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# Meshfree Methods for Nonlocal Diffusion

## Radial Basis Functions in Nonlocal Mechanics

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# What is Nonlocal Diffusion?

- An alternative to classical diffusion
  - Nonlocal diffusion model covers many examples such as fractional Laplacian
- Relies on integral equations
  - Classical methods use partial differential equations
- Volume constraints instead of boundary constraints
- Analysis and Approximation of Nonlocal Diffusion with Volume Constraints, SIAM Review, Vol. 54, No. 4, pp. 667–696 Qiang Du,Max GunzburgerR. B. Lehoucq,Kun Zhou

# The Problem

- $Lu(x) = \int_{\Omega \cup \Omega_I} \big(u(x) - u(y)\big) \gamma(x, y\,) dx\, dy = f(x)$
  - Holds for $x \in \Omega$
  - $\Omega_I$ is the interaction domain
- Volume constraints enforced in the interaction domain
  - $u(x) = g(x)$ for $x \in \Omega_I$
- This equation is  a ‘nonlocal’ problem
  - The value of $Lu(x)$ depends on points around $x$
- The data and the solution can be very irregular
  - Discontinuities can occur in $\gamma(x, y)$, u(x), and f(x)

# Radial Basis Functions

- Radial Basis Functions (RBFs) were built to interpolate arbitrarily scattered data sets in arbitrary dimensions
- Start with a fixed, one variable function $\varphi(r)$
  - Given data $(x_j, y_j)$ find interpolant $s(x_j) = y_j$
  - $s(x) = \sum_{j=1}^{N} c_j \varphi(||x - x_j||)$
- Does not require a mesh and is dimension blind
- Established theory proves error estimates for approximation of a function by interpolation at nodes

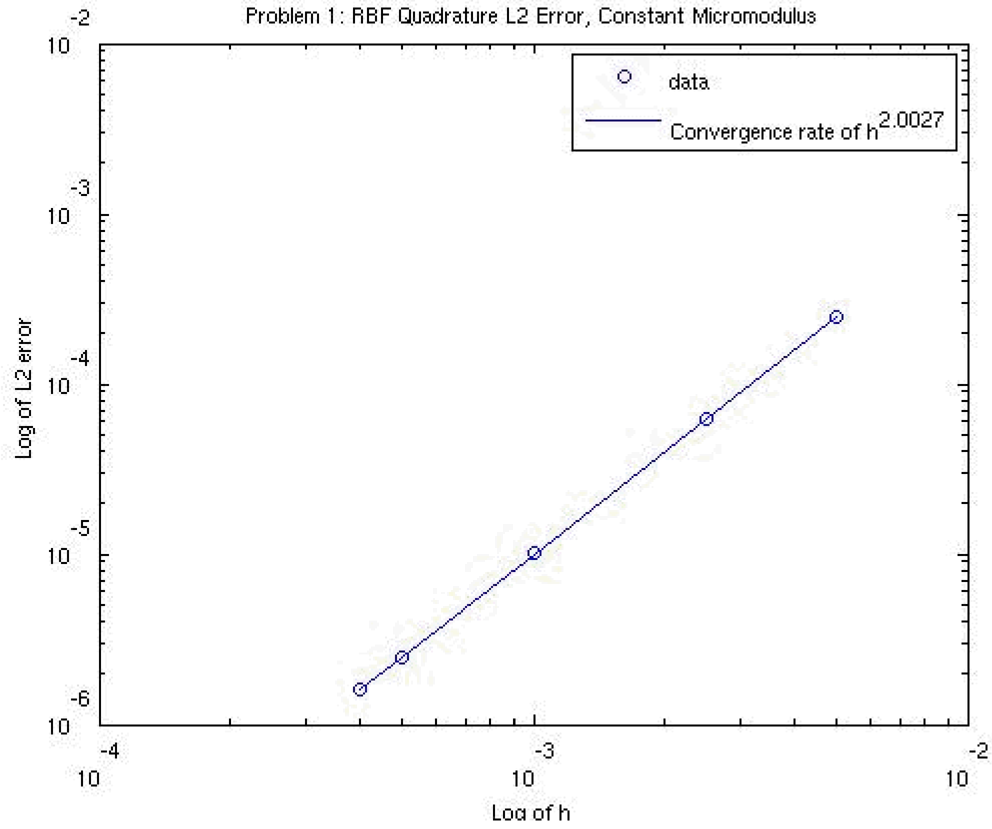
# Radial Basis Collocation Method

- We want to solve
  $Lu(x) = \int_{\Omega \cup \Omega_I} \big(u(x) - u(y)\big)\gamma(x, y\,)dx\,dy = f(x)$
- We assume $\hat{u}(x) = \sum_{j=1}^{N} c_j \varphi(||x - x_j||)$
  - First construct collection of nodes $\{x_j\}_{j=1}^{N}$
  - Substitute $\hat{u}(x)$ into equation for $L$
  - Enforce $L\,\hat{u}\big(x_j\big) = f(x_j)$ for each $x_j \in \Omega$
  - Enforce Volume Constraint $\hat{u}(x_k) = g(x_k)$ for $x_k \in \Omega_I$
- One point quadrature enables fast matrix assembly
- Works with spatially varying micromodulus and multiple horizons
- No mesh!

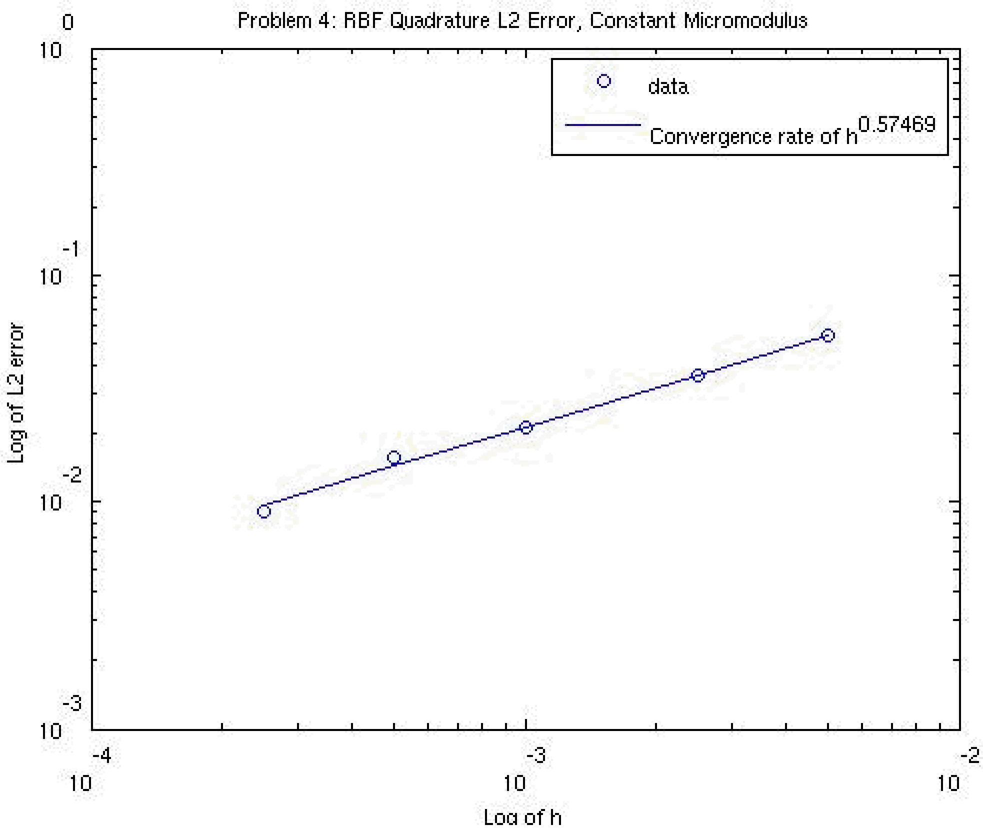
# My Goal for the Summer

- How can we apply radial basis functions to nonlocal diffusion?
  - We consider collocation using Radial Basis Functions and also a meshfree Galerkin method
- Develop and implement a numerical algorithm to solve the integral equation
- Experiment in 1D and 2D with various examples of $\gamma(x, y)$
- Check convergence rates against known interpolation error estimates
- Explore applications and understand numerical behavior of Radial Basis Functions in Collocation method

# Example of Numerical Result



# Numerical Result for Discontinuous Function



# Future Work

- Developing further 2D work for heterogeneous micromodulus
- Developing 3D code
- Exploring further a 1D and 2D meshfree Galerkin method
- Experimenting with modified basis functions
- Error estimates need to be established

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