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Additional Examination of An Analytically Enriched XFEM for Cohesive Crack Modeling

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Meshfree and Generalized/Extended Finite Element Methods

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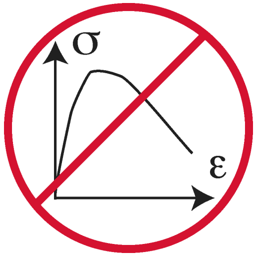
Preview

- Introduction
- PUFEM/XFEM Displacement Field Enrichment
- Past Results
  - Model Mode I Problem
  - Mixed Mode Problems
- Recent Results
  - Orthotropic material – errors in the fields
  - Attempts to solve a "model glass seal problem"
- Observations and Conclusions

Introduction

Objective: A "valid" means of modeling material localization in finite element analyses.



Key Goals of Recent Work:

- Analytical enrichment for a cohesive crack that can capture response gradients "not represented by the mesh"
- Examination of the bounds of effective application
- Implementation in a Sandia code (Aria)

PUFEM Displacement Field Enrichment

- Standard FEM
- PUFEM/XFEM

Global displacement approximations

Standard FEM:

$$\mathrm{u}(\mathrm{x}) = \sum_{i=1}^{N_\Phi} \Phi_i(\mathrm{x})\mathrm{u}_i$$

PUFEM/XFEM:

$$\mathrm{u}(\mathrm{x}) = \sum_{i=1}^{N_\Phi} \Phi_i(\mathrm{x})\mathrm{u}_i + \sum_{j=1}^{N_\Lambda} \sum_{i=1}^{N_{\Phi^*}} \Lambda_j(\mathrm{x})\Phi_i^*(\mathrm{x})\alpha_{ij}$$

Element displacement approximations

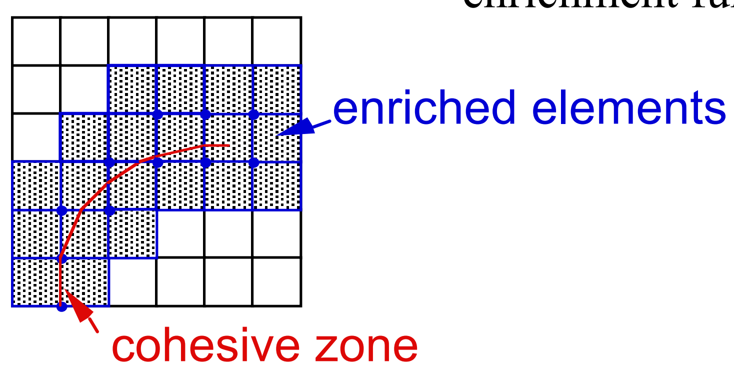
Standard FEM:

$$\mathrm{u}(\mathrm{x}) = \sum_{i=1}^{N_N} \mathrm{N}_i(\mathrm{x})\mathrm{u}_i$$

PUFEM/XFEM:

$$\mathrm{u}(\mathrm{x}) = \sum_{i=1}^{N_N} \mathrm{N}_i(\mathrm{x})\mathrm{u}_i + \sum_{j=1}^{N_\Lambda} \sum_{i=1}^{N_{N^*}} \Lambda_j(\mathrm{x})\mathrm{N}_i^*(\mathrm{x})\alpha_{ij}$$

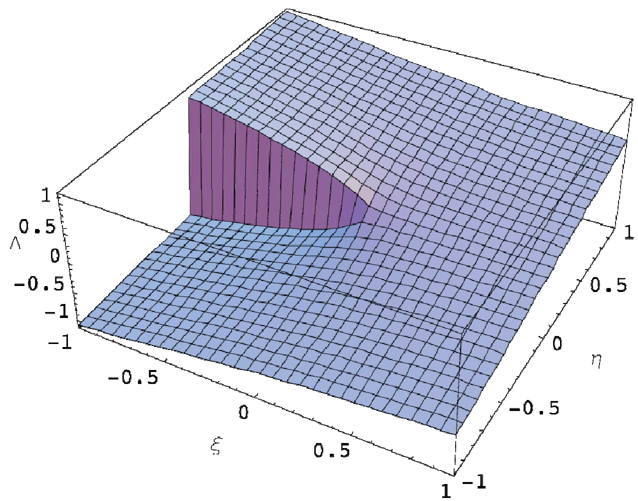
enrichment functions

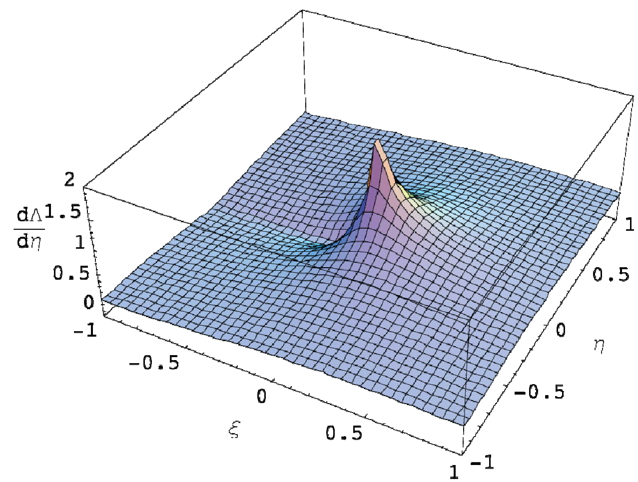


Earliest Enrichment Functions for Fracture

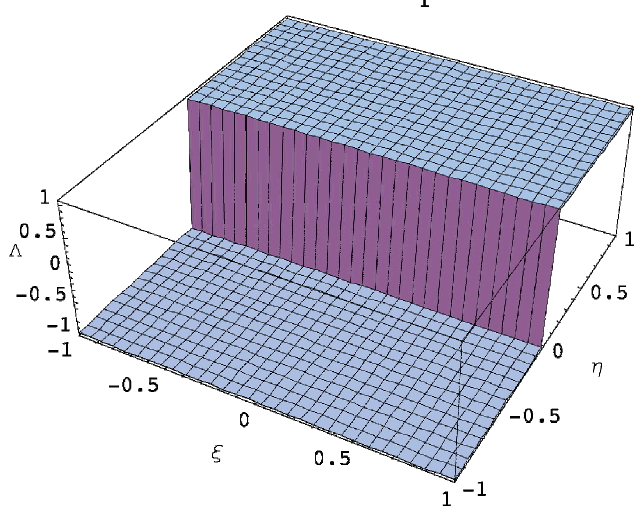
u_2 | du_2/dx_2

LEFM





Heaviside



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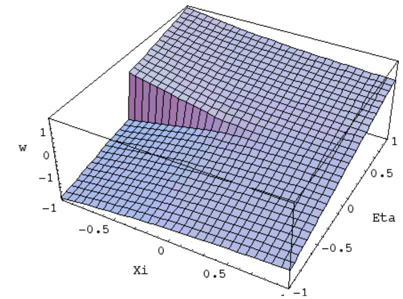
“My Path to Enrichment”

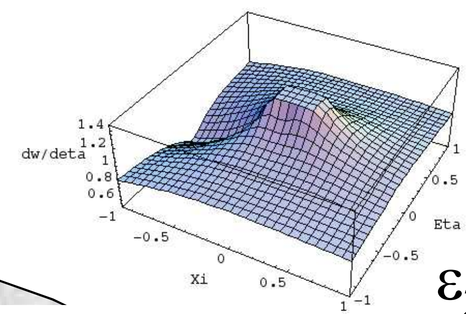
“I have not failed. I’ve found 10,000 ways that won’t work.” – Thomas Edison

Formulated simple series that incorporated a discontinuity.



u_2

Formulated simple functions that had key features of accurate numerical results.



ε_{22}

Analytically derived enrichment functions based upon the Muskhelishvili formalism.

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Enrichment Functions: An Analytical Source

Muskhelishvili formalism (1953)

Hong & Kim (2003) obtained a series solution to the inverse problem

Zhang & Deng (2007) obtained "asymptotic solutions"

– both assumed linear elastic isotropic material (except for cohesive zone)

Additional analysis was used to:

verify the proposed solutions

extend them for field variables required by the XFEM

Displacements

$$u_1 + iu_2 = \frac{1}{2\mu}\left\{\kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)}\right\}$$

where φ and ψ are analytic functions, and $z = x+iy$.

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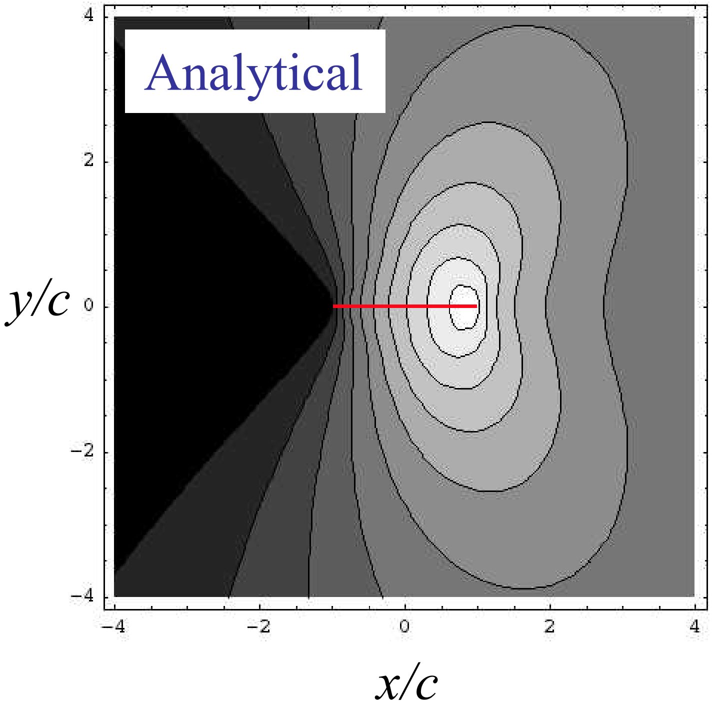
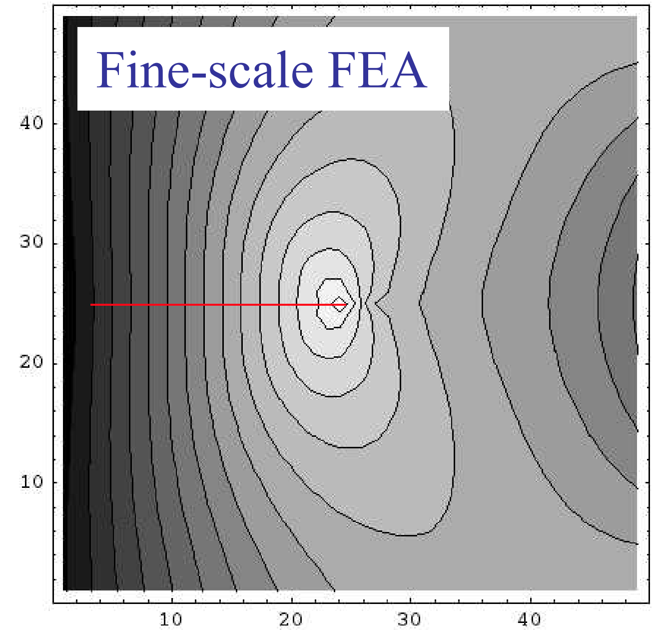
Enrichment Functions: An Analytical Source

Qualitative comparison of σ_{22} with fine-scale FEA

- Analytical ~ First terms in series for Hong & Kim solution
- "Fine-scale" FEA ~ results for finely meshed FEA with interface el.

Note: problems differ and CZ sizes are not to the same scale.



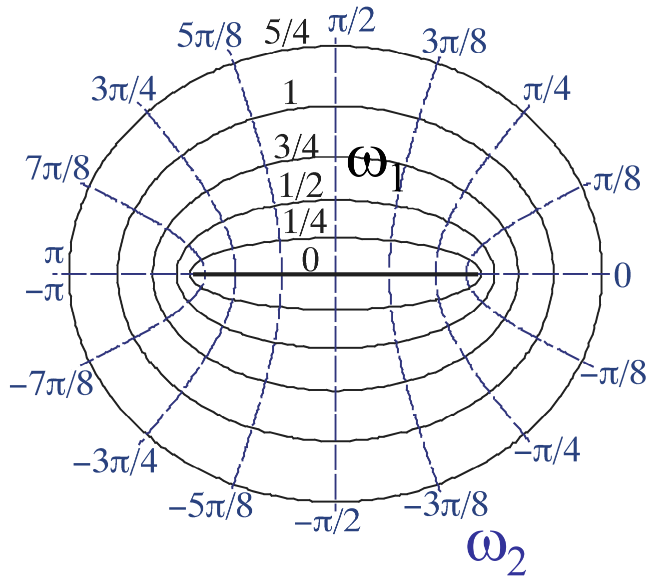


Cohesive zone length = $2c$

Enrichment Functions: An Analytical Source

Zhang & Deng (2007) solve the problems in terms of elliptic coordinates (ω)

$$z = c\cosh(\omega)$$



Symbolically the inverse map is give by

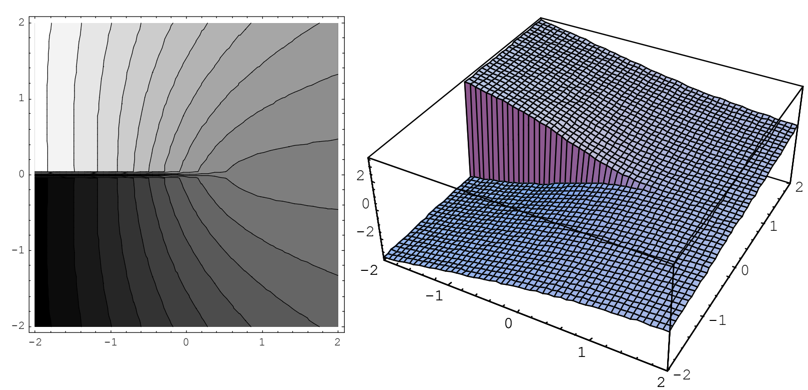
$$\omega = \cosh^{-1}(z/c)$$

complex analysis → useful forms.

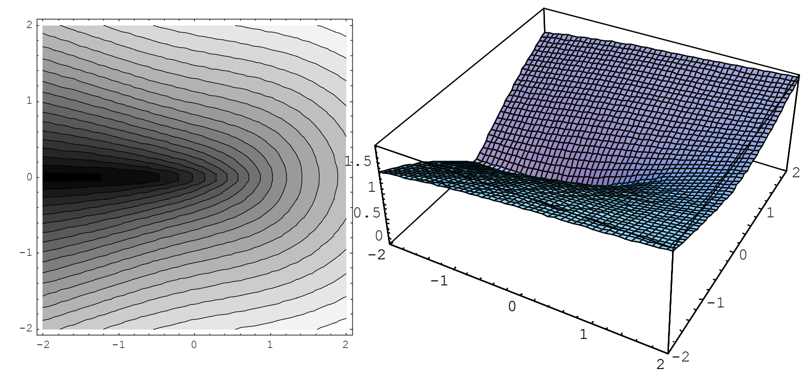
They (1) adopt a Westergard stress function,
→ one unknown analytic function,
(2) express this in a series, and
(3) define one term of the series to be the asymptotic solution.

Mode-I Enrichment Functions

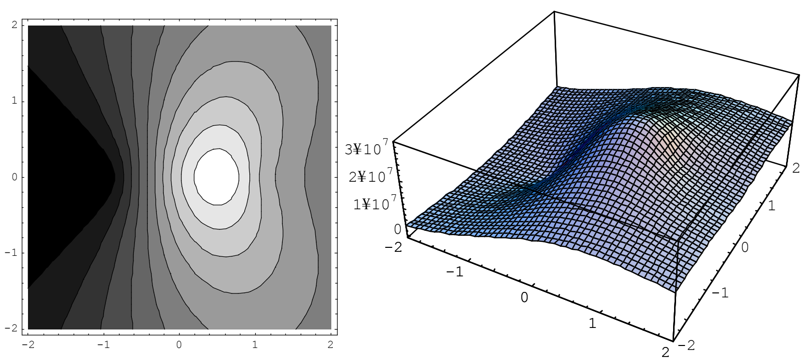
- Based upon the asymptotic solutions of Zhang & Deng



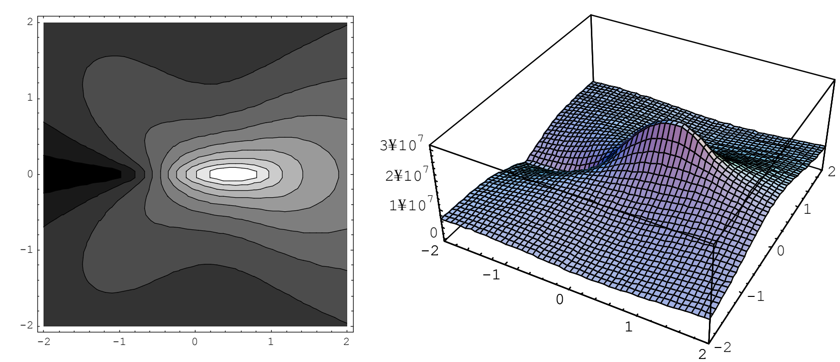
u_2



u_1



σ_{22}

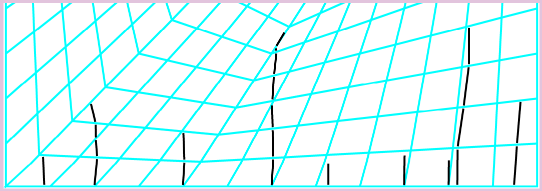


σ_{11}

Solver Approach

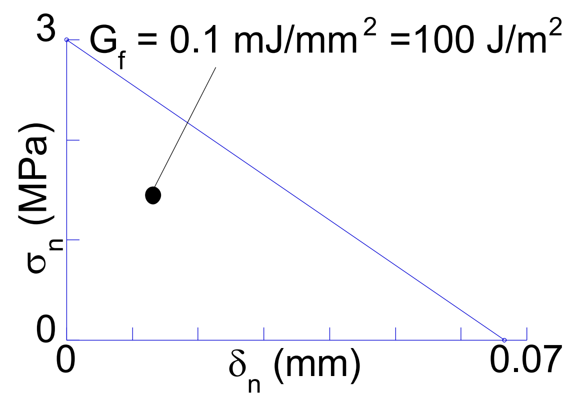
Assign equation numbers;   Determine storage for **K**;

Repeat (* time increment loop *)
| **Repeat (* outer level solver loop -- aka localization loop *)**
| | Update **K** & R
| | Reset penalty number to large value when entering a new element, else 0
| | …
| | **Repeat (* penalty reduction loop *)**
| | | Relax the penalty number
| | | Reset line search
| | | **Repeat (* nonlinear iteration loop *)**
| | | | Factor **K**
| | | | Forward eliminate & back substitute to obtain dUiter
| | | | **Repeat (* line search loop *)**
| | | | | Search line for dU_{iter}
| | | | | …
| | | | **Until ($||R||<R_{toler}$) OR ($||R||<||R_{old}||$)**
| | | | …
| | | **Until $||R||<R_{toler}$**
| | **Until penalty number is reduced to zero**
| | …
| **Until localization is complete**
| $U := U + dU_{step}$; $dU_{step} := 0$; $U_{old} := U$
| …
Until time stepping is complete

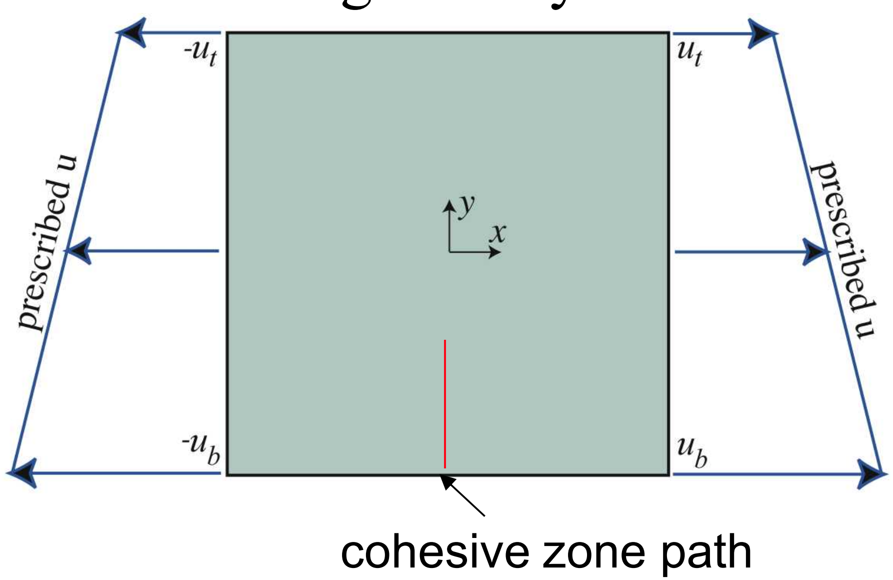


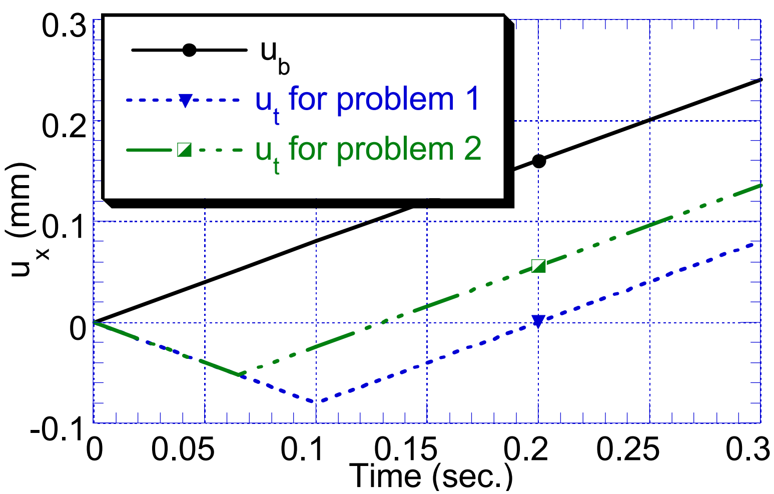
Initial Simple Test Problems

- Concrete test problems
  - relevant to HDBT
  - domain 1 m x 1 m
  - process-zone size ~ O(250 mm)
  - representative concrete tensile properties (except for simplified linear softening)
  - mode I quasistatic crack propagation



Problem geometry





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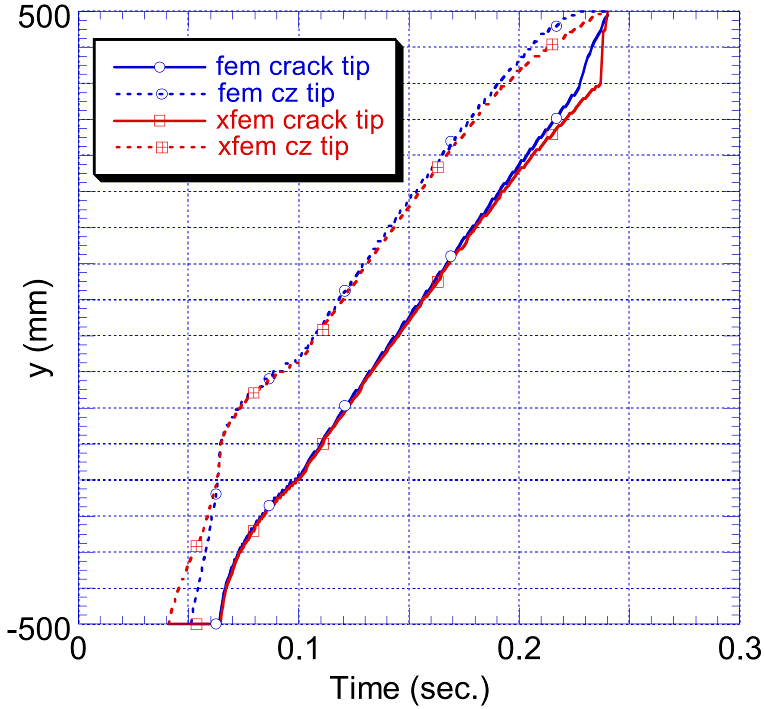
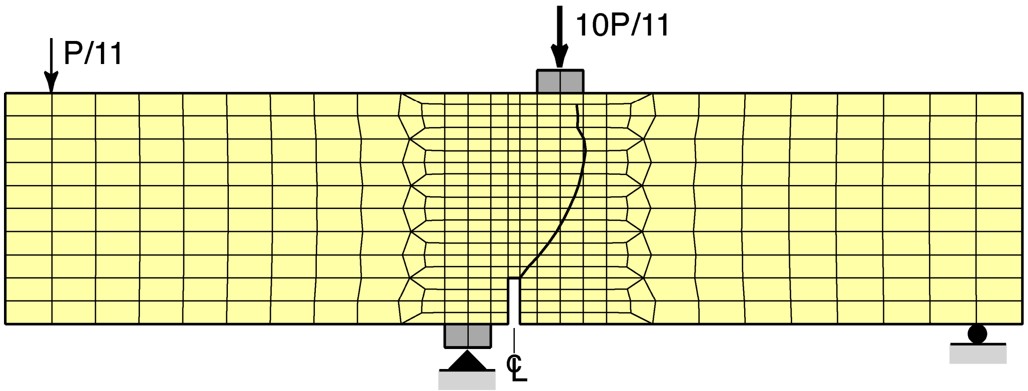
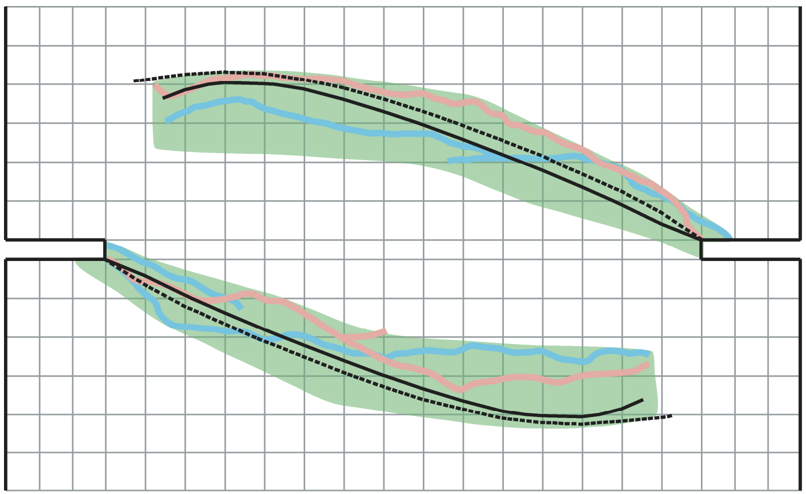
Past Results

- Quasi-brittle, quasi-static crack propagation







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Key Lingering Questions

- To what extent are the enrichment functions affecting the accuracy?

- What other classes of problems can these enrichment functions be beneficially applied to?

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Recent Results

- Field accuracy for an orthotropic material
  - Based on earlier model problem
- Fracture of glass seals
  - LMOS glass-fiber seal
  - Model problem for glass-metal seal

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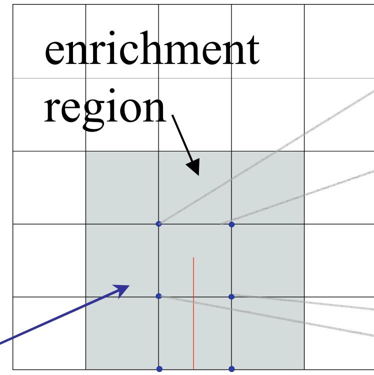
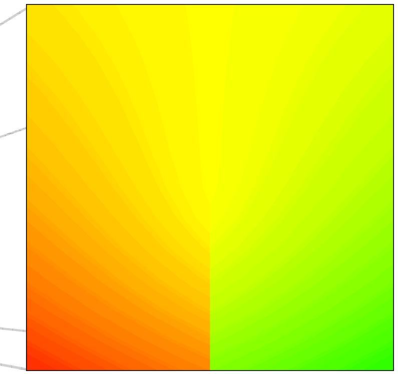
Examination of the Model Problem Field Results

Example response in the "tip-element"



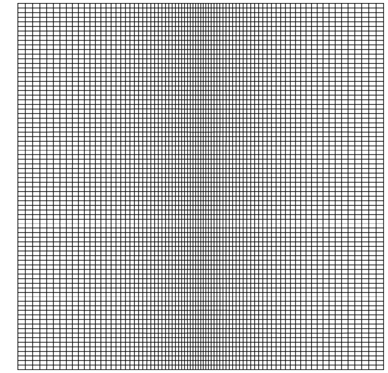
Surface plot view

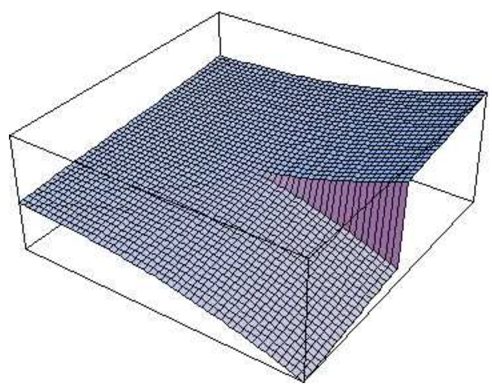
Fine-scale



u_x for tip-element

81x80





XFEM

u_x for tip-element

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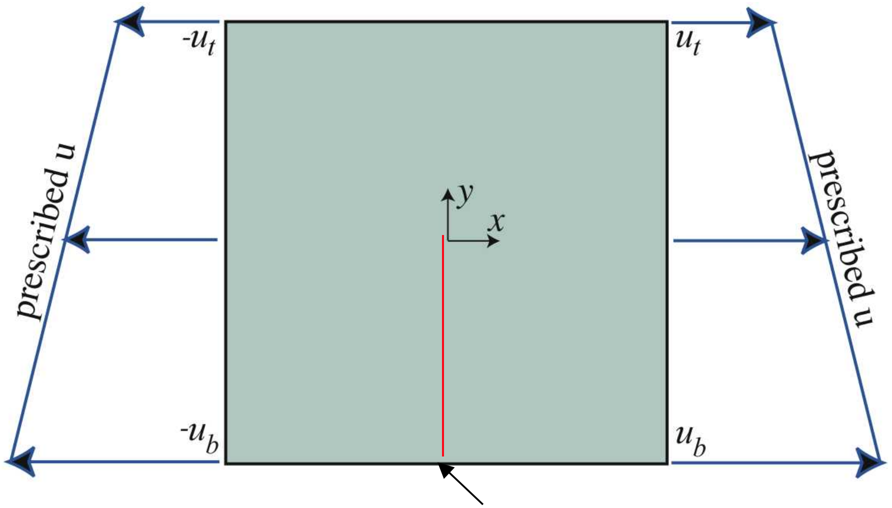
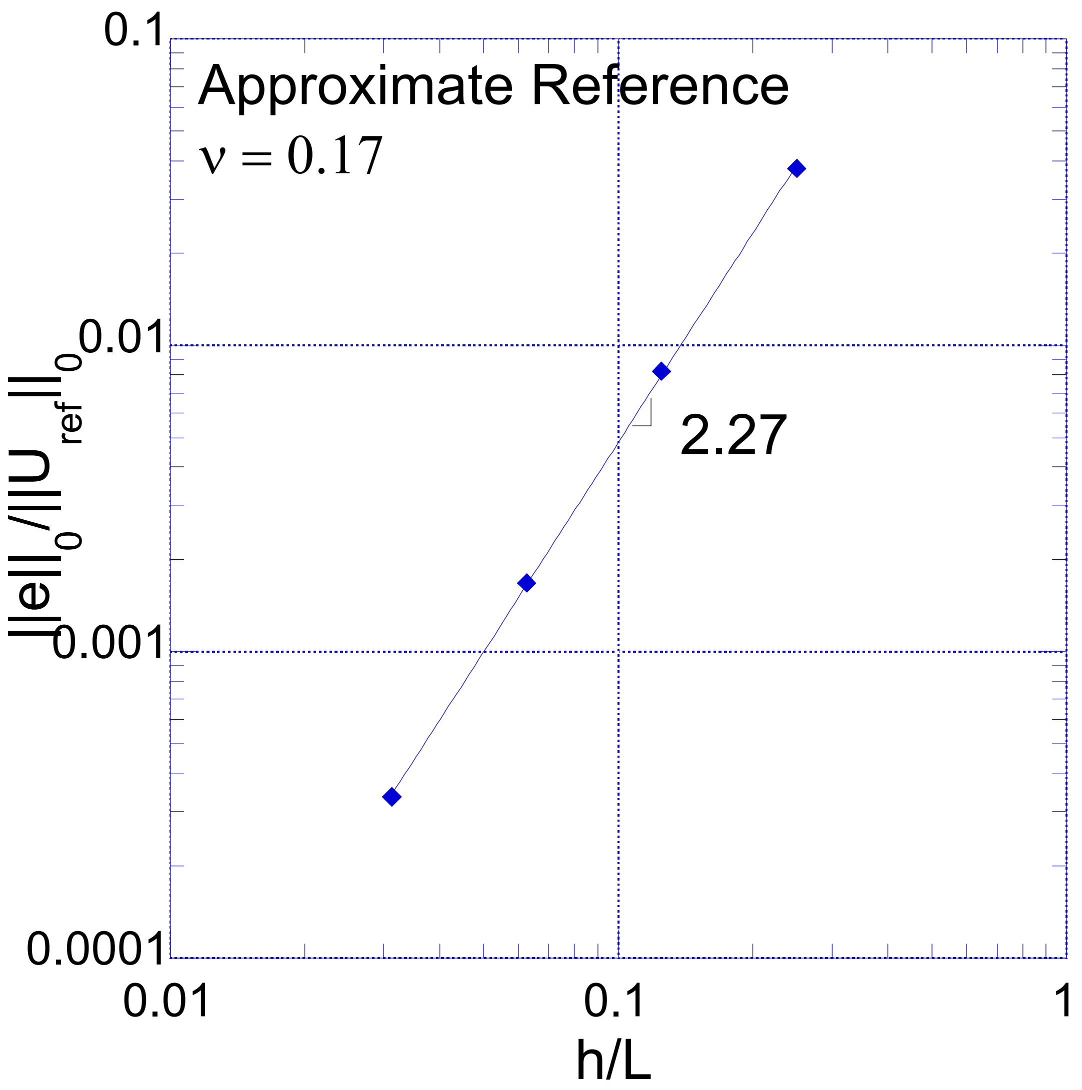
Model Test Problem with a Stationary Crack



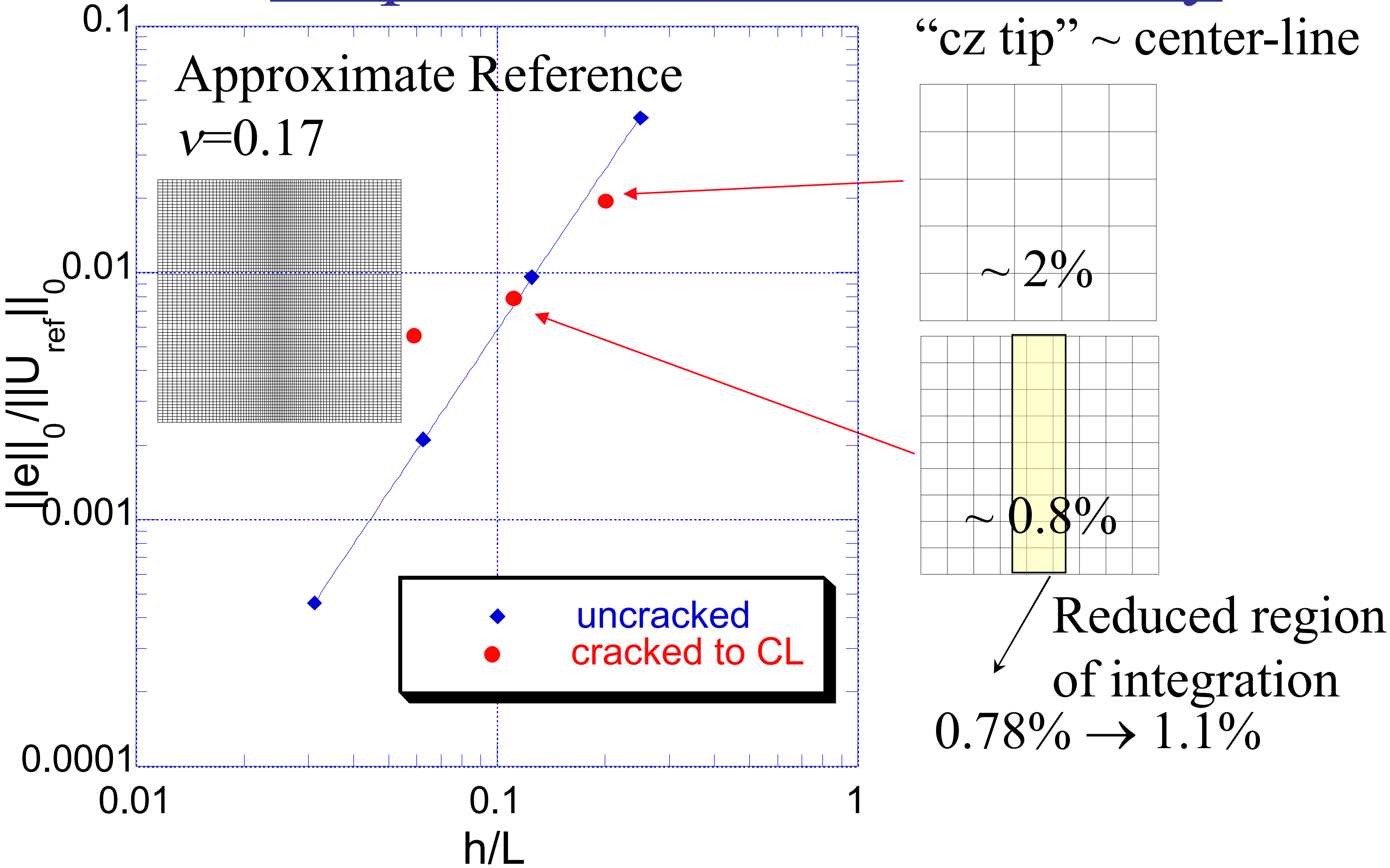
Initial tests excluded crack -- to establish limitations of approximate reference solution

- Exact reference solution (ν=0)
- Approximate reference solution (ν=0)
- Approximate reference solution (ν=0.17)

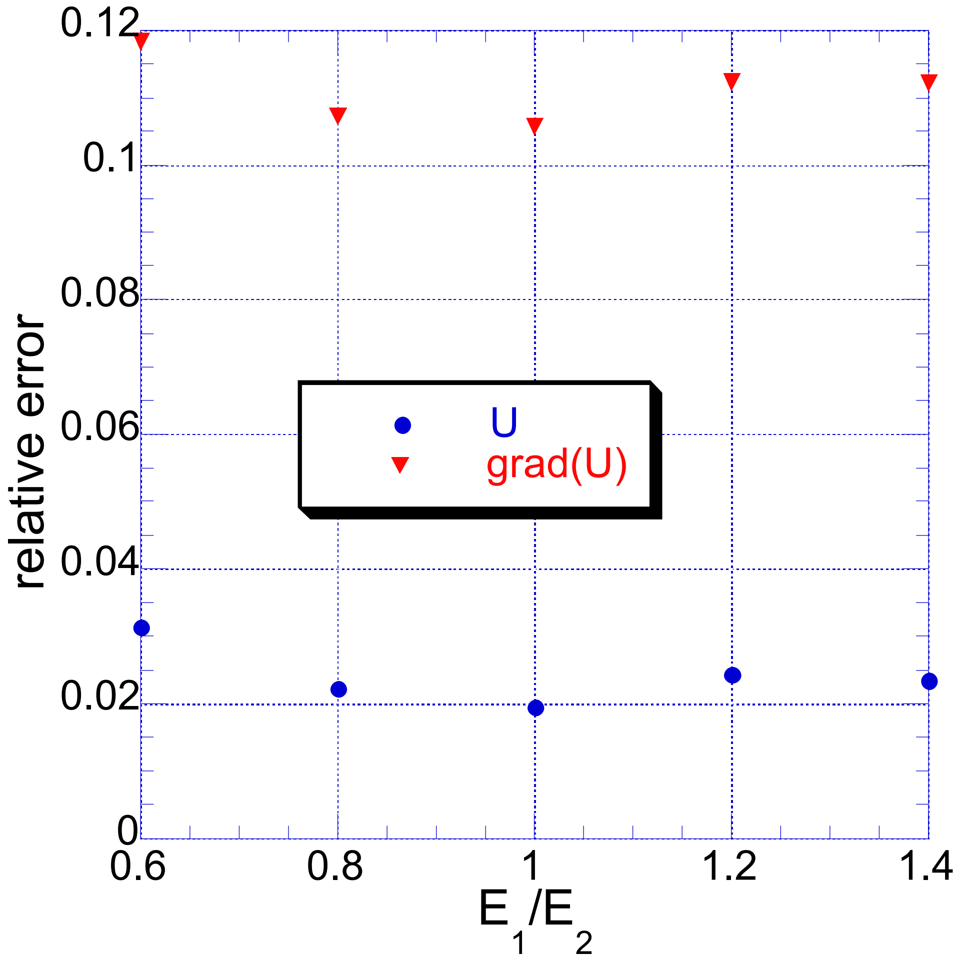
Displacement Field Accuracy



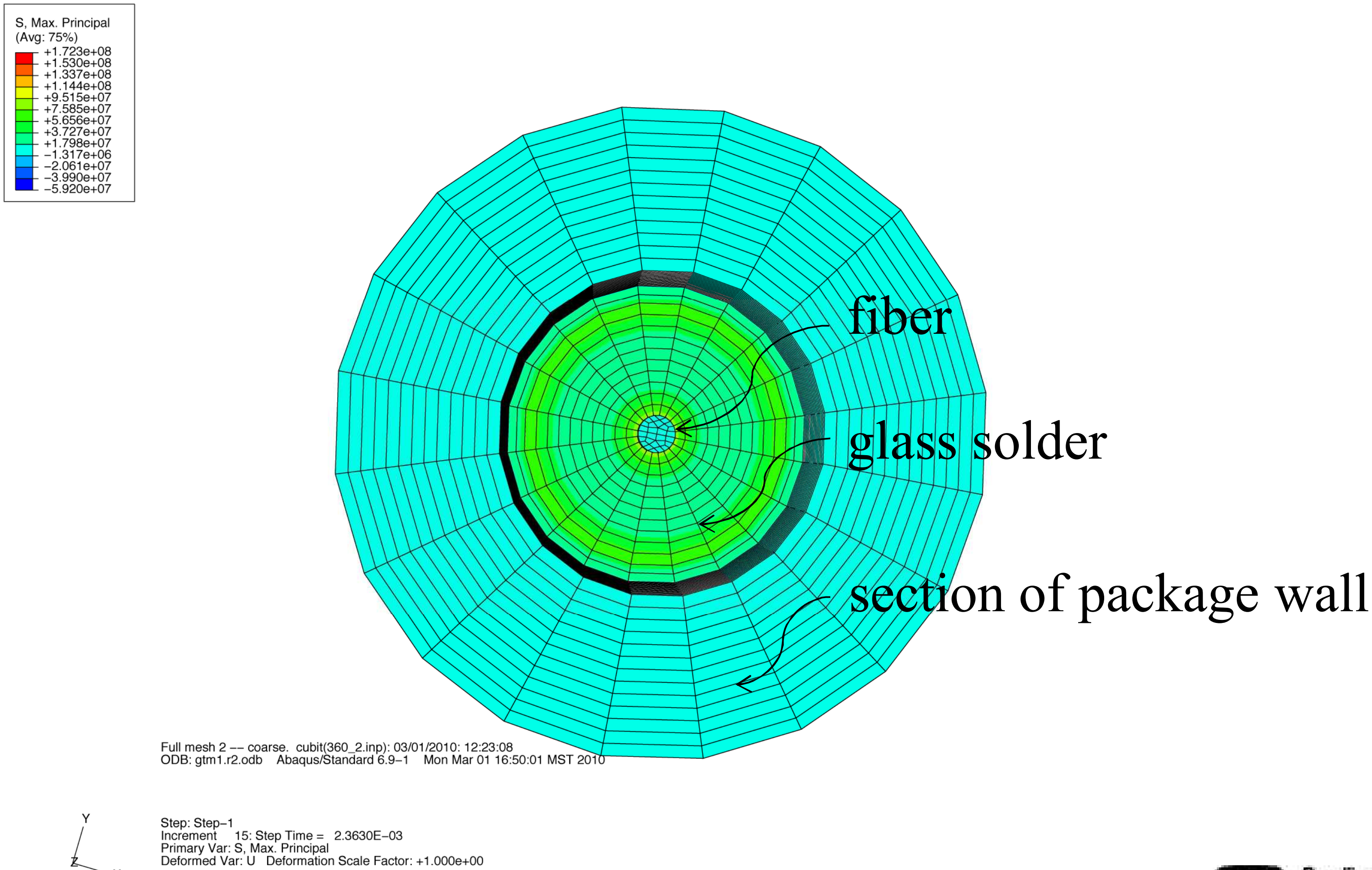
Displacement Field Accuracy



Relative Errors for Orthotropic Elastic Materials



LMOS Fiber Seal Problem



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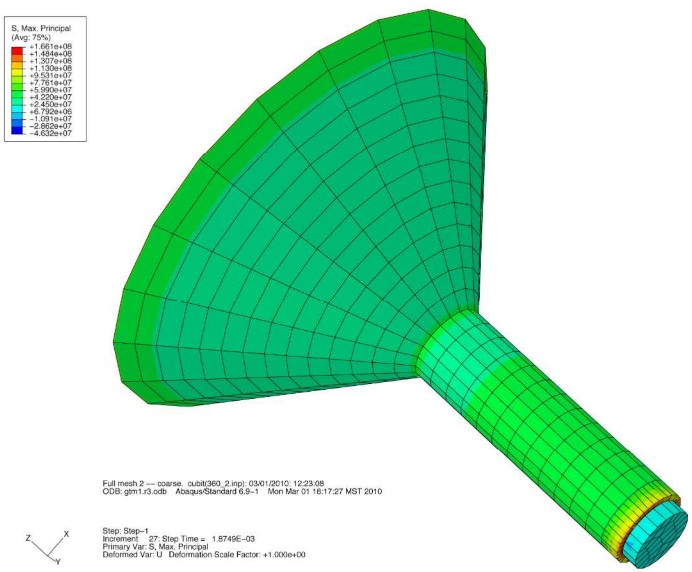
Numerical Formulation Issues

Abaqus results which do not “compete cracks”

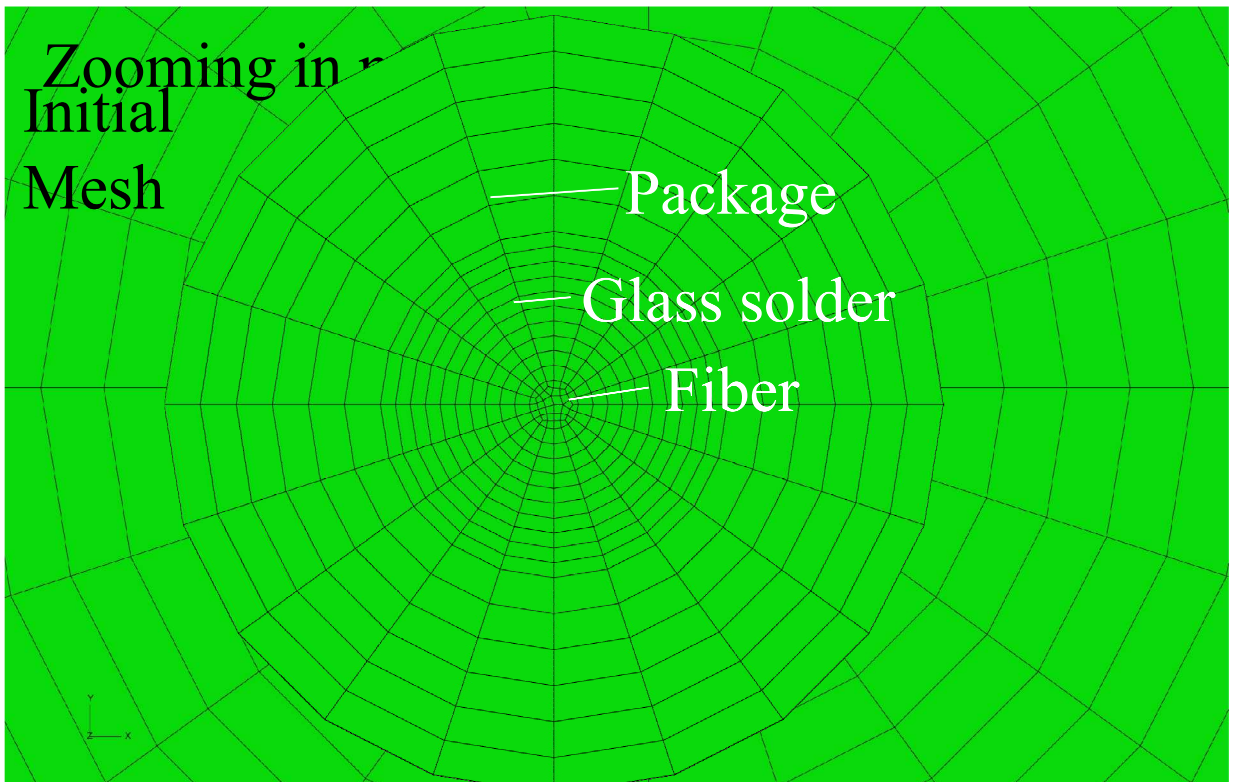
Fiber optic seal within a package -- 3D



Numerical Formulation Issues

Abaqus results which do not "compete cracks"

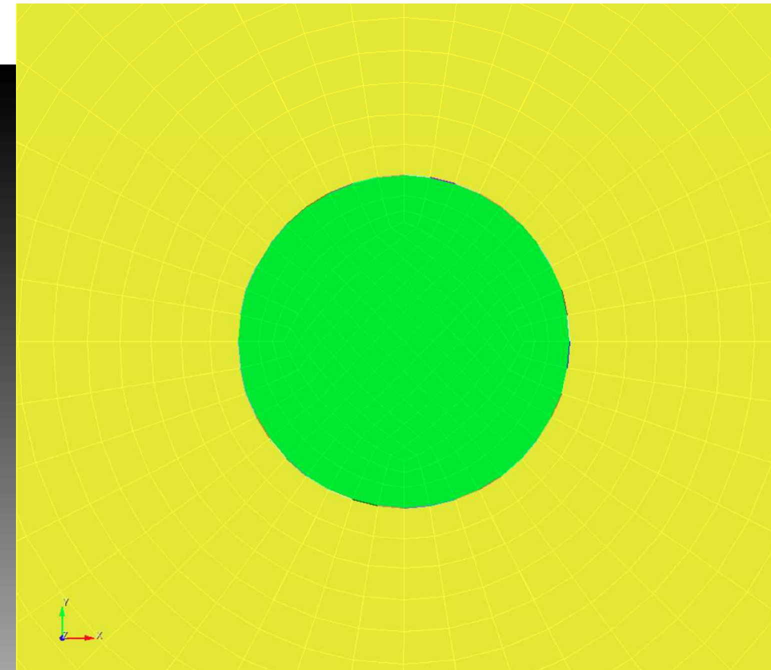
Fiber optic seal within a package -- 2D (plane strain)



Glass-Metal Seal **Model** Problem

Work in Progress!

Mesh 2:



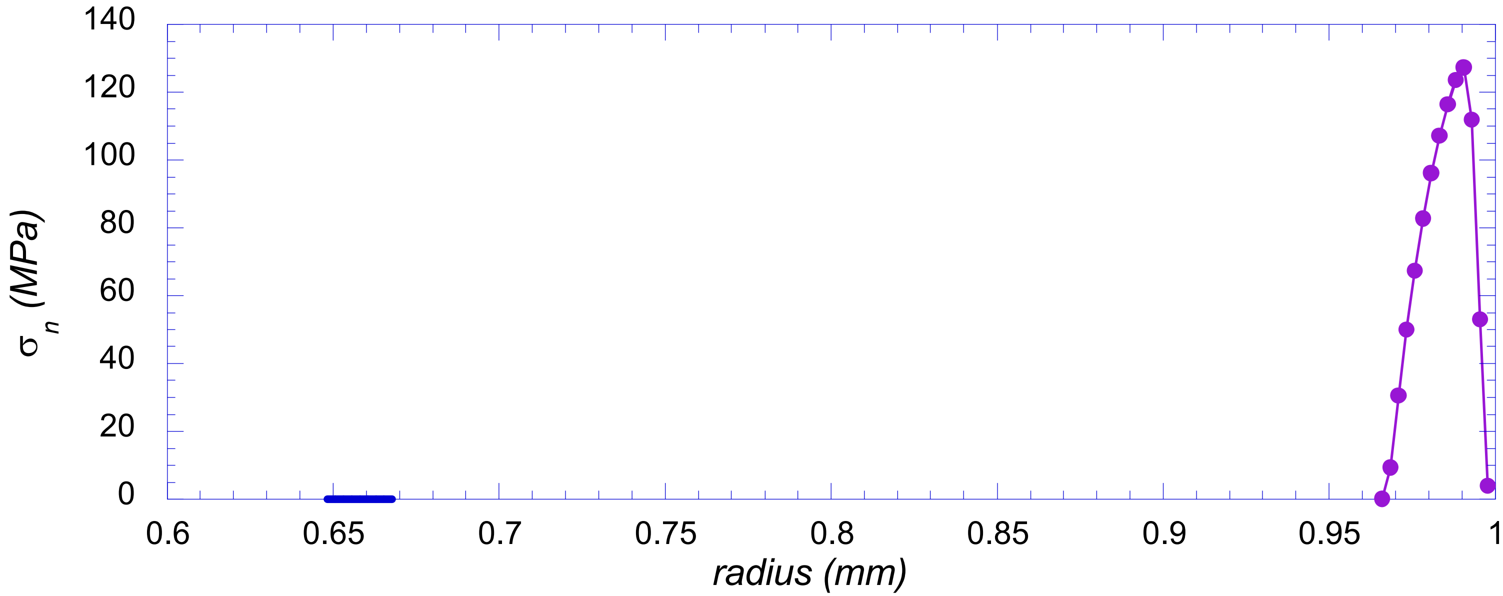
Interface elements used around pin

Glass-Metal Seal Example Analysis

- Feasibility questions to address:
  - Can we run problems with a relatively small cohesive zone?
  - Can we make the cohesive zone small enough to provide results for brittle fracture without requiring an extremely fine mesh?
  - Can we seed a crack (provide a “defect”) and determine the temperature that it might go unstable at?

Glass-Metal Seal Model Problem Example

- Temperature drop to produce unstable crack ~ 376° C
- Cohesive zone traction just prior to unstable crack growth



- Cohesive zone length is ~ 1/11 crack length
  ~ 1/4 element length
- "Resolution of cohesive zone" appears to be in terms of increments across an element – not elements.

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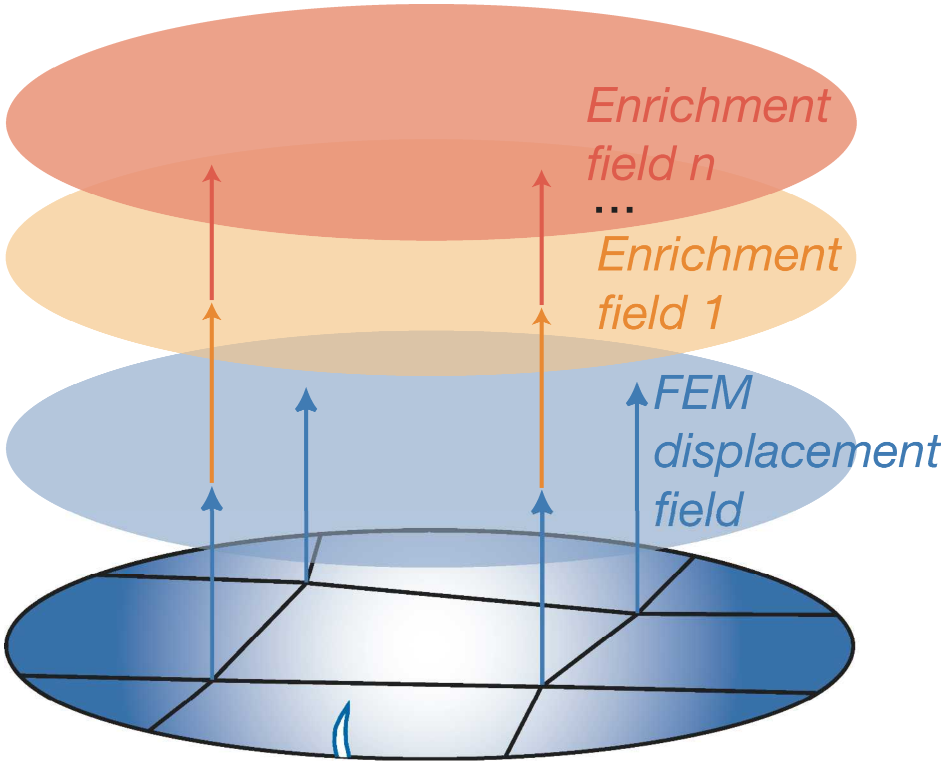
A “Global Issue” for the Aria Implementation

Management of enrichment DOFs



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Observations & Conclusions

- No free-lunch -- algorithm complexity ↑ with analytical enrichment
- Several open issues and opportunities, *e.g.*:
  - Value of *c* and its possible adjustment
  - How useful is this analytical enrichment for materials that are not homogeneous isotropic elastic?
    - Preliminary results for an orthotropic elastic material did not preclude their application.
    - Clearly not the best approach for all classes of problems.
- Appears to allow relatively coarse meshes in some cases
- Resolution of cohesive zone appears to be in measured in terms of increments across an element – not elements
- Refinements are still needed to improve the accuracy
- Analytically enriched XFEM for cohesive zone modeling of localization has potential.

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Acknowledgements

- Initial funding was provided by the Materials Directorate, Army Research Laboratory.
- Current funding is from the *Engineering Science Research Foundation*, Sandia National Laboratories.

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Also ref. Cox [2008] IJNME.

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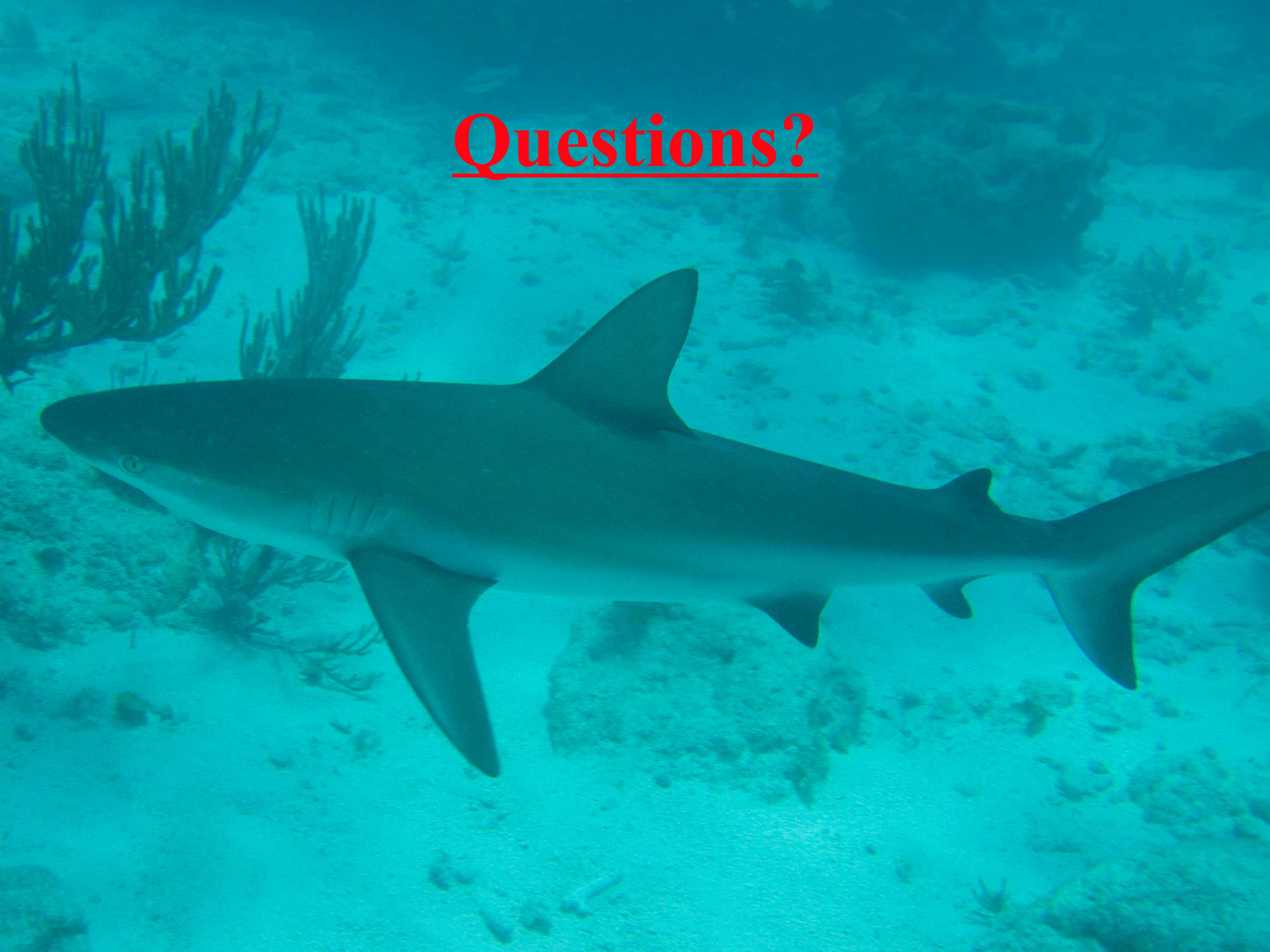
Questions?

Questions?



Back-up slides referenced:

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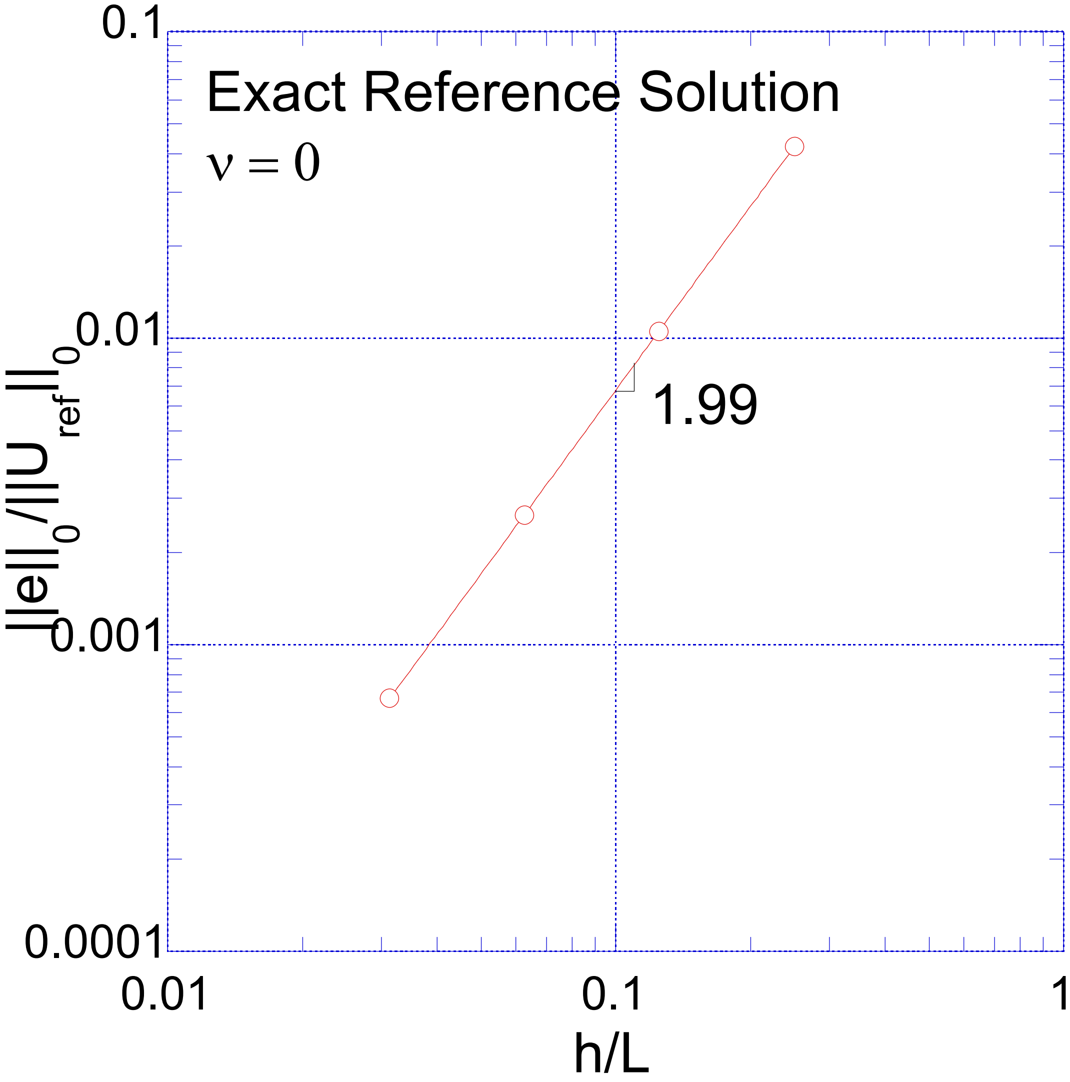
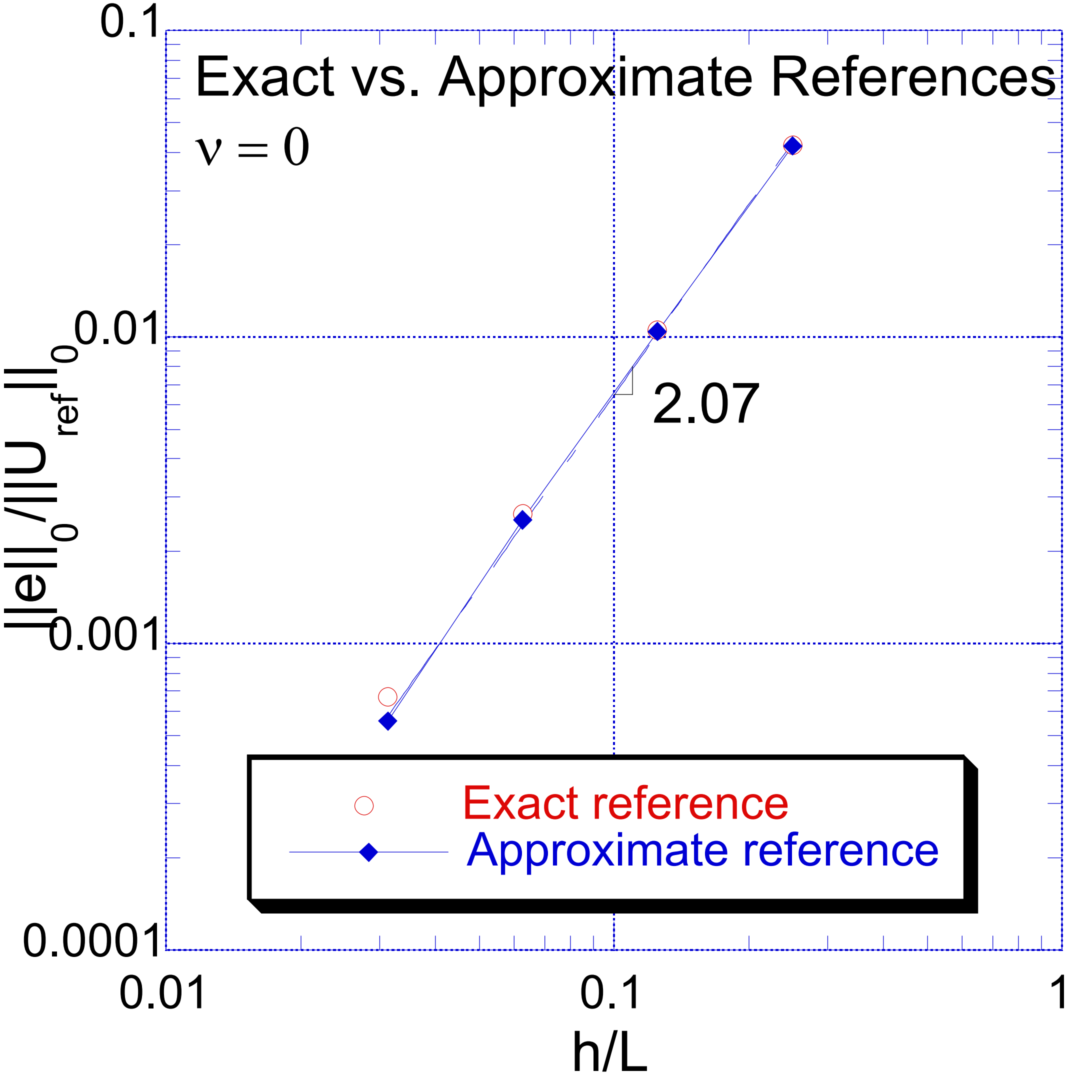
Displacement Field Accuracy



Meshes: 4x4, 8x8, 16x16, 32x32

Displacement Field Accuracy



Approximate reference: 81x80

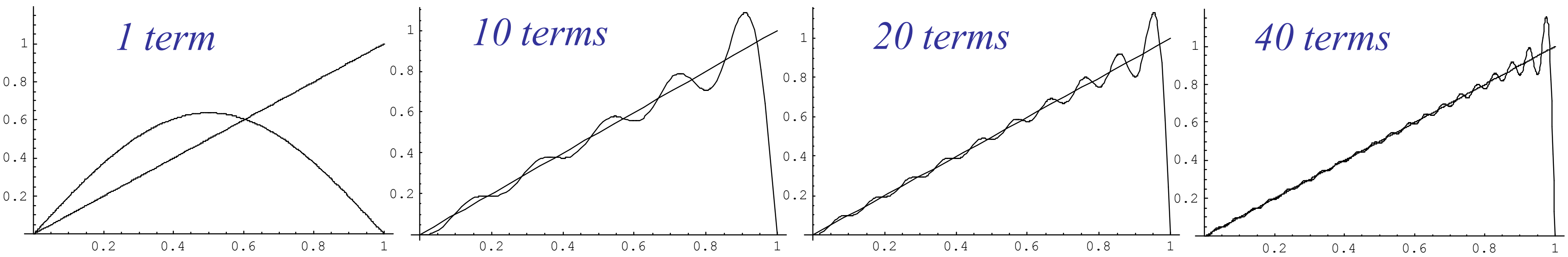
Value of c for 1 series term

Analogy to illustrate the point that if only one term of the series is used, adjusting another parameter of the single basis function can improve the solution.

Consider the approximation of the function $y=x$, on the interval [0,a].
In this case let the basis be an orthonormal sine series of the form:

$$\left\{ \sqrt{\frac{2}{a}} Sin\left(\frac{i\pi x}{a}\right) \right\}_{i=1}^{\infty}$$

Approximation of *y/a vs. x/a* when keeping a finite number of terms:



Having a basis for the function we can approximate it as closely as desired in the sense of the L_2-norm, but 1 term is not very accurate.

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Value of c for 1 series term

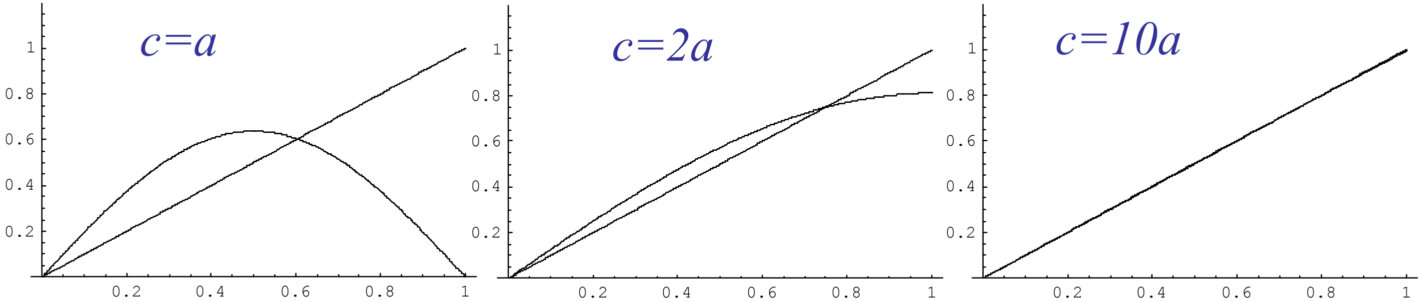
Analogy to illustrate the point that if only one term of the series is used, adjusting another parameter of the single basis function can improve the solution.

Consider the approximation of the function $y=x$, on the interval [0,a].
If we can only keep one term of the series, consider changing a to c and treating it as a parameter that can be adjusted. Our approximate solution then takes the form:

$$\hat{y} = b \sin\left(\frac{\pi x}{c}\right)$$

b ~ of the nodal unknown in the FEA. Here it is determined by a least squares fit. c ~ c of the cz analytical solution

Approximation of *y/a vs. x/a* when adjusting *c*:



When only 1 term is retained, increasing c improves the accuracy in the sense of the L_2-norm -- obvious from a Taylor series point of view.

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