SAND2011-5040C

7/20/2011

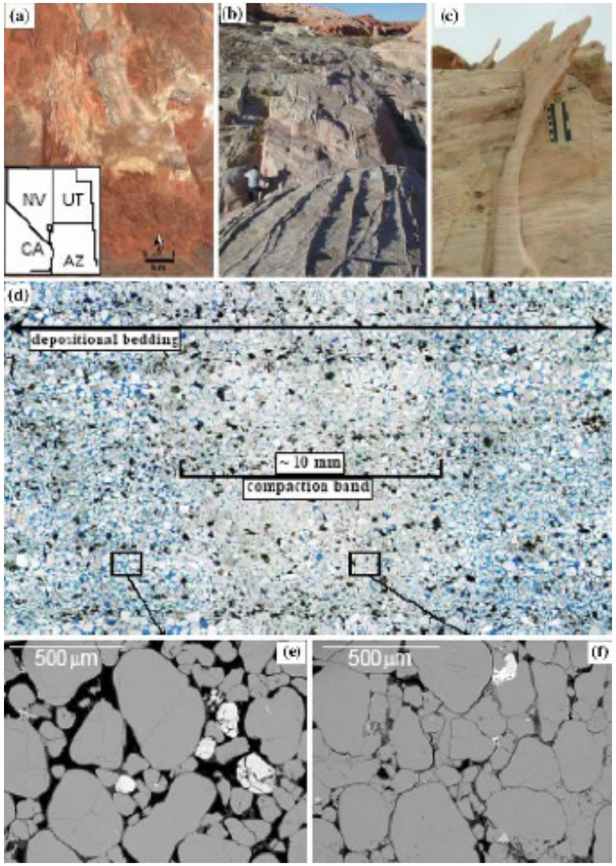
Capturing Micro-Structural Attributes and Macroscopic Fluid Transport Properties of Porous Media

WaiChing Sun[1], Jose E. Andrade[2], John W. Rudnicki[3]

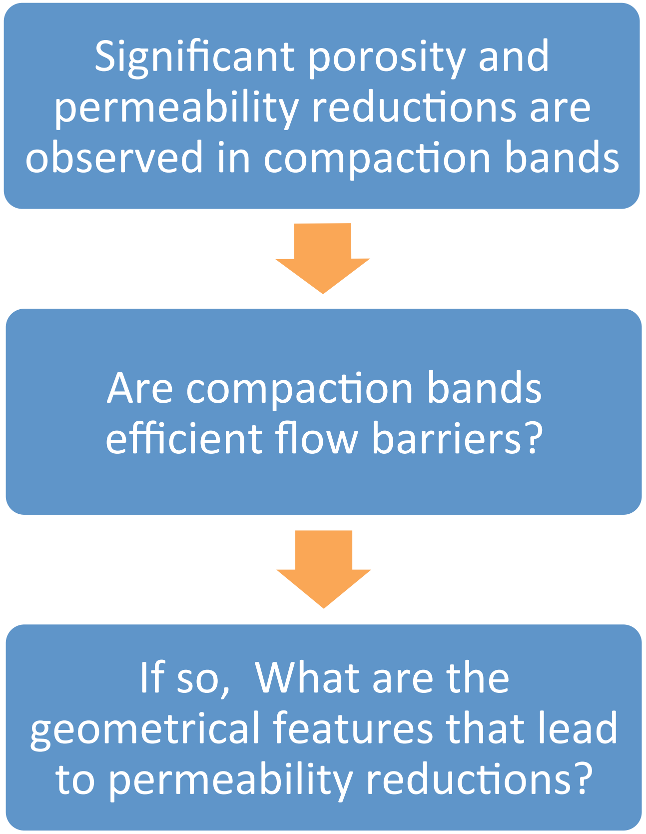
1. Mechanics of Material, Sandia National Laboratories, Livermore, CA
2. Engineering and Applied Science, Caltech, Pasadena, CA
3. Civil and Environmental Engineering, and Mechanical Engineering, Northwestern University, Evanston, IL

Sandia National Laboratories is a multi program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. SAND2011-0439P

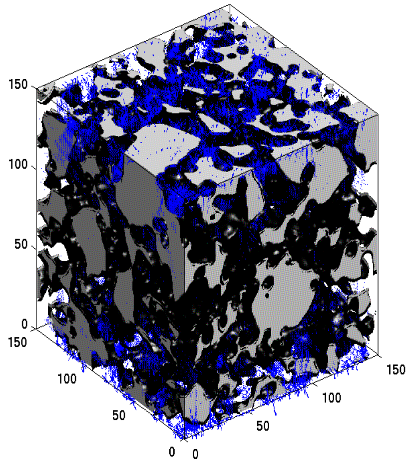
Motivation



Aztec Sandstone, Holcomb et al 2007



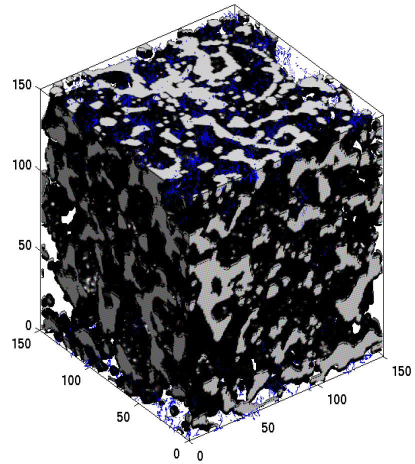
Effective Permeability Measurement via Lattice Boltzmann method



Flow outside CB

$$k_{11} = 8.6\times10^{-13} m^2$$

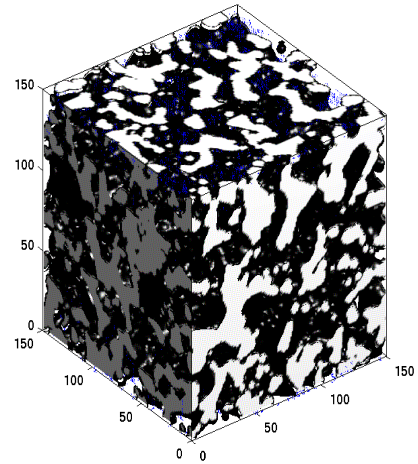
$$\phi^f = 0.21$$



Flow in transition zone

$$k_{11} = 1.8\times10^{-13} m^2$$

$$\phi^f = 0.13$$



Flow inside CB

$$k_{11} = 2.4\times10^{-13} m^2$$

$$\phi^f = 0.15$$

- intrinsic permeability tensor of REV is computed by finding volume average velocity of Lattice Boltzmann cube with prescribed hydraulic gradient on a 150x150x150 voxels

$$k_{ij} = -\langle v_i(x)\rangle \frac{\mu}{\nabla h_j}$$

- Reynold's number must be less than 1 to ensure the validity of Darcy's law

Sandia National Laboratories

Question:

1. How does the micro-structural change cause permeability reduction?

2. How about scale effect?

3. How can we verify our results?

Sandia National Laboratories

Outline

1. Background
2. Tools used to study characteristics of compaction band
   i. Medial Axis (use level set)
   ii. Tortuosity (use weight graph)
   iii. Isolated Pore Space (use graph)
   iv. Effective permeability (use FEM/LBM)
3. Results on Aztec sandstone
4. Conclusion

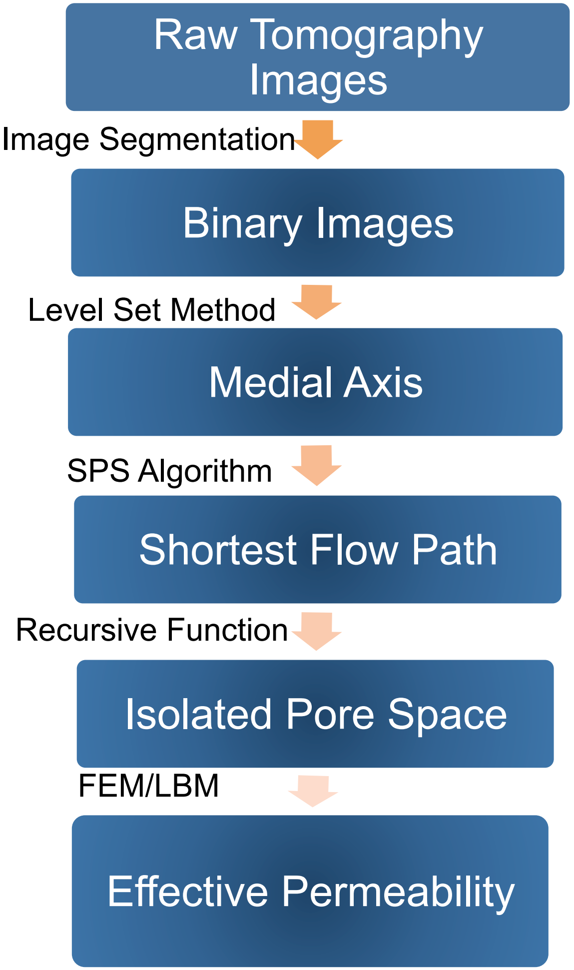
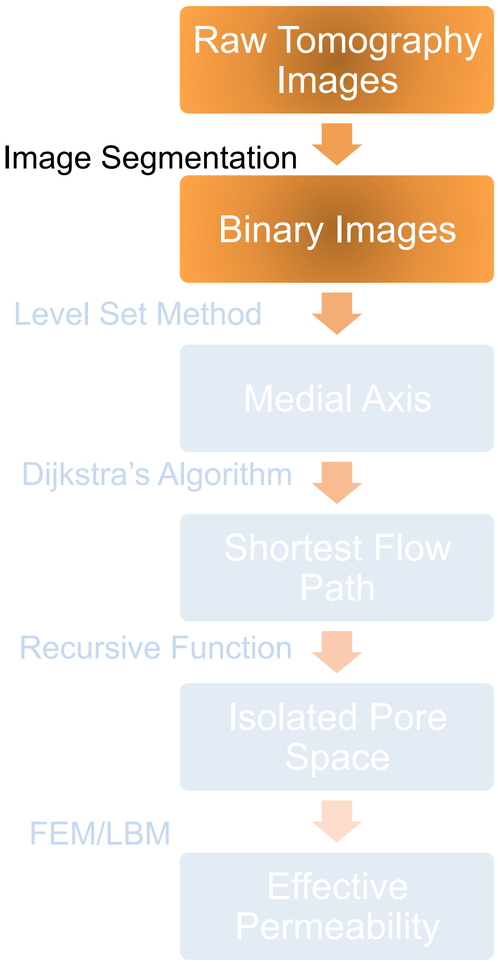
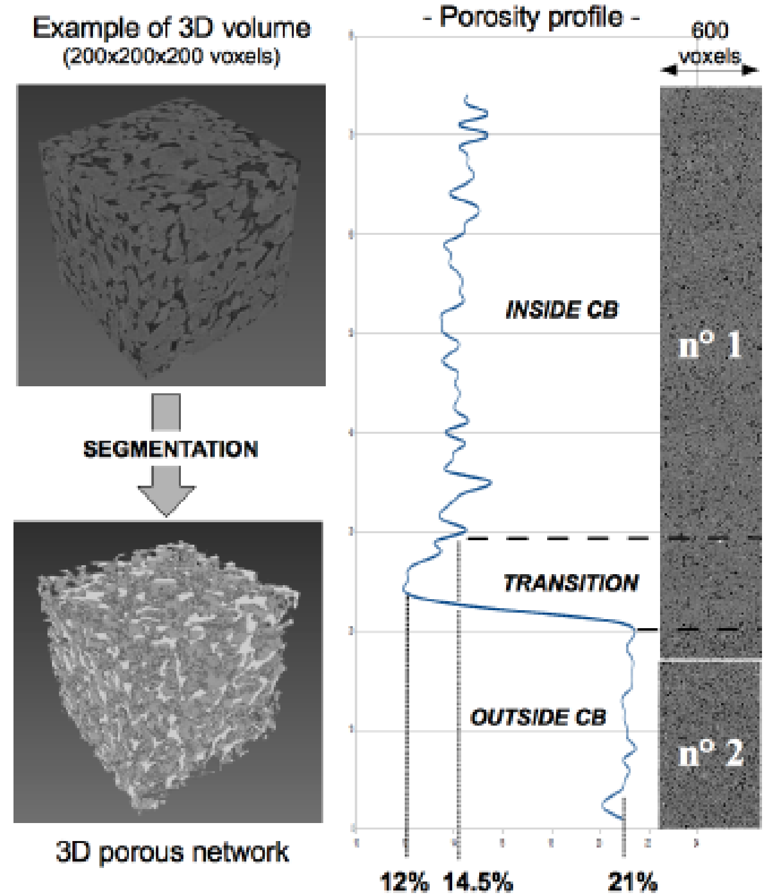


Image Segmentation



Aztec Sandstone Specimen

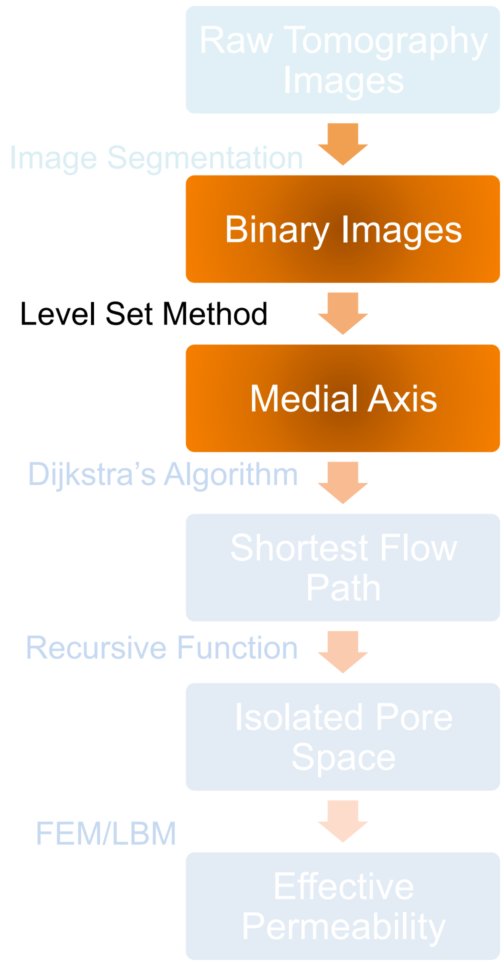
- 3D tomographic images are taken from Aztec sandstone collected at Valley of Fire State Park, Nevada.
- A threshold is defined to distinguish pore space and skeleton.
- A binary 3D images are crated from the original 3D images.
- Connected and isolated pore space are not distinguished.



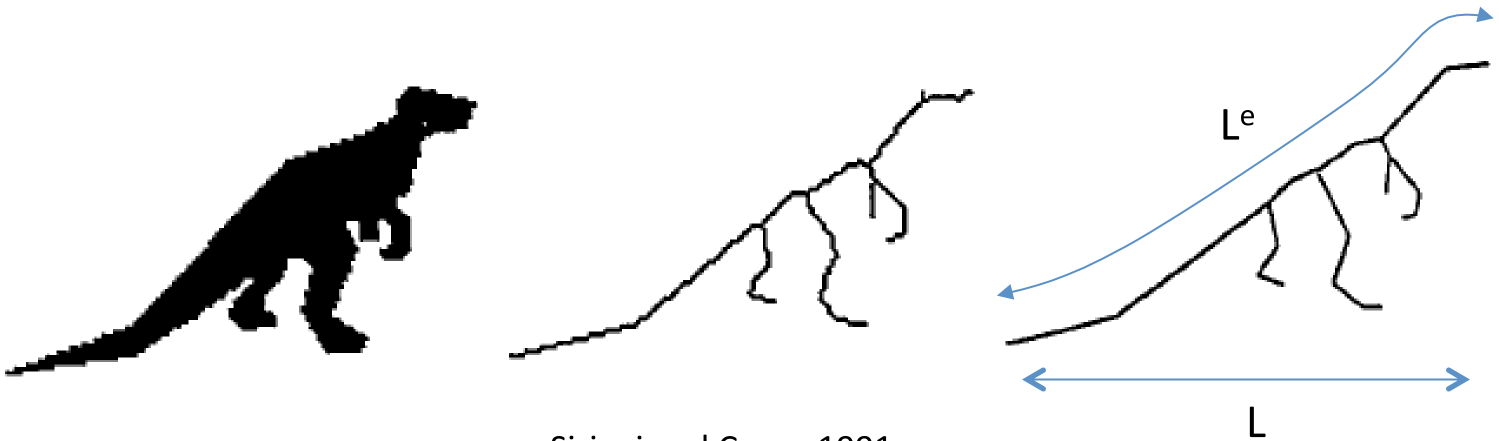
Work conducted by Dr. Leonir

Aztec Sandstone Image, Leonir, et al 2010

Level Set Method



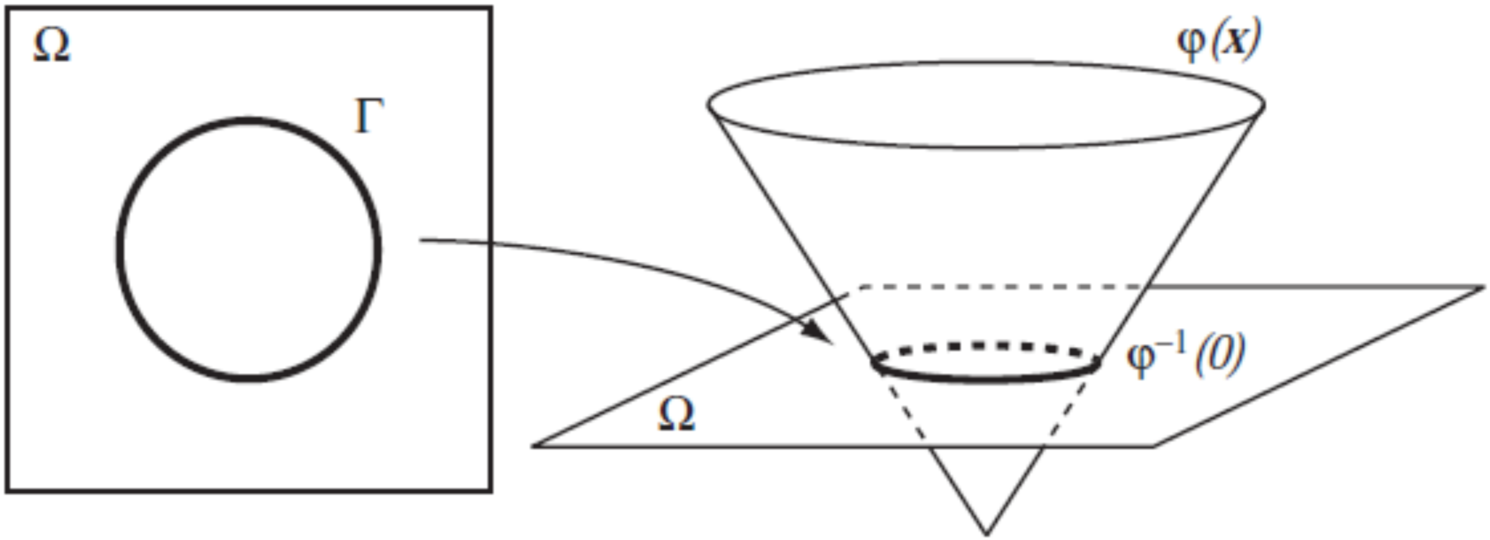
Medial Axis of Flow Paths



Sirjani and Cross, 1991

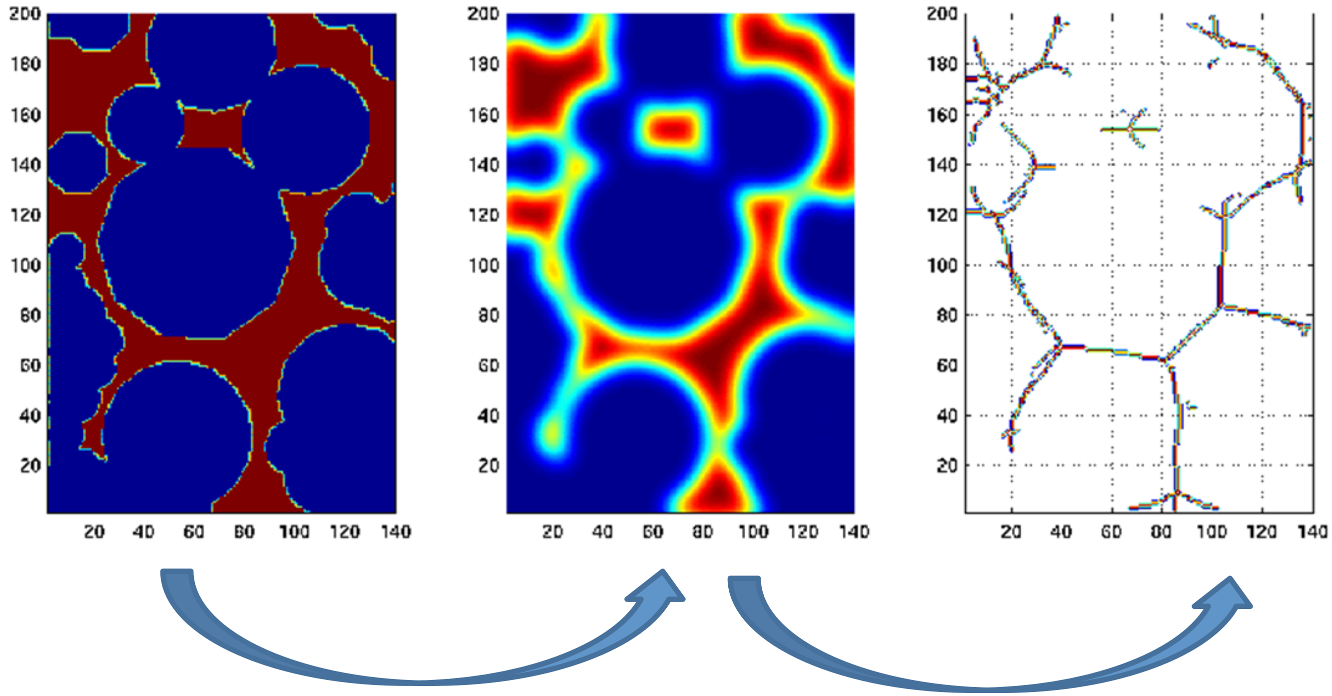
- Medial axis is the spine of a volume filling object
- Media axis is a union of curves that represent the topology and geometry of the volume

Relation between Level Set Function and Medial Axis



- Local minimum of the signed distance function φ are located at medial axis.
- Use level set scheme to obtain signed distance function

Locating Medial Axis of Flow Path via Level Set



Convert binary image into level set via semi-implicit scheme

Extract Local minimum of level set function

Formulation of Variational Level Set Method

Penalize the deviation of ϕ from a signed distance function φ

- Action functional (Li et al 2005)

$$\mathcal{E}(\phi) = \mu \mathcal{P}(\phi) + \lambda \mathcal{L}_g(\phi) + \nu \mathcal{A}_g(\phi)$$

Drive $\phi = 0$ at the object boundaries

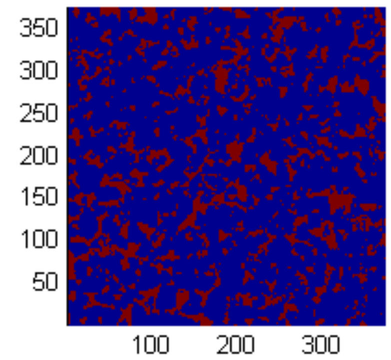
- Governing equatio

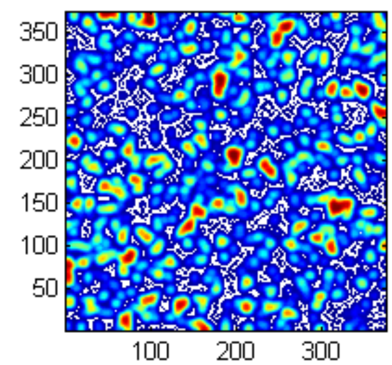
$$\frac{\partial \phi}{\partial t} = \mu[\Delta^x \phi - \nabla^x \cdot (\frac{\nabla^x \phi}{|\nabla^x \phi|})] + \lambda \delta(\phi) \nabla^x \cdot (g \frac{\nabla^x \phi}{|\nabla^x \phi|}) + \nu g \delta(\phi)$$

- Semi-Implicit Finite Dif

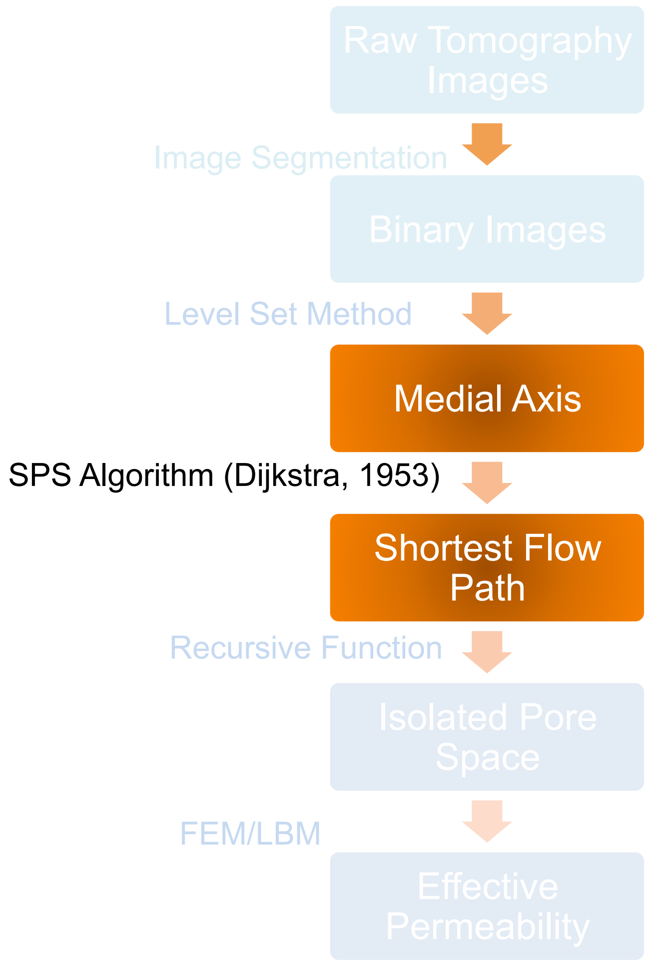
The discrete Laplacian term is treated implicitly

$$\frac{\phi^{n+1} - \phi^n}{t_{n+1} - t_n} = \mu \Delta^c \phi^{n+1} - \mu \nabla^c \cdot \frac{\nabla^c \phi}{||\nabla^c \phi||} + \lambda \delta(\phi^n) \nabla^c \cdot (g \frac{\nabla^c \phi}{||\nabla^c \phi||}) + \nu g \delta(\phi^n)$$

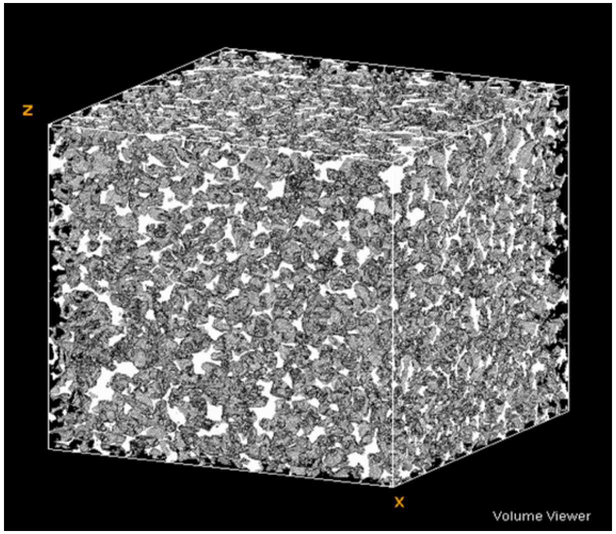


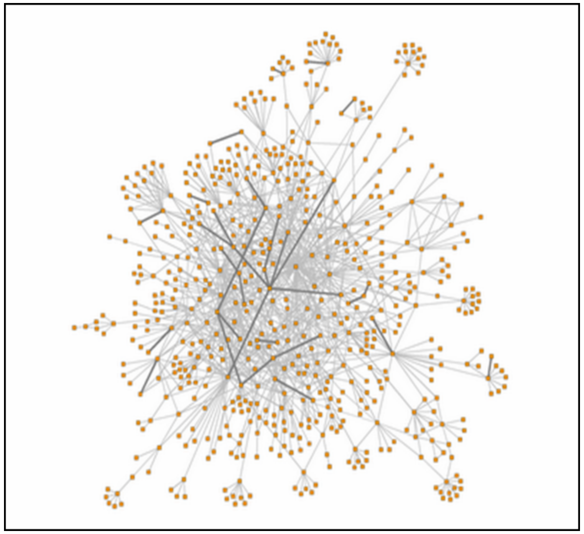


Shortest Path Searching Algorithm



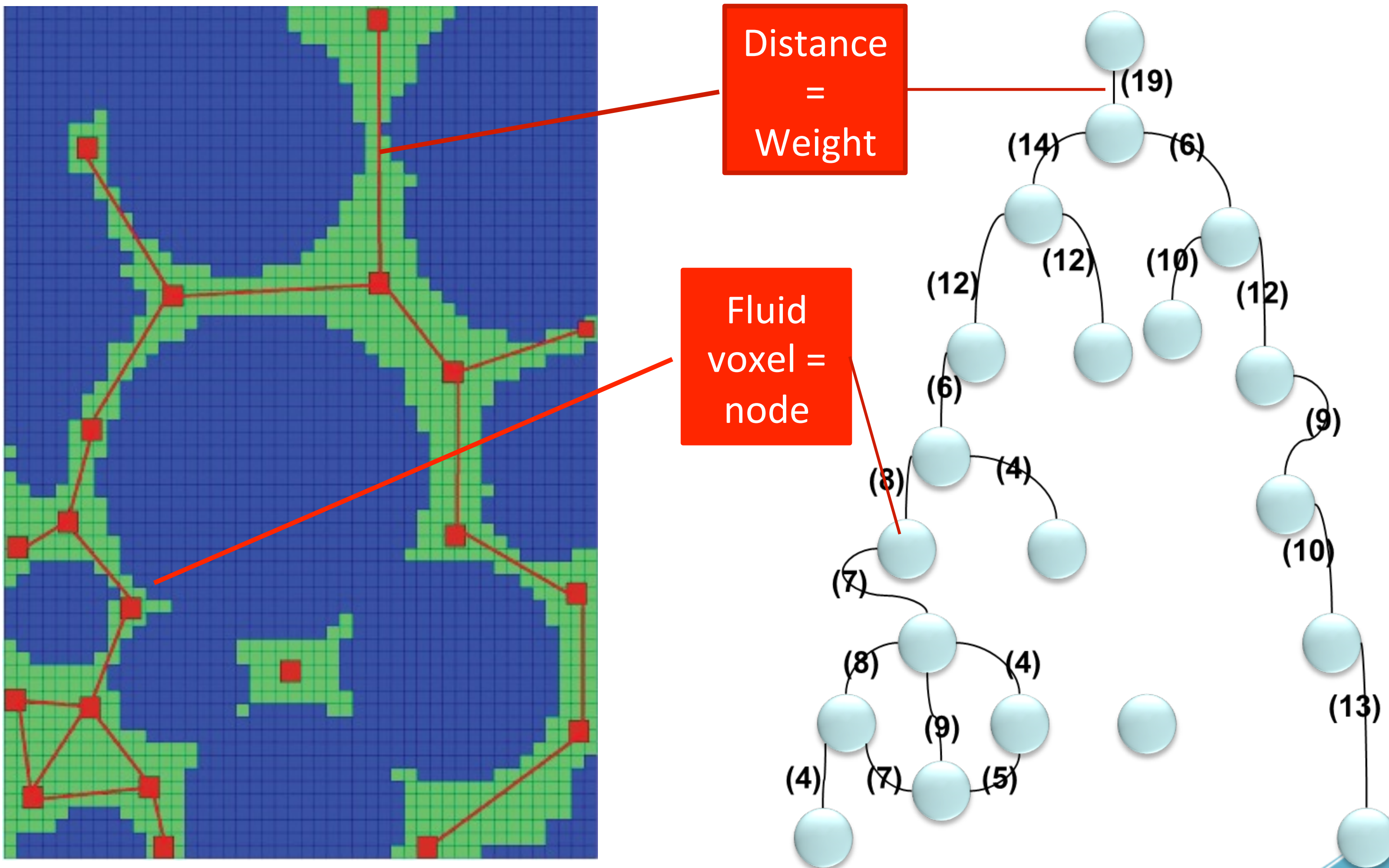
Graph Theory



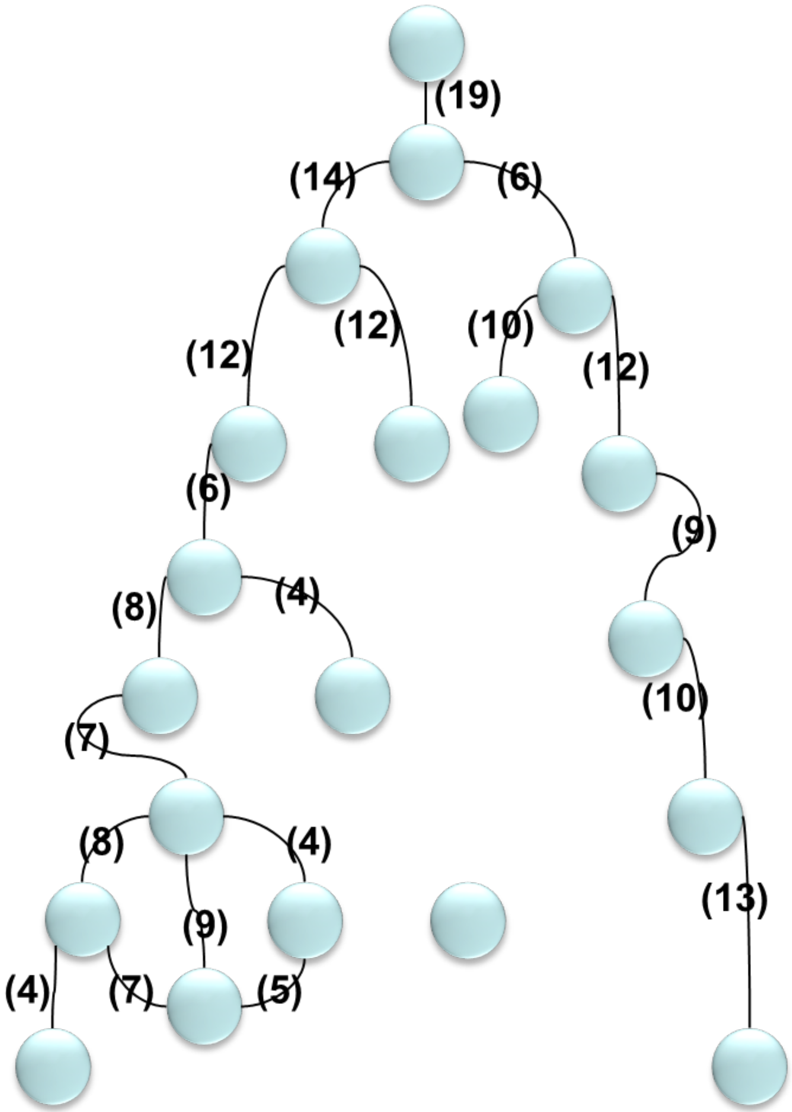


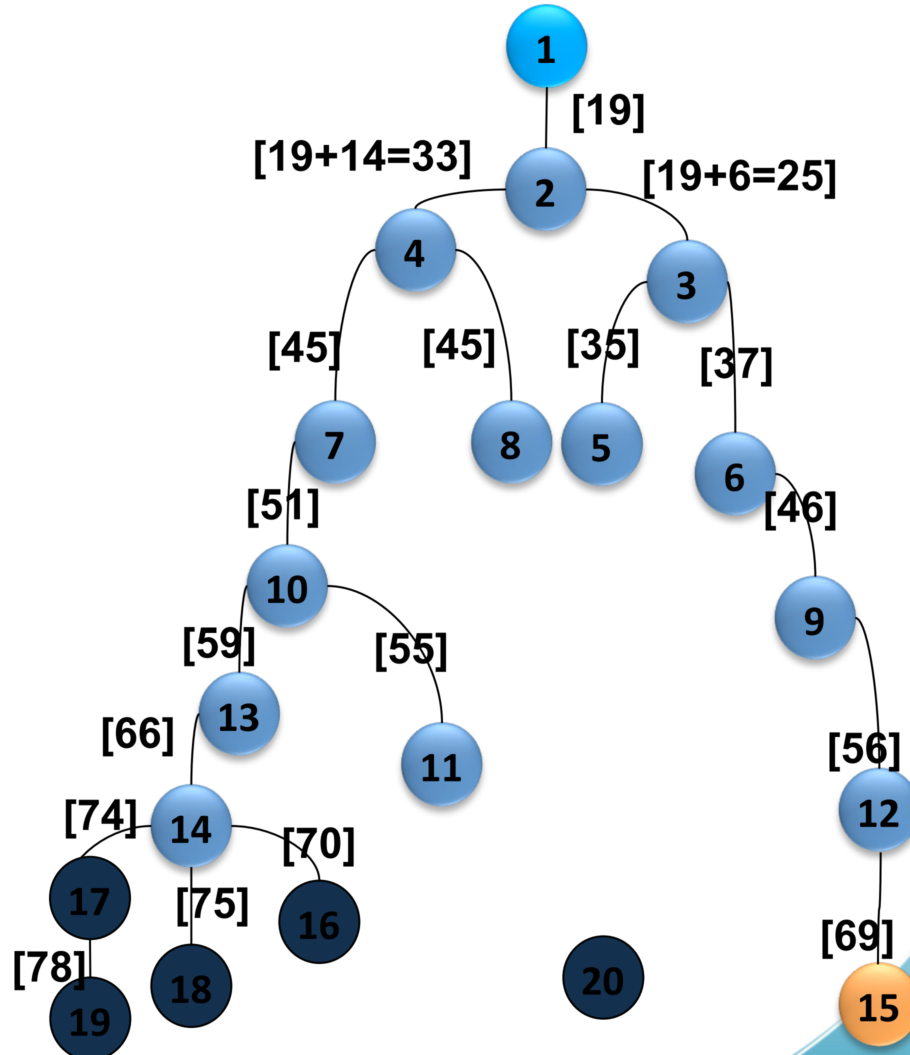
Extract topology info → Each fluid voxel as nodes → Connected nodes with edges → Assign weight to edges → Form weighted graph → Apply SPS algorithm → Determine shortest Flow Path → Determine Geometrical tortuosity

Weighted Graph



Shortest Path Searching Algorithm

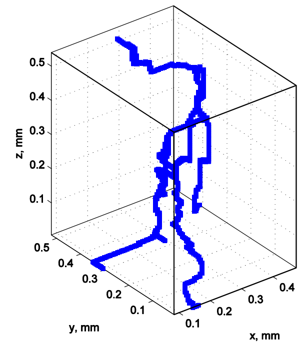




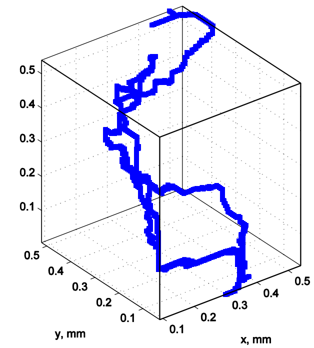
Shortest Flow Path Inside and Outside Compaction Bands

INSIDE CB

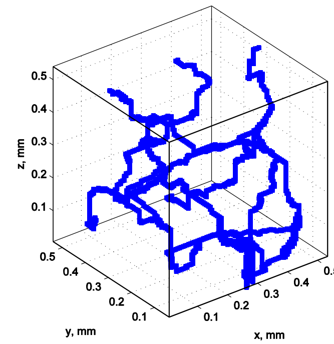
$\phi = 0.14$



$\tau = 2.79$
K= 3.4e-13 m^2



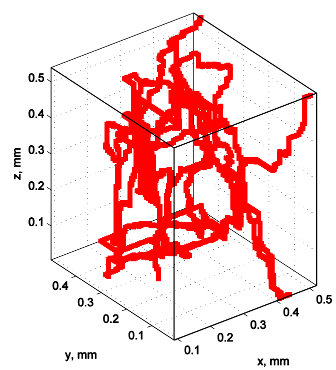
$\tau = 2.15$
K= 5.3e-13 m^2



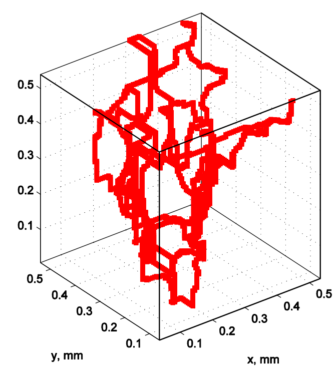
$\tau = 2.56$
K= 4.4e-13 m^2

OUTSIDE CB

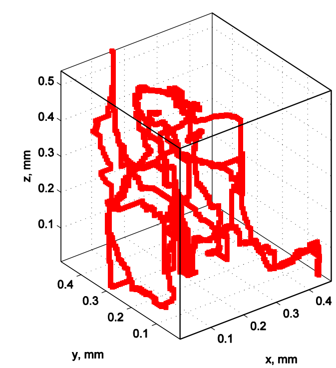
$\phi = 0.21$



$\tau = 1.77$
K= 1.3e-12 m^2

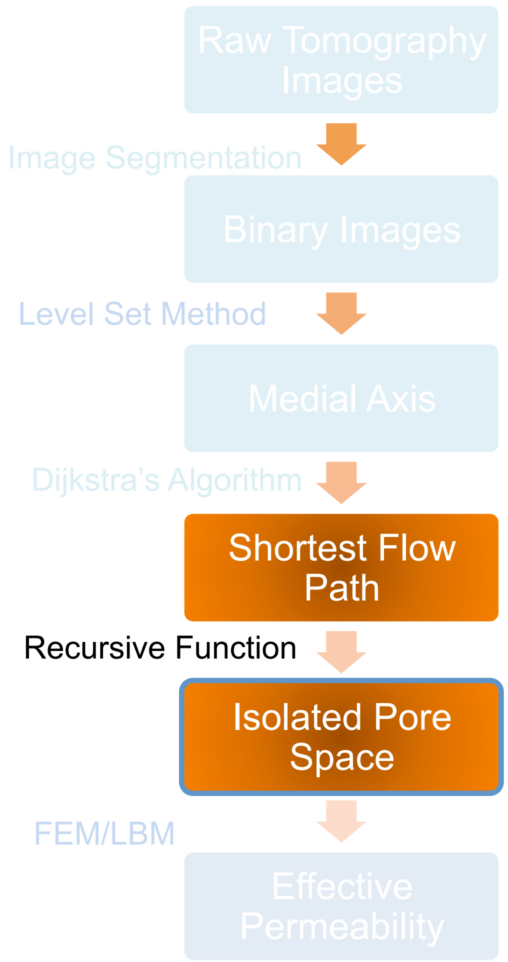


$\tau = 1.76$
K= 1.2e-12 m^2



$\tau = 1.81$
K= 1.3e-12 m^2

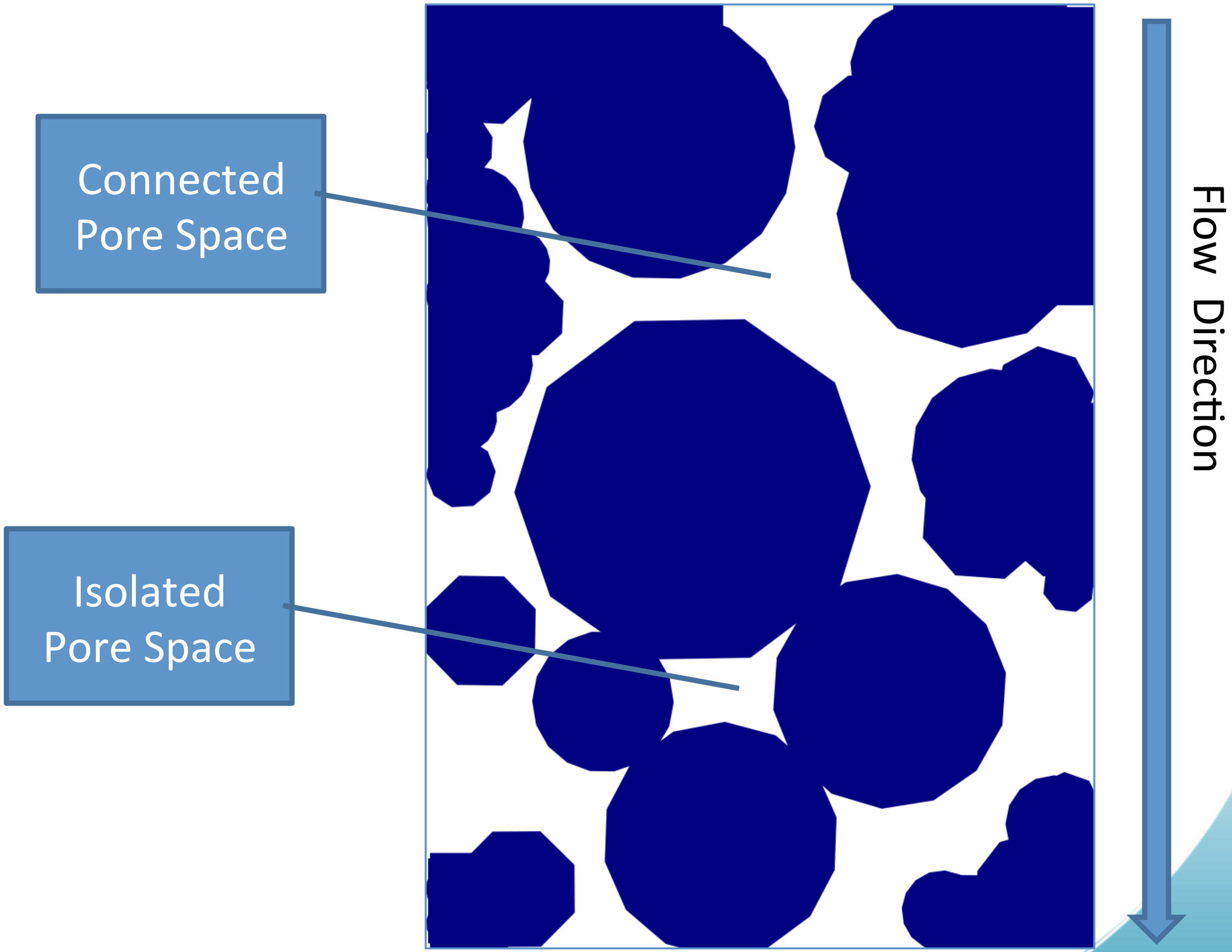
Recursive Function



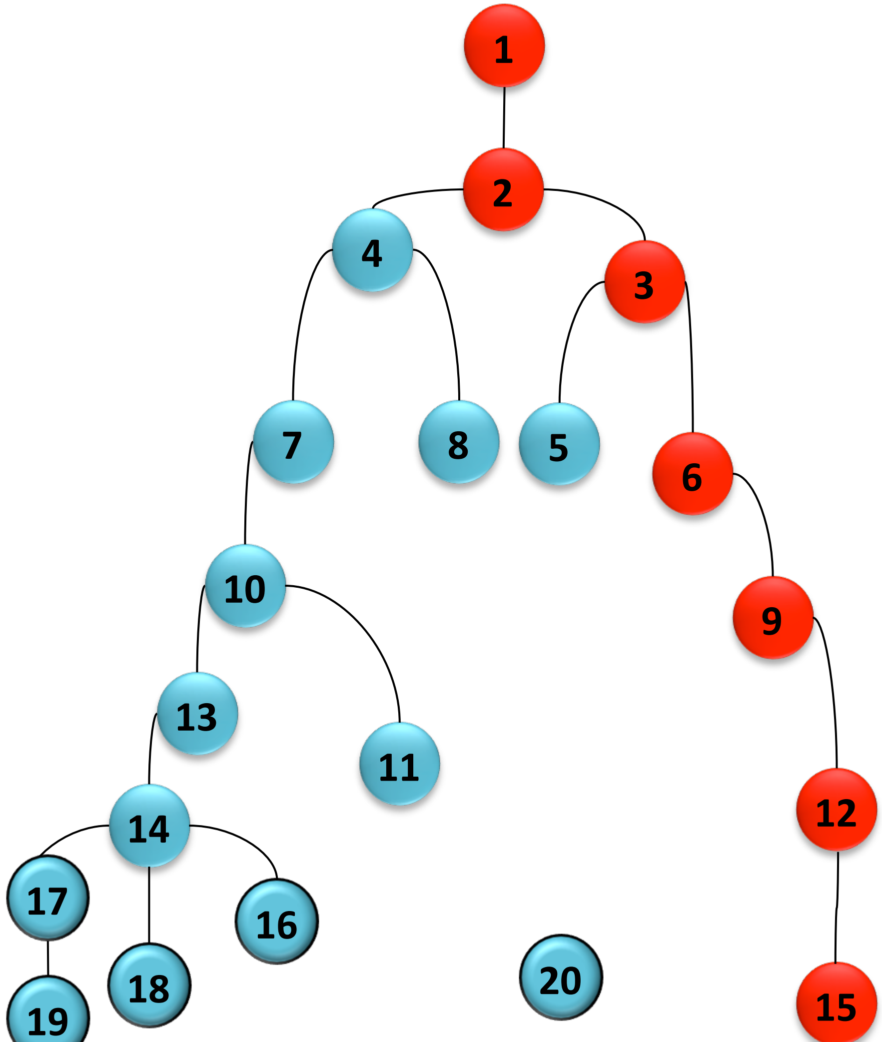
Connected Porosity vs. Porosity

- Only the connected pore space affects the effective permeability

- Isolated pore space should be treated as inactive voxels in LBM simulation to reduce computational cost



Recursive Functions



PROGRAM MAIN

1. **Activate all vertices along the flow path as active nodes and mark them as visited vertices**
2. **While there exists at least one active node**
3. **call the recursive function MARKNEIGHBOR**

EXIT

FUNCTION MARKNEIGHBOR

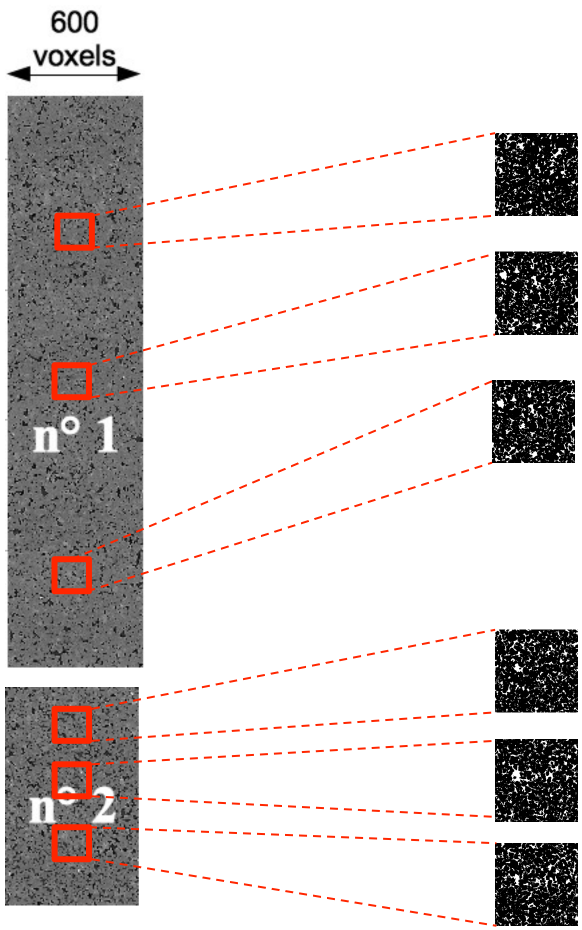
1. **IF at least one neighbors of the active nodes has not yet been visited**
    1. Activate the unvisited neighbor vertices
    2. Mark them as visited vertices.
    3. Deactivate the old active nodes with unvisited neighbor(s).
    4. Call the recursive function MARKNEIGHBOR
2. **ELSE**
    1. Deactivate the active nodes with no unvisited neighbor.
3. **EXIT**

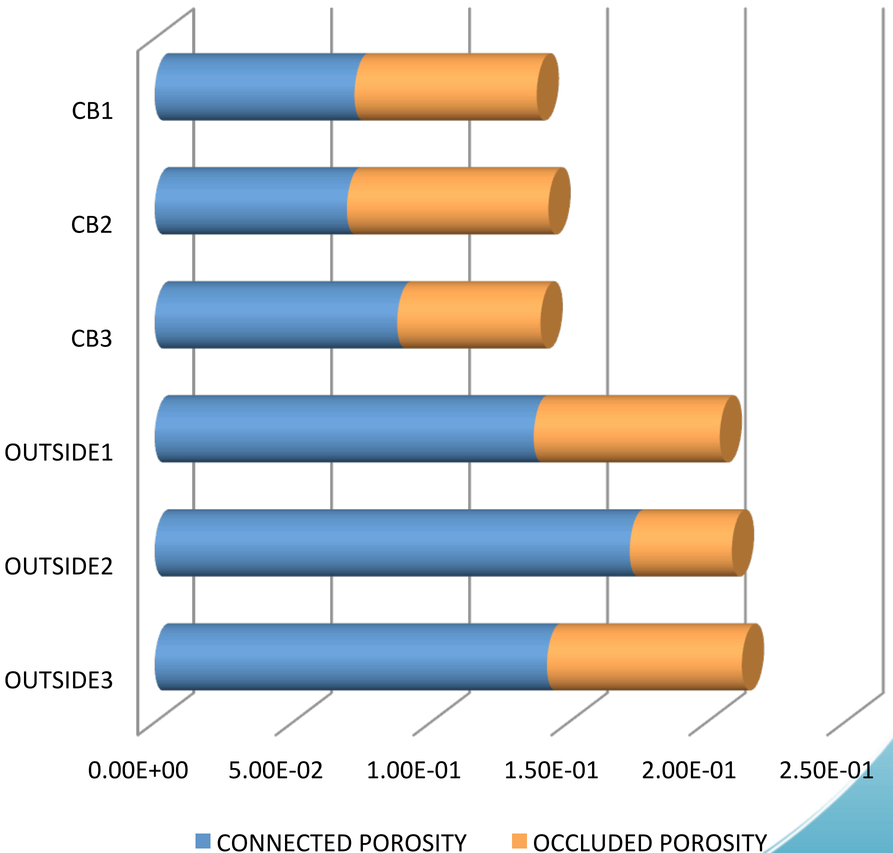
EXIT

Connected and Isolated Pore Space of Compaction Bands

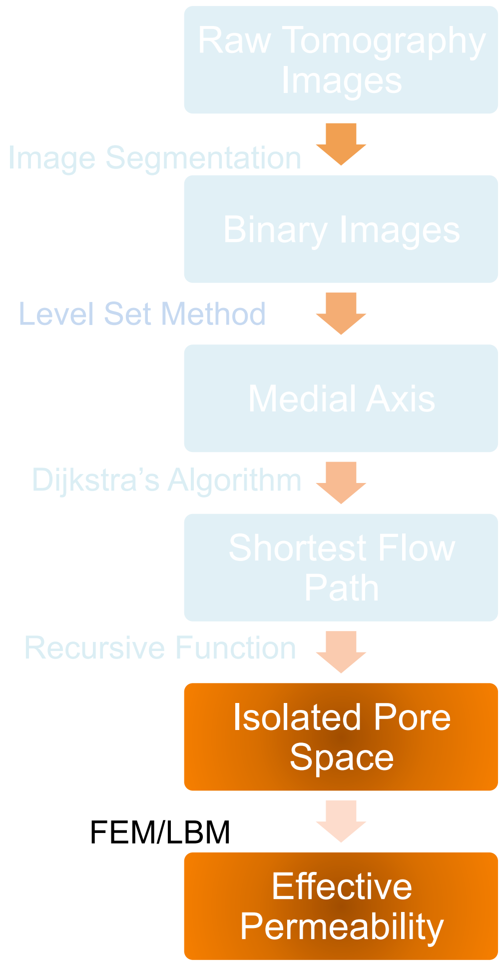
Inside CB

Outside CB

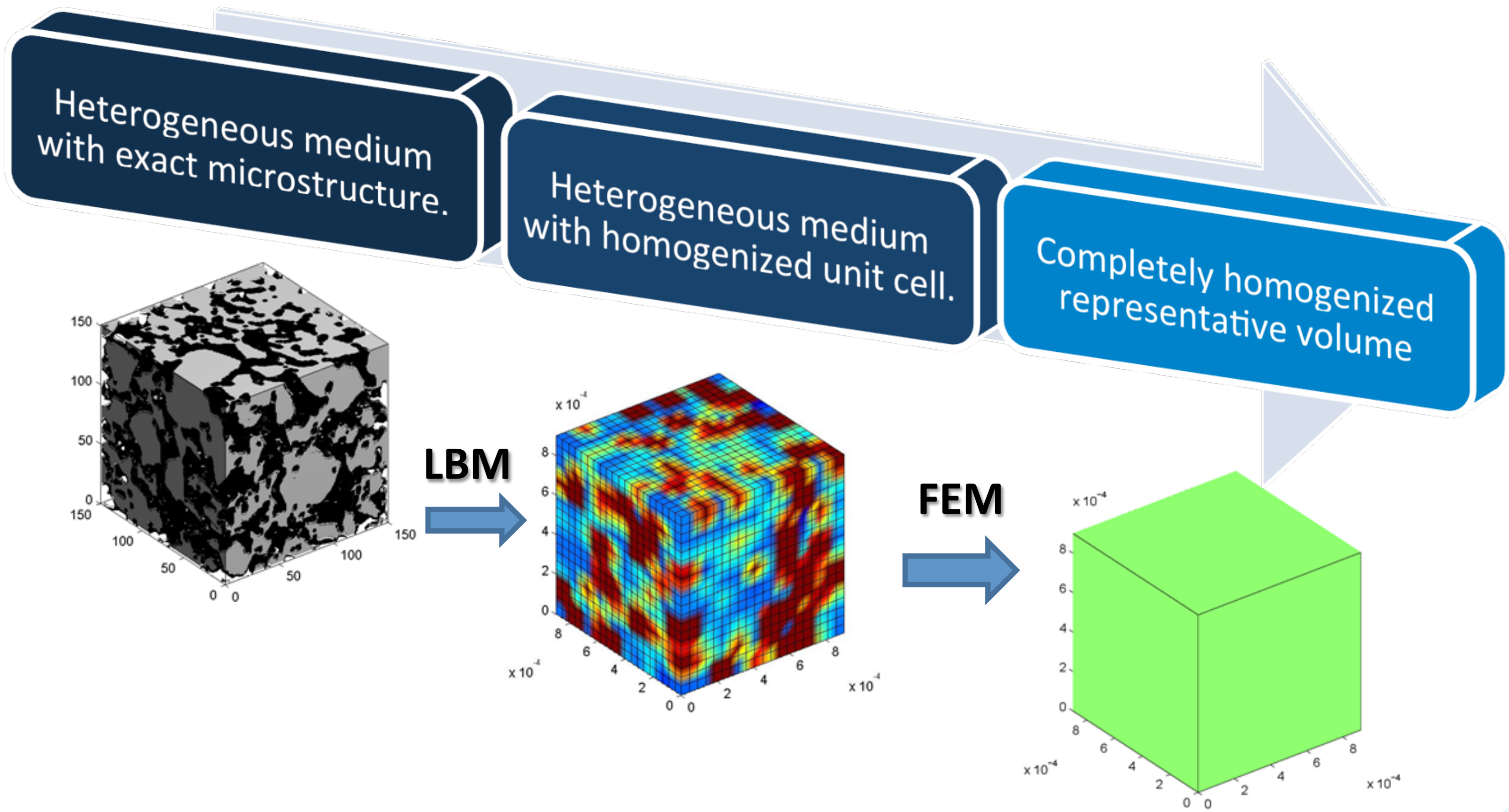




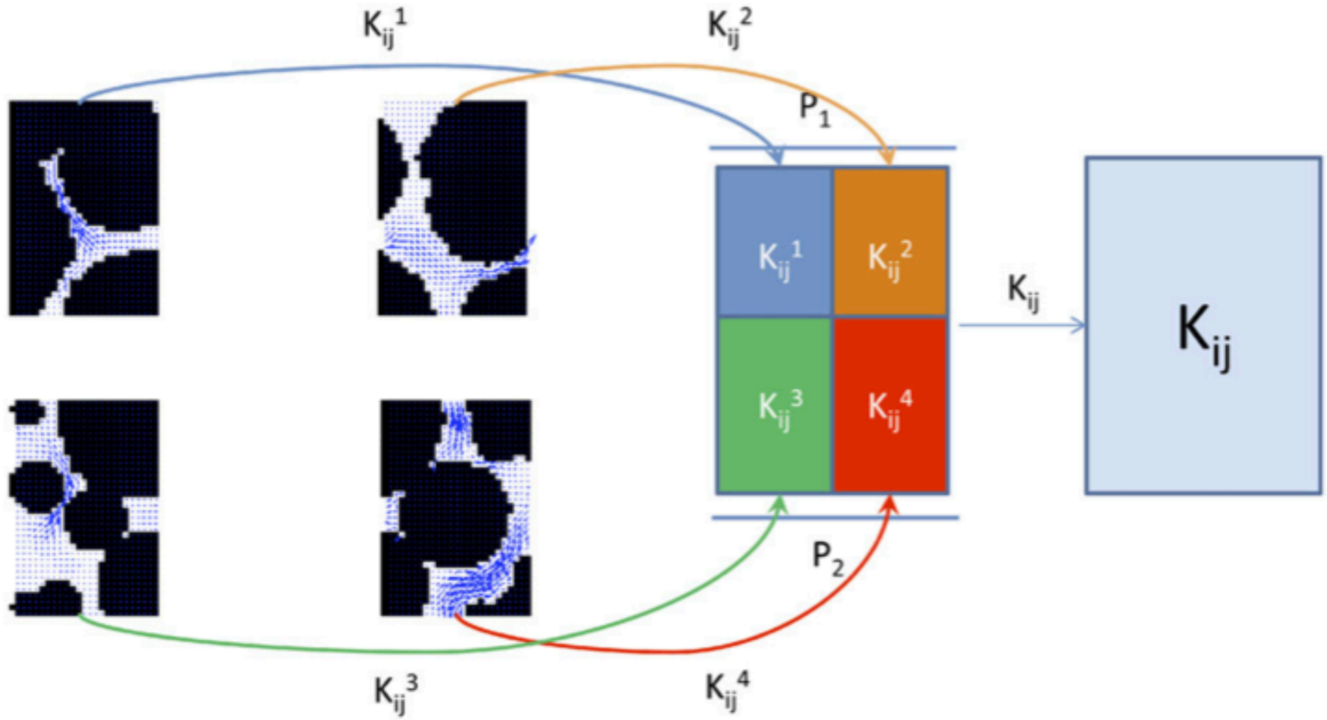
Finite Element/Lattice Boltzmann Hybrid Method



Homogenization of Effective Permeability Across scale



Lattice Boltzmann/Finite Element Flow Transport Simulation



Lattice Boltzmann Simulation

Finite Element Simulation

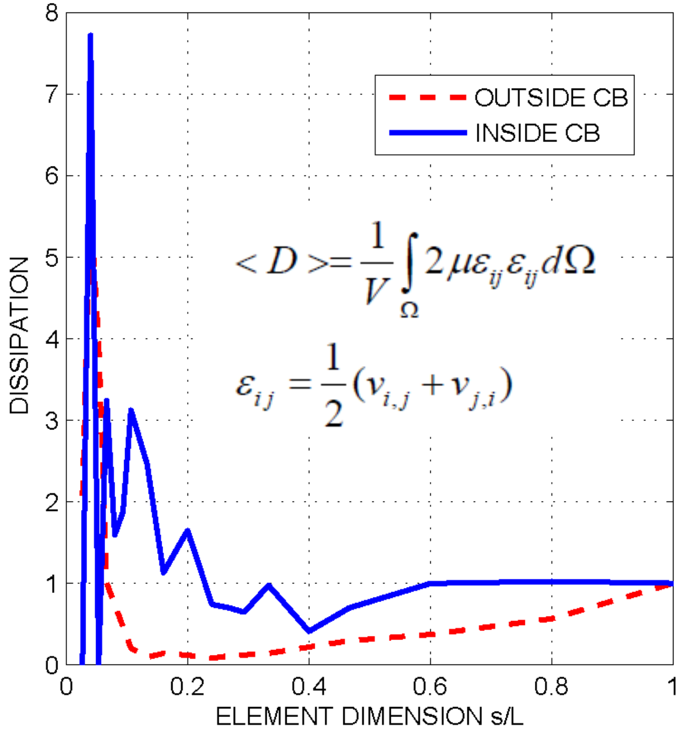
- Equation of State

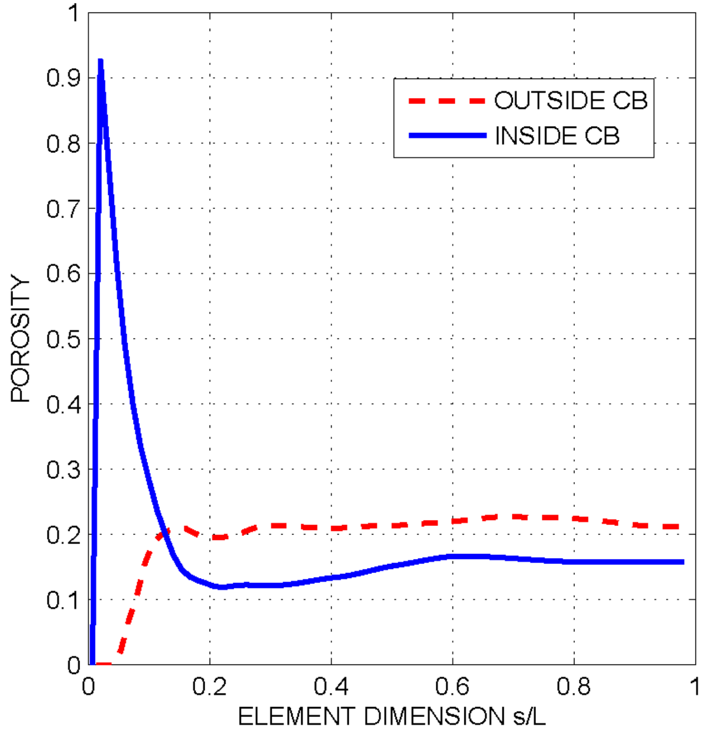
$$\frac{\partial}{\partial t}\rho + v \cdot \nabla \rho = 0 \qquad \frac{\partial}{\partial t} f + v \cdot \nabla f = 0$$

- Balance of Linear Momentum

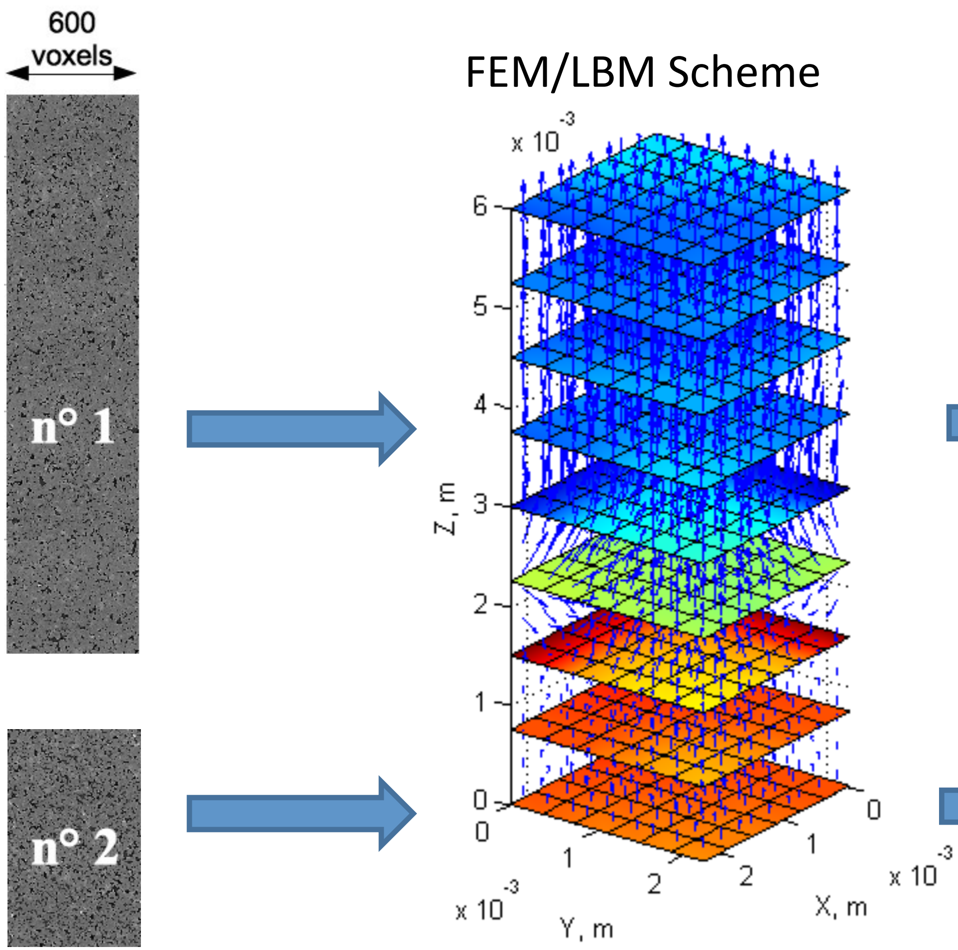
$$\frac{\partial}{\partial t}u + (u \cdot \nabla)u + \frac{1}{\rho}\nabla p = 0 \qquad \left(\frac{\partial}{\partial t} + \frac{p}{m} \cdot \nabla + \vec{F} \cdot \partial_p\right) f = 0$$

Size of Representative Elementary Volume





Effective Permeabilities Inside and Outside Compaction Bands



Inside CB → K_{zz} = 3.5e-13 m^2

Outside CB → K_{zz} = 14.0e-13 m^2

Conclusion

- The geometrical changes of micro-structure due to the formation of compaction bands are examined.
- Level set, graph theory, lattice Boltzmann and finite element methods are used as tools in this study.
- Increased tortuosity, reduction of connected porosity are main factors that lead to permeability reduction of compaction band.

Acknowledgement

- **Dr. Nicolas Leonir,Ecole des Ponts Paris Tech, Université Paris-Est**
- **Dr. Peter Eichhubl, University of Texas at Austin**
- **Dr. Cheng Chen, University of Southern California**
- **Prof. Aaron Packman, Northwestern University**
- **Prof. David Salac, SUNY Buffalo**

Thank you for your attention!