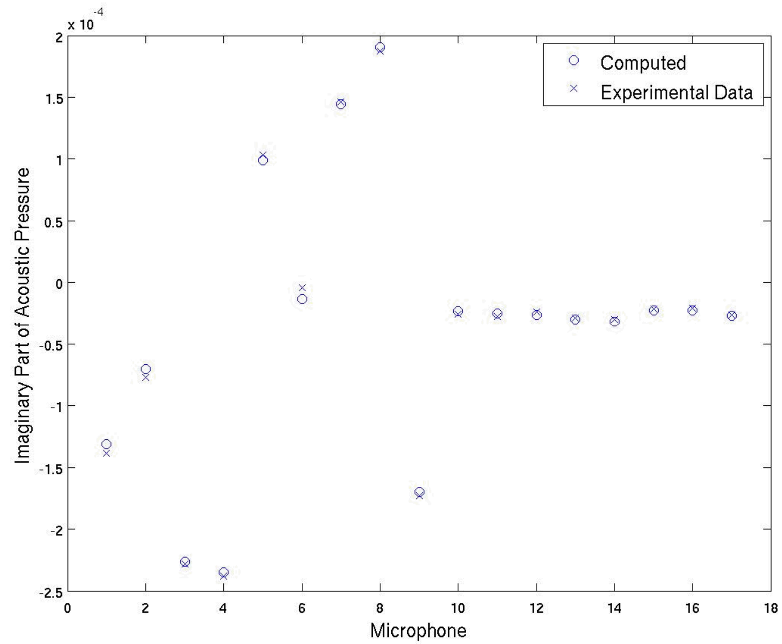


Frequency Domain Source Inversion

Single Frequency Results



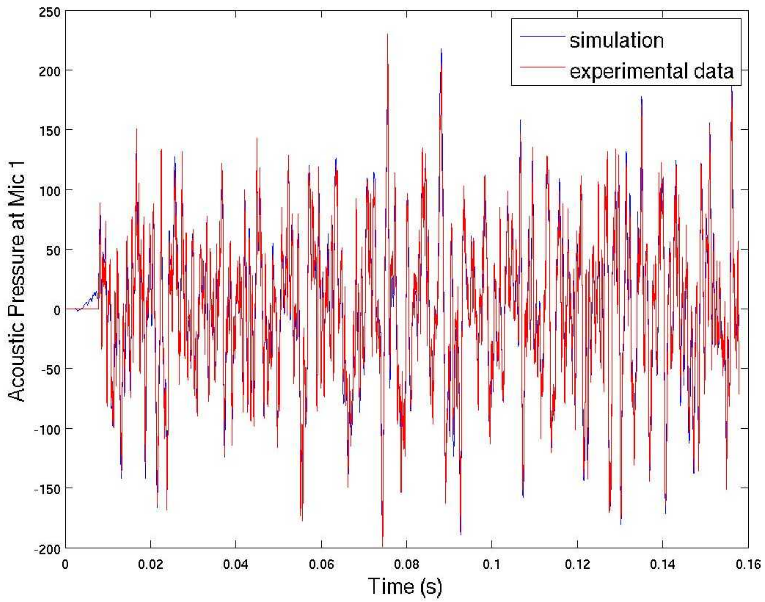
Real part of acoustic pressure



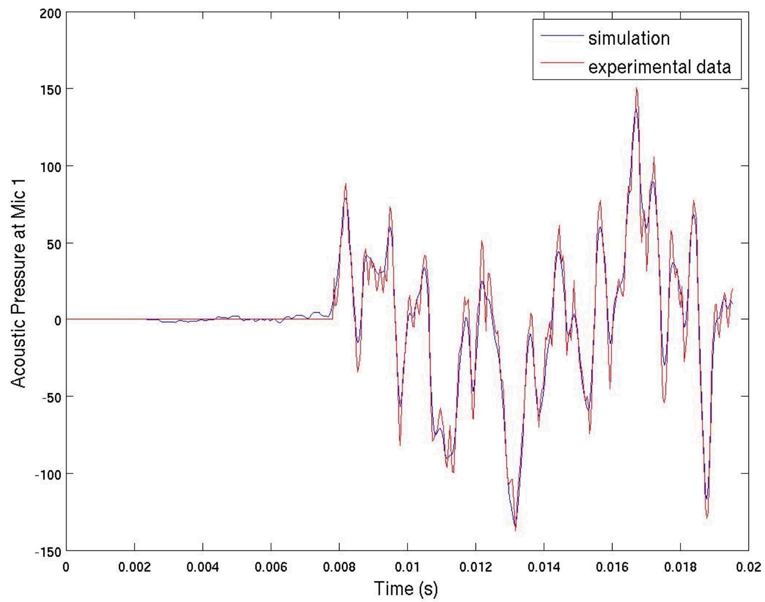
Imaginary part of acoustic pressure

Time Domain Source Inversion

Results for Microphone 1 (other mics were similar)



Full time history



Blow-up near origin of wave

Conclusions

- **Massively parallel finite element structural acoustics and optimization codes have been loosely coupled for the solution of source and material inversion problems.**
- **Adjoint methods have been implemented in Sierra-SD in both time and frequency domains.**
- **Applicable to large-scale models with many degrees of freedom.**
- **The method allows flexibility to work with both time and frequency domain, and nonlinear problems.**
- **Method has been applied to solving both source and material inversion on problems of interest.**