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Modeling Braided Shields via Multipole Representations for the Braid Charges and Currents

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Motivation

- Goal : predict voltages and currents induced by external environments into electronics and electric devices.
  - Cable coupling is an integral part of this goal by describing the penetration of EM fields from the outside of the cable to the inside for a variety of cables (with High to low optical coverage).
  - Field penetration through the braid translates to transmission line voltage and current sources for propagation along the cable.
- *Goal : develop 1st principles model based on solutions to electrostatic and magnetostatic integral equations for braid penetration*
  - *Use a doubly infinite "planar braid" model to simplify the problem*
  - *Use multipole filament source representations to simplify the integral equations*
  - Use Ewald techniques to increase efficiency in the computationally-intensive calculations of the Green's functions and their gradients
- The *principles models are used to compare to semi-empirical formulas when applicable as well as providing solutions for braids where these semi-empirical models are not available.*

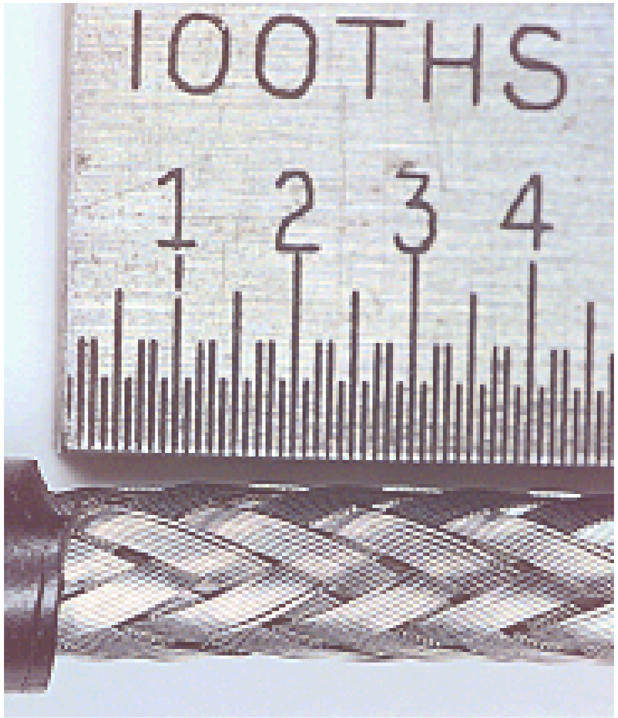
Extension of previous work

- Modeling of Braided Shields, Johnson, Warne, Basilio,Coats, Kotulski, Jorgenson , ICEAA 2011, Torino
  - Full electrostatic integral equation solutions
  - Comptationally intense simulations
  - Difficulties in extending to the finitely conducting magnetostatic braid penetration
    - Led us to suggest modal series solutions for the the wires
- The multipole approach of the current work provides a far simpiler soultion to that suggestion in the above reference

Outline

- Transmission Line models for cable braid
  - Pec for now
  - magnetic field diffusion through braid (future)
- Transfer capacitance and Inductances
- Infinite 2d periodic (nearly planar braid)
- Electrostatic formulation for the transfer capacitance
  - Integral equation formulations for filament and multi-pole source unknowns
    - Uniform field solution
    - Unit potential on braid solution
    - Final solution by superposition
  - Ewald acceleration for $G, \nabla G, \nabla\nabla G, \nabla\nabla\nabla G$
- Magneto-static formulation of the transfer Inductance
  - Uniform current approximate distribution
  - Self-consistent non-uniform current distribution

A commercially available Beldon cable



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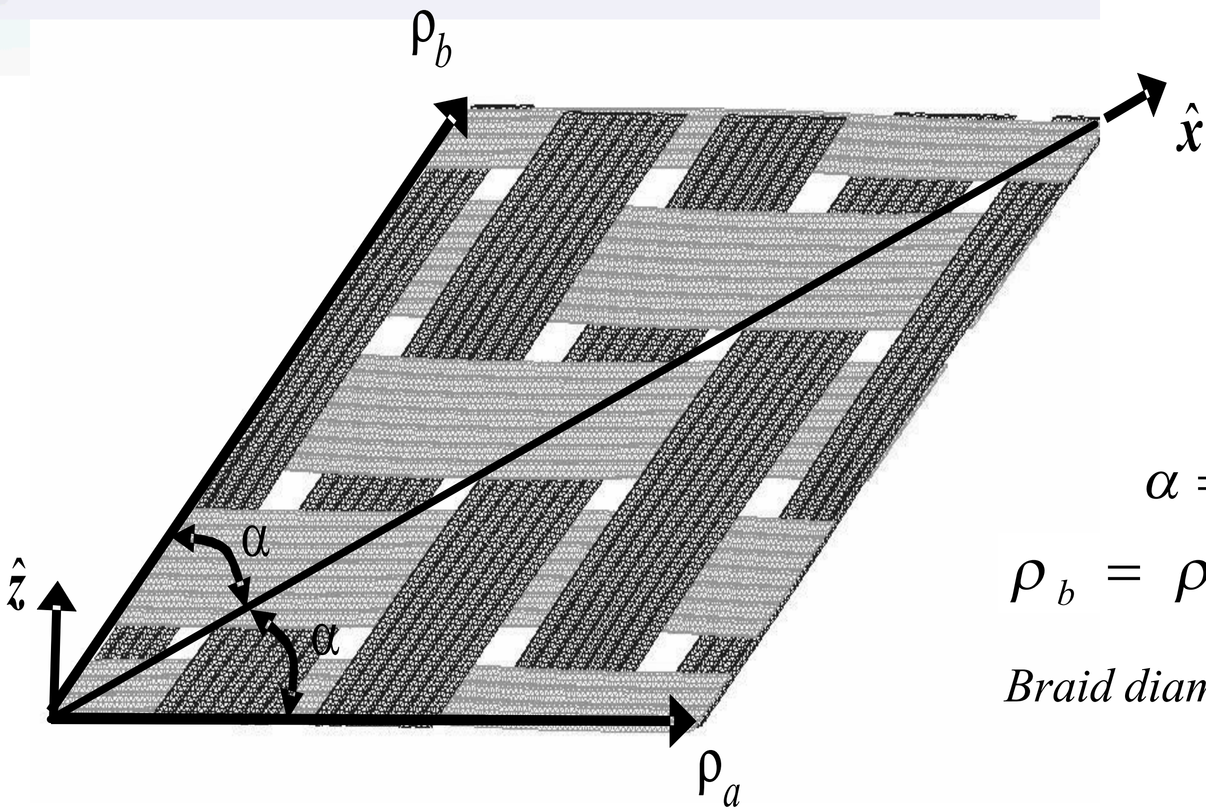
Infinitely periodic (nearly planar) unit Cell



$$\alpha = 24.4^{0}$$

$$\rho_b = \rho_a = .244''$$

$$Braid\ diameter = .005''$$

The unit cell of the two-dimensional infinitely periodic braid. The x direction points along the cylinder axis of the coaxial cable .

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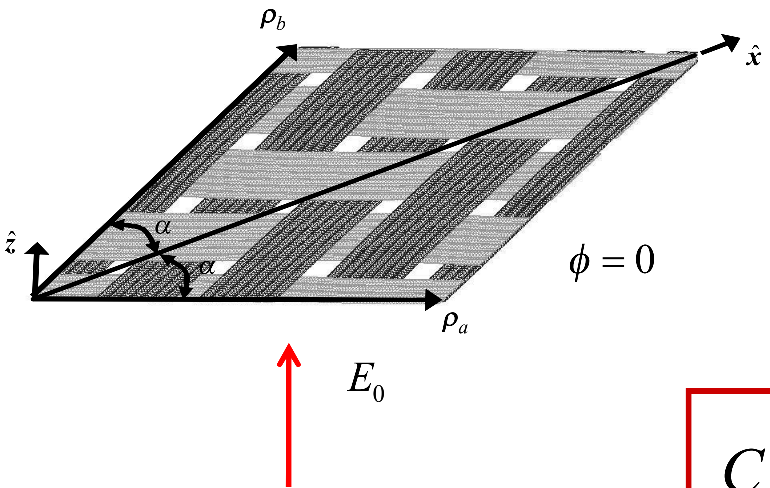
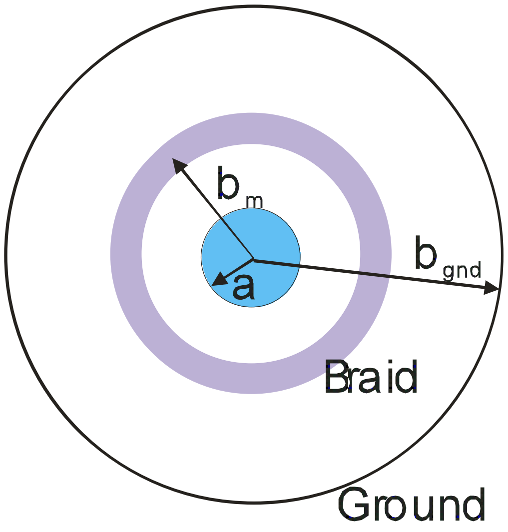
Connection of Coax to "Planar" Braid - Electric Field Formulation

- Scalar reciprocity formulation yields
  - Correction to self capacitance per unit length
  - Transfer capacitance per unit length

$$\phi \approx \phi_c$$





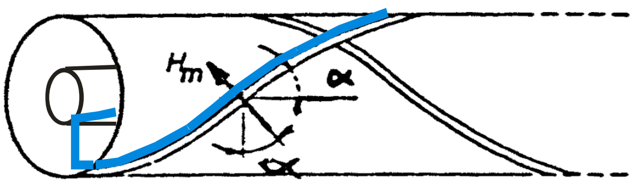
$$C_t = \frac{2\pi\varepsilon_0}{\ln\left[\left(b_m + \phi_c / E_0\right) / a\right]} \quad C_{sh} = \frac{2\pi\varepsilon_0}{\ln\left(b_{gnd} / b_m\right)}$$

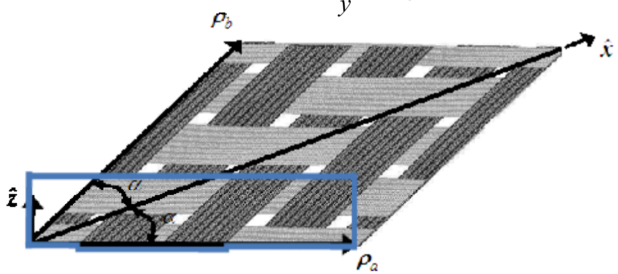
$$\Rightarrow C_T = -\frac{\phi_c / E_0}{2\pi b_m \varepsilon_0} C_{sh} C$$

Connection of Coax to “Planar” Braid - Magnetic Field Formulation

Flux path

- Difference flux path near braid surface





$$L = \frac{1}{H_0 l_{axis}} \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$L_T = L/(2\pi b_m)$$

Braid Shield Circuit Model

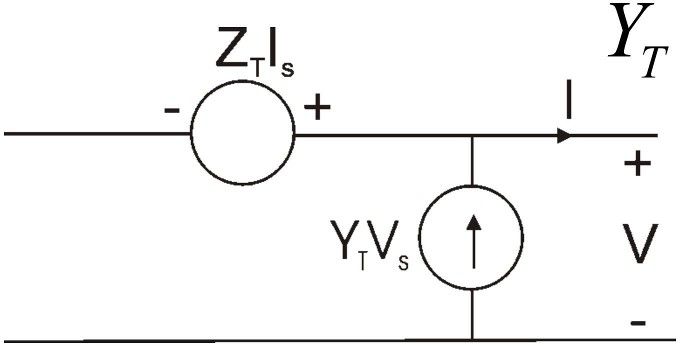
- Transfer impedance and cancellation of inductance terms in 78% cable

cancellation

$$Z_T = Z_R + j\omega\left(M_L - L_G\right) + Z_S$$

Shell diffusion term — *Hole inductance* — *Interweave inductance* — *Internal impedance term*

- Transfer admittance (capacitance)



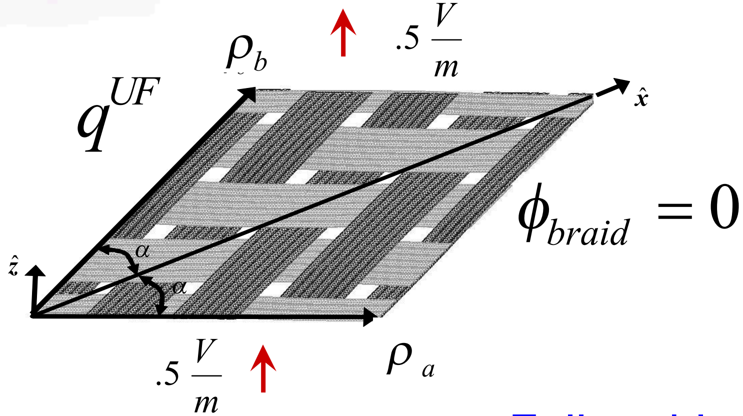
$$Y_T = j\omega C_T$$

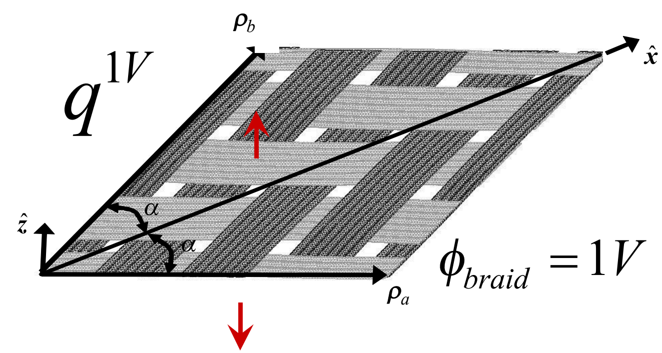
Hole capacitance

All parameters related to geometry and material properties (although some fixed empirical constants present)

Electrostatic Problem

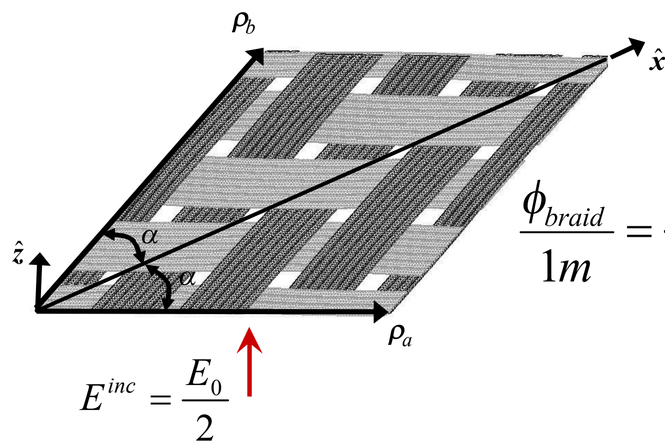
$\phi_{sc}^{UF}(\mathbf{r})$ Two preliminary problems $\phi^{1V}(\mathbf{r})$





Full problem by superposition

$$\phi_c \approx \phi(\mathbf{r}) - \phi_{braid} = \frac{E_0}{(1V/m)}\left(\phi_{sc}^{UF}(\mathbf{r}) - .5\, r_z + (\phi^{1V} - 1)\phi_{braid}\right)$$



$$\frac{\phi_{braid}}{1m} = -E_0\left(\varepsilon_0 A \frac{1\mathrm{V}}{m} + q^{UF}\right)/q^{1V}$$

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Formulation of the electrostatic problem

Local coordinates for nth segment

$$\phi^{tot}(\mathbf{r}) = \phi^{inc}(\mathbf{r}) + \phi^{sc}(\mathbf{r})$$

$$\phi^{sc}(\mathbf{r}) = \frac{1}{4\pi}\sum_{n=1}^{N_{seg}} \sigma_n^{(i,j)} \int_{seg_i} \left(-\frac{\partial}{\partial x_n}\right)^i \left(-\frac{\partial}{\partial y_n}\right)^j G^P(\mathbf{r}-\mathbf{r}'(s))ds$$

Filamentary charge density coefficients	$\sigma_n^{(0,0)}$	
Dipole coefficients	$\sigma_n^{(1,0)}$	$\sigma_n^{(0,1)}$
Quadrupole coefficients	$\sigma_n^{(2,0)}$	$\sigma_n^{(1,1)}$
Octopole coefficients	$\sigma_n^{(3,0)}$	$\sigma_n^{(2,1)}$

Due to the near linear dependency of

$\sigma_n^{(2,0)}$ and $\sigma_n^{(0,2)}$

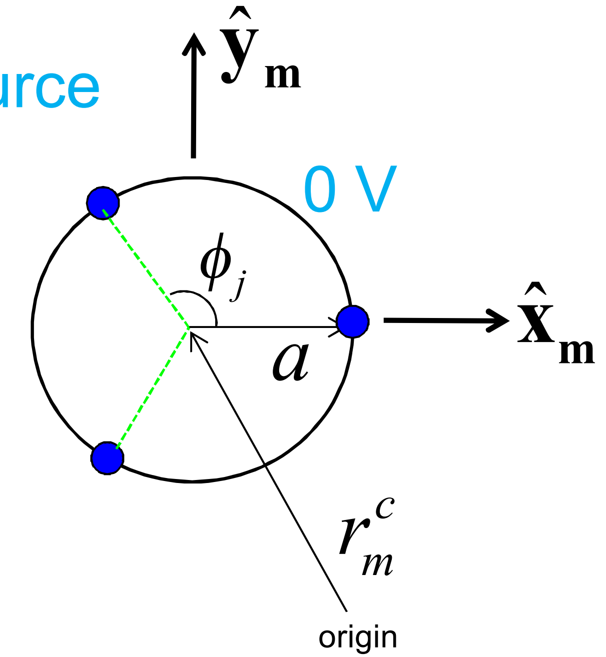
Corresponding terms are omitted

Uniform Field and Zero Potential Problem

$$\phi^{tot}(\mathbf{r}_{m,j}) = 0,\ m = 1,\ldots N^{seg}\ \ j = 1,2,\ldots,N_{seg}^{unk}$$

$$N_{seg}^{unk} = \begin{bmatrix} 1(\text{filaments}) \\ 3(\text{dipoles}) \\ 5(\text{quadrupoles}) \\ 7(\text{octopoles}) \end{bmatrix}$$

Dipole Source



$$\phi^{sc}(\mathbf{r}_{m,j}) = \frac{z_{m,j}}{2},\ m = 1,\ldots N^{seg}$$

$$j = 0,1,2,\ldots,N^{unkn,seg} - 1$$

$$r_{m,j} = r_m^c + a(\cos\phi_j \hat{x}_m + \sin\phi_j \hat{y}_m)$$

$$\phi_j = \frac{2\pi j}{N^{u,seg}}$$

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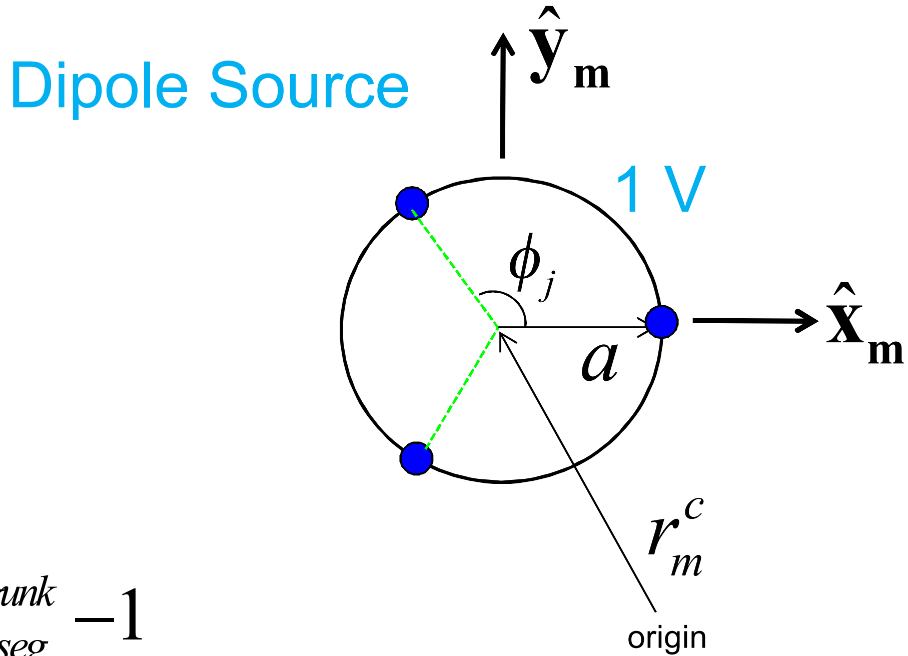
Zero Incident Field and Unit Potential Problem

$$N_{seg}^{unk} = \begin{bmatrix} 1(\text{filaments}) \\ 3(\text{dipoles}) \\ 5(\text{quadrupoles}) \\ 7(\text{octopoles}) \end{bmatrix}$$

$$\phi^{sc}(\mathbf{r}_{m,j}) = 1,\ m = 1, \ldots N^{seg}$$

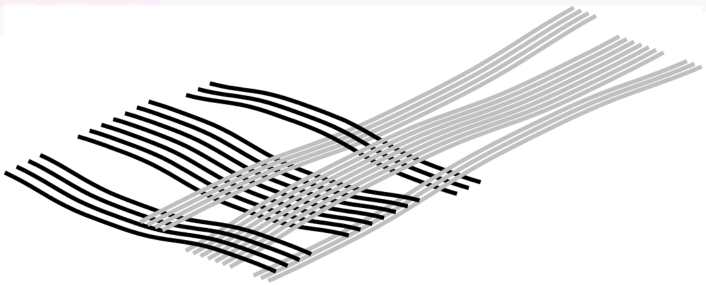
$$j = 0, 1, 2, \ldots, N_{seg}^{unk} - 1$$



$$r_{m,j} = r_m^c + a(\cos\phi_j \hat{x}_m + \sin\phi_j \hat{y}_m)$$

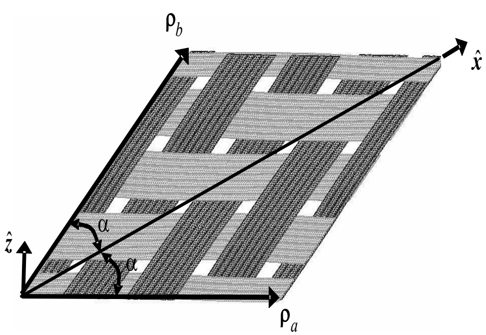
$$\phi_j = \frac{2\pi j}{N^{u,seg}}$$

Results Electro-static Problem



Highest multipole	ϕ^c/E_0
Filament	5.26×10^{-5}
Dipole	5.10×10^{-5}
Quadrupole	4.86×10^{-5}
Octopole	4.87×10^{-5}

Cable braid model consisting of half (14x14) the wires in the unit cell of the Beldon cable.
The individual lines correspond to the center lines of the wires.
The wire thickness due to the radius a is not shown.



Highest multipole	ϕ^c/E_0
Filament	-2.60×10^{-7}
Dipole	3.60×10^{-7}
Quadrupole	-7.66×10^{-8}
Octopole	-5.72×10^{-8}

Full Eiger Run
12 around the circum $9.11\ 10^{-8}$
16 around the circum $8.67\ 10^{-8}$

Semi-emperical 1.3e-7

Evaluation of the periodic Green's Function

$$G(r) = \frac{1}{4p} \sum_{\substack{m=-\infty \\ n=-\infty}}^{\infty} \left[\frac{1}{R_{mn}} - \frac{(1-\delta_{m0}\delta_{n0})}{R^0_{mn}} \right] \qquad R_{mn} = \left| r - (m\boldsymbol{r}_a + n\boldsymbol{r}_b) \right| \qquad R^0_{mn} = \left| m\boldsymbol{r}_a + n\boldsymbol{r}_b \right|$$

If |z| <.1 ρ_a Ewald acceleration methods are used

$$G(\mathbf{r}) = \frac{1}{4\pi}\frac{1}{R_{m_0,n_0}} + \widehat{G}^{E,spatial}(\mathbf{r}) + \widehat{G}^{E,spectral}(\mathbf{r}) - \widehat{G}^{E,spatial}(\mathbf{0}) - \widehat{G}^{E,spectral}(\mathbf{0})$$

$$\mathbf{k}_{tmn} = \frac{2\pi}{A} m\boldsymbol{\rho}_b \times \hat{z} + n\hat{z} \times \boldsymbol{\rho}_a$$

$$\widehat{G}^{E,spectral}(\mathbf{r}) = \frac{1}{4A}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} S^{E,spectral}_{m,n} \qquad S^{E,spectral}_{0,0} = -2\left[|z|\operatorname{erf}(|z|E) + \frac{e^{-(|z|E)^2}}{E\sqrt{\pi}} \right] \qquad S^{E,spectral}_{m,n} = \left(e^{-k_{tmn}|z|}\operatorname{erfc}\left(\frac{k_{tmn}}{2E} - |z|E\right) + e^{k_{tmn}|z|}\operatorname{erfc}\left(\frac{k_{tmn}}{2E} + |z|E\right) \right)\frac{\cos(\mathbf{k}_{tmn}\cdot\boldsymbol{\rho})}{k_{tmn}}$$

$$\widehat{G}^{E,spatial}(\mathbf{r}) = \frac{1}{4\pi}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} S^{E,spatial}_{m,n} \qquad S^{E,spatial;}_{m,n} = \frac{\operatorname{erfc}(R_{m,n}E)}{R_{m,n}} \qquad S^{E,spatial}_{0,0} = \frac{-\operatorname{erf}(R_{m_0,n_0}E)}{R_{m_0,n_0}}$$

$$\nabla G(\mathbf{r}) = \frac{1}{4\pi}\nabla\frac{1}{R_{m_0,n_0}} + \nabla\widehat{G}^{E,spatial}(\mathbf{r}) + \nabla\widehat{G}^{E,spectral}(\mathbf{r})$$

$$\nabla\widehat{G}^{E,spectral}(\mathbf{r}) = \frac{1}{4A}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\left[k_{tmn,x}\hat{x} + k_{tmn,y}\hat{y}\right]S^{E,spectral}_{m,n} + \hat{z}T^{E,spectral}_{m,n} \qquad S^{E,spectral}_{m,n} = -\left[e^{-k_{tmn}|z|}\operatorname{erfc}\left(\frac{k_{tmn}}{2E} - |z|E\right) + e^{k_{tmn}|z|}\operatorname{erfc}\left(\frac{k_{tmn}}{2E} + |z|E\right) \right]\frac{\sin(\mathbf{k}_{tmn}\cdot\boldsymbol{\rho})}{k_{tmn}}$$

$$T^{E,spectral}_{m,n} = (-e^{-k_{tmn}|z|}\operatorname{erfc}\left(\frac{k_{tmn}}{2E} - |z|E\right) + e^{k_{tmn}|z|}\operatorname{erfc}\left(\frac{k_{tmn}}{2E} + |z|E\right))\cos(\mathbf{k}_{tmn}\cdot\rho)\operatorname{sgn}(z) + \frac{2E}{k_{tmn}\sqrt{\pi}}e^{-k_{tmn}|z|}e^{-\left[\frac{k_{tmn}}{2E} - |z|E\right]^2} - \frac{2E}{k_{tmn}\sqrt{\pi}}e^{k_{tmn}|z|}e^{-\left[\frac{k_{tmn}}{2E} + |z|E\right]^2}$$

$$T^{E,spectral}_{0,0} = -\frac{\operatorname{sgn}(z)\,erf(|zE|)}{2A}$$

,

We need terms up to $\nabla\nabla\nabla G(\mathbf{r})$

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Evaluation of the periodic Green's Function

If |z| > .1 ρ_a Spectral series are used

$$\widehat{G}^{spectral}(\mathbf{r}) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{\cos(\mathbf{k}^{s}_{tmn}\cdot\boldsymbol{\rho})S^{spectral}_{m,n}}{2A} \qquad S^{spectral}_{m,n} = \frac{e^{-k_{tmn}|z|}}{k_{tmn}} \qquad S^{spectral}_{0,0} = |z|$$

$$\nabla\widehat{G}^{spectral}(\mathbf{r}) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\left(\frac{\left[k^{s}_{tmn,x}\widehat{x} + k^{s}_{tmn,y}\widehat{y}\right]U^{spectral}_{m,n} + \widehat{z}V^{spectral}_{m,n}}{2A}\right)$$

$$U^{spectral}_{m,n} = \frac{-e^{-k_{tmn}|z|}}{k^{s}_{tmn}}\sin(k^{s}_{tmn}\cdot\rho) \qquad V^{spectral}_{m,n} = -\mathrm{sgn}(z)\cos(\mathbf{k}^{s}_{tmn}\cdot\boldsymbol{\rho})e^{-k^{s}_{tmn}|z|} \qquad V^{spectral}_{0,0} = -\mathrm{sgn}(z)$$

We need terms up to $\nabla\nabla\nabla G(\mathbf{r})$

Formulation of the magneto-static problem uniform current approximation

Local coordinates for nth segment

$$\mathbf{A}^{tot}(\mathbf{r}) = \mathbf{A}^{inc}(\mathbf{r}) + \mathbf{A}^{sc}(\mathbf{r})$$

Circumferential currents are neglected

$$\mathbf{A}^{sc}(\mathbf{r}) = \frac{\mu_0}{4\pi} \sum_{m=1}^{N^{wires}} \sum_{n=1}^{N^{seg}_{wire}} I^{(i,j)}_{n,m} \overset{\frown}{s}_{n,m} \int_{seg_{n,m}} \left(-\frac{\partial}{\partial x_n}\right)^i \left(-\frac{\partial}{\partial y_n}\right)^j G^P(\mathbf{r} - \mathbf{r}'(s))\,ds$$

Filamentary charge density coefficients	$I^{(0,0)}_{n,m} =$	$I^{(0,0)}_{m}$
Dipole coefficients	$I^{(1,0)}_{n,m}$	$I^{(0,1)}_{n,m}$
Quadrupole coefficients	$I^{(2,0)}_{n,m}$	$I^{(1,1)}_{n,m}$
Octopole coefficients	$I^{(3,0)}_{n,m}$	$I^{(2,1)}_{n,m}$

Due to the near linear dependency of

$I^{(2,0)}_{n,m}$ and $I^{(0,2)}_{n,m}$

Corresponding terms are omitted

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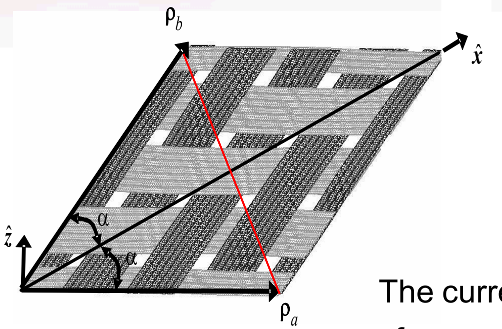
Magneto-static problem : Uniform Current Approximation



Uniform current approximation

Ampers law at x= ρ_a sin(α)

$$I_m \approx \mathrm{sgn}(\hat{s}_{1,m} \cdot \hat{x}) \frac{H_0 2\rho_a}{N^{wire}} \tan(\alpha)$$

The currents may be renormalized by setting for a point far above the braid

$$\mu_0 H_y^{sc}(\mathbf{r}) = \frac{\partial A_x^{sc}(\mathbf{r})}{\partial z} - \frac{\partial A_z^{sc}(\mathbf{r})}{\partial x} = \frac{\mu_0 H_0}{2}$$

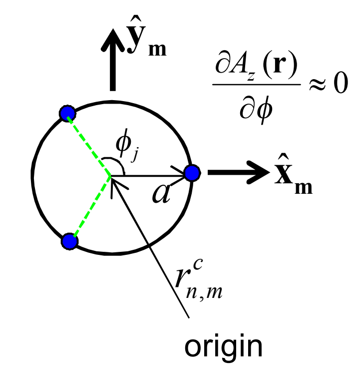
$$B_\rho(\mathbf{r}) = \frac{1}{a}\frac{\partial A_z(\mathbf{r})}{\partial \phi} - \frac{\partial A_\phi(\mathbf{r})}{\partial z} \approx \frac{1}{a}\frac{\partial A_z(\mathbf{r})}{\partial \phi} \qquad B_\rho(\mathbf{r}) = 0$$

$$\mathbf{A}_z^{sc}(\mathbf{r}_{n,m,j}) - \mathbf{A}_z^{sc}(\mathbf{r}_{n,m,0}) = \mathbf{A}_z^{inc}(\mathbf{r}_{n,m,0}) - \mathbf{A}_z^{inc}(\mathbf{r}_{n,m,j})$$

$$j = 0,1,2,..,N^{unkn,seg} - 1; m = 1,...N^{seg}$$

$$N_{seg}^{unk} = \begin{bmatrix} 0(\text{filaments}) \\ 2(\text{dipoles}) \\ 4(\text{quadrupoles}) \\ 6(\text{octopoles}) \end{bmatrix}$$

Dipole Source



$$\frac{\partial A_z(\mathbf{r})}{\partial \phi} \approx 0$$

$$r_{n,m,j} = r_m^c + a(\cos\phi_j \hat{x}_m + \sin\phi_j \hat{y}_m)$$

Formulation of the magneto-static problem: Non-uniform current

Local coordinates for nth segment

$$\mathbf{A}^{tot}(\mathbf{r}) = \mathbf{A}^{inc}(\mathbf{r}) + \mathbf{A}^{sc}(\mathbf{r})$$

Circumferential currents are neglected

$$\mathbf{A}^{sc}(\mathbf{r}) = \frac{\mu_0}{4\pi} \sum_{ng=1}^{N^{groups}} \sum_{m=1}^{N^{wires}_{group}} \sum_{n=1}^{N^{seg}_{wire}} I^{(i,j)}_{n,m} \hat{s}_{n,m,ng} \int_{seg_{n,m,ng}} \left(-\frac{\partial}{\partial x_n}\right)^i \left(-\frac{\partial}{\partial y_n}\right)^j G^P(\mathbf{r} - \mathbf{r}'(s))\, ds$$

Filamentary charge density coefficients	$I^{(0,0)}_{n,m} =$	$I^{(0,0)}_{m}$
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Octopole coefficients	$I^{(3,0)}_{n,m}$	$I^{(2,1)}_{n,m}$

Due to the near linear dependency of

$I^{(2,0)}_{n,m}$ and $I^{(0,2)}_{n,m}$

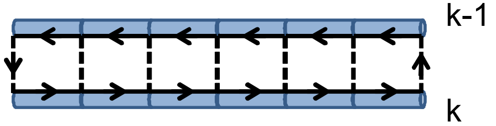
Corresponding terms are omitted

Magneto-static problem : Non-uniform Current

$$I_1 \approx \text{sgn}(\hat{s}_{1,1} \cdot \hat{x})$$

$$\int_S \mathbf{B} \cdot d\mathbf{S} = \oint_C \mathbf{A} \cdot d\mathbf{l} = 0$$

Wires in Group 1



$$\int_{W_k^1} \mathbf{A}^{sc} \cdot d\mathbf{l} - \int_{W_k^1} \mathbf{A}^{sc} \cdot d\mathbf{l} = \int_{W_k^1} \mathbf{A}^{inc} \cdot d\mathbf{l} - \int_{W_k^1} \mathbf{A}^{inc} \cdot d\mathbf{l} \quad m = 2, \ldots, N_{group}^{wires}$$

The currents may be renormalized by computing

$$\mu_0 H_y^{sc}(\mathbf{r}) = \frac{\partial A_x^{sc}(\mathbf{r})}{\partial z} - \frac{\partial A_z^{sc}(\mathbf{r})}{\partial x}$$

for a point far above the braid

$$I_m = \frac{H_0}{2H_y^{sc}(\mathbf{r})} \quad m = 1, 2, \ldots, N_{group}^{wires}$$

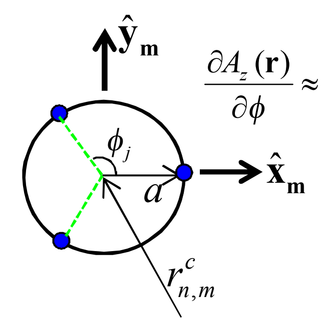
$$B_\rho(\mathbf{r}) = \frac{1}{a}\frac{\partial A_z(\mathbf{r})}{\partial \phi} - \frac{\partial A_\phi(\mathbf{r})}{\partial z} \approx \frac{1}{a}\frac{\partial A_z(\mathbf{r})}{\partial \phi} \qquad B_\rho(a) = 0$$

$$\mathbf{A}_z^{sc}(\mathbf{r}_{n,m,j}) - \mathbf{A}_z^{sc}(\mathbf{r}_{n,m,0}) = \mathbf{A}_z^{inc}(\mathbf{r}_{n,m,0}) - \mathbf{A}_z^{inc}(\mathbf{r}_{n,m,j})$$

$$j = 0, 1, 2, .., N^{unkn,seg} - 1; m = 1, \ldots N^{seg}$$

$$N_{seg}^{unk} = \begin{bmatrix} 0 \text{(filaments)} \\ 2 \text{(dipoles)} \\ 4 \text{(quadrupoles)} \\ 6 \text{(octopoles)} \end{bmatrix}$$

Dipole Source



$$\frac{\partial A_z(\mathbf{r})}{\partial \phi} \approx 0$$

$$r_{n,m,j} = r_m^c + a(\cos\phi_j \hat{x}_m + \sin\phi_j \hat{y}_m)$$

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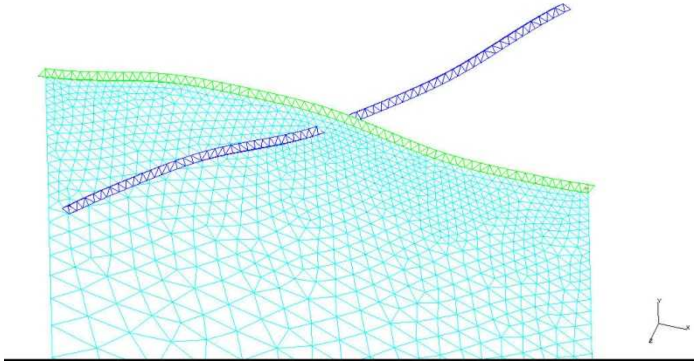
Meandering-Wire Test Case (Magnetostatic)

- This test case is a simplified mesh which contains all of the topology of the full cable braid mesh.

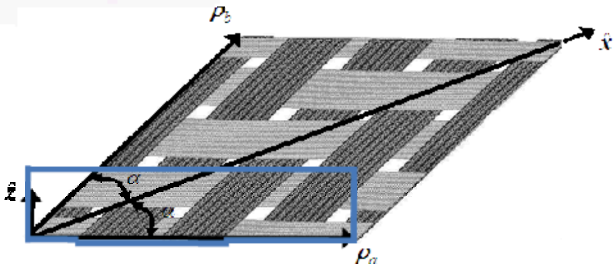
$L = 1.3\, nH$ (Multipole Simulation, *Current Filaments Only*)

$L \approx 1.3\,\text{nH}$ (EIGER_S Simulation)

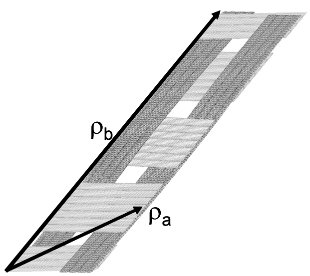
$L \approx 3.0\,\text{nH}$ (Semi-Empirical Result)

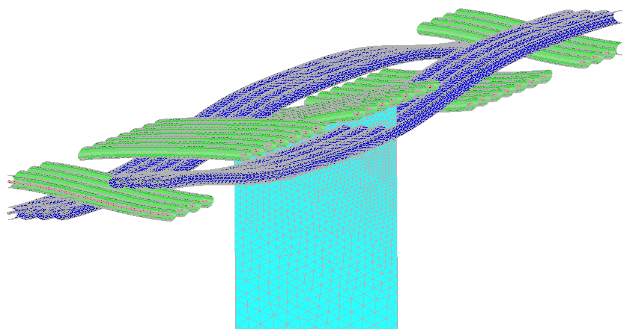


Magneto Static results



$$L = \frac{1}{H_0 l_{axis}} \int_S \mathbf{B} \cdot d\mathbf{S} = \oint_C \mathbf{A} \cdot d\mathbf{l}$$





$L \approx -0.008\,\text{nH}$ (Semi-empirical)

$L \approx -.081\,\text{nH}$ (filament)

Leiger = -0.14 nH

I will put results here from the filament ,dipole,quadrapole,and octopole for the inductance L

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Conclusions

- A first principles approach to modeling based on multipoles for field penetration through a cable braid has yielded sucessful results for a perfectly conducting braid.
- Magnetic diffusion into a finitely conducting cable braid will be next be attempted using these methods.