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General control landscape structure shared by open-loop and closed-loop quantum control approaches

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Reduced Density Matrices in Quantum Chemistry and Physics

Motivation: Quantum Control Applications

- ✔ Photochemistry: Ultrafast laser control of molecular reactions (photoionization; photodissociation; photoisomerization; photosynthesis)
- ✔ Non-linear spectroscopy (selective excitation of molecular vibrational modes)
- ✔ Non-linear microscopy (selective excitation of multiphoton fluorescence)
- ✔ Atomic physics (control of multiphoton absorption; control of photoelectron spectral and angular distributions; control of atomic wave function shape)
- ✔ High-harmonic generation (selective generation of coherent X-ray pulses in atomic vapors by intense laser fields)
- ✔ Solid-state physics: Coherent control of quantum processes in semiconductors (electrons/holes in quantum dots; photocurrents; nonlinear optical responses)
- ✔ Quantum information sciences (optimal control of quantum logic operations; protection against environment and instrumental noise)
- ✔ Quantum biology: Coherent control of quantum transport in biomolecules (energy transfer in photosynthetic complexes; proton transfer in enzymes)

C. Brif et al., "*Control of quantum phenomena: past, present and future*,"
New Journal of Physics **12**, 075008 (2010)

Optimal Quantum Control

In quantum control applications, often both scientific and engineering goals are pursued:

- Scientific goals: understand properties and dynamical behavior of quantum systems, as well as mechanisms by which physical and chemical processes can be managed
- Engineering goals: achieve the control objectives in the best possible way (e.g., highest yield of a chemical reaction; highest fidelity of a quantum computation)

Better physical understanding ⟷ Optimal control performance

How to achieve optimality (find optimal control solutions)? $J_{\mathrm{opt}} = \max_{\varepsilon(t)} J[\varepsilon(t)]$

Open-loop control

- Find optimal controls for a theoretical model (optimal control theory)
- Apply theoretical optimal control designs to the actual system in the laboratory

Closed-loop control

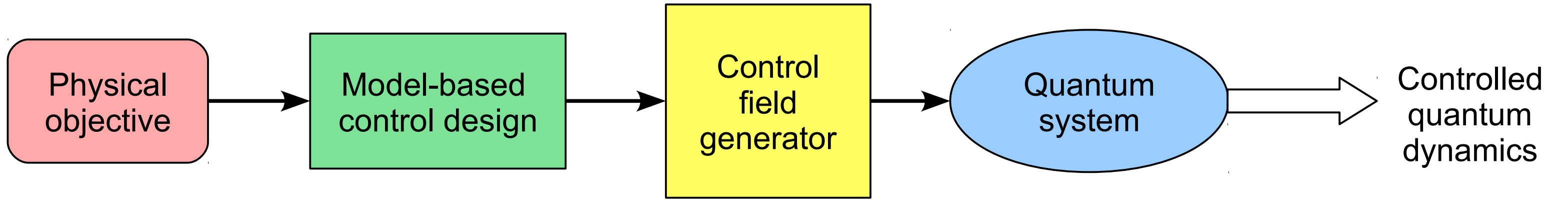
Find optimal controls directly in the laboratory, using feedback from a signal from the controlled system

Adaptive feedback control
Measure the control objective to guide optimization by a learning algorithm. Reset the system after each measurement.

Real-time feedback control
Measure a signal from the system and use it in real time to select the next control action. Measurements affect the system dynamics

Coherent feedback control
Coherently process a signal from the controlled quantum system ("plant") by an auxiliary quantum system ("controller")

Open-loop control (OLC)

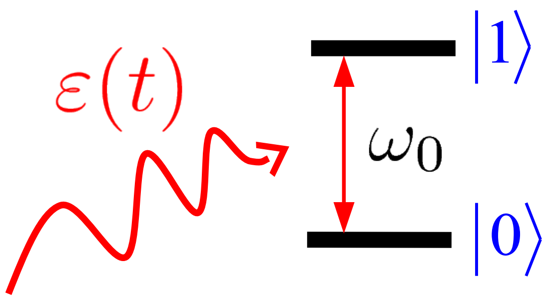


- A **theoretical model** of the quantum system dynamics is used to design the controls
  - Using knowledge of spectroscopic data (transition frequencies)
  - Using knowledge of system symmetry (allowed and forbidden transitions)
  - Using optimal control theory (iterative optimization employing numerical solution of system evolution equations)
- The obtained control design is directly applied in the laboratory

Advantages:
- ✔ A simple model usually helps to understand the control mechanism
- ✔ It is helpful to do feasibility analysis before building the lab

Drawbacks:
- ✔ A simple model does not adequately describes the reality
- ✔ It is fundamentally difficult to simulate many-body quantum dynamics
- ✔ It is difficult to reliably implement theoretical designs in the laboratory

Example of OLC: Single-qubit gates with improved robustness to errors



The qubit (a two-level quantum system) is encoded in two hyperfine levels of a single laser-cooled, trapped ion $^{171}Yb^{+}$
The qubit is controlled by microwave radiation

$$\omega \approx 2\pi \times 12.6 \text{ GHz} \qquad \Omega \approx 2\pi \times 10 \text{ kHz}$$

The goal is to enact the transformation

$$|0\rangle \rightarrow |\theta, \phi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi} \sin(\theta/2)|1\rangle$$

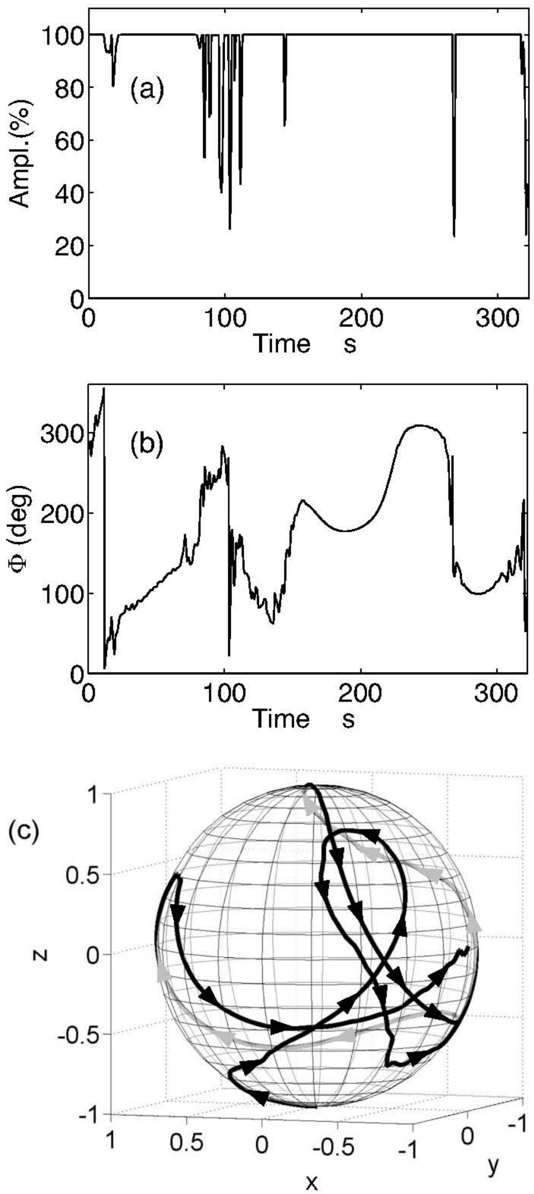
In reality, frequency and amplitude noise of the applied radiation will produce a different transformation, resulting in a different final state.

$$F = |\langle \theta, \phi | \tilde{\theta}, \tilde{\phi} \rangle|^2$$

Optimal control theory was used to design the control field that maximizes the fidelity F in the presence of control noise.

N. Timoney et al., Phys. Rev. A **77**, 05234 (2008)

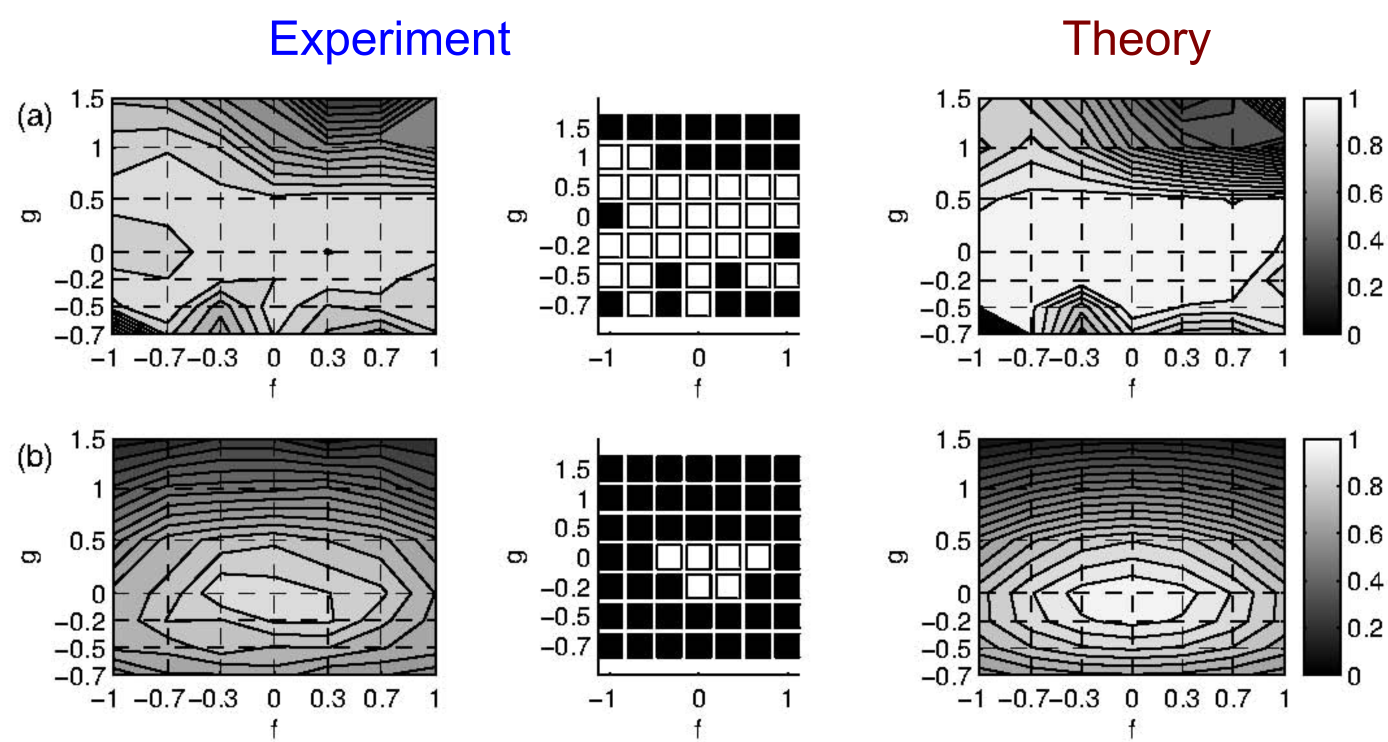
Example of OLC: Single-qubit gates with improved robustness to errors



$\theta = \pi/2,\ \phi = -\pi/2 \qquad |\theta,\phi\rangle = (|0\rangle - i|1\rangle)/\sqrt{2}$

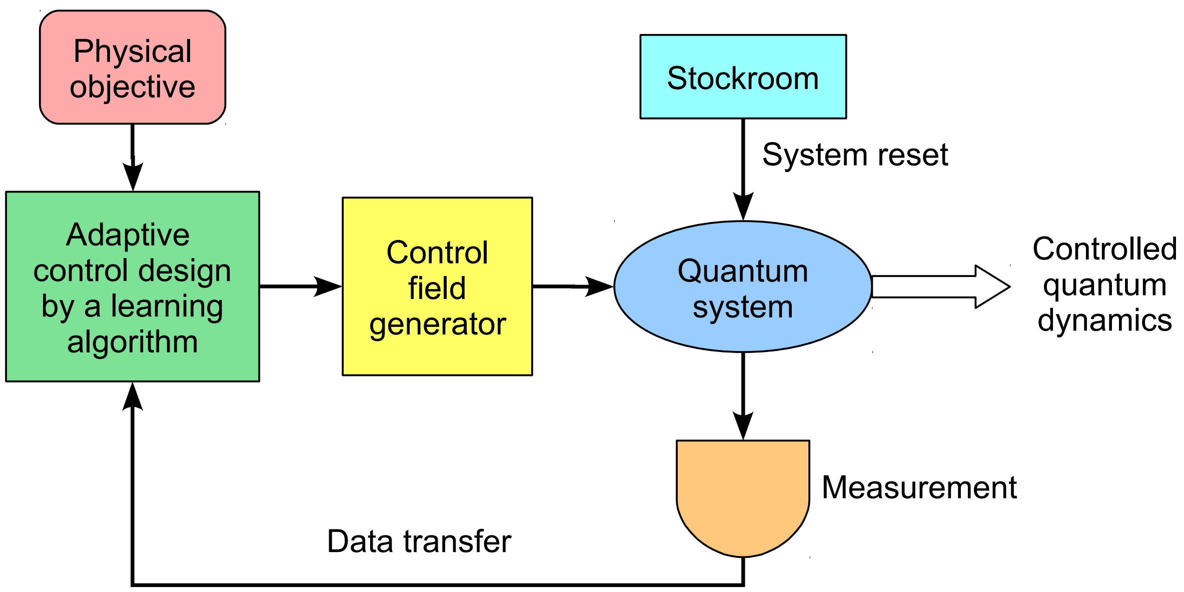
f – frequency error, *g* – amplitude error (scaled)

Fidelity for (a) optimal control pulse, (b) squared pulse



N. Timoney et al., Phys. Rev. A **77**, 05234 (2008)

Adaptive feedback control (AFC)

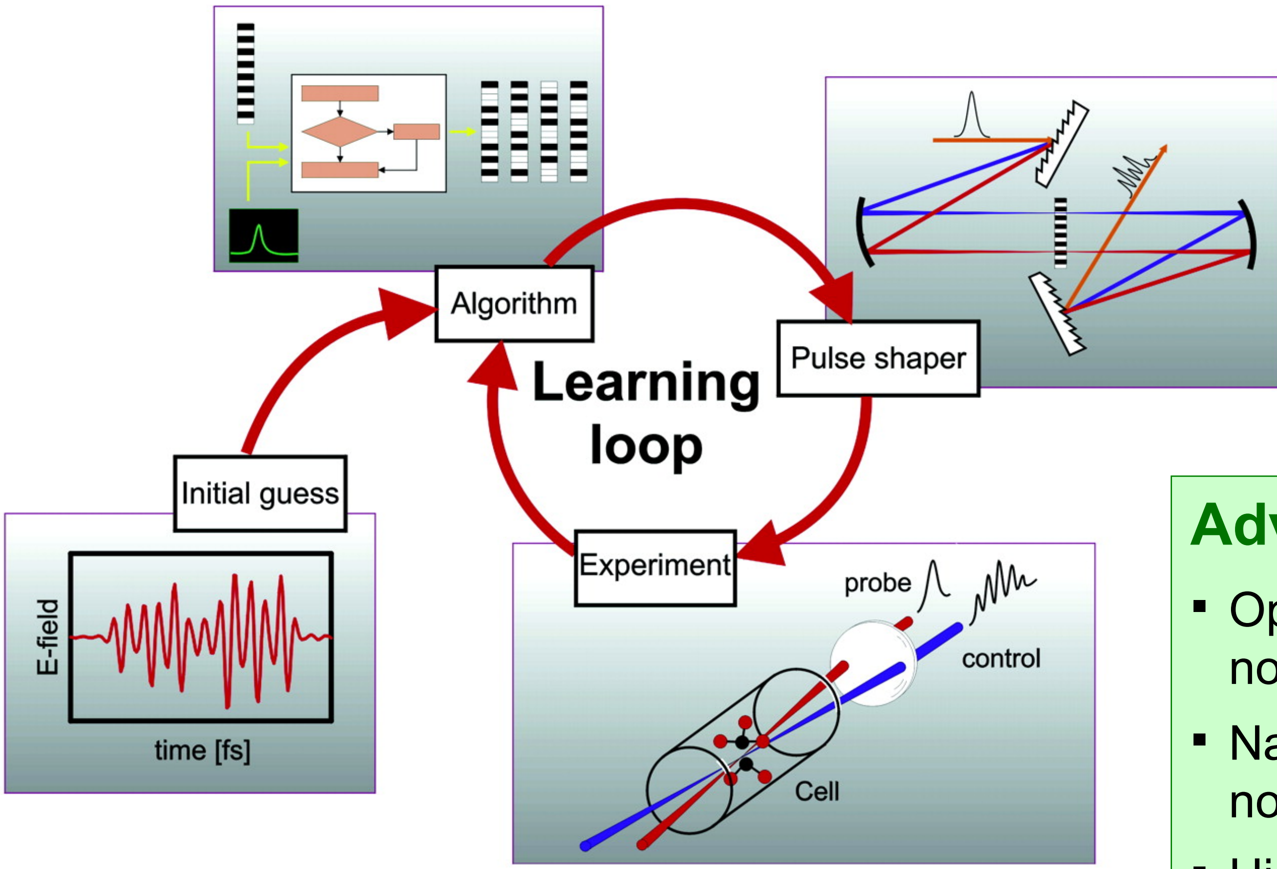


A loop is closed in the laboratory, with results of measurements on the quantum system used to evaluate the success of the applied control and to refine it, until the control objective is reached as best as possible.

At each iteration of the feedback loop:

- The external control is applied to the system.
- The signal is measured and fed back to the learning algorithm.
- The algorithm evaluates each control based on its measured outcome with respect to a predefined objective, and searches for an optimal solution.
- The system is reset to the initial state at the end of each trial.

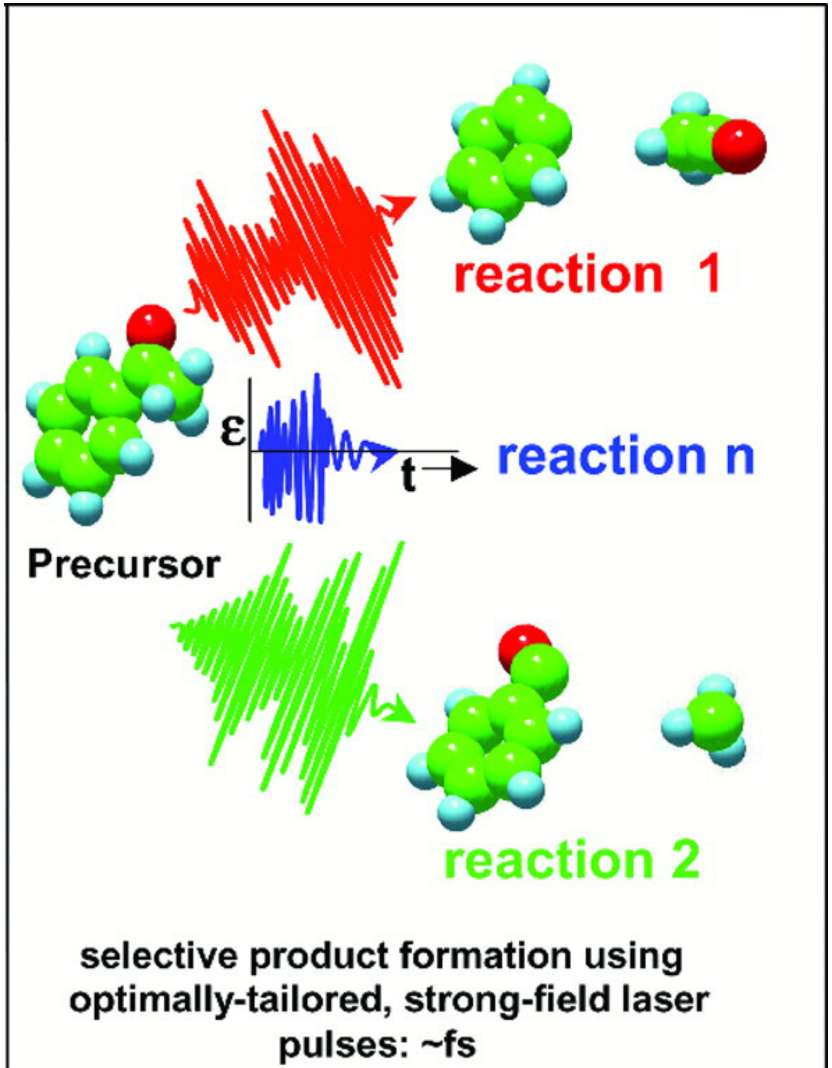
Laboratory AFC of quantum phenomena using ultrafast shaped laser pulses

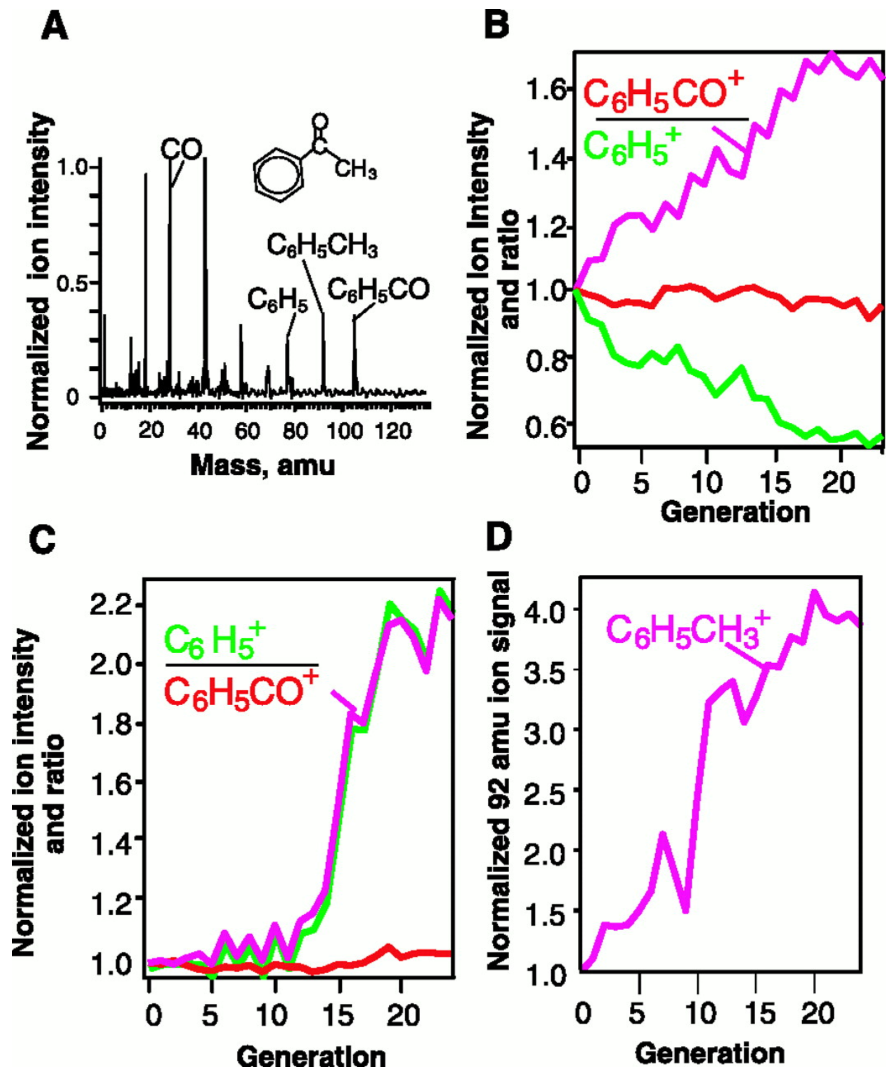


Advantages of AFC:

- Optimization for the actual system, not for a model
- Natural robustness to instrumental noise
- High-duty cycle (thousands of experiments per second)
- Fully automated
- A fresh sample at each cycle (no back-action effect from the measurements)

Example of AFC: Control of photodissociation with strong-field femtosecond laser pulses

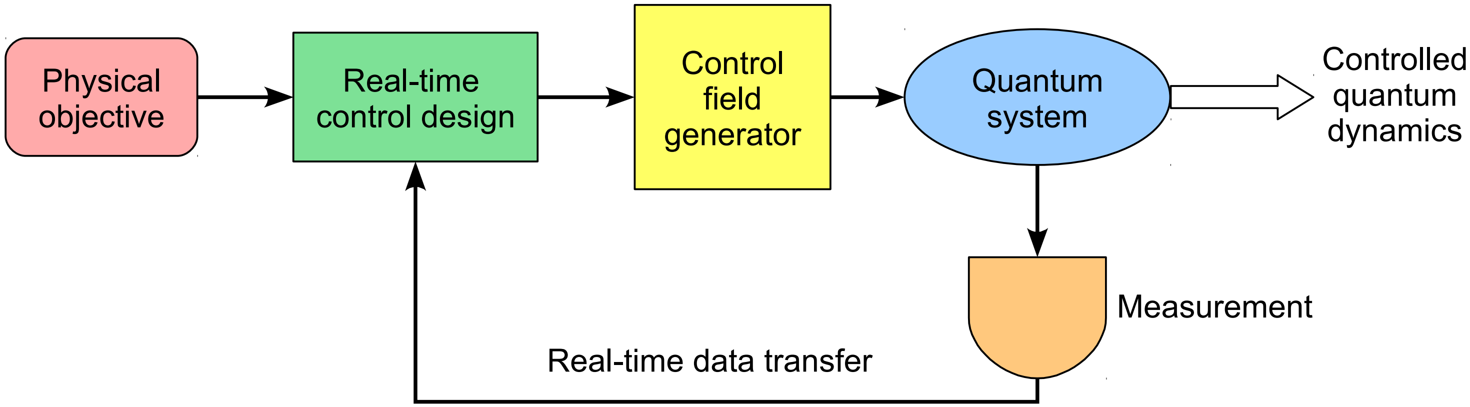




R. J. Levis et al.,
Science **292**, 709 (2001)

Selective control of photodissociation of **acetophenone**
Controlled products: **phenyl**, **phenylcarbonyl**, **toluene**

Real-time feedback control (RTFC)



In RTFC, measurements are used to probe the quantum system, and the obtained information is processed classically in real time to select the next control action.

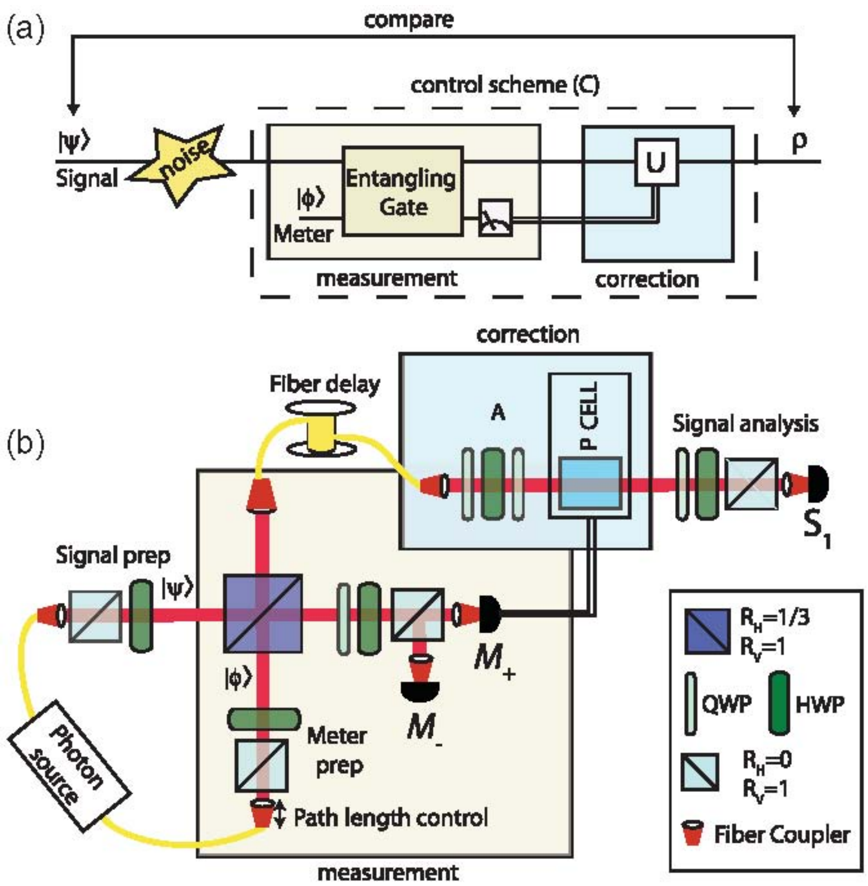
The system evolution during each feedback iteration is affected not only by the coherent control action (conditioned upon the measurement outcome), but also by incoherent back-action exerted by the measurement.

Advantages:
- ✔ Real-time feedback is ubiquitous in classical engineering control
- ✔ Quantum measurement back-action is a useful non-unitary control

Drawbacks:
- ✔ Simulating non-unitary quantum dynamics is computationally hard
- ✔ Loop latency: interesting quantum phenomena are usually much faster than data processing by classical electronics

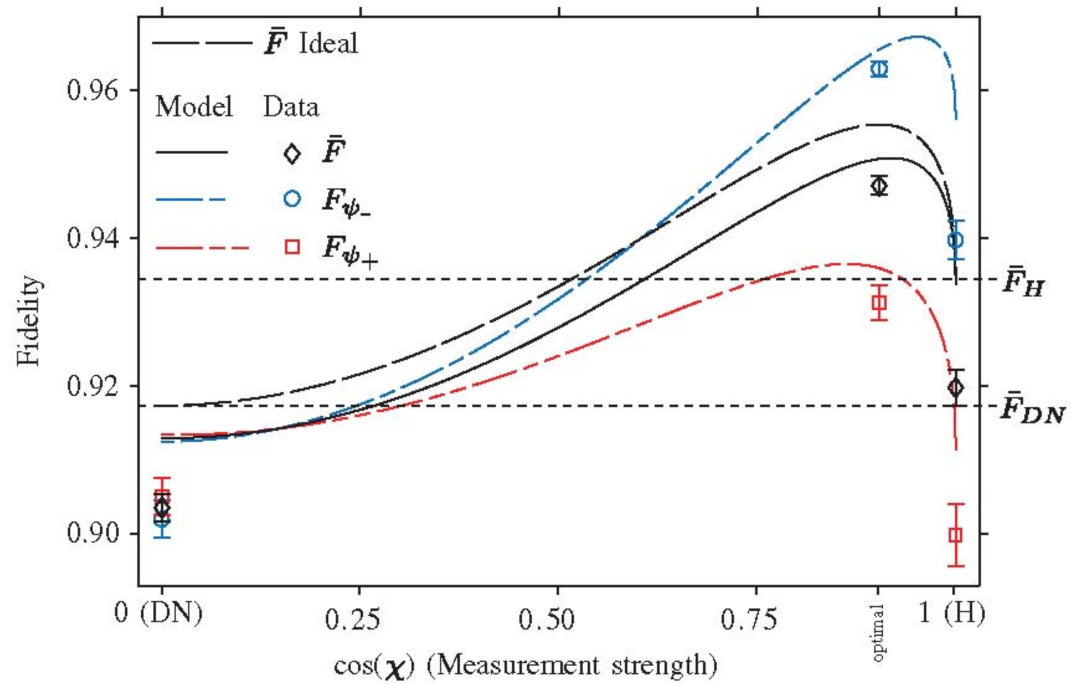
Example of measurement-based RTFC: Stabilization of a photon qubit state

A qubit is encoded in the polarization of single photons: $|0\rangle = |V\rangle$, $|1\rangle = |H\rangle$. The objective is to preserve the initial qubit state (two non-orthogonal states are used). The measurement is on an auxiliary photon that is entangled with the qubit photon, and the measurement strength may be varied by tuning the degree of entanglement.



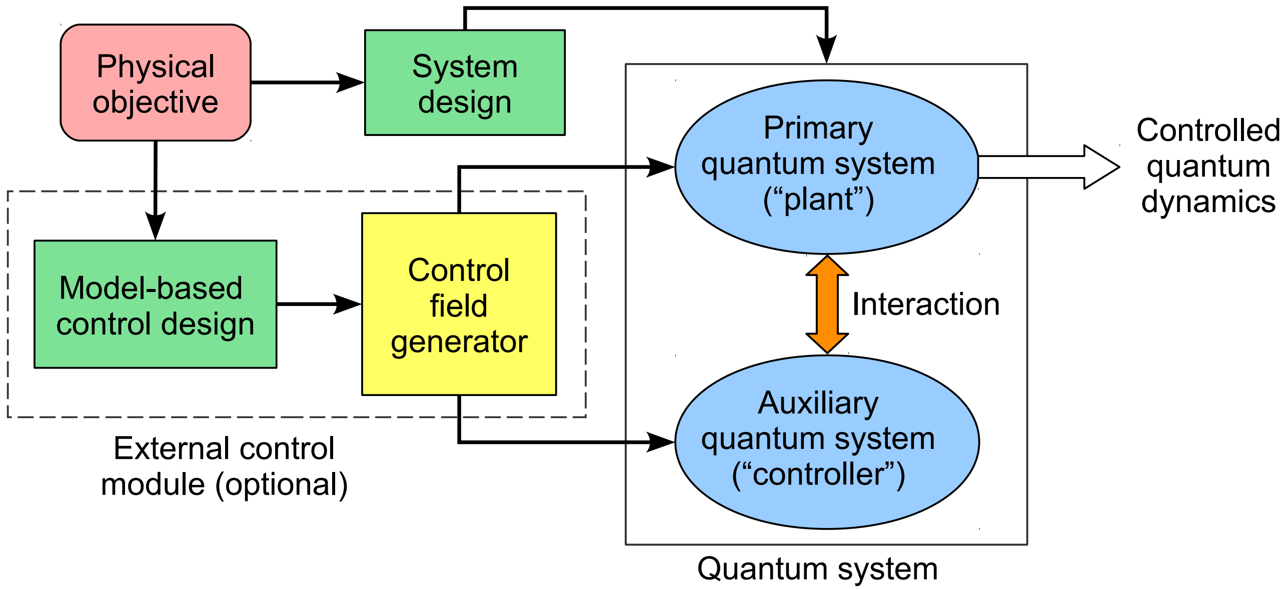
$$|\psi_\pm\rangle \overset{\text{noise}}{\rightarrow} \rho^{\text{n}}_\pm \overset{\text{meas}}{\rightarrow} \rho^{\text{m}}_\pm \overset{\text{corr}}{\rightarrow} \rho^{\text{c}}_\pm$$

$$F_{\psi_\pm} = \langle \psi_\pm | \rho^{\text{c}}_\pm | \psi_\pm \rangle$$



G. G. Gillett et al., Phys. Rev. Lett. **104**, 080503 (2010)

Coherent RTFC

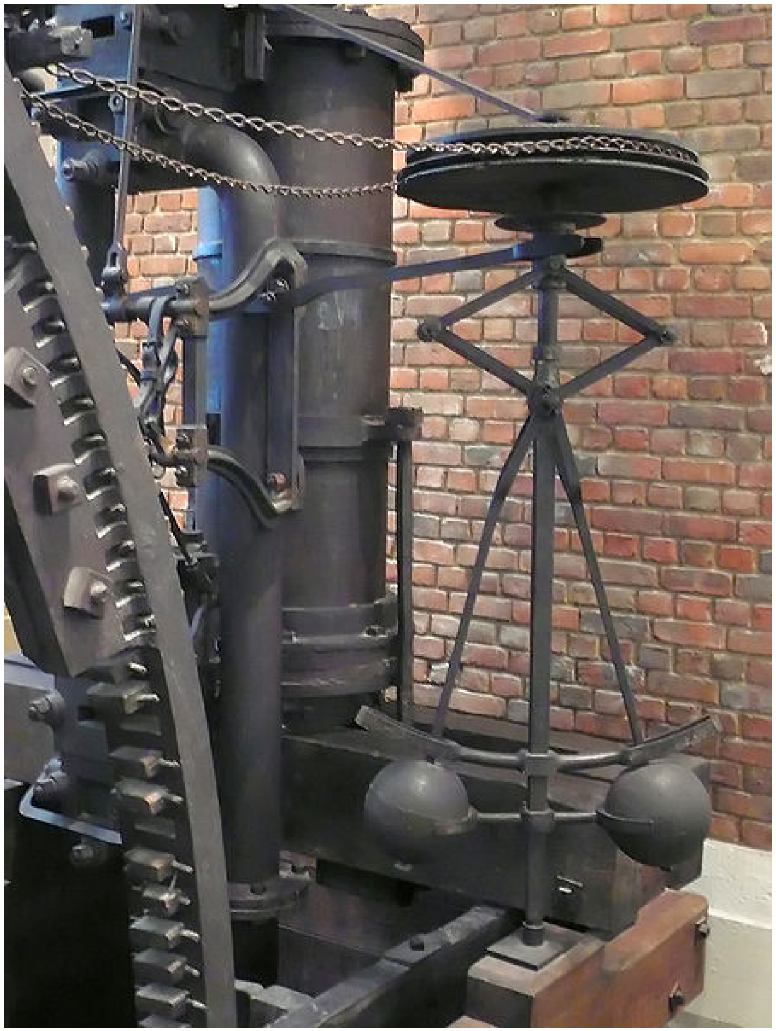


- In coherent RTFC, the evolution of the primary quantum system (the "plant") is manipulated through its interaction with an auxiliary quantum system (the "controller").
- The evolution of the composite quantum system which consists of the plant and controller is unitary (assuming that environmental effects are neglected).
- Optionally, the plant, controller, or both can be also coherently driven by external classical control fields (e.g., designed based on a theoretical model, as in OLC).

Advantage: Coherent RTFC can overcome the issue of loop latency

Challenge: The quantum controller itself may require precise engineering to assure quality control performance

Coherent RTFC: A quantum analog of Watt's flyball governor



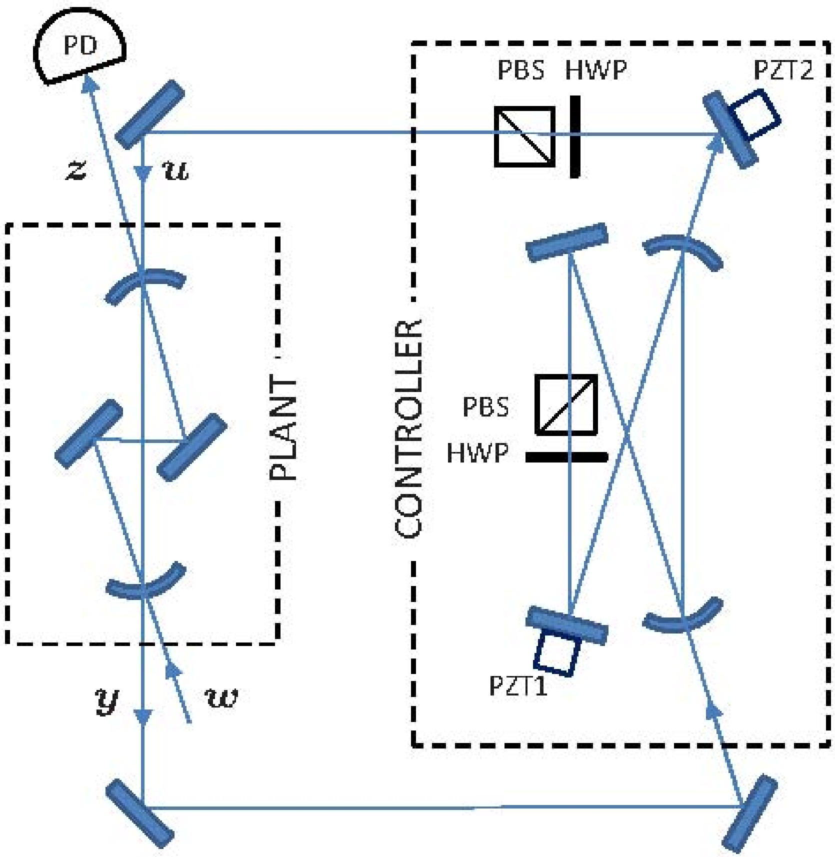
Coherent RTFC is a quantum analog of a classical control system employing Watt's flyball (centrifugal) governor.

Such a machine is self regulating, with one part of the composite system (the “controller”) used to control/stabilize the function of the other part (the “plant”)

The working machine: M. Boulton and J. Watt, 1788
Classical theory: J. C. Maxwell, “On Governors,” Proc. Roy. Soc. Lond. **16**, 270 (1868)
Quantum theory: S. Lloyd, Phys. Rev. A 62, 022108 (2000)

Example of coherent RTFC: Dynamic compensator of laser noise in a resonator

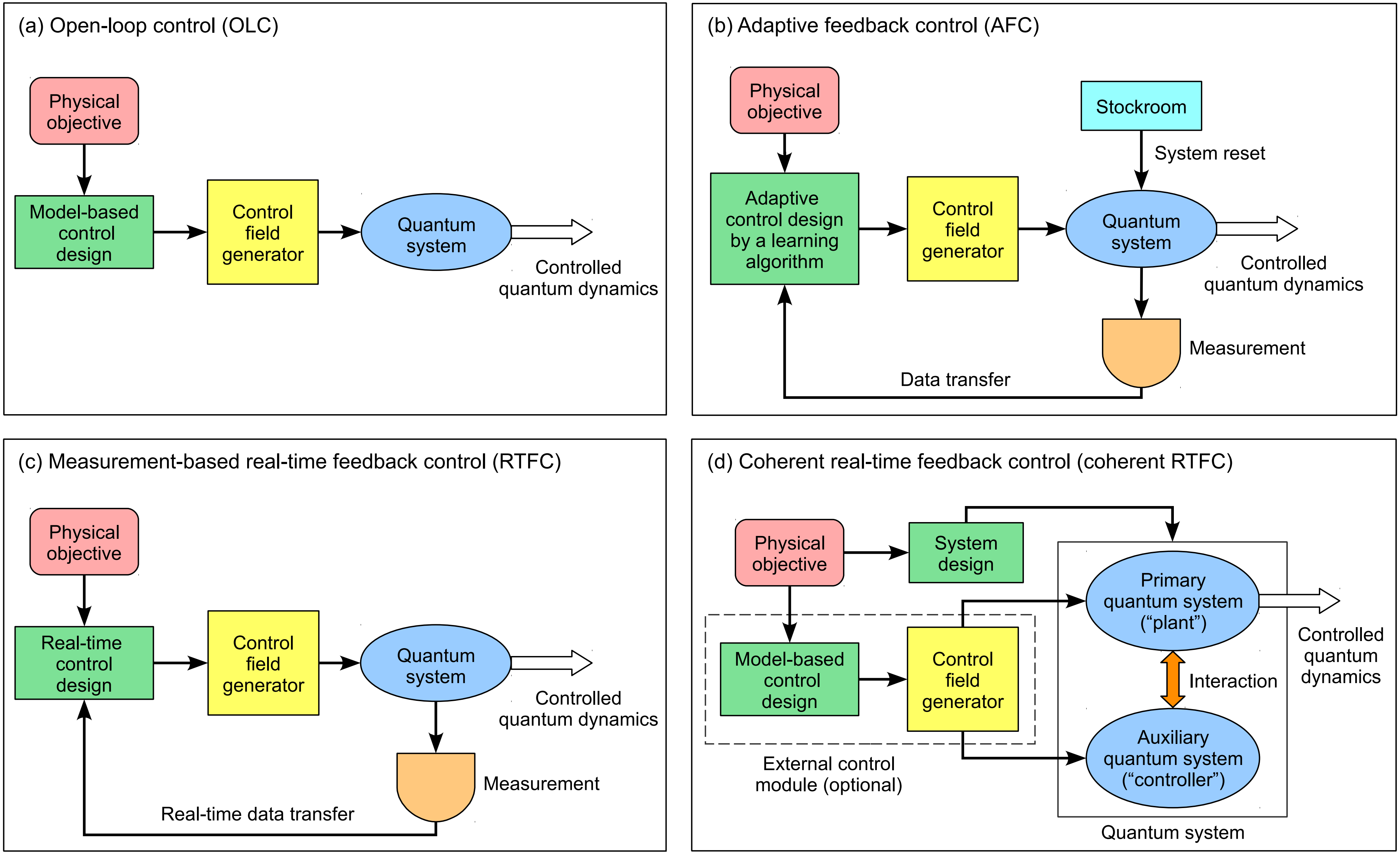
- Two optical ring resonators represent the “plant” and “controller” quantum systems.
- The objective is to tailor the properties of the controller so as to minimize the optical power detected at output z when a “noise” signal (optical coherent state with arbitrary time-dependent complex amplitude) is injected at the input w.



- The component y of the noise beam that reflects from the plant input coupler is treated as the error signal.
- The error signal y is coherently processed by the controller to produce the feedback signal u.
- The feedback signal u is fed back into the plant resonator via the output coupler (matched spatially to the same resonant mode driven by the noise input w).

H. Mabuchi, Phys. Rev. A **78**, 032323 (2008)

Summary: Four principal types of quantum control



Unitary and non-unitary quantum dynamics

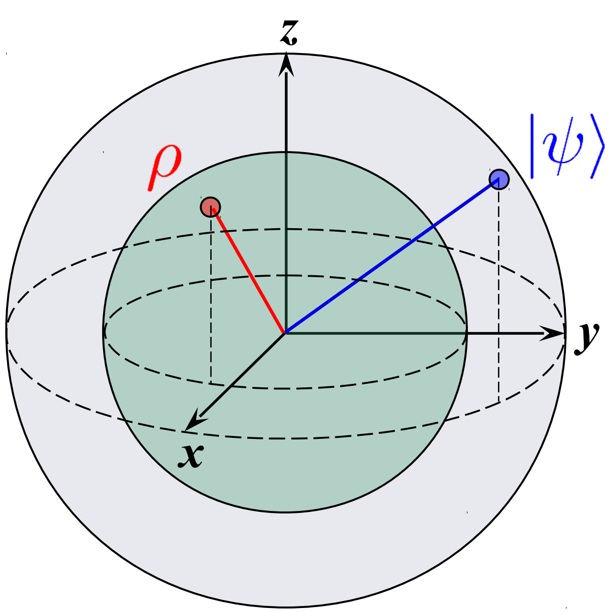
Closed quantum systems

A pure state: $|\psi\rangle$

Unitary (coherent) evolution:

$$|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle$$

$$UU^\dagger = U^\dagger U = I$$



Open quantum systems

A mixed state: $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

Non-unitary evolution:

$$\rho(t) = \Phi[\rho(t_0)]$$

Φ is a completely positive, trace-preserving linear map (Kraus map or quantum channel):

$$\rho(t) = \sum_\alpha K_\alpha \rho(t_0) K_\alpha^\dagger$$

$$\sum_\alpha K_\alpha^\dagger K_\alpha = I$$

Unified description of controlled quantum dynamics

Dynamics of the controlled quantum system in **all** types of quantum control (OLC, AFC, RTFC) can be generally described by a **Kraus map**

Proof for measurement-based RTFC

A generalized quantum measurement with outcomes $\{O_a\}$ is characterized by a set of Kraus operators $\{K_{a,b}\}$

$$\rho(t) \to \rho_a(t+\tau_m) = (1/p_a) \sum_b K_{a,b} \rho(t) K_{a,b}^\dagger$$

The feedback action is generally described by a Kraus map:

$$\rho_a(t+t_m) \to \rho_a(t+t_m+t_f) = \Lambda_a[\rho_a(t+t_m)]$$

The measurement is a stochastic process; we should average over all possible outcomes:

$$\rho(t) \to \rho(t+t_m+t_f) = \sum_a p_a \rho_a(t+t_m+t_f)$$

The explicit form of this map:

$$\rho(t) \to \Phi[\rho(t)] = \sum_a \Lambda_a \Big[\sum_b K_{a,b} \rho(t) K_{a,b}^\dagger \Big] = \sum_{c,a,b} Z_{c,a,b} \rho(t) Z_{c,a,b}^\dagger$$

$$Z_{c,a,b} = L_{c,a} K_{a,b}, \quad \sum_{c,a,b} Z_{c,a,b}^\dagger Z_{c,a,b} = I$$

This is a Kraus map!

Unified control landscape topology

A general control process C can include: coherent control (application of external fields), measurements, unitary and non-unitary feedback actions, interactions with an auxiliary system. The evolution of the controlled system is described by a Kraus map:

$$\rho(T) = \Phi_{\mathcal{C}}[\rho(t_0)] = \sum_{\alpha} K_{\alpha}^{\mathcal{C}} \rho(t_0) (K_{\alpha}^{\mathcal{C}})^{\dagger}$$

A typical control objective is to maximize the expectation value of a target observable A at a final time T:

$$J_{\mathrm{opt}} = \max_{\mathcal{C}} J[\mathcal{C}]$$

$$J[\mathcal{C}] = \langle A(T) \rangle = \mathrm{Tr}[A\rho(T)] = \mathrm{Tr}\Big[A \sum_{\alpha} K_{\alpha}^{\mathcal{C}} \rho(t_0) (K_{\alpha}^{\mathcal{C}})^{\dagger}\Big]$$

The control landscape of an open system is *equivalent* to an auxiliary landscape of a closed composite system, which includes the controlled system and an "environment" [see R. B. Wu et al., J. Math. Phys. **49**, 022108 (2008) for details].

If the system is *controllable* (controls are sufficient to generate any Kraus map), then for *regular* controls (the tangent map from C to $\{K_{\alpha}^{C}\}$ is surjective):

no local traps exist in the open-system control landscape

(all extrema, except for the global maximum and global minimum, are saddles).

“Ultimate quantum control machine”

A prospective hybrid scheme including all existing quantum control approaches