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A Nonlocal, Ordinary, State-Based Plasticity Model for Peridynamics

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Introduction and Background

- *Ordinary* and *non-ordinary* state-based materials
- Peridynamics treatment of momentum equation
- Brief introduction to peridynamics kinematics

Related Works

- State-based peridynamics and flow rule [2, Silling, 2007]
- *Bond*-based and *Non-ordinary* constitutive models [Foster, 2009]

A nonlocal-ordinary-state-based plasticity model for peridynamics

- Rate equations and constraints
- Practical yield condition

Demonstration Calculations

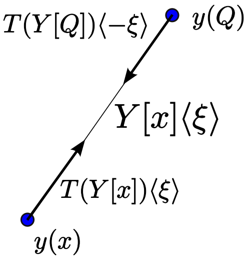
- Integration of a single bond
- Expanding ring

Key References

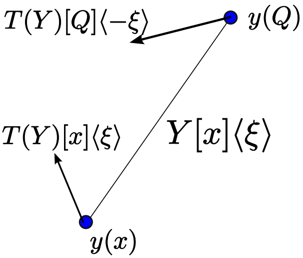
John A. Mitchell.
A nonlocal, ordinary, state-based plasticity model for peridynamics.
Sandia Report SAND2011-3166, Sandia National Laboratories, Albuquerque, NM, May 2011.

S. A. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari.
Peridynamic states and constitutive modeling.
J Elasticity, (88):151–184, 2007.

Ordinary

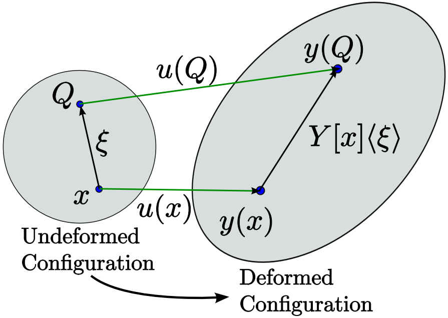


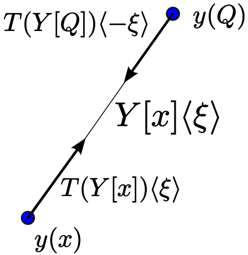
Non-ordinary



Momentum equation and internal force f for particle x

$$\begin{aligned}\rho(x)\ddot{u}(x,t) &= f(x,u(x,t),t)+b(x,t)\\ f(x,u(x,t),t) &= \int_H \{T(Y)[x]\langle\xi\rangle - T(Y)[Q]\langle-\xi\rangle\}\, dV_Q\end{aligned}$$

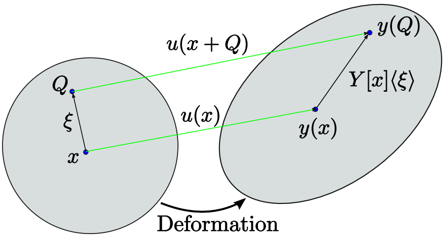




The vector force state T is given as:

$$T(Y) = t(Y)M(Y) \quad \text{where} \quad M(Y) = \frac{Y}{|Y|}$$

The scalar force state $t(Y)$ is the subject matter of this talk.



Bond: ξ

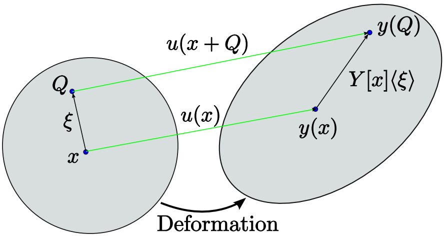
$$\xi = Q - x$$

Scalar Reference State: $|X|$

$$|X|\langle\xi\rangle = |Q - x| = |\xi|$$

Scalar Deformation State: $|Y|$

$$|Y|\langle\xi\rangle = \left[Y[x]\langle\xi\rangle \cdot Y[x]\langle\xi\rangle\right]^{\frac{1}{2}}$$



Weighted Volume: m

$$m = \int_H \omega |\xi|^2 \, dV_Q$$

Scalar Extension State: e

$$e = |Y| - |X|$$

Dilatation: θ

$$\hat{\theta}(e) = \frac{3}{m}(\underline{\omega x}) \bullet \mathbf{e} = \frac{3}{m}\int_H \omega |X|\langle\xi\rangle |Y|\langle\xi\rangle \, dV_Q - 3$$

Deviatoric Extension State: e^d

$$e^d(Y) = e(Y) - \frac{\theta(Y)|X|}{3}$$

Summary of Rate Equations and Constraints

- Additive decomposition of extension state: $\underline{e}^{d} = \underline{e}^{de} + \underline{e}^{dp}$
- Elastic force state relations: $t = -\frac{3p}{m}\underline{\omega x} + \alpha\underline{\omega}(\underline{e}^{d} - \underline{e}^{dp})$
- Elastic force states domain defined by a yield surface/function that depends upon the deviatoric force state: $f(t^{d}) = \psi(t^{d}) - \psi_0 \leq 0$, where $\psi(t^{d}) = \frac{\|t^{d}\|^2}{2}$
- Flow rule which describes rate of plastic deformation: $\dot{e}^{dp} = \lambda \nabla^{d} \psi$
- Loading/un-loading conditions (Kuhn-Tucker constraints): $\lambda \geq 0, \;\; f(t^{d}) \leq 0, \;\; \lambda f(t^{d}) = 0$
- Consistency condition: $\lambda \dot{f}(t^{d}) = 0$

Problem definition

- Given: $\{e^d_{n+1}, e^d_n, e^{dp}_n\}$, where n denotes the time step
- Find: e^{dp}_{n+1} and t^d_{n+1}

Exact solution and algorithm for backward Euler approximation

- Compute trial deviatoric force state: $t^d_{trial} = \alpha \underline{\omega}(e^d_{n+1} - e^{dp}_n)$
- if $f(t^d_{trial}) \leq 0$, then step is elastic, $\Delta\lambda = 0$, and $t^d_{n+1} = t^d_{trial}$
- else

$$t^d_{n+1} = \sqrt{2\psi_0}\,\frac{t^d_{trial}}{\|t^d_{trial}\|}, \quad e^{dp}_{n+1} = e^{dp}_n + \frac{1}{\alpha}\left[\frac{\|t^d_{trial}\|}{\sqrt{2\psi_0}} - 1\right] t^d_{n+1}$$

Additional Comments

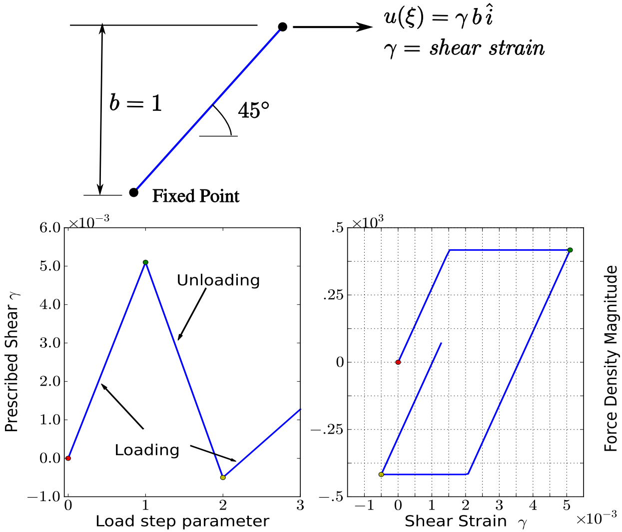
- Evolution of plasticity is driven by incoming scalar deviatoric extension state e^d
- Above algorithm is conceptually similar to radial return

A Practical Definition for ψ_0.

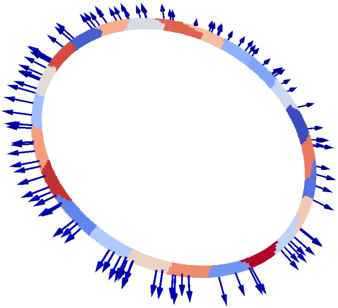
- Assume pure shear conditions
- Expand scalar deformation state in a taylor series
- Dilation is zero under linearized conditions
- Scalar extension state and deviatoric extension state are identical

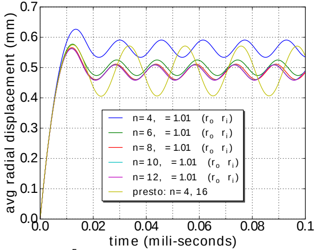
$$\begin{aligned} \psi_0 &= \frac{1}{2}\left[\frac{15\mu}{m}\right]^2 \|e^d\|^2 \\ &= \frac{75}{8\pi}\frac{E_y^2}{\delta^5} \end{aligned}$$

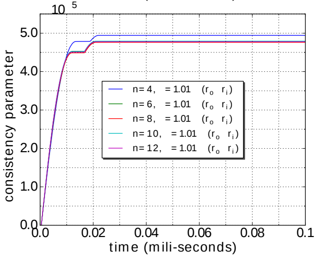
where E_y is the shearing yield stress and δ is the *horizon*


$u(\xi) = \gamma\, b\, \hat{i}$
$\gamma = shear\ strain$
$b = 1$
45°
Fixed Point
×10^−3
Prescribed Shear γ
Unloading
Loading
Load step parameter
×10^3
Force Density Magnitude
Shear Strain γ
×10^−3

- cylindrical ring
- square cross-section
- IC: outward radial velocity
- non-local solution *peridigm*
- compare local *presto* solution

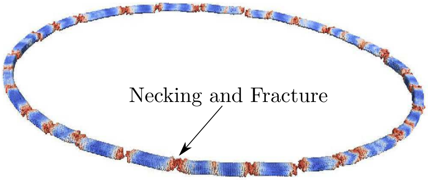






Peridigm Simulation
Ductile failure of expanding aluminum ring

Plasticity model inherits all the advantages of peridynamics for modeling fracture; Color indicates damage.



Peridigm is (soon to be) an open source Sandia peridynamics code.
More Info: talk to David Littlewood, Michael Parks, John Mitchell, Stewart Silling

Closing Remarks and Summary

- Presented new nonlocal, ordinary-state-based plasticity model
- Presented *force-based* yield condition
- Not presented but shown in report: model satisfies 2nd law of thermodynamics
- Not presented but developed in report: Linearization
- Damage is easy to incorporate
- Presented demonstration calculations

Detailed report is available: [1, Mitchell,2011]