

# **Towards Eliminating Element Size Dependence in Ductile Tearing**

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# Outline

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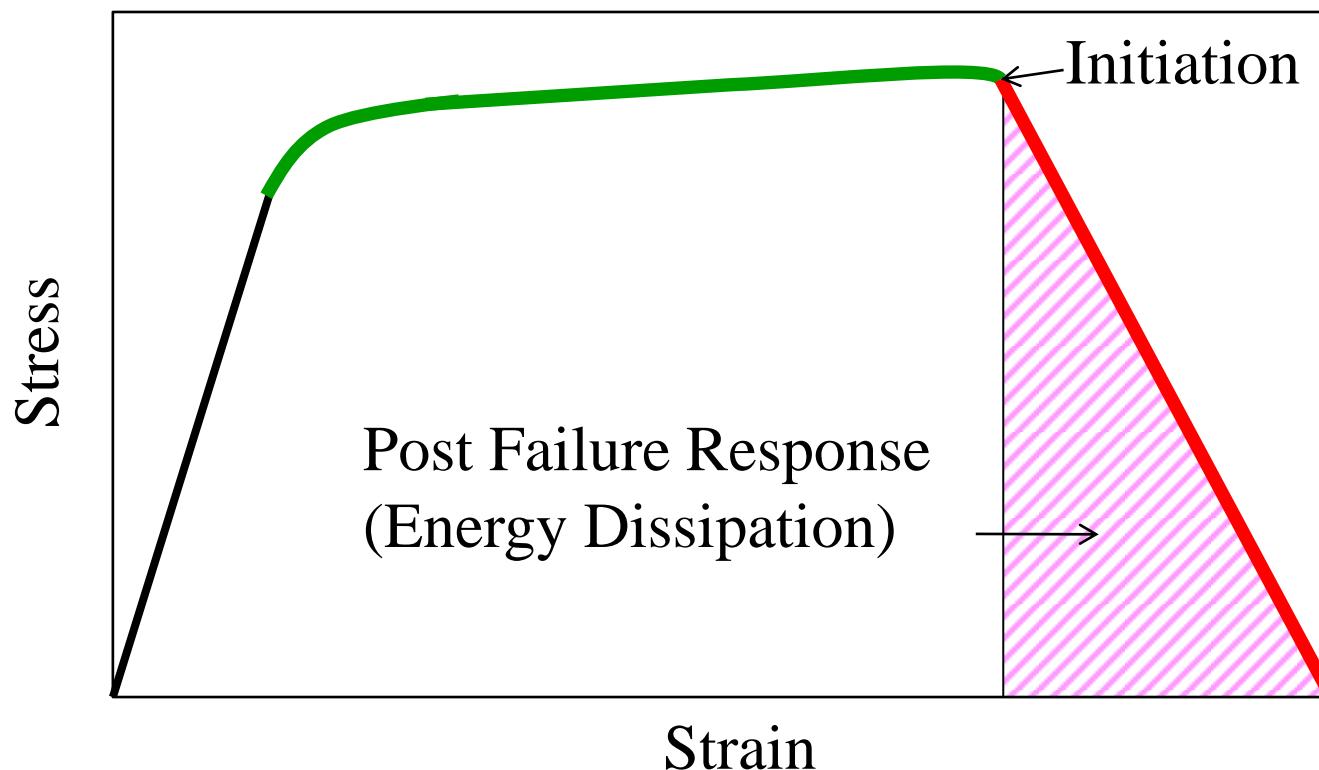
- Multi-Linear Elastic-Plastic Failure (MLEPF) Material Model Basics
- Tearing Parameter an Initiation Criteria
- Critical Crack Opening Strain (CCOS)
- Motivating Analysis Results
- Setting CCOS to Eliminate Mesh Dependence
- Application to the Motivating Analysis
- 3 Hole Test Results
- Questions for Future Work
- Conclusions



# MLEPF Material Model Basics

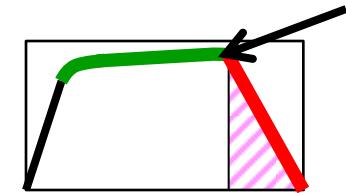
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Piece-wise Linear Elastic-Plastic Constitutive Model



# Tearing Parameter an Initiation Criteria

$$TP = \int_0^{\varepsilon_f} \left\langle \frac{2\sigma_T}{3(\sigma_T - \sigma_m)} \right\rangle^N d\varepsilon_p$$



$\sigma_T$  = maximum principal stress

$\varepsilon_p$  = equivalent plastic strain

$\sigma_m$  = mean or hydrostatic stress

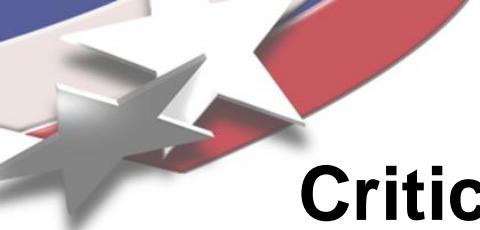
$\varepsilon_f$  = plastic strain at failure

$N$  = power fit

$\langle \rangle$  Macaulay Brackets, accumulates if quantity is positive

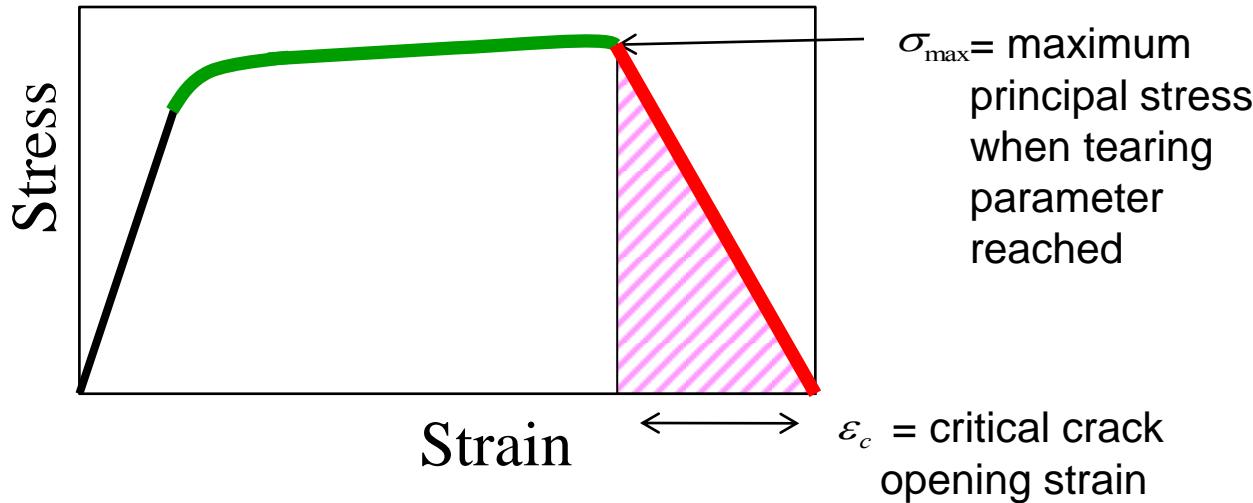
- Failure Criterion is Local Integral of Stress State over Plastic Strain<sup>1</sup>
- The critical value of the tearing parameter can be determined from a tensile test.

<sup>1</sup> Brozzo, P., Deluca, B. And Rendina, R., *A New Method for the Prediction of the Formability Limits of Metal Sheets, Proceedings of the 7th Biennial Congress of International Deep Drawing Research Group, 1972.*



# Critical Crack Opening Strain (CCOS)

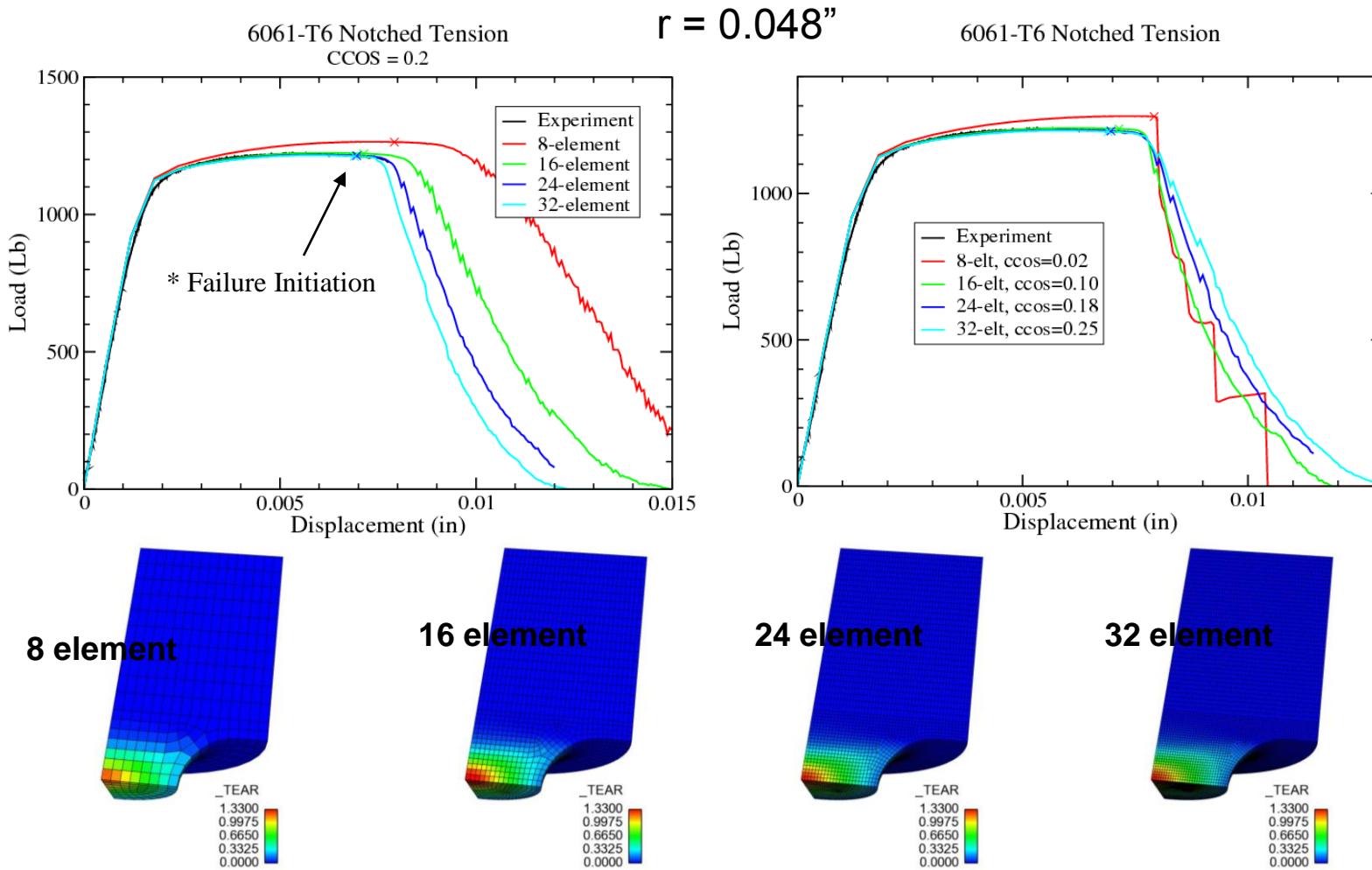
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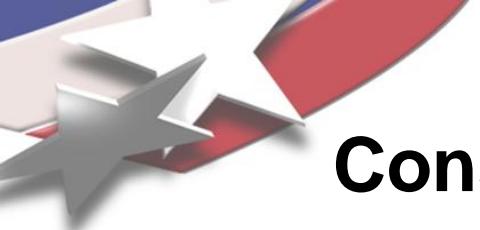
- Critical crack opening strain set with material properties.
- Failure direction set with corresponding eigenvector to maximum principal stress at onset of failure.
- Decay behaves like cohesive surface model.
- Only decayed when strained in failure direction.
- Unloads along elastic modulus.

# Motivating Analysis Results

## Mesh dependence and Critical Crack Opening Strain (CCOS)



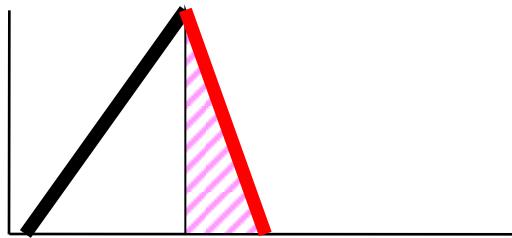
Analysis original performed by Nicole L. Breivik.



# Constant Decay Energy for Eliminating Mesh Size Dependence ?

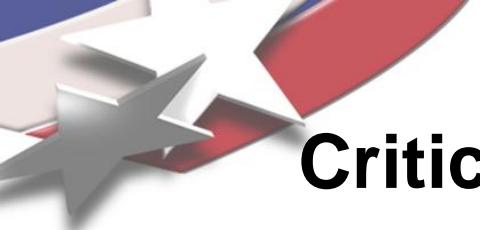
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- Mesh size dependence shown to be eliminated in smeared crack approach where decay set with constant energy per unit surface.<sup>2</sup>
- Caveats: No plasticity. Decay energy is the only unrecoverable energy.

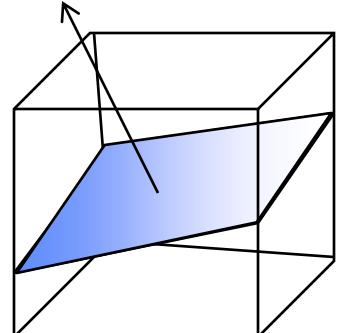
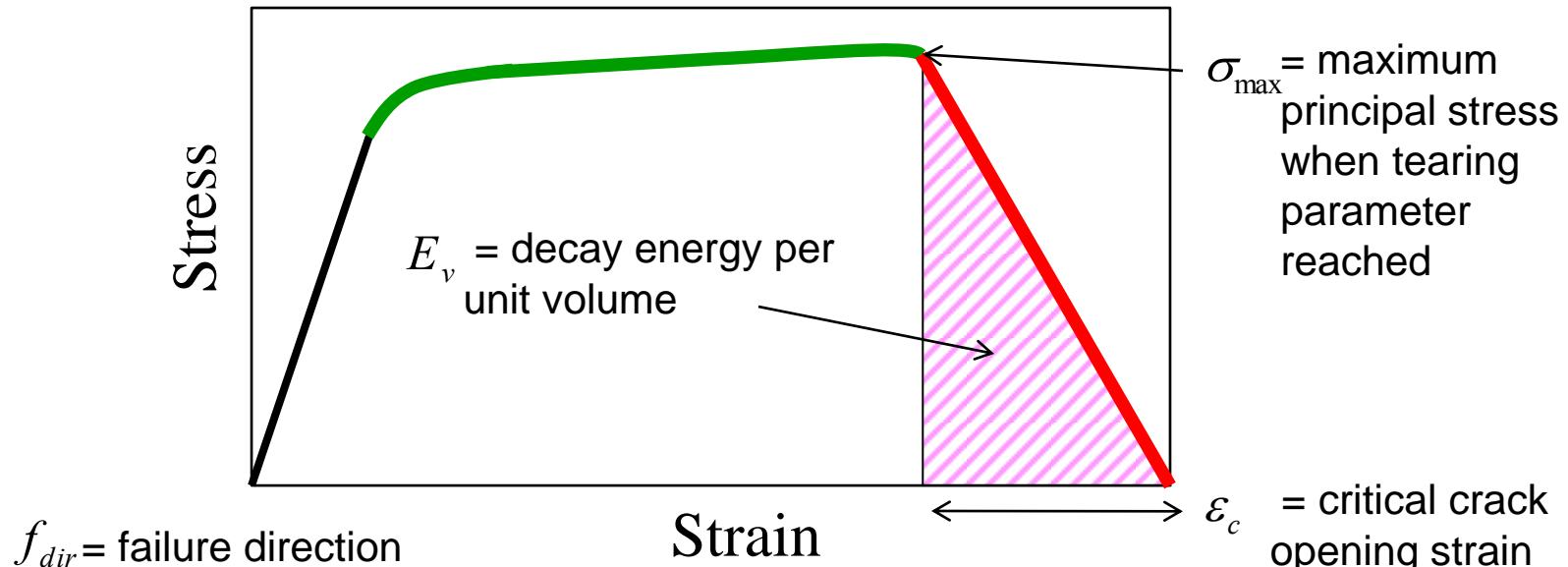


- Will this work in the presence of plasticity and with the tearing parameter criterion?

<sup>2</sup> Bazant, Zednek P. and Oh, B. H., *Crack Band Theory for Fracture of Concrete, Materials and Structures 16*, 1983.



# Setting Critical Crack Opening Strain (CCOS) for Constant Decay Energy

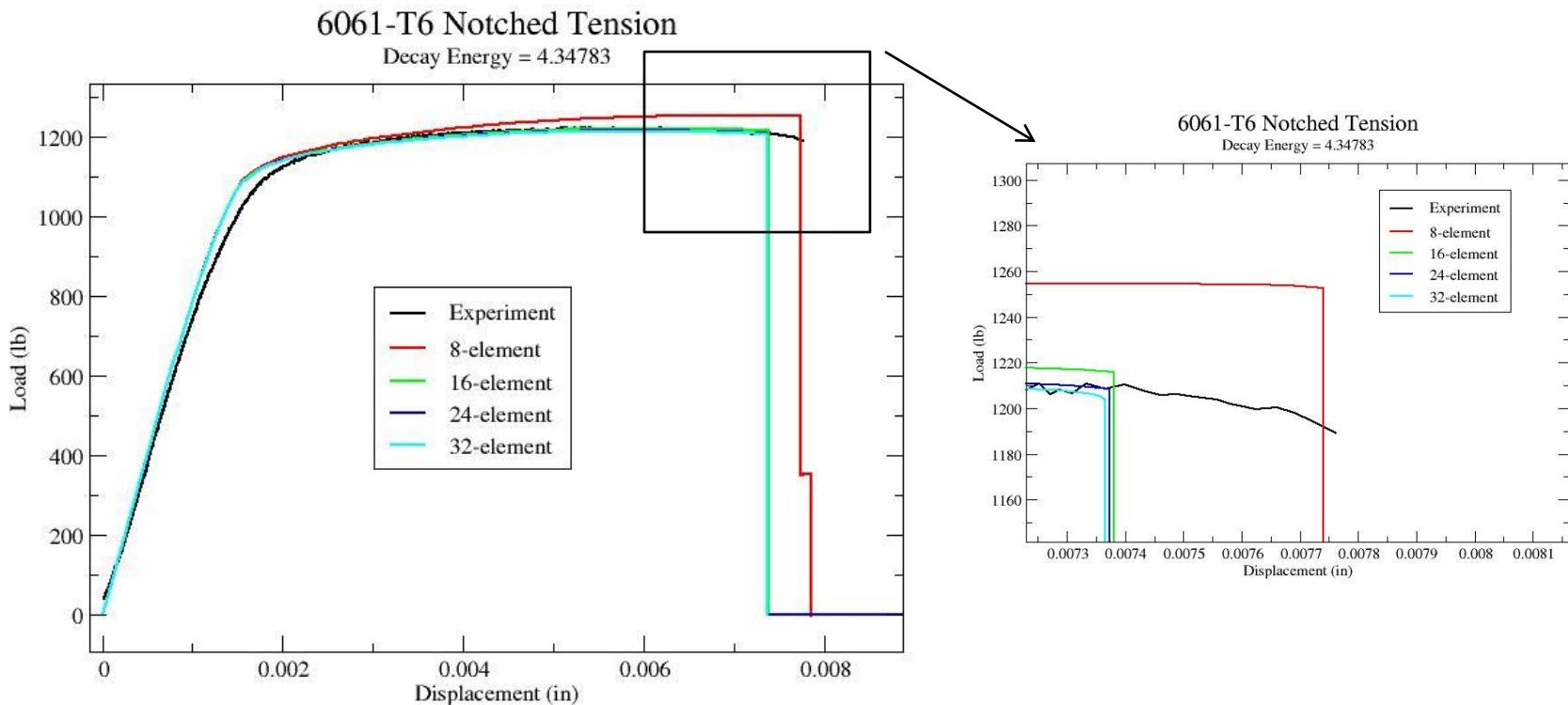


$$E_v = \frac{1}{2} \sigma_{\max} \varepsilon_c$$

$$E_A = \text{energy per unit area}$$

$$\varepsilon_c = \frac{2E_A A}{V \sigma_{\max}}$$

# Constant Decay Energy Applied to Motivating Analysis



Decay portion not a major factor in this test.  
Tearing Parameter and no decay may have been sufficient.

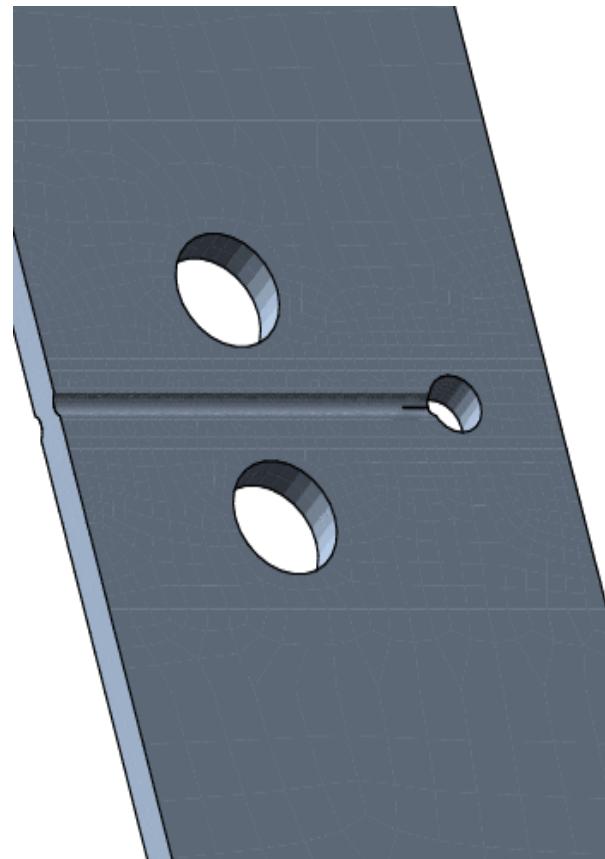


# Newman's<sup>5</sup> 3-Hole Tension Specimen

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- Tension dominated – starter hole/notch precludes cracking to near free surface, minimizing bending.
- Two large holes provide a “shadow” for the tensile stress providing for extensive stable crack growth.
- Shallow side grooves provide for direction control of crack and reduce required load.

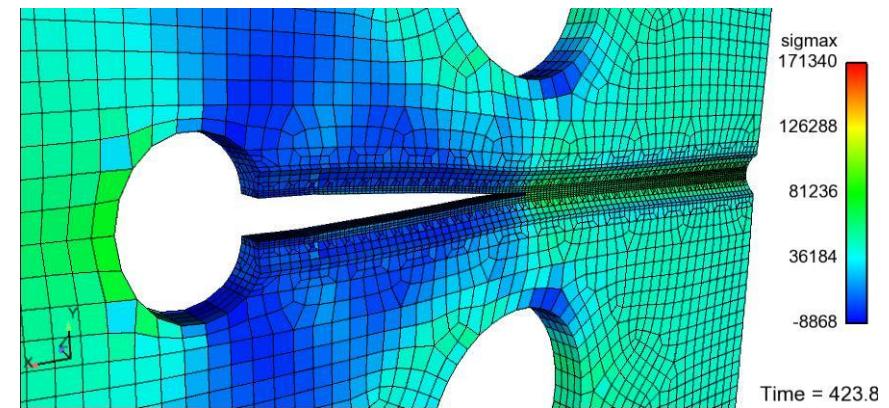
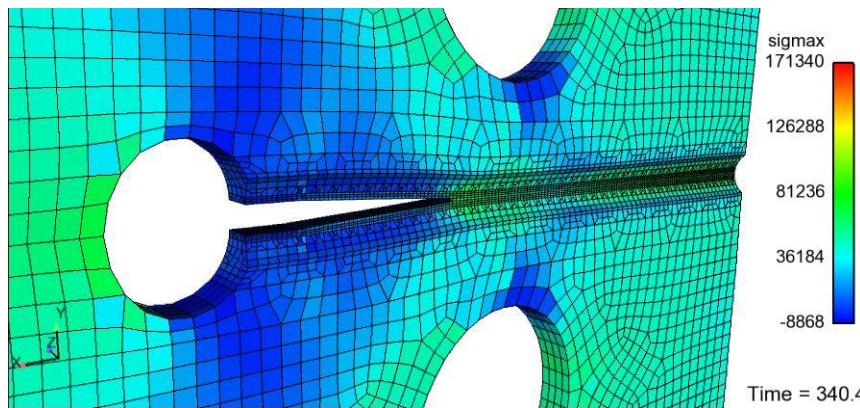
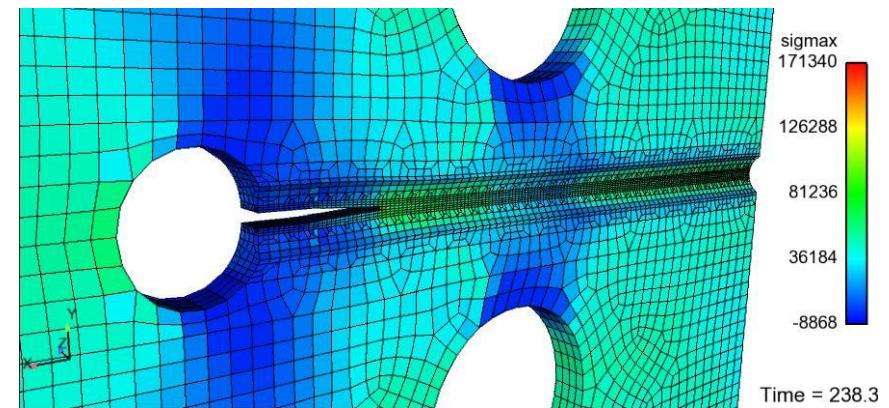
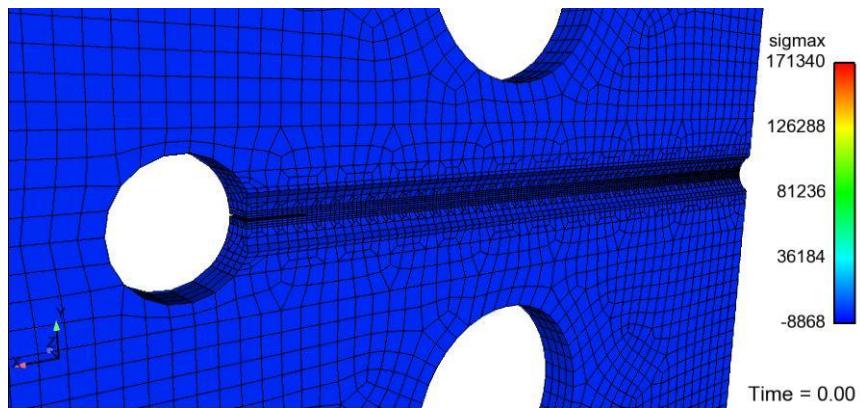
<sup>5</sup>Newman and Loss, 1985





# Stable Crack Growth

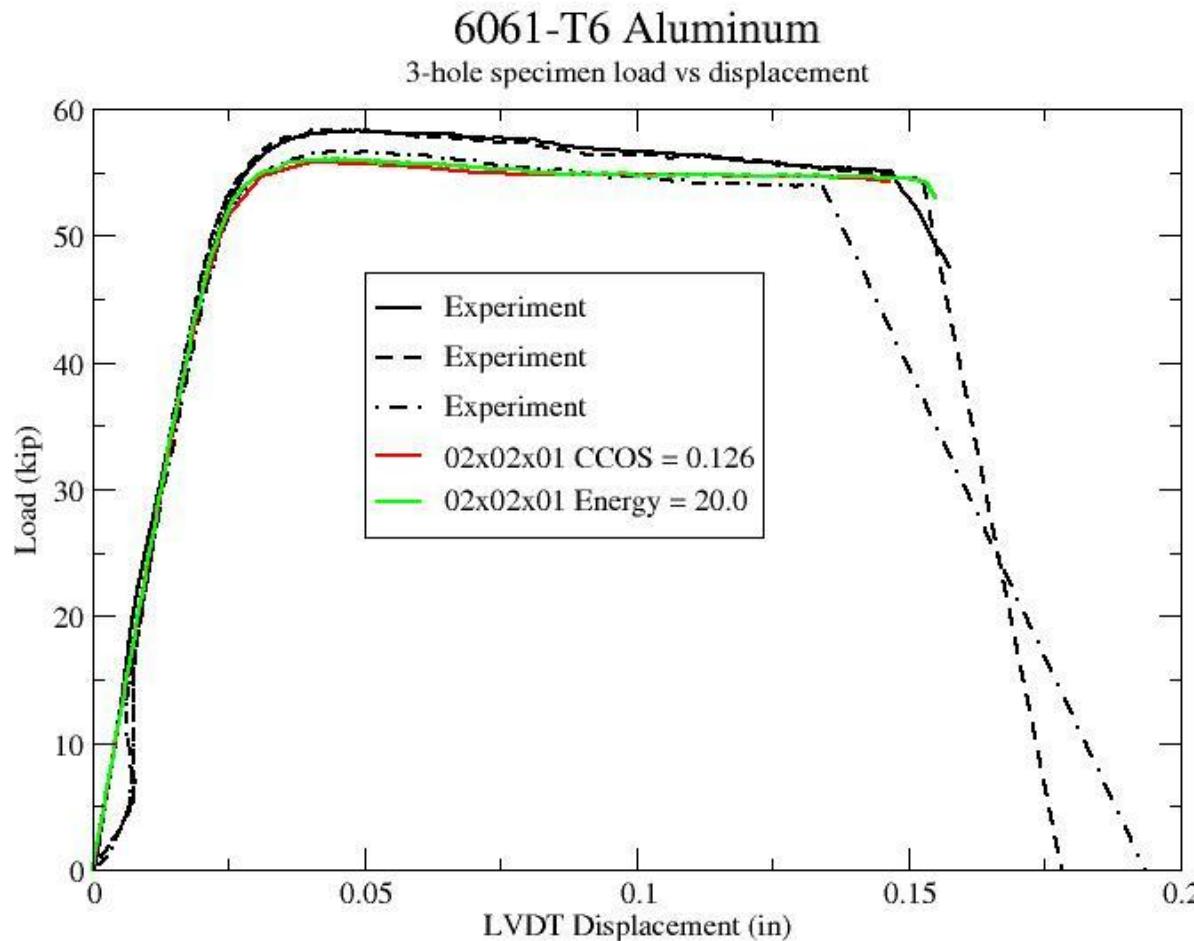
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# 3-Hole Tension Specimen Load vs. Displacement

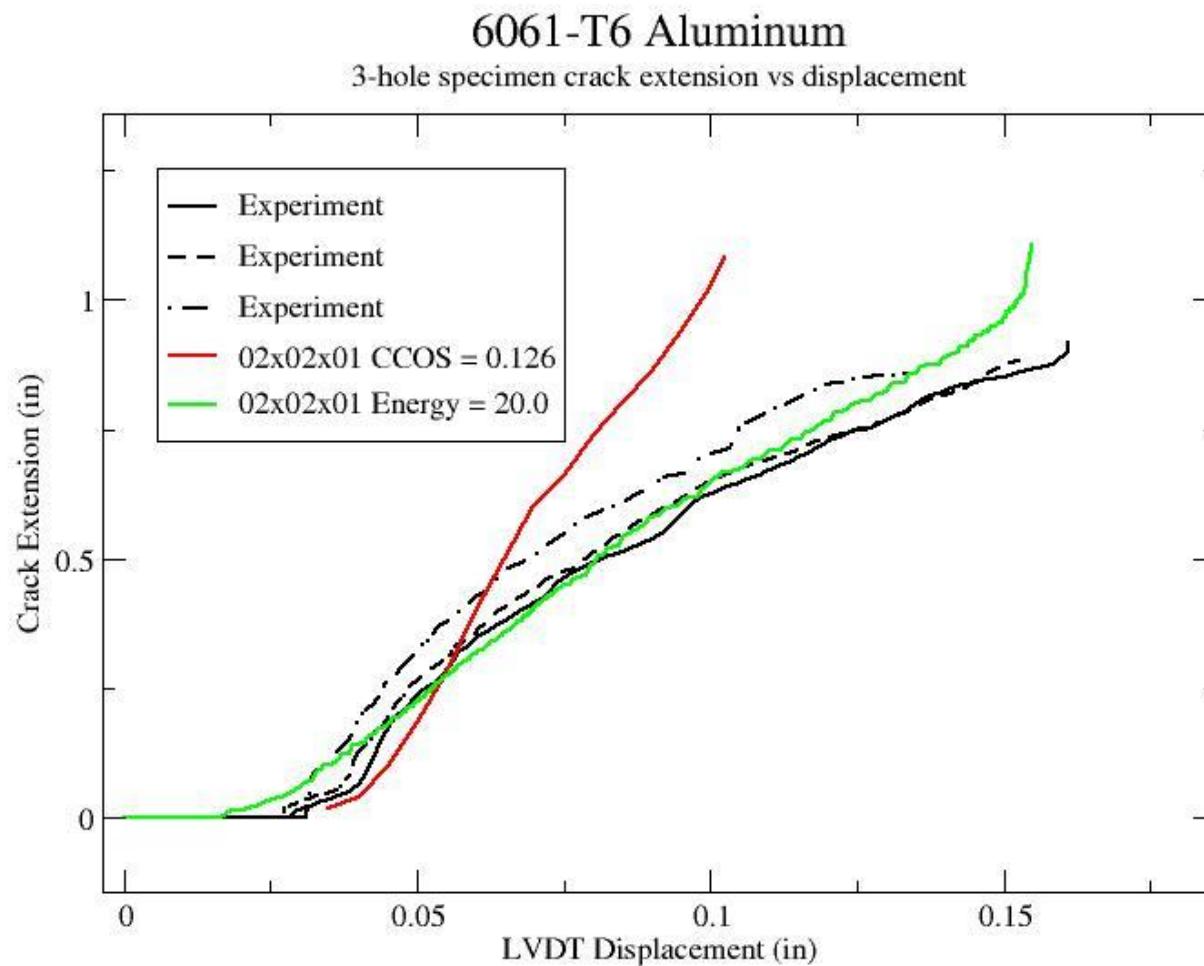
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# 3-Hole Tension Specimen Crack Extension vs. Displacement

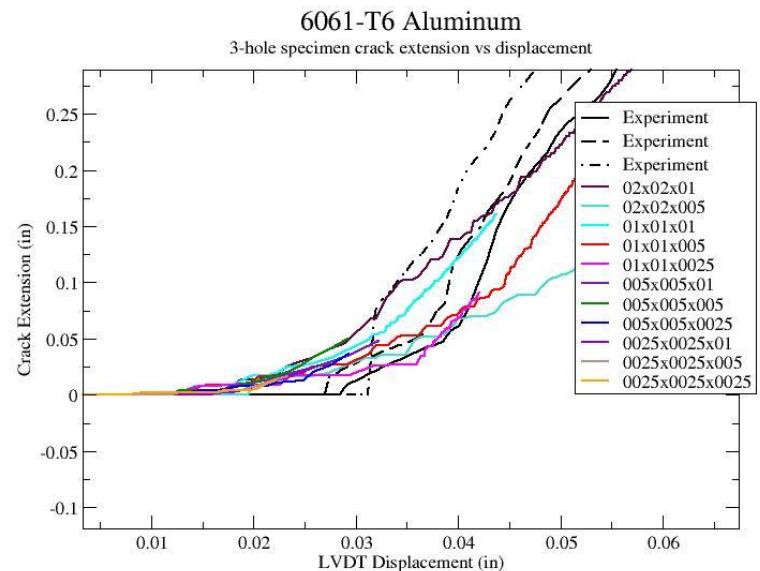
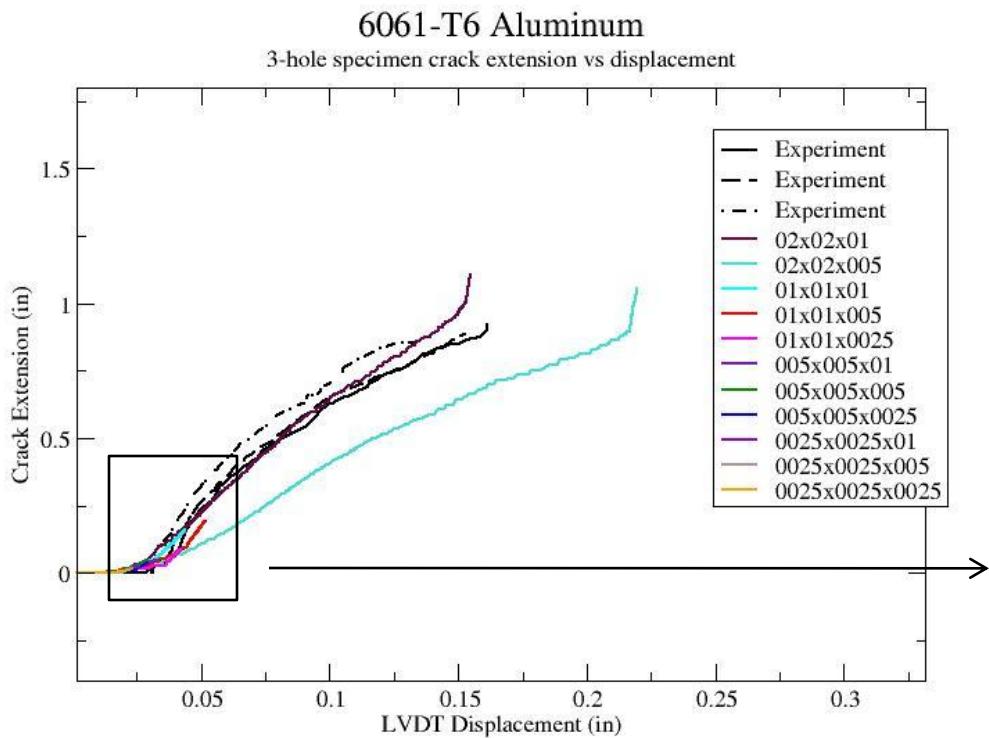
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# 3-Hole Tension Specimen

## Crack Extension vs. Displacement

### Multiple Mesh Sizes

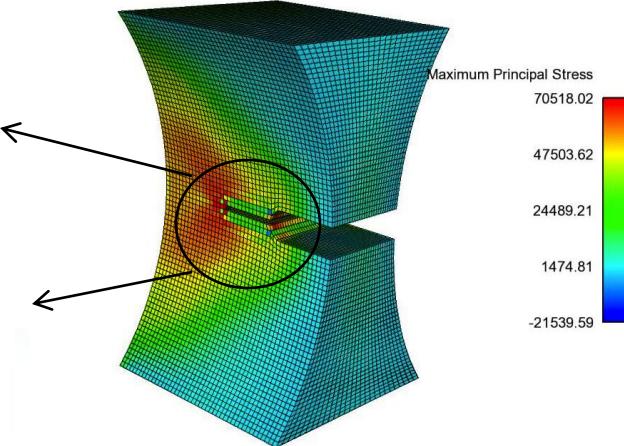
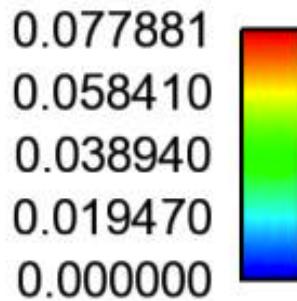
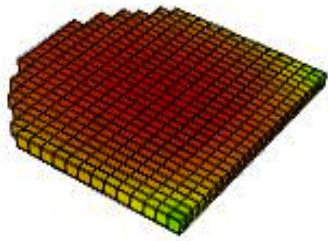




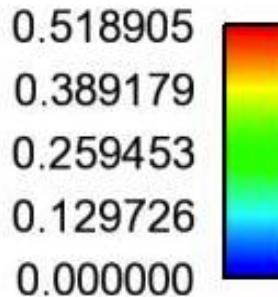
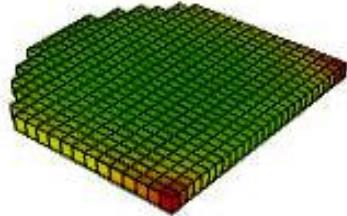
# Competing Energies: Possible Reason Mesh Dependence Still Exist in Stable Crack Growth

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Critical Crack Opening Strain



Equivalent Plastic Strain



Un-deformed Cracked Elements



## Questions for Future Exploration

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- Should a crack band width be included in determining the critical crack opening strain (CCOS)?
- Should we consider the amount of plastic energy dissipated when setting the CCOS with the decay energy?
- Is this convergent in a problem that has stable crack growth?
- How do we determine the decay energy?



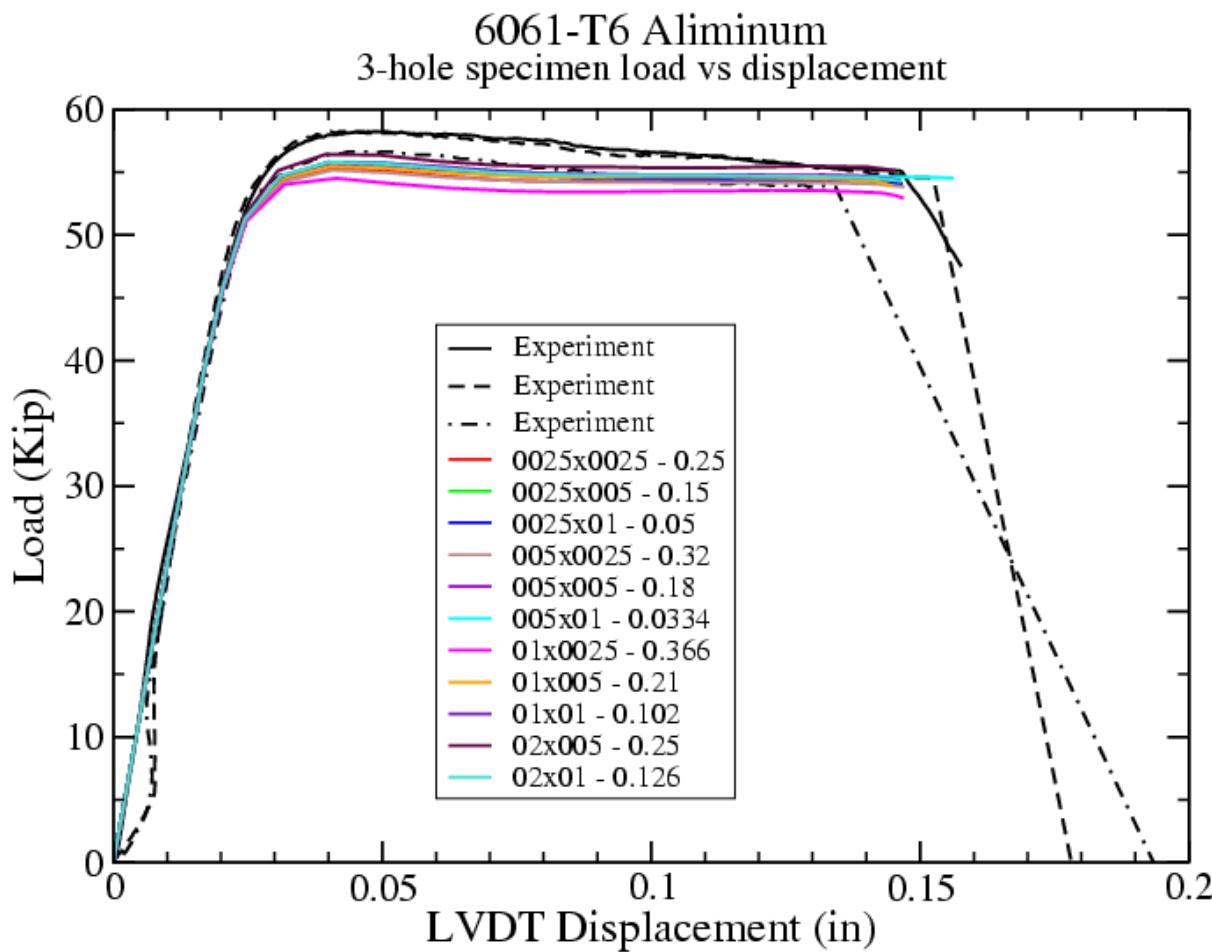
## Conclusions

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- We have a simple approach to crack initiation and propagation that shows promise
- We are making progress towards mesh independence
- A framework is in place for exploration in setting the CCOS with knowledge of the element dimensions
- We have existing experimental data to validate the model against

# Backup 1

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# Backup 2

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