

Proposed Approach for Admittance Testing of a Complex Aerospace Structure

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Nomenclature

FBS	Frequency Based Substructuring
DOF	Degree of freedom
H	Frequency response function
FRF	Frequency Response Function
PL	Payload
TS	Transmission Simulator
DS	Delivery System
INT	Interim System
Φ_{SC}^{TS}	Transmission Simulator mode shape matrix
SC	Sensor Connection
Subscripts	
O	Output Response
C	Connection Response
I	Force Input degree of freedom
T	Transpose
+	Pseudo-inverse
Q	Modal connection degree of freedom
X	Vector

1) ABSTRACT

It is common practice to utilize lower fidelity payloads to represent a complex aerospace payload during delivery system-with-payload, ground testing. Typically, the high fidelity payload hardware is not only costly but hard to acquire. Admittance testing can theoretically be used to experimentally model responses of a payload due to interface forces. In this paper we will consider the question, "Can the response of high fidelity payload hardware be predicted from an environmental test on a delivery system with low fidelity payloads and admittance data on all the substructures?" In theory, by acquiring the admittance models of the high and low fidelity payloads as well as the delivery system, one can adjust the measured interface responses and predict the response as if the high fidelity unit had been present in the ground test instead of the low fidelity payload. In this paper, work in progress is described to demonstrate the payload substitution capability for a specific complex aerospace structure.

¹ Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

2) INTRODUCTION

One of the tests in a current program applies a random environment to a delivery system with low fidelity payloads mounted on it. The purpose of this work is to use admittance modeling theory, which will now be referred to as frequency based substructuring (FBS), to analytically replace the low fidelity payload with a high fidelity payload in order to predict full system response as if the high fidelity payload had been attached to the delivery system in the environment.

As in any program, the first year has given greater insight into the magnitude of the testing that needs to be performed to investigate the frequency based substructuring theory. Logistics and availability of hardware play an important role in coordinating testing in stages that allows not only analysis of data but collaboration with finite element modeling in developing the process. Truth models need to be developed to which predictions with the FBS method can be compared to quantify its uncertainty.

This approach utilizes FBS with a fixture at the connection called the transmission simulator. The transmission simulator method can be used with either modal based substructures or FBS as has been previously published [3]. Typically, the transmission simulator is a specially made and easy to model fixture on which connection equilibrium and continuity between multiple substructures is satisfied. The transmission simulator is used as the common interface between the substructures. The transmission simulator is a substructure in itself, and inherently includes the effects of moments and rotations even though one may only measure translational accelerations and forces. In this approach the transmission simulator is actually a subset of the true mounting hardware. As depicted in Figure 1, the three mounting pedestals which connect a payload to the delivery system will be used as the transmission simulator.

The objective of the work is to take measured acceleration responses from an environmental test of the delivery system with low fidelity payloads attached, and mathematically calculate what the response would have been if a high fidelity payload had been attached. One task is to measure the FBS substructures for the delivery system, high fidelity payload and low fidelity payload.

3) PAYLOAD, DELIVERY SYSTEM AND TRANSMISSION SIMULATOR SUBSTRUCTURES

The aerospace payload consists of an exterior shell structure surrounding interior bracing, which supports several brackets holding payloads. Figure 1 shows a simplified illustration of the payload. Rather than developing other fixtures for transmission simulators to mount to each substructure, we elected to use the three mounting pedestals as the transmission simulator for both the payload substructure and delivery system substructure.

Before any testing began, a finite element model of one pedestal was developed and its natural frequencies were compared to test data.

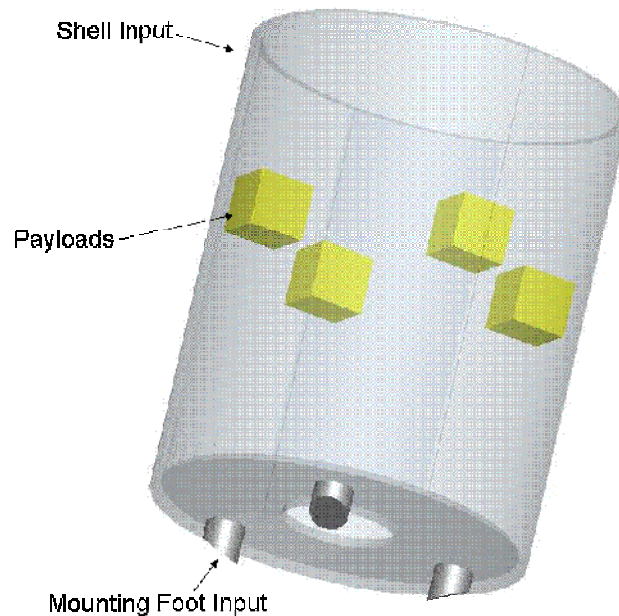


Figure 1: Simplified Illustration of Payload

The modes and frequency response functions (FRFs) of the transmission simulator (pedestal) were calculated using Salinas, a massively parallel structural dynamics code developed at Sandia National Laboratories [1]. Ten locations were selected on each pedestal to mount accelerometers and apply input forces. This allowed a least squares fit to six rigid body modes and one elastic mode of each pedestal to provide seven modal constraints for each pedestal to connect one substructure to another.

4) FBS TESTING

By all accounts, the FBS approach requires the care and attention similar to a very detailed modal test.

The basic approach is to develop a FBS of three different payloads with varying hardware fidelity and a FBS of the delivery system so that any combination of payload and delivery system could be modeled.

A truth test was performed which includes a low fidelity payload attached to a representative structure of the delivery system. In this test a hammer input was imparted at a location on the delivery system and FRFs were gathered on the payload system. Modes were extracted from the measured FRFs, up to 1000 Hz, using the SMAC algorithm [2] to give confidence that the FRFs were meaningful. In the future, FBS substructures will be developed for the representative delivery system and the low fidelity payload. These will be added together to see how well the truth FRFs can be represented from the combined FBS substructures.

For the program purposes, testing of a medium and high fidelity payloads with the transmission simulators attached have been conducted to develop FBS substructures for those payloads. In the future, data would also be collected on low and high fidelity delivery systems with the transmission simulators attached.

The following list describes the test series for documenting the FBS testing which will be performed in stages and Figure 2 shows simplified illustrations of the payload and delivery system shown with the transmission simulators.

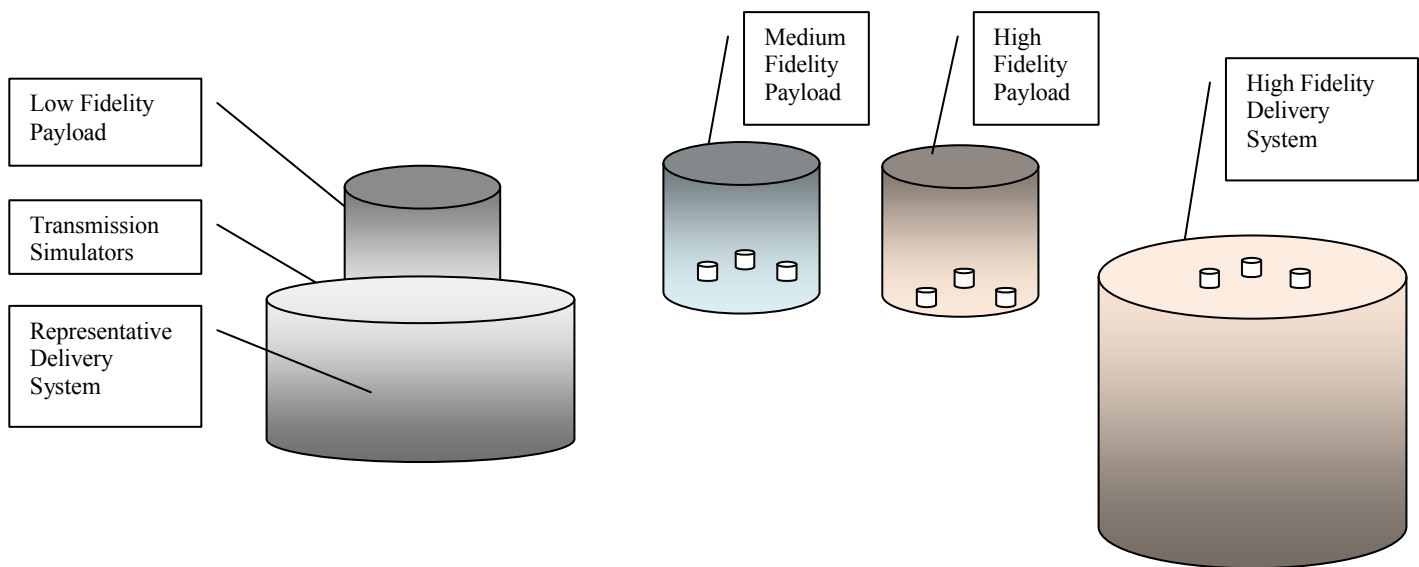


Figure 2: Simplified Illustration of Payload and Delivery System (Truth Hardware on Left)

- Transmission simulator characterization
- Truth testing of low fidelity payload attached to representative delivery system
- Low fidelity payload characterization
- Representative delivery system characterization

- Medium fidelity payload characterization
- High fidelity payload characterization
- High fidelity delivery system characterization

The transmission simulators act as the common interface so they are instrumented with enough fidelity to capture motion and forces that can be represented with the modal motion and forces associated with the six rigid body modes and one elastic mode. One needs to have more sensors on the transmission simulator than modes of each transmission simulator in the bandwidth in order to reduce the effects of experimental errors. Co-linear force inputs to each accelerometer location with the use of the impact hammer at all simulator locations are required to obtain the connection FRF matrix on any substructure. Additional cross FRFs are gathered from accelerometers on components of interest inside a payload.

For testing up to this time on the payloads, data acquisition systems were set to measure FRFs in the 0 to 2000 Hz range. No window was used with the modal hammer force inputs. Six averages were collected at each input location.

5) THEORY

The theory is developed in two sections with the first section addressing the FBS theory to connect two substructures and then subtract another substructure. The second section addresses the derivation of equations to determine the resulting connection degrees of freedom motion if an unknown force is applied on a payload and delivery system, measurements are made at the connection DOF on the transmission simulators, and then the payload is analytically removed and replaced by a different payload.

A) FBS THEORY TO CONNECT TWO SUBSTRUCTURES AND THEN SUBTRACT ANOTHER SUBSTRUCTURE

The FRFs for the payload (PL) will be designated as \mathbf{H}^{PL} with the understanding that there is an FRF matrix for every frequency line in the FRF. The designation denoting these FRFs as a function of frequency will be omitted for brevity. The mounting feet will be a substructure which we denote as the transmission simulator (TS) has FRFs designated as \mathbf{H}^{TS} . The delivery system FRFs will be denoted as \mathbf{H}^{DS} . The responses of each substructure can be further subdivided into groups of degrees of freedom where subscript O is output response, subscript C is a connection response (on the mounting feet) and subscript I is a force input degree of freedom. There can be inputs and outputs on any substructure, but to limit the scope, here we will only have an input degree of freedom (DOF) on the delivery system substructure and outputs on the payload substructure.

The standard FBS equations will be modified with the free mode shapes of the transmission simulator (mounting feet). This matrix needs to have more measurement DOF than modes to provide the least squares averaging effect. A model or test data can be utilized to provide these mode shapes. As a minimum one would generally include all 6 rigid body mode shapes, or for 3 mounting feet this would be 18 rigid body mode shapes. If a mode shape is not exercised in the actual assembly motion, it is not necessary. One can visualize that the roll mode shape of each mounting foot might not be very important, so this may be one that could be neglected for each mounting foot. In addition as many elastic modes as are desired may be added in. In previous work, it appeared that any elastic modes up to the frequency of desired accuracy should be included. Let us denote the transmission simulator mode shapes as

$$\Phi_{SC}^{TS}$$

which will only be measured at specific DOFs that we will call the sensor connection (SC) DOF. With this approach, the transmission simulator, or mounting feet, will be installed on the payload to get \mathbf{H}^{PL} and on the delivery system support substructure to get \mathbf{H}^{DS} . Then we will connect \mathbf{H}^{PL} and \mathbf{H}^{DS} together and subtract one set of mounting feet, \mathbf{H}^{TS} , to get the final system. The standard FBS equation for input on one system (A) with output on another system (B) giving response on total system (C) would be

$$\mathbf{H}_{OI}^C = \mathbf{H}_{OC}^B (\mathbf{H}_{CC}^B + \mathbf{H}_{CC}^A)^{-1} \mathbf{H}_{CI}^A \quad (1)$$

or for our system

$$\mathbf{H}_{OI}^{INT} = \mathbf{H}_{OC}^{PL} (\mathbf{H}_{CC}^{PL} + \mathbf{H}_{CC}^{DS})^{-1} \mathbf{H}_{CI}^{DS} \quad (2)$$

where the superscript INT denotes the interim system with the payload and delivery system connected together that has 2 sets of mounting feet, one of which still needs to be subtracted. In the subscripts, the first subscript represents an acceleration response DOF

and the second subscript represents a force input DOF. Now we modify equation (2) to get modal connection DOF using the pseudo-inverse of the mode shape matrix giving

$$\mathbf{H}_{OI}^{INT} = \mathbf{H}_{OC}^{PL} \Phi_{SC}^{TS^{T+}} (\Phi_{SC}^{TS+} \mathbf{H}_{CC}^{PL} \Phi_{SC}^{TS^{T+}} + \Phi_{SC}^{TS+} \mathbf{H}_{CC}^{DS} \Phi_{SC}^{TS^{T+}})^{-1} \Phi_{SC}^{TS+} \mathbf{H}_{CI}^{DS} \quad (3)$$

Where the superscript T represents transpose and the superscript + represents the pseudo-inverse. From this one can see that bookkeeping is paramount, e.g. if there are 30 connection measurements they MUST be in the same order in the mode shape matrix and the FRF matrices for ALL substructures. One will need several FRFs between inputs, outputs and modal connection DOF for the final step. Starting with

$$\mathbf{H}_{CC}^{INT} = (\mathbf{H}_{CC}^{PL^{-1}} + \mathbf{H}_{CC}^{DS^{-1}})^{-1} \quad (4)$$

and performing a similar transformation with the transmission simulator mode shapes yields

$$\mathbf{H}_{QQ}^{INT} = ((\Phi_{SC}^{TS+} \mathbf{H}_{CC}^{PL} \Phi_{SC}^{TS^{T+}})^{-1} + (\Phi_{SC}^{TS+} \mathbf{H}_{CC}^{DS} \Phi_{SC}^{TS^{T+}})^{-1})^{-1} \quad (5)$$

in which the sensor connection DOF have now been replace with modal connection DOF which are denoted with subscript Q. The output on the INT system with inputs at the Q DOF is analogous to equation (3) with

$$\mathbf{H}_{OQ}^{INT} = \mathbf{H}_{OC}^{PL} \Phi_{SC}^{TS^{T+}} (\Phi_{SC}^{TS+} \mathbf{H}_{CC}^{PL} \Phi_{SC}^{TS^{T+}} + \Phi_{SC}^{TS+} \mathbf{H}_{CC}^{DS} \Phi_{SC}^{TS^{T+}})^{-1} \Phi_{SC}^{TS+} \mathbf{H}_{CC}^{DS} \Phi_{SC}^{TS^{T+}}, \quad (6)$$

and the modal connection response on the INT system with inputs at the I DOF is

$$\mathbf{H}_{QI}^{INT} = \Phi_{SC}^{TS+} \mathbf{H}_{CC}^{PL} \Phi_{SC}^{TS^{T+}} (\Phi_{SC}^{TS+} \mathbf{H}_{CC}^{PL} \Phi_{SC}^{TS^{T+}} + \Phi_{SC}^{TS+} \mathbf{H}_{CC}^{DS} \Phi_{SC}^{TS^{T+}})^{-1} \Phi_{SC}^{TS+} \mathbf{H}_{CI}^{DS}. \quad (7)$$

The transmission simulator modal connection FRF is

$$\mathbf{H}_{QQ}^{TS} = \Phi_{SC}^{TS+} \mathbf{H}_{CC}^{TS} \Phi_{SC}^{TS^{T+}}. \quad (8)$$

The standard equation for adding two systems together with the input and output on system A is

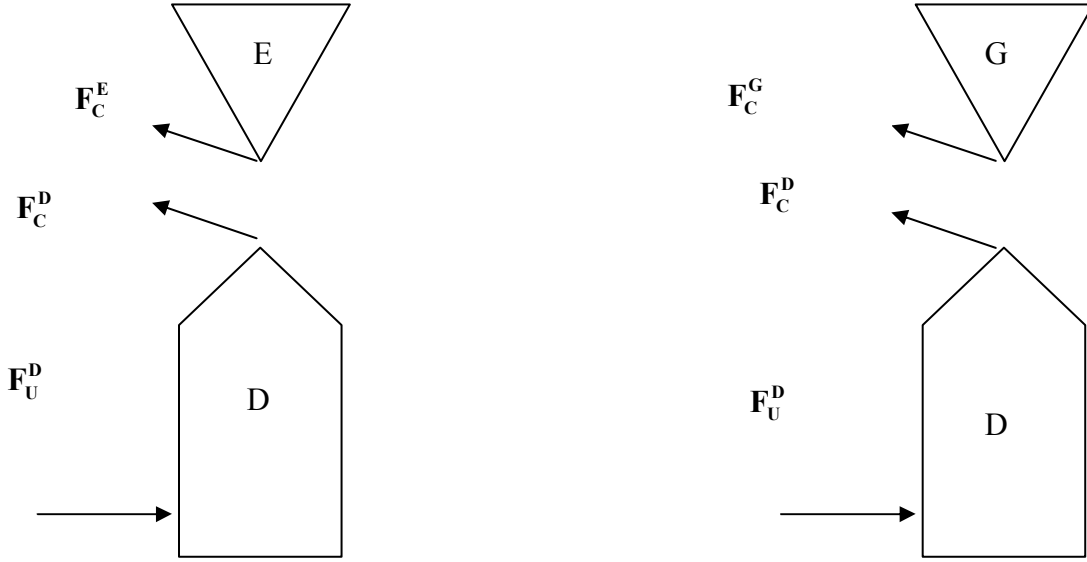
$$\mathbf{H}_{OI}^C = \mathbf{H}_{OI}^A - \mathbf{H}_{OC}^A (\mathbf{H}_{CC}^A + \mathbf{H}_{CC}^B)^{-1} \mathbf{H}_{CI}^A. \quad (9)$$

The final step is to subtract the transmission simulator (one set of mounting feet) using the form of equation (9) with both the input and output on INT and modal connection Q DOF as

$$\mathbf{H}_{OI}^{FINAL} = \mathbf{H}_{OI}^{INT} - \mathbf{H}_{OQ}^{INT} (\mathbf{H}_{QQ}^{INT} - \mathbf{H}_{QQ}^{TS})^{-1} \mathbf{H}_{QI}^{INT}. \quad (10)$$

B) PREDICTING CONNECTION POINT RESPONSE BASED ON A MULTI-SUBSTRUCTURE TEST WITH UNKNOWN BUT CONSTANT FORCE INPUT ON THE UPSTREAM SUBSTRUCTURE AFTER ANALYTICALLY CHANGING THE DOWNSTREAM SUBSTRUCTURE

Given substructure D which is excited by an unknown but constant force and attached to substructure E on which responses are desired, the connection point responses between D and E are measured. Remove substructure E and attach a new substructure G and predict the new connection point response using FBS. Here substructure D is denoted as the upstream substructure on which force is applied and substructures E or G as downstream substructures on which responses are desired, but no external force is applied. For our acoustic environmental test, substructure D would represent the delivery system which is being excited and substructures E or G might be different payloads. In the figures below, \mathbf{F}_U^D is the unknown applied force and \mathbf{F}_C^D , \mathbf{F}_C^E and \mathbf{F}_C^G are the connection forces between the substructures denoted by their superscript.



Consider the structure on the left side which will ultimately have substructure D coupled to substructure E, and will be excited by \mathbf{F}_U^D in an environmental test. For this system the connection force vectors

$$\mathbf{F}_C^D = -\mathbf{F}_C^E \quad (11)$$

where we denote the connection DOF as subscript C. We can write the following equations of motion in terms of FRFs as

$$\mathbf{X}_C^D = \mathbf{H}_{CU}^D \mathbf{F}_U^D + \mathbf{H}_{CC}^D \mathbf{F}_C^D \quad (12)$$

and

$$\mathbf{X}_C^E = \mathbf{H}_{CC}^E \mathbf{F}_C^E \quad (13)$$

where the superscripts are related to the particular substructure and subscripts are the response or input DOF. The \mathbf{H} systems are matrices and the \mathbf{X} responses are vectors. When the system is connected

$$\mathbf{X}_C^E = \mathbf{X}_C^D \quad (14)$$

So equate the right hand side of equation (12) and (13) to give

$$\mathbf{H}_{CU}^D \mathbf{F}_U^D + \mathbf{H}_{CC}^D \mathbf{F}_C^D = \mathbf{H}_{CC}^E \mathbf{F}_C^E . \quad (15)$$

Substituting equation (11) into equation (15) gives

$$\mathbf{H}_{CU}^D \mathbf{F}_U^D - \mathbf{H}_{CC}^D \mathbf{F}_C^E = \mathbf{H}_{CC}^E \mathbf{F}_C^E . \quad (16)$$

Gathering terms yields

$$\mathbf{H}_{CU}^D \mathbf{F}_U^D = (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^E) \mathbf{F}_C^E . \quad (17)$$

From equation (13) we can replace \mathbf{F}_C^E to give

$$\mathbf{H}_{CU}^D \mathbf{F}_U^D = (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^E) \mathbf{H}_{CC}^{E^{-1}} \mathbf{X}_C^{DE} \quad (18)$$

where \mathbf{X}_C^{DE} is the connection response of DOF C on either system D or E or the combined system DE. We specified in the beginning that the unknown force was constant. In this case \mathbf{H}_{CU}^D is also unknown but constant. Applying exactly the same approach to the system on the right hand side of the figure which has a different substructure G yields a similar result with

$$\mathbf{H}_{CU}^D \mathbf{F}_U^D = (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^G) \mathbf{H}_{CC}^{G^{-1}} \mathbf{X}_C^{DG} \quad (19)$$

To eliminate the unknown force and unknown FRF on the left hand side, equating the right hand side of equation (18) to equation (19) gives

$$(\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^E) \mathbf{H}_{CC}^{E^{-1}} \mathbf{X}_C^{DE} = (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^G) \mathbf{H}_{CC}^{G^{-1}} \mathbf{X}_C^{DG} \quad (20)$$

where \mathbf{X}_C^{DE} is known from measured test data and all other terms are measured FBS substructure matrices. Now we find the desired predicted response at the connection DOF as

$$\mathbf{X}_C^{DG} = \mathbf{H}_{CC}^G (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^G)^{-1} (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^E) \mathbf{H}_{CC}^{E^{-1}} \mathbf{X}_C^{DE} \quad (21)$$

This theory is based on an assumption that no external forces (other than the connection force to substructure D) are applied to substructures E or G. We can now also predict the responses of various measured internal DOF in substructure G. First determine equivalent connection point forces on system G from

$$\mathbf{F}_C^G = \mathbf{H}_{CC}^{G^{-1}} \mathbf{X}_C^{DG} \quad (22)$$

The final goal of determining output responses at DOF O on substructure G can be predicted from

$$\mathbf{X}_O^G = \mathbf{H}_{OC}^G \mathbf{F}_C^G \quad (23)$$

These can also be predicted with the modal responses of the transmission simulator applied to equations (22) and (23). Equation (22) in transmission simulator modal coordinates is

$$\mathbf{F}_Q^G = (\Phi_{SC}^{T+} \mathbf{H}_{CC}^G \Phi_{SC}^{+})^{-1} \Phi_{SC}^{T+} \mathbf{X}_C^{DG} \quad (24)$$

\mathbf{X}_C^{DG} , which is needed in equation (24), can be calculated from the more easily inverted transmission simulator modal matrices and \mathbf{X}_C^{DE} to modify equation (21) to give

$$\begin{aligned} \mathbf{X}_C^{DG} = & \mathbf{H}_{CC}^G \Phi_{SC}^{TS T+} (\Phi_{SC}^{TS+} \mathbf{H}_{CC}^D \Phi_{SC}^{TS T+} + \Phi_{SC}^{TS+} \mathbf{H}_{CC}^G \Phi_{SC}^{TS T+})^{-1} \times \\ & (\Phi_{SC}^{TS+} \mathbf{H}_{CC}^D \Phi_{SC}^{TS T+} + \Phi_{SC}^{TS+} \mathbf{H}_{CC}^E \Phi_{SC}^{TS T+}) (\mathbf{H}_{CC}^E \Phi_{SC}^{TS T+})^+ \mathbf{X}_C^{DE} \end{aligned} \quad (25)$$

Equation (23) in modal connection coordinates is

$$\mathbf{X}_O^{DG} = \mathbf{H}_{OC}^G \Phi_{SC}^{+} \mathbf{F}_Q^G \quad (26)$$

6) DISCUSSION

As one goes about planning, scheduling, and implementing a test series there are always hurdles that must be overcome before the actual data can even be analyzed.

Upon project initiation, came the reality of dealing with instrumentation issues and funding issues. As mentioned previously there must be a collaboration and availability of hardware in order to instrument, assemble, and perform these experiments that will then allow one to exercise the theory outlined in this paper. The funding, hardware and scheduling issues have delayed the completion of the project.

Preliminary data has been collected on the representative delivery system with the low fidelity payload which is to be used as a truth test as well as the medium and high fidelity payloads. Current work involves capturing data to characterize the high fidelity delivery system with the use of the transmission simulators. Further work involves the characterization of a low fidelity payload that was utilized to capture truth test data in the beginning of the program. Additional work revolves around the use of acoustical input as the excitation source and its impact upon the admittance testing theory being exercised in this paper.

7) CONCLUSION

Admittance testing of a complex aerospace structure or as we prefer the name frequency based substructuring continues to move into new areas of exploration. There are still many ideas or approaches that may be considered to perform additional experiments. As the model and structure move forward in areas of study, other areas of interest may arise.

Currently we have only gathered portions of the data to exercise the theory presented in this paper. Acoustic environmental tests in various configurations are being scheduled.

Consideration in areas such as instrumentation selection, setup, data acquisition setup, and noise levels should also be investigated as part of any large experiment as one relies on data being collected for supporting the admittance testing theory.

Future work will focus on implementing the theory outlined in the paper with more supporting data results from the original truth testing performed in order to verify the theory in a step by step fashion. The objective as testing moves forward in the program is to be able to predict output responses for each subsequent test in the series of tests for various excitation inputs with the use of admittance test theory for any payload or combination of payloads.

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