

# Fundamental Limits of Force and Transductions in Nano-Optomechanics.

SAND2011-3703C

## Phononics 2011

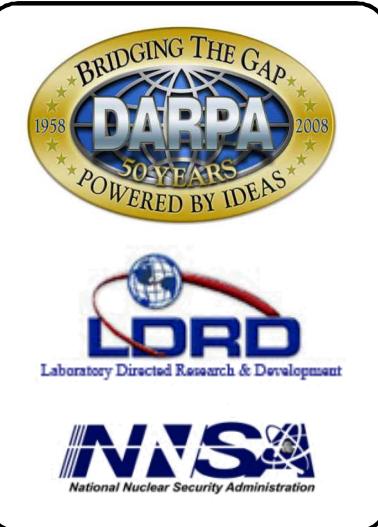
Peter T. Rakich<sup>1</sup>, Zheng Wang<sup>2</sup>, Ryan Camacho<sup>1</sup>,  
Charles Reinke<sup>1</sup>, Paul Davids<sup>1</sup>.

<sup>1</sup>*Applied Photonic Microsystems Group, Sandia National Labs, NM, USA*

<sup>2</sup>*Research Laboratory of Electronics MIT, Cambridge MA, USA*

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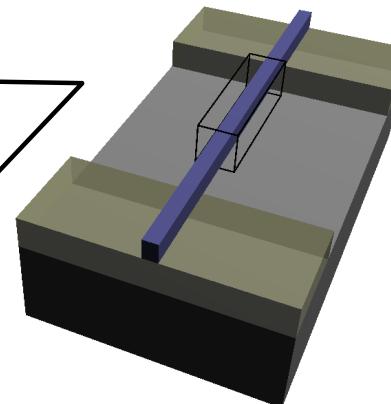
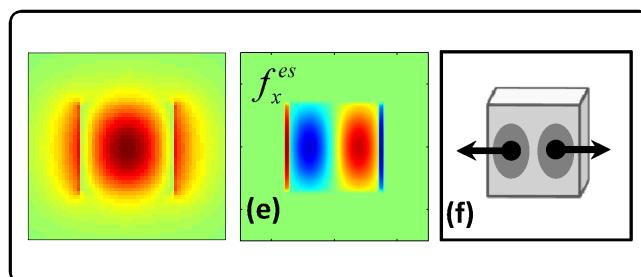


# Outline of Presentation:

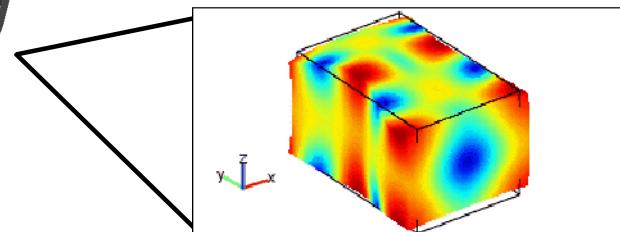
## Outline of Presentation:

1. Overview of photon-phonon coupling at micro- and nanoscales.
2. Examine origins of optical forces within nanoscale materials and geometries.
3. Explore scaling of stimulated photon-phonon coupling via new framework.

Material induced optical forces



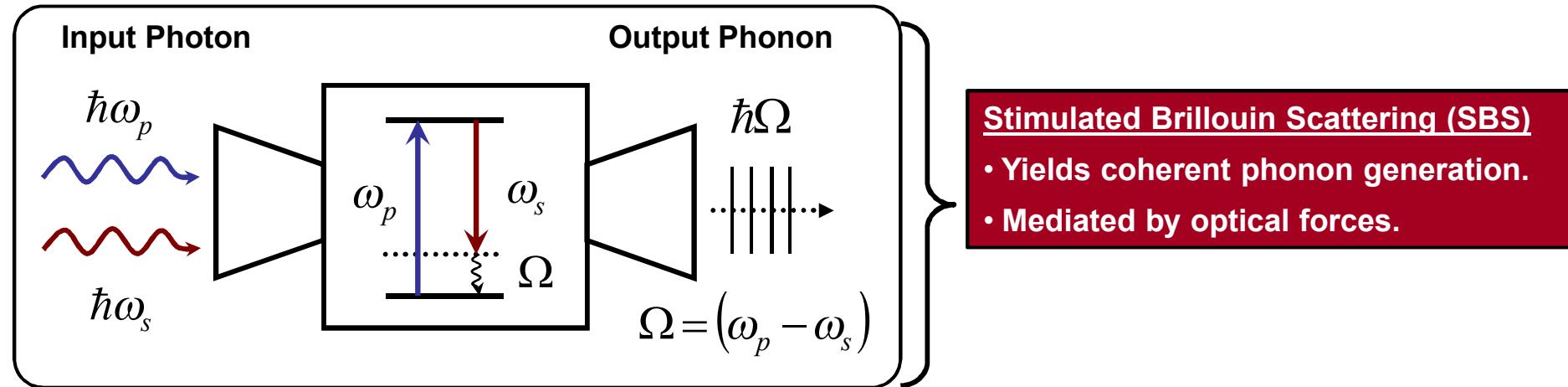
Nanoscale photon-phonon coupling



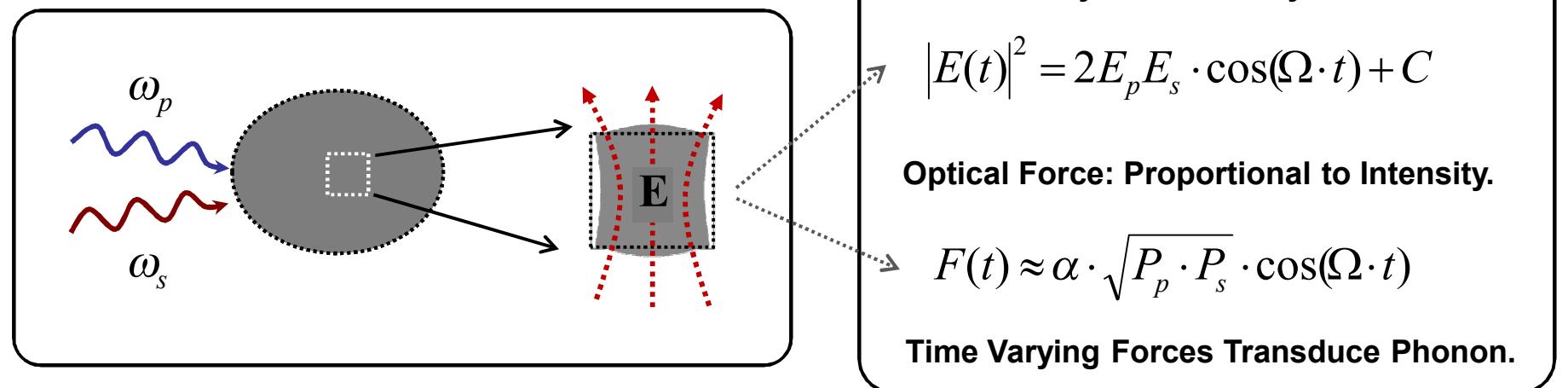
**Result: Giant Enhancement of Stimulated Brillouin Scattering at Nanoscales**

# How Does Stimulated Brillouin Scattering Work?

## Physics of stimulated phonon generation:

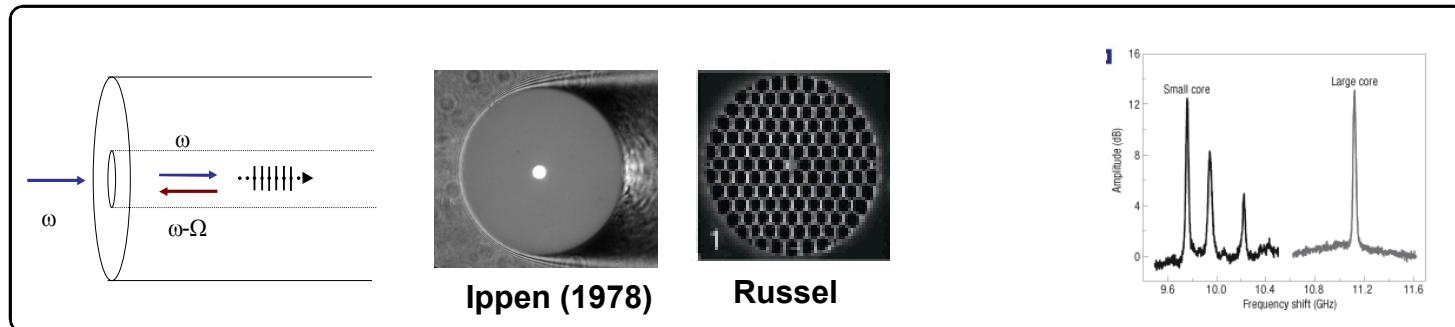


## Micro-scale origins of parametric process:

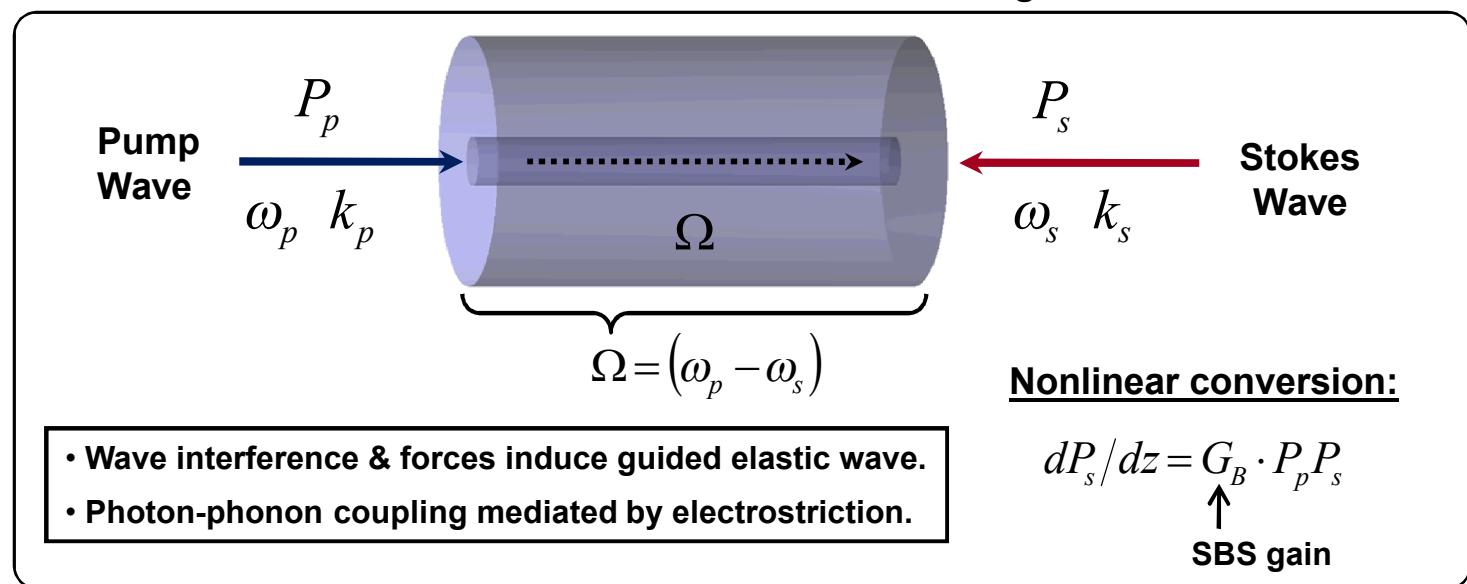


# Guided-Wave Stimulated Brillouin Scattering:

## Microscale Guided-Wave Stimulated Brillouin Scattering :

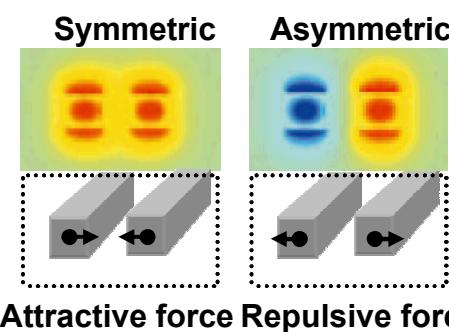


## How microscale stimulated Brillouin scattering works:



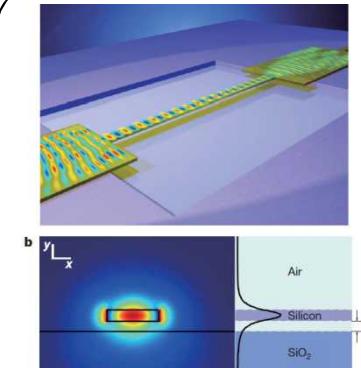
Nanoscales: light-matter interactions change in a fundamental way....

# Enhanced Radiation Pressure at Micro and Nanoscales



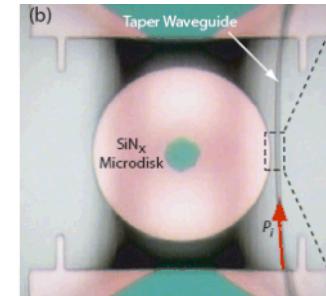
Attractive force Repulsive force

Povinelli, et. al. Opt. Lett. 30, 3042 (2005).

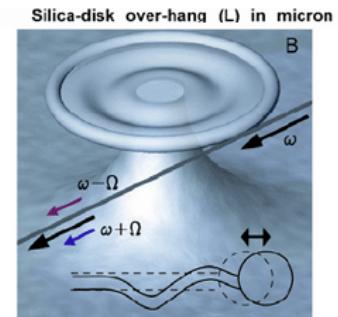


Mo Li et al. Nature 1, 480–484 (2008).

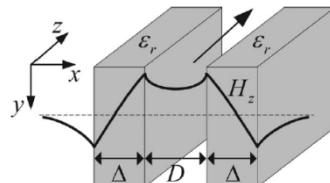
## Optical microdisk



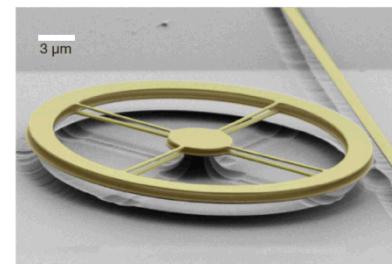
Eichenfield, M. et al. Nat. Photon. 1, 416–422 (2007).



T. Carmon, H. Rokhsari, L. Yang, T. Kippenberg, and K. Vahala, Phys. Rev. Lett. 94, 223902 (2005).



A. Mizrahi & L. Schächter Opt. Lett. 32, (2007), p.692-694

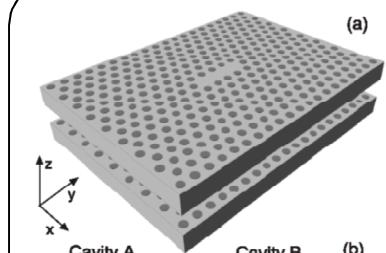


G. S. Wiederhecker, L. Chen et al. Nature 462 2009, p.633

## Dual microring geometry:



Rakich, Popovic et al. Nat. Photon. 1, 685-669 (2007).

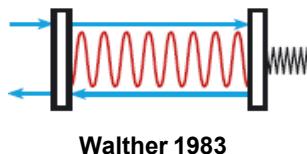


Notomi, M. and Taniyama, H. Phys. Rev. Lett. 94, p23903 (2006)

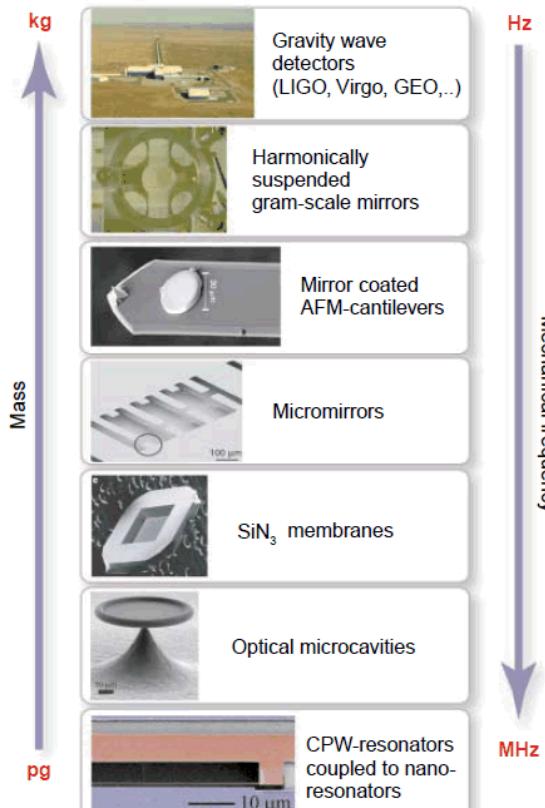
- Most of studies have focused on forces between discrete bodies.
- We will see that optical forces within bodies become very important to consider as well...

# Nano Optomechanics: Stimulated Scattering

## Cavity Optomechanics

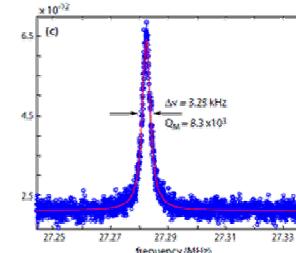
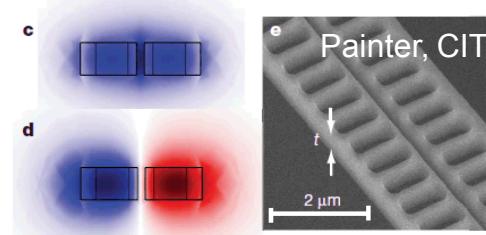


Walther 1983



Kippenberg, Vahala, Science 2008

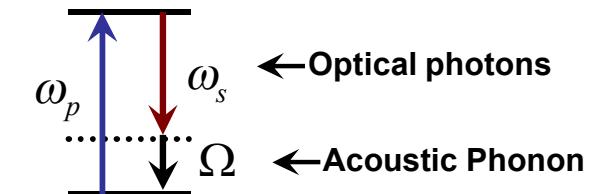
## Optomechanical Parametric Oscillation



## Radiation Pressure

Eichenfield, et. al. Nature, 459, 550 (2009).

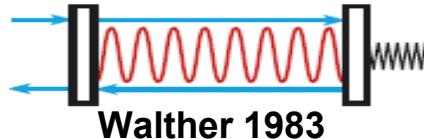
Radiation pressure mediates photon-phonon coupling.



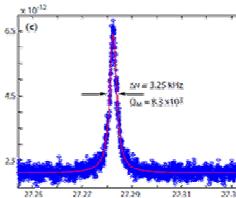
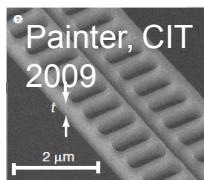
- Formally equivalent to Brillouin process.
- A key distinction: mediated by radiation pressure.

# Traveling-wave SBS at nanoscales:

## Cavity Optomechanics

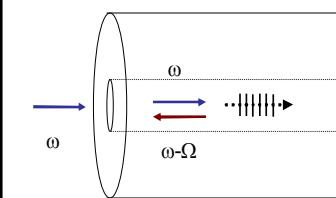


Walther 1983

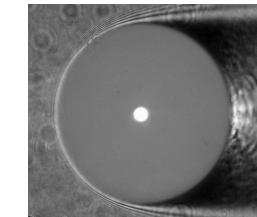


Painter (2009)

Nanoscales:  
Radiation pressure  
Mediated Stimulated  
Brillouin processes

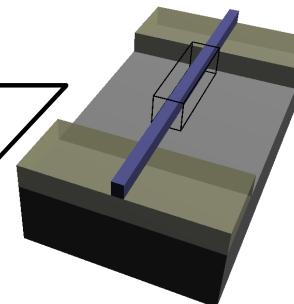
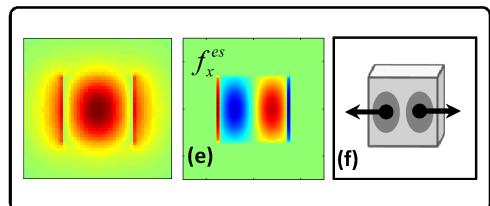


Microscales:  
Electrostrictively  
Mediated Stimulated  
Brillouin processes

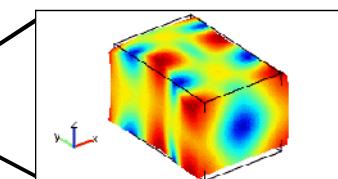


## Result: Giant Enhancement of Stimulated Brillouin Scattering at Nanoscales

Electrostriction + Radiation Pressure



10-40 GHz Phonons



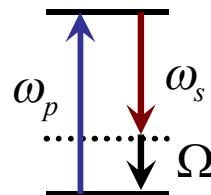
Radically enhanced SBS.

Electrostriction  
& Radiation  
Pressure  
mediate SBS

# How to Formulate SBS Gain in Nanoscale Systems?

For a time-harmonic force,  $F(t) = F_{opt} \cdot \cos(\Omega \cdot t)$ , produced by modal beating:

## Brillouin Process



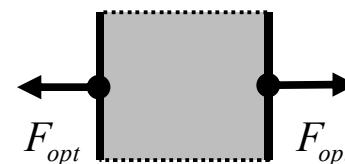
## Acoustic power

## Optical Force

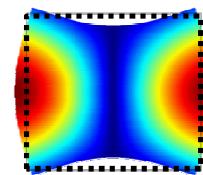
$$P_{ac} = \frac{F_{opt}^2}{2Z}$$

Mechanical Impedance

## Optical Force



## Displacement



“Mechanical Transduction in Periodic Media,”  
R. Camacho, (Paper 0130), Next.

“Analysis of Forces in Nanophotonic  
Waveguides,” C. Reinke. (Paper 0170),  
2:40-3:00 PM

$$dP_s/dz = G_B \cdot P_p P_s$$

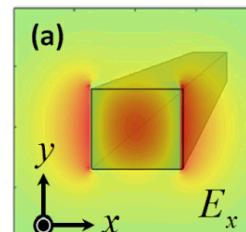
SBS gain

Exact solution:  $G_B = \left( \frac{\omega_s}{\Omega} \right) \cdot \frac{1}{P_s P_p} \frac{F_{opt}^2}{Z} \alpha$  ← Geometric scale-factor

Strong photon-phonon coupling requires: (1) large optical force, (2) small mechanical impedance.

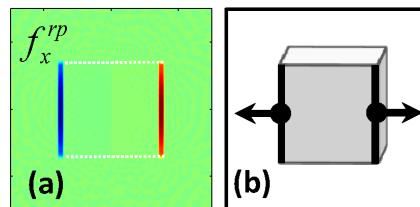
# Optical Forces at Nano-scales

## Silicon Waveguide.

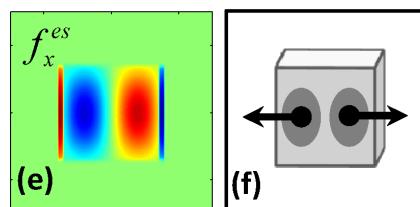


$300\text{nm} \times 300\text{nm}$

## Radiation Pressure



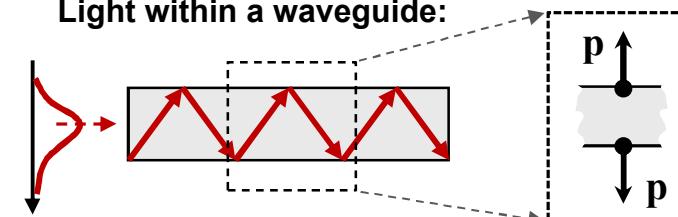
## Electrostrictive Forces



## Radiation Pressure.

### Radiation Pressure: Scattering from boundaries.

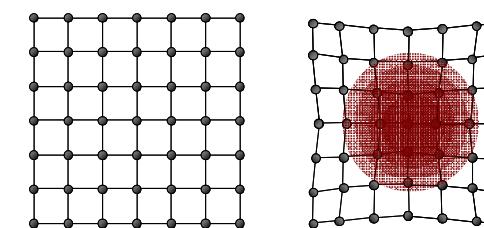
Light within a waveguide:



Entirely depends on geometry.

## Electrostrictive Forces

### Electrostriction: dynamic response of media to light.

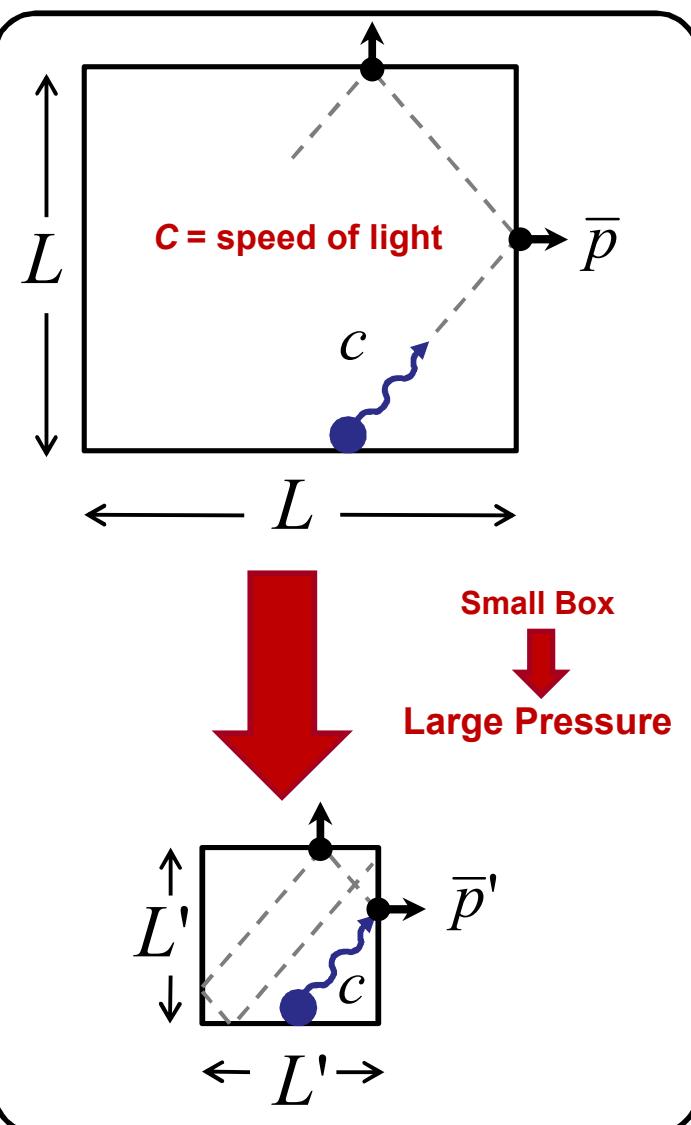


Depends primarily on material properties.

Both: Radically enhanced at nanoscales.

How to understand origin and enhancement?

# Why is Radiation Pressure Enhanced at Nano-scales?



## Equation of State for an Ideal Photon Gas:

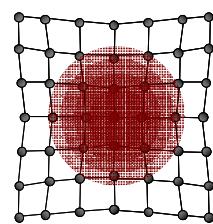
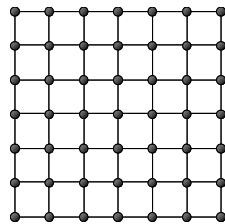
## Pressure Per Photon:

$$\bar{p} = \frac{1}{3} \cdot \frac{\hbar\omega}{V} = \frac{1}{3} \cdot \frac{\hbar\omega}{L^3}$$

**For  $L/L' = 1000$ ,  $p'/p = 1,000,000,000$ .**

Each photon packs more punch at nanoscales!

# Optical Forces Within Dielectric Media



$$S = \alpha \cdot E_i + \beta \cdot E_i E_j$$

Strain  
Piezo Coeff.  
Electrostrictive Coeff.

**Electrostriction = Material induced optical forces.**

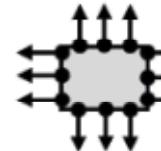
- All dielectrics exhibit electrostriction (not piezo electricity).
- Sign and magnitude are tailorable by choice of material



Germanium



Silicon

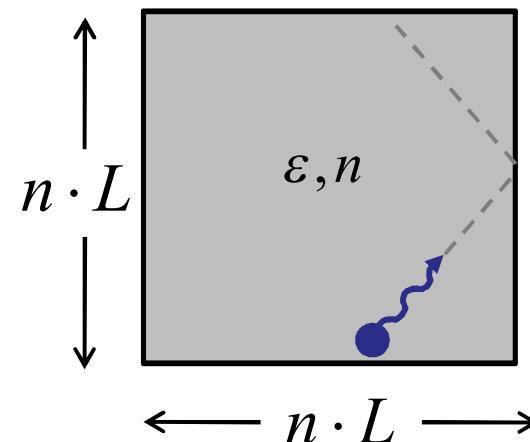
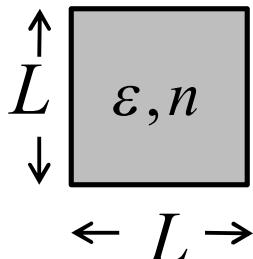


GaAs

**What's going on?**

## Box: Photon's Perspective

### Box: Real Space



### From Photon's Perspective:

- Space is now quite different.

$$V \Rightarrow V \cdot n^3$$

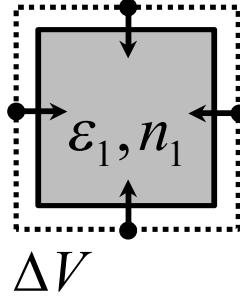
Box seems much bigger.

Oddities don't end here...

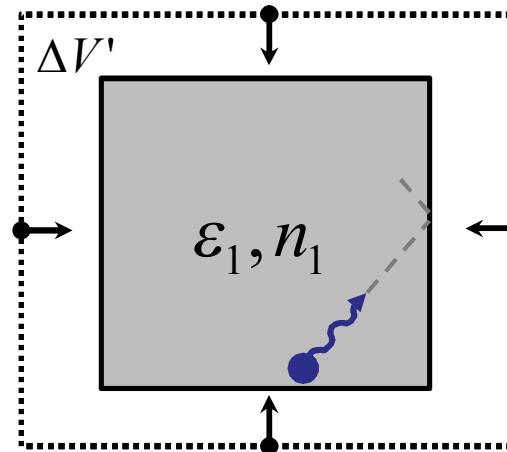
# Electrostriction: Material Induced Forces

Since  $\partial n / \partial V \neq 0$ , distortion of the box is perceived quite differently by a photon:

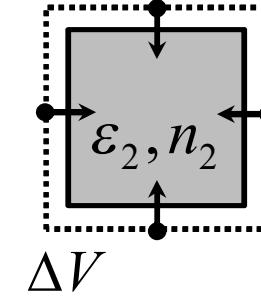
Box: Real Space



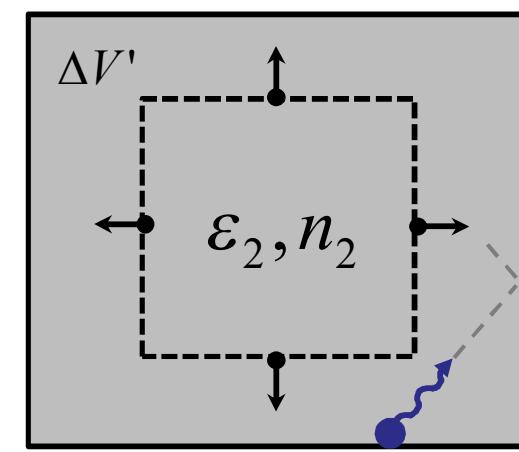
Box: Photon's Perspective



Box: Real Space



Box: Photon's Perspective



Photon gas within a dielectric:

$$\Delta V' \Rightarrow n^3 \Delta V + 3n^2 V \cdot \left( \frac{\partial n}{\partial V} \right) \cdot \Delta V$$

Correction due to dynamic response

Photon gas within a dielectric:

$$\bar{p} = \frac{1}{3} \cdot \frac{N \cdot \hbar \omega}{V} \left[ 1 + \frac{3}{n} \frac{\partial n}{\partial V} \right]$$

Electrostriction

# Alternative: Electrostrictive Stress & Force.

## Dependence on material properties:

$$\left\{ \begin{array}{ll} \epsilon_{kl} & \text{(Dielectric Tensor)} \\ p_{jkmn} & \text{(Elasto-optic Tensor)} \end{array} \right\}$$

### Electrostrictive Stress in Cubic Crystal (Si):

$$\sigma_{kl}^{es} = -\frac{1}{2} \epsilon_0 \cdot n^4 \cdot p_{ijkl} \cdot E_i E_j$$

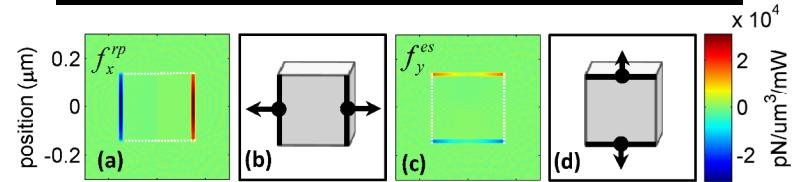
Force Density:  $\mathcal{F}_j = -\partial_i \sigma_{ij}$

### Important Properties of Stress/Force

1. Increases as  $n^4$ .
2. Proportional to  $p_{ijkl}$ .
3. Sign & magnitude depends on  $p_{ijkl}$

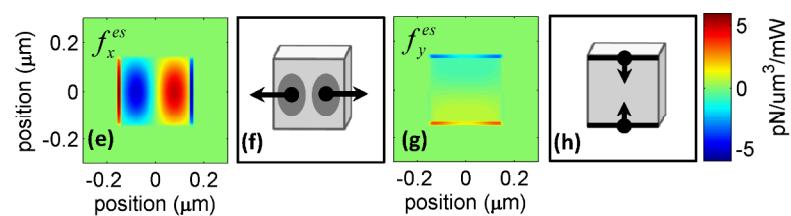
## Example: Silicon waveguide.

### Radiation Pressure: Si waveguide



1. Force localized to boundary.
2. Directed outward.

### Electrostriction: Si waveguide

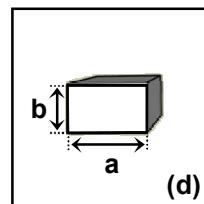


1. Force distributed within volume.
2. Directed outward or inward.

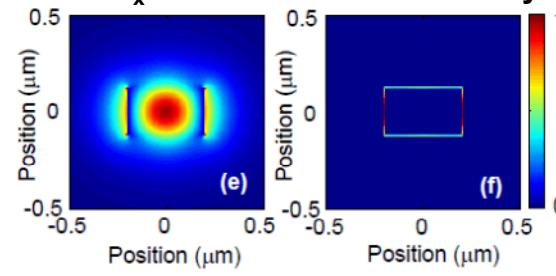
[1] P. T. Rakich, P. Davids, and Z. Wang, "Tailoring Optical Forces in Waveguides Through Radiation Pressure and Electrostriction," Opt. Express 18, 14439-14453 (2010)

# Key to Radiation Pressure: Dispersion.

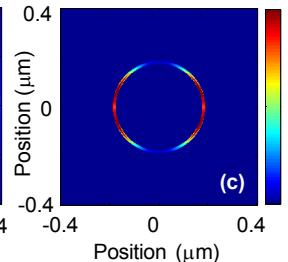
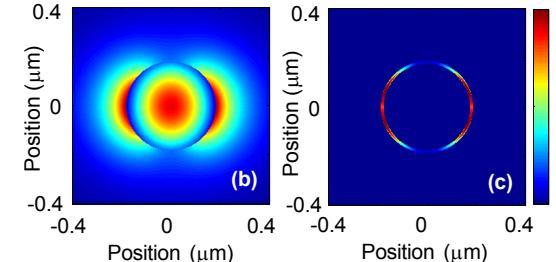
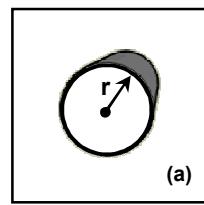
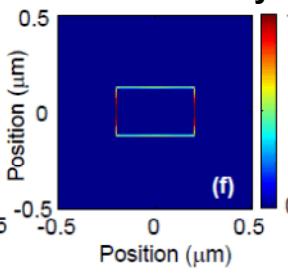
Geometry



$E_x$ -field



Force Density



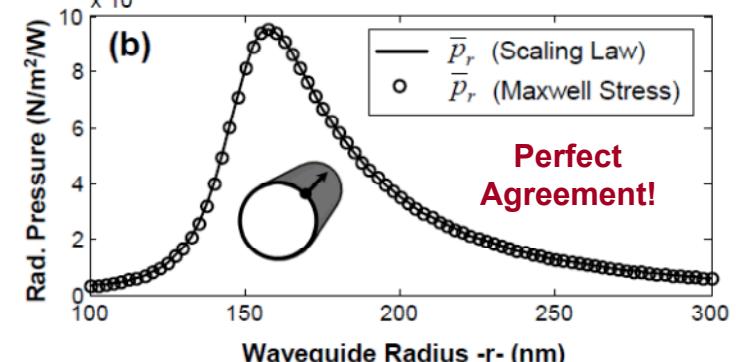
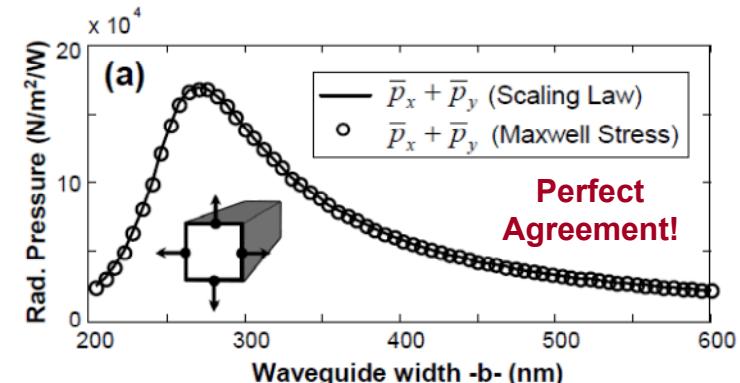
## Radiation Pressure:

$$\bar{p}^{rp} = \frac{P_{opt}}{c \cdot A} \cdot (n_g - n_p) \leq \frac{P_{opt}}{c \cdot A} \cdot (2\Delta n)$$

Relation Holds for Any Dielectric Waveguide and Any Guided Mode!

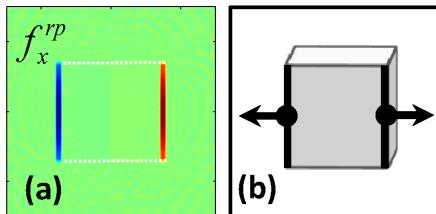
[1] P. T. Rakich, Z. Wang, and P. Davids "Scaling of Optical Forces in Dielectric Waveguides: Rigorous Connection Between Dispersion and Radiation Pressure," *Optics Letters*.

## Comparison of Maxwell Stress & Scaling Law



# Nanoscales: The Neglected Optical Force Dominates!

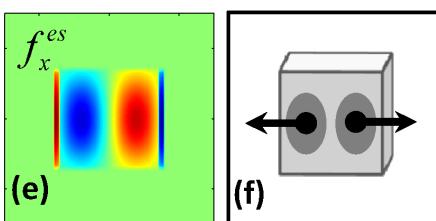
## Radiation Pressure



**Rad. Pressure:**  $\bar{p}^{rp} = \frac{P_{opt}}{c \cdot A} \cdot (n_g - n_p) = \frac{P_{opt}}{c \cdot A} \cdot \alpha^{rp}$

**Electrostriction:**  $\bar{p}^{es} = \frac{P_{opt}}{c \cdot A} \cdot n_g n^2 (p_{11} + 2p_{12}) / 2 = \frac{P_{opt}}{c \cdot A} \cdot \alpha^{es}$

## Electrostrictive forces



Material	Symmetry	$p_{11}$	$p_{12}$	$p_{11} + 2p_{12}$	$n$	$\alpha^{rp}$	$\alpha^{es}$
Si	cubic	-0.09	+0.017	-0.056	3.5	-5	-1.7
Ge	cubic	0.27	0.235	0.74	4.2	-6.4	+40
GaAs	cubic	-0.165	-0.14	-0.445	3.4	-4.8	-12
Silica	amorphous	0.121	0.27	0.661	1.45	-0.89	+1.0
As <sub>2</sub> S <sub>3</sub>	amorphous	0.25	0.24	0.73	2.4	-2.8	+6.5
As <sub>2</sub> Se <sub>3</sub>	amorphous	-	-	-	2.8	-3.6	-

## How Large are Forces?

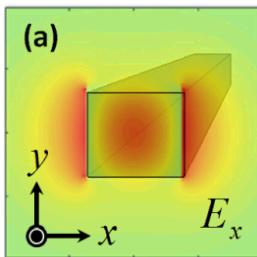
Material	Pressure (Pwr = 100mW)
Si	$\sim 5 \times 10^4 \text{ N/m}^2$
Ge	$\sim 10^6 \text{ N/m}^2$

5-50 People standing on manhole cover

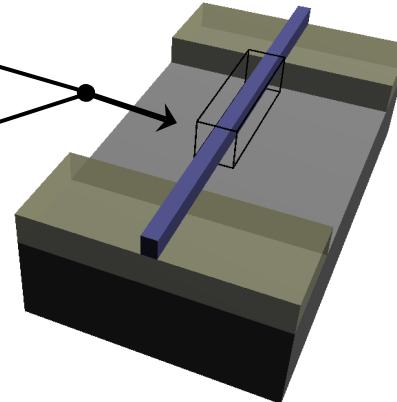
Material	Pressure (Pwr = 1kW)
Si	$\sim 5 \times 10^8 \text{ N/m}^2$
Ge	$\sim 10^{10} \text{ N/m}^2$

Stresses Approach Material Yield Strength

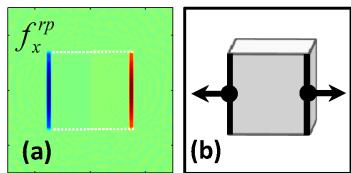
# Next: Examine photon-phonon coupling.



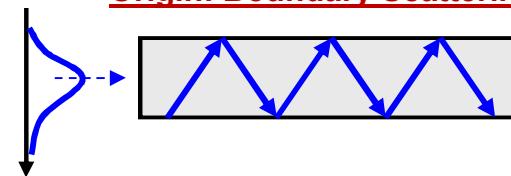
Guided mode within suspended dielectric waveguide. (300x300nm)



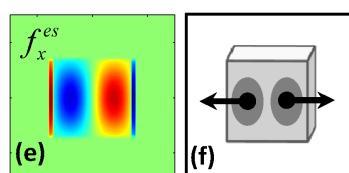
## Radiation Pressure



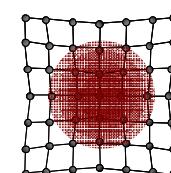
## Origin: Boundary Scattering



## Electrostrictive forces



## Origin: Dynamic Material Response

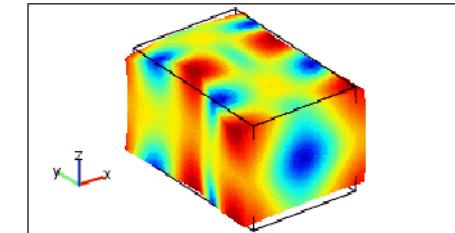


Akin to piezoelectricity

$$S = \alpha \cdot E_i + \beta \cdot E_i E_j$$

Strain      Piezo Coeff.      Electrostrictive Coeff.

Generalized theory of photon-phonon coupling:

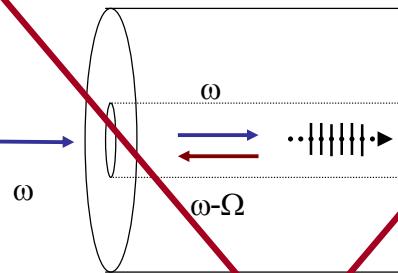


Reveals powerful new photon-phonon coupling processes at nanoscales.

# How to Formulate SBS Gain in Nanoscale Systems?

## Unified Treatment of SBS: Valid at any Length-Scale

### Microscale SBS Theory:



$$g_B = \frac{2\pi n^2 p_{12}^2}{c \lambda_p^2 \rho v_a \Delta V_B}$$

- Neglects radiation pressure.
- Simplified elastic-wave mode.

Not valid at nanoscales.

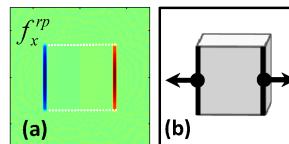
### Canonical relation for SBS gain:

$$dP_s/dz = G_B \cdot P_p P_s$$

SBS gain

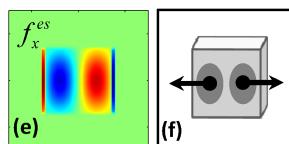
$$G_B(\Omega) = \frac{1}{\delta z} \frac{\omega_s}{\Omega} \cdot \frac{1}{P_p \cdot P_s} \int_{\delta V} \langle \mathbf{f}_\Omega(\mathbf{r}, t) \cdot \dot{\mathbf{u}}(\mathbf{r}, t) \rangle \cdot dV,$$

#### Radiation Pressure:

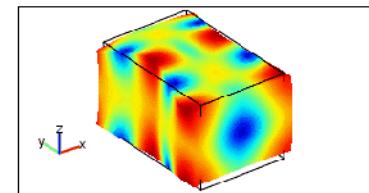


#### Time harmonic force

#### Electrostriction:



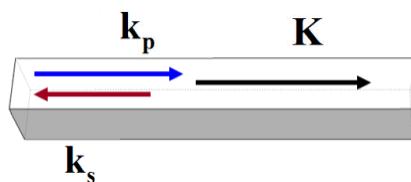
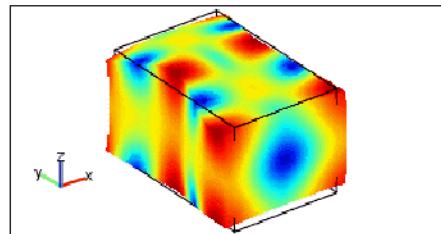
#### Velocity distribution



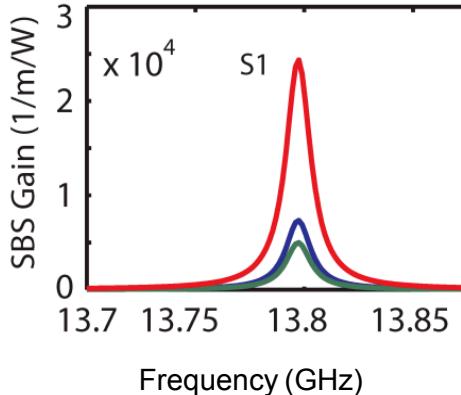
# SBS Radically Enhanced at Nanoscales:

The image cannot be displayed. Your computer may not have enough memory to open the image. If this is the case, you may need to free up memory by closing other programs, and then open the file again. If you still cannot open the image, you may need to delete the image and then insert it again.

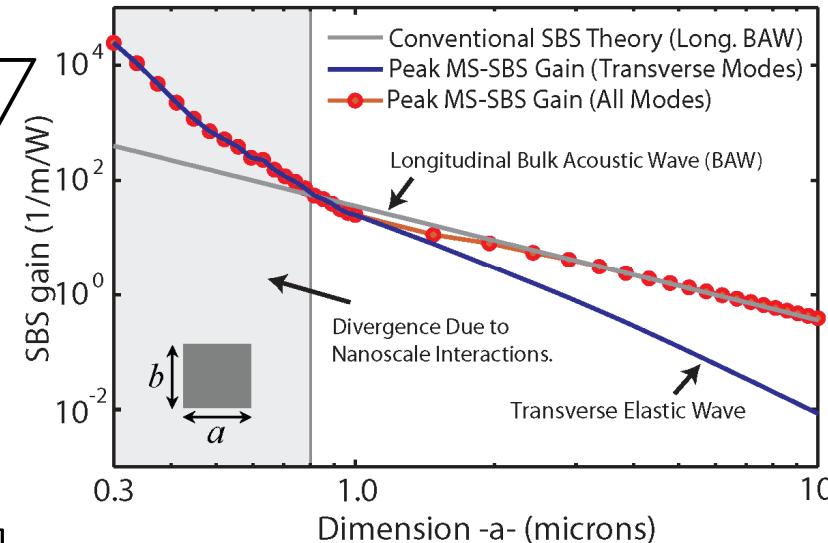
## Nanoscale Stimulated Brillouin Scattering (SBS)



- Electrostriction
- Radiation Pressure
- Electrostriction & Radiation Pressure



## Photon-phonon coupling vs dimension.



## Conventional Treatment of SBS.

$$g_B = \frac{2\pi n^7 p_{12}^2}{c \lambda_p^2 \rho v_a \Delta \nu_B}$$

### Conventional theory:

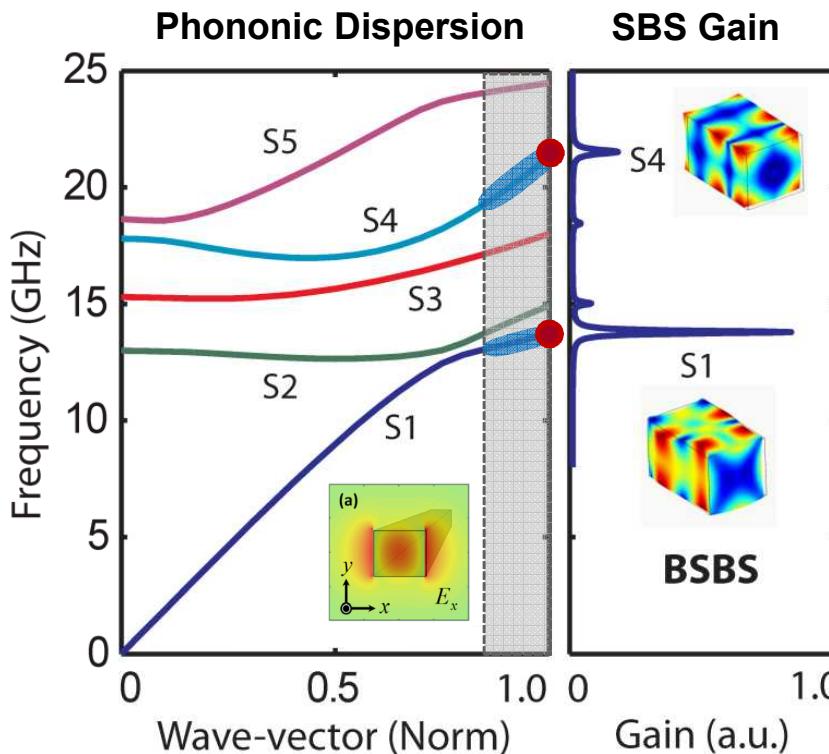
1. Silicon material properties used.
2. Perfect agreement from 2-10 microns.

### Nanoscale SBS:

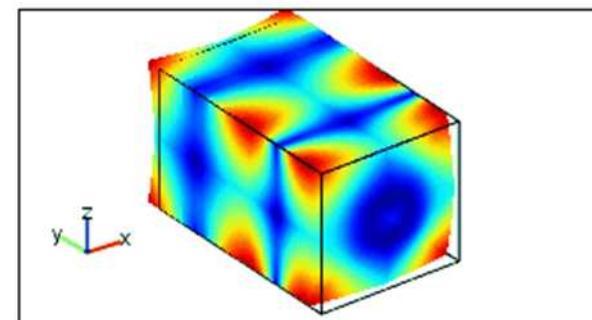
1. For dimensions  $< 800\text{nm}$ , conventional SBS theory breaks down.
2. Rigorous model reveals 100x enhanced SBS at small dimensions.

Coherent combination of electrostriction and radiation pressure produce further enhancement.

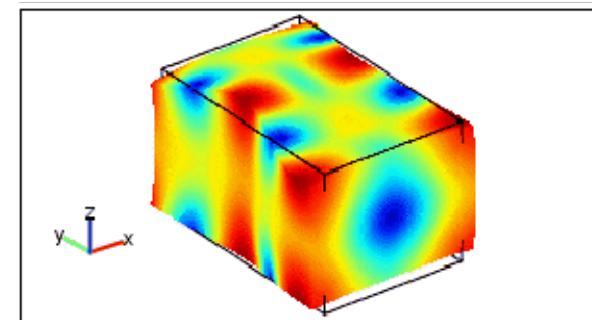
# Nanoscale Backward-SBS: Tunable Phonon Emission.



## Excitation: 21.6 GHz Phonons

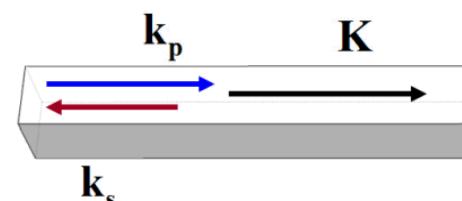


## Excitation: 13.8 GHz Phonons

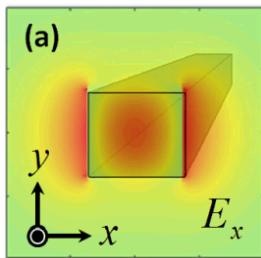


### Nano-optomechanical backward-SBS:

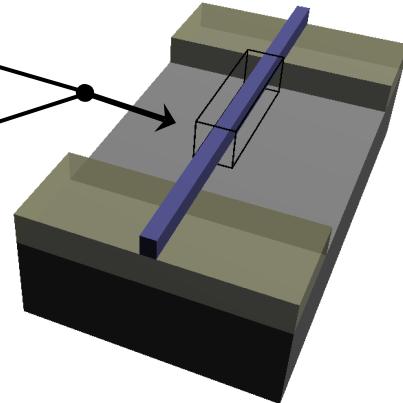
1. Gain is  $10^6$  x Larger than in Fibers.
2. 20% frequency tunable phonon emission.



# How Strong is SBS:

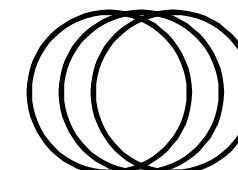
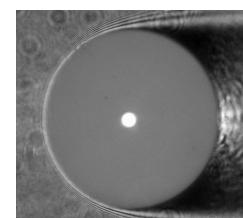


Guided mode within suspended dielectric waveguide. (300x300nm)

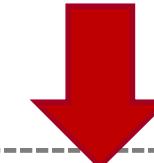


Suspended waveguide:  $L = 100$  microns

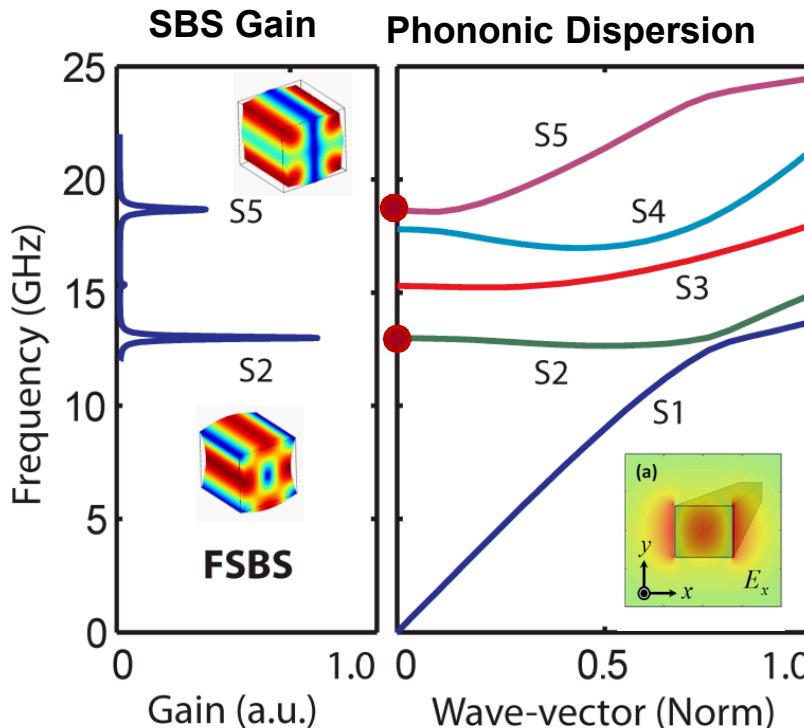
= SBS nonlinearity of 10-100 meters of fiber



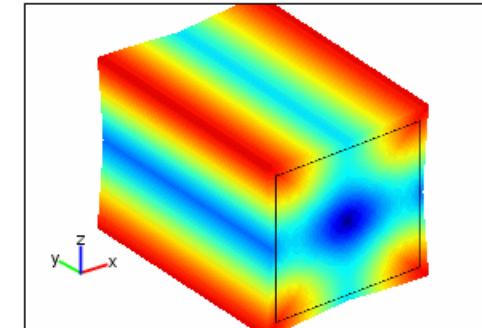
Fiber optic:  $L = 10-100$  meters



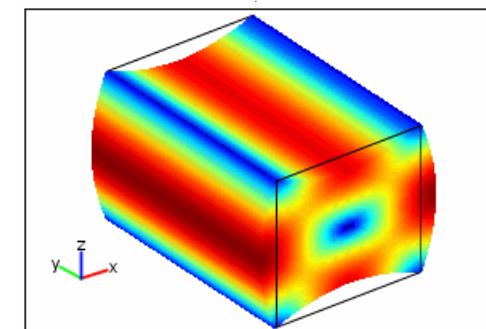
# Forward-SBS: Fixed Frequency Resonances.



Excitation: 18.6 GHz Phonons

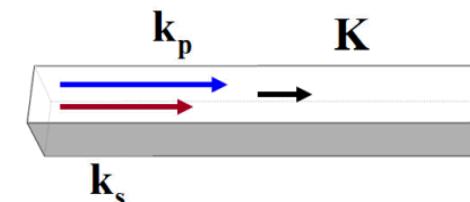


Excitation: 13.0 GHz Phonons



## Nano-optomechanical Forward-SBS:

1. Generally forbidden in guided wave-systems.
2. Ultra-low threshold parametric oscillation possible.



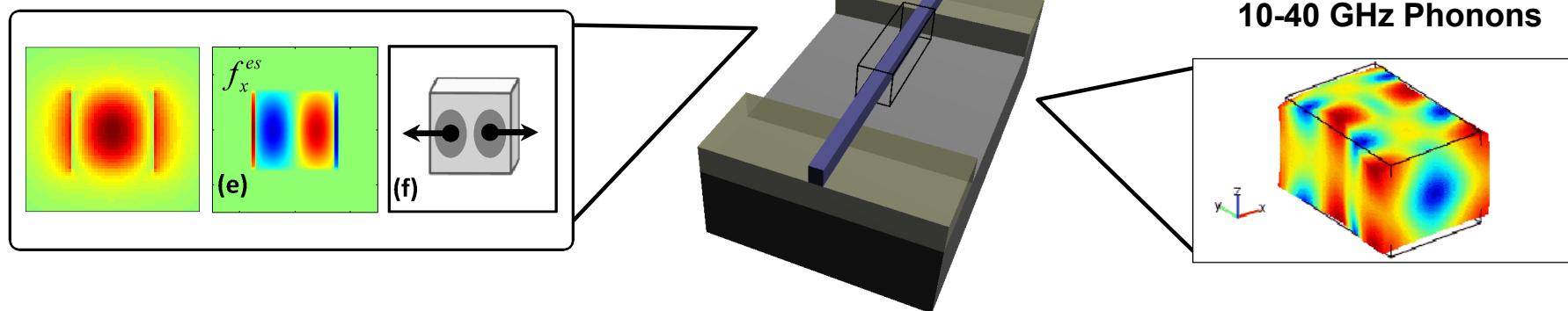
# Conclusions:

Developed unified treatment of SBS at nanoscales:

1. Valid at any lengthscale (micro- to nano-scales).
2. Radically enhanced SBS processes found  
    → Resulting from electrostriction & radiation pressure.
3. Forward SBS: Excitation of ultra-high frequency modes.
4. Backward SBS 1E6 x stronger than in fiber.
5. Important step towards highly tailorable chip-scale SBS.

“Giant Enhancement of Stimulated Brillouin Scattering at Nanoscales,” P. Rakich, C. Reinke, R. Camacho, P. Davids, Z. Wang, (submission to PRL).

## Material induced optical forces





# Acknowledgments:

## Special thanks:

**Ihab El-kady, Troy Olsson, Milos Popovic**

## Funding Agencies:

**DARPA—MTO (PMs: Mike Haney, Scott Rodgers, Jeff Rogers).**

**DOE—Laboratory Directed Research and Development funding.**

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

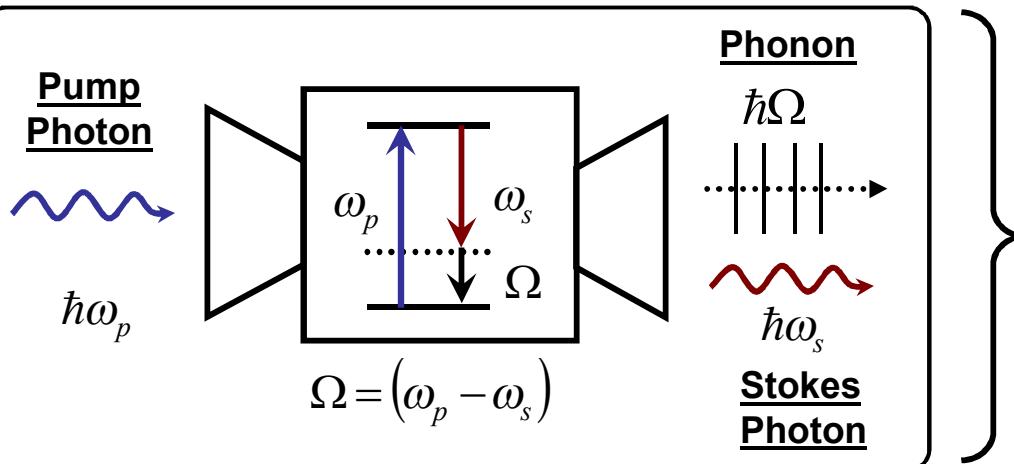
This work was supported in part by the office of the Director of Defense Research and Engineering under Air Force contract FA8721-05-C-0002.



# Backup Slides Follow:

# What is a Brillouin Process?

## Brillouin Scattering Energy Level Diagram:



### Brillouin Scattering:

3<sup>rd</sup> order Parametric process by which photons couple to acoustic phonons.

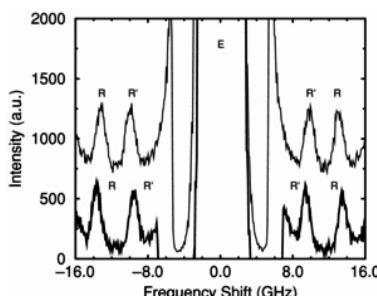
Bulk Media

Micro-Scales

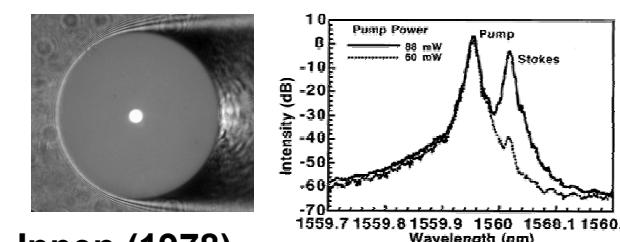
Nano-Scales



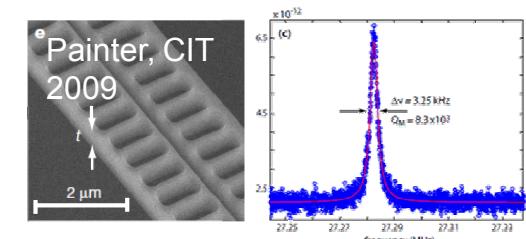
### BLS in Bulk Media



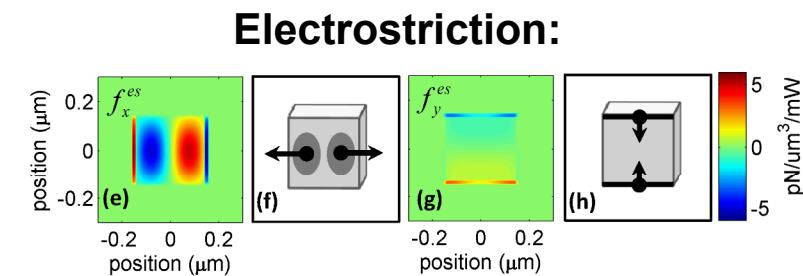
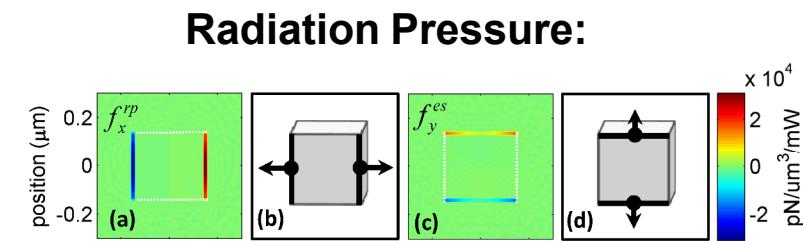
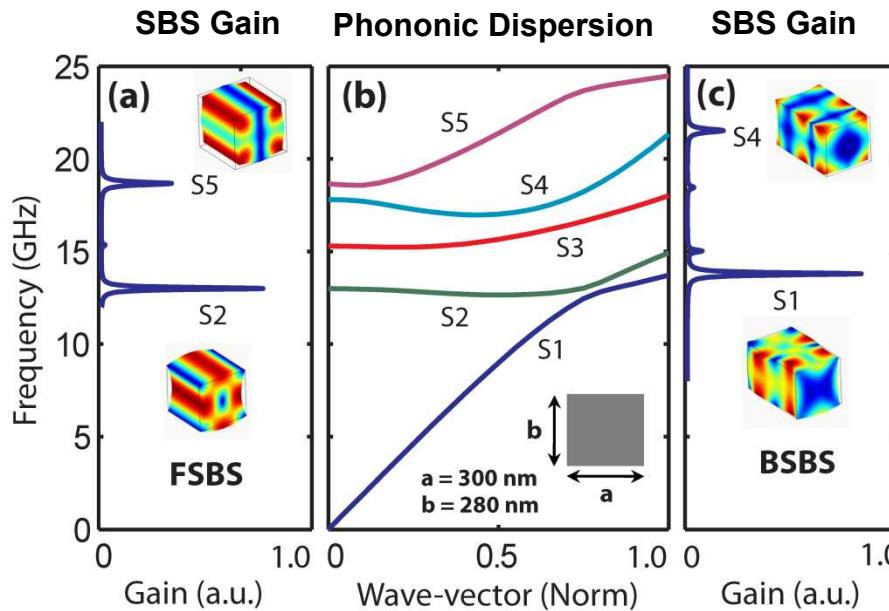
### SBS bulk media and optical fibers



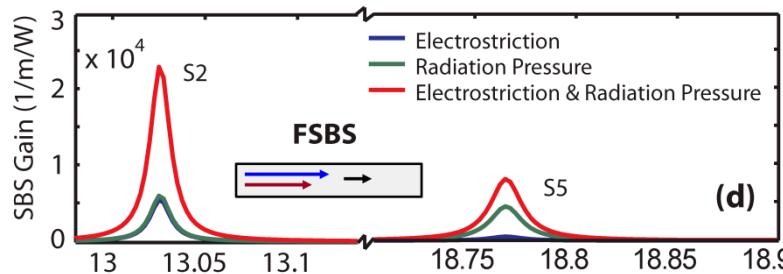
### Nano-scale SBS and BLS



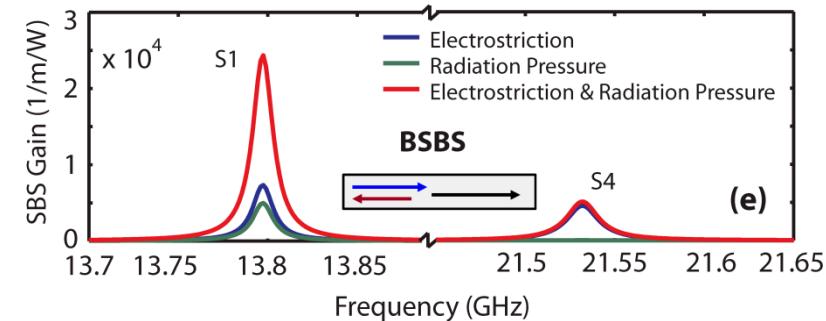
# Contributions of Different Forces at Nanoscales:



## Forward SBS (FSBS):

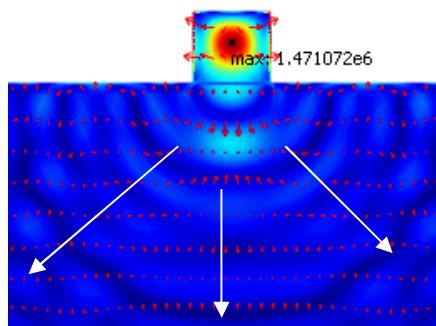


## Backward SBS (BSBS):



# SBS in Silicon Photonics?

## Transduced Waves:

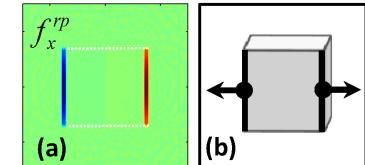


Canonical Treatment of SBS is not valid for Si waveguides!

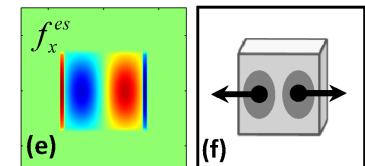
### What's Different?

- Elastic Wave Leakage:  
→ Makes SBS extremely weak.
- Must treat combination of:
  1. Radiation Pressure
  2. Electrostriction

### Radiation Pressure



### Electrostrictive forces

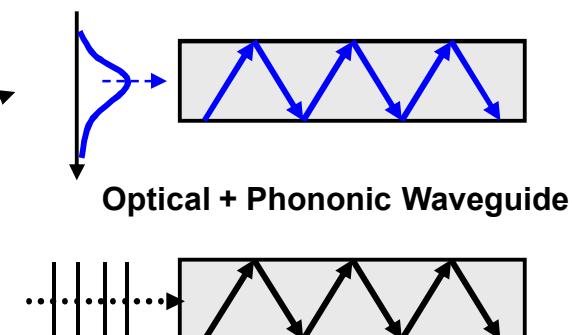
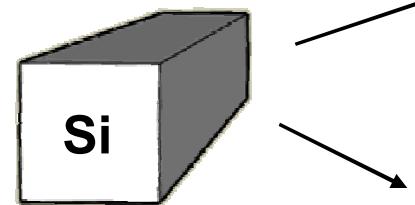


## Simplest Way to Control Photons and Phonons for Enhanced SBS:

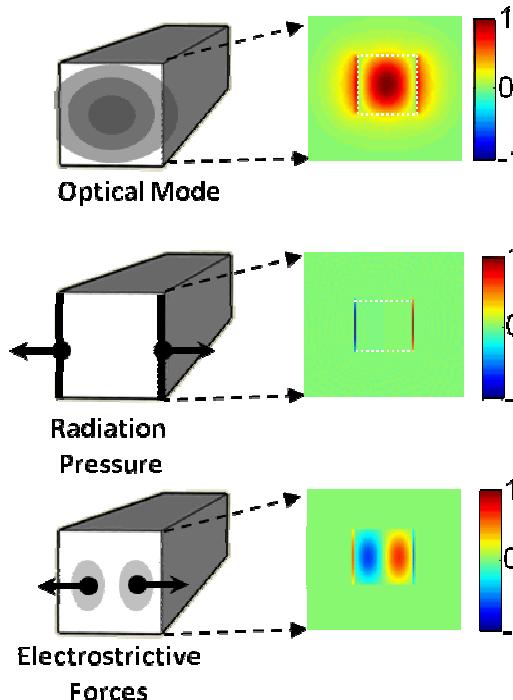
### Nano-wire Supports:

- Guided *Optical Wave*
- Guided *Elastic Wave*

### Suspended Waveguide



# Nanoscales: The Neglected Optical Force Dominates!



**Rad. Pressure:**  $\bar{p}^{rp} = \frac{P_{opt}}{c \cdot A} \cdot (n_g - n_p) = \frac{P_{opt}}{c \cdot A} \cdot \alpha^{rp}$

**Electrostriction:**  $\bar{p}^{es} = \frac{P_{opt}}{c \cdot A} \cdot n_g n^2 (p_{11} + 2p_{12})/2 = \frac{P_{opt}}{c \cdot A} \cdot \alpha^{es}$

Material	Symmetry	$p_{11}$	$p_{12}$	$p_{11} + 2p_{12}$	$n$	$\alpha^{rp}$	$\alpha^{es}$
Si	cubic	-0.09	+0.017	-0.056	3.5	-5	-1.7
Ge	cubic	0.27	0.235	0.74	4.2	-6.4	+40
GaAs	cubic	-0.165	-0.14	-0.445	3.4	-4.8	-12
Silica	amorphous	0.121	0.27	0.661	1.45	-0.89	+1.0
As <sub>2</sub> S <sub>3</sub>	amorphous	0.25	0.24	0.73	2.4	-2.8	+6.5
As <sub>2</sub> Se <sub>3</sub>	amorphous	-	-	-	2.8	-3.6	-

## How Large are Forces?

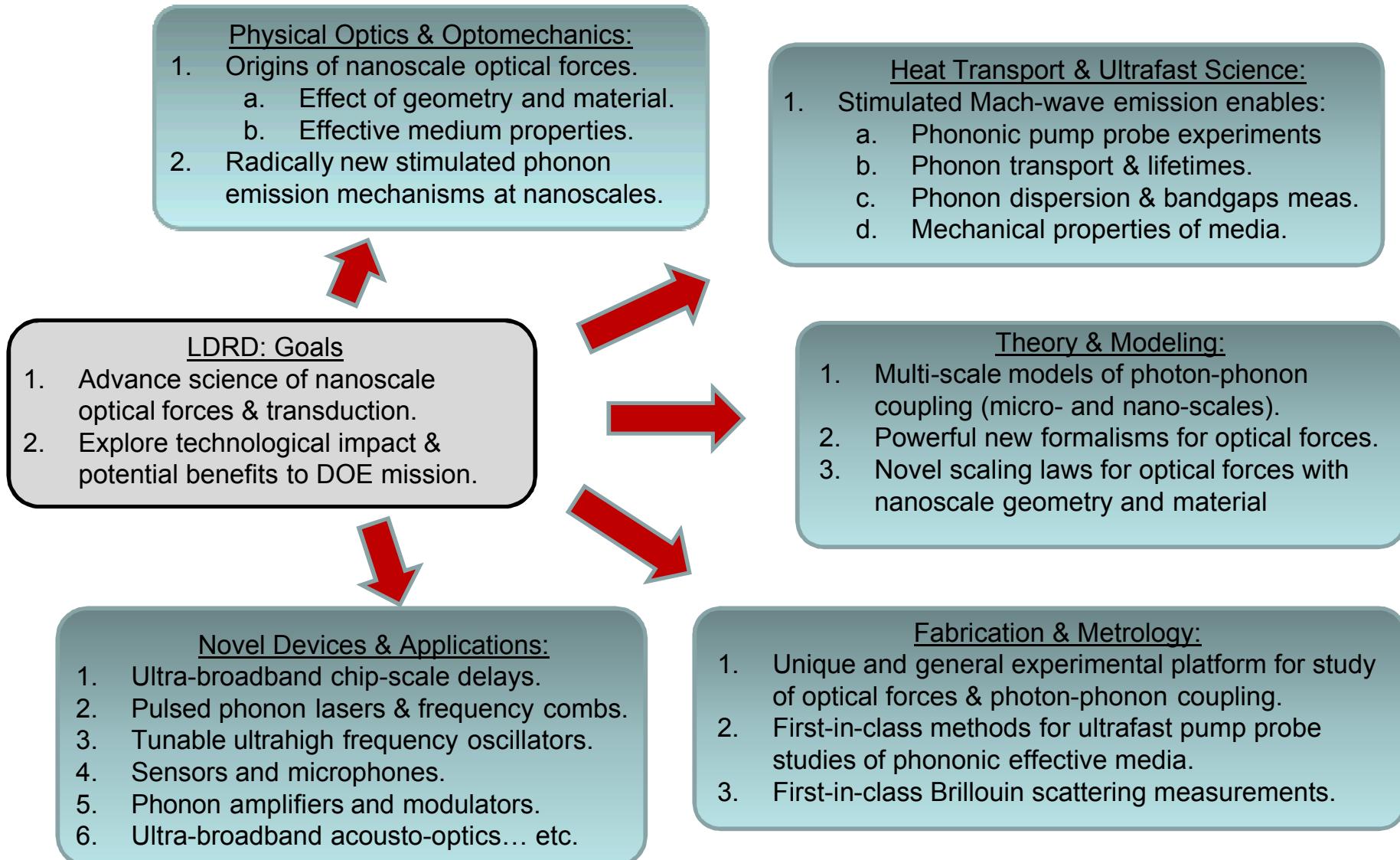
Material	Pressure (Pwr = 100mW)
Si	$\sim 5 \times 10^4 \text{ N/m}^2$
Ge	$\sim 10^6 \text{ N/m}^2$

5-50 People standing on man-hole cover

Material	Pressure (Pwr = 1kW)
Si	$\sim 5 \times 10^8 \text{ N/m}^2$
Ge	$\sim 10^{10} \text{ N/m}^2$

Stresses Approach Material Yield Strength

# Significance of LDRD: Big Picture & Impact.



# How are Phonons (acoustic waves) Generated by Light?

## Physics of stimulated phonon generation:

Input Photon

$$\begin{aligned} \hbar\omega_p \\ \hbar\omega_s \end{aligned}$$

Output Phonon

$$\hbar\Omega$$

$$\Omega = (\omega_p - \omega_s)$$

**Stimulated Brillouin Scattering (SBS)**

Describes all optomechanical photon-phonon coupling processes (including cavity optomechanics).

## Micro-scale origins of parametric process:

$$\begin{aligned} \hbar\omega_p \\ \hbar\omega_s \end{aligned}$$

Interference yields intensity “Beat Note”.

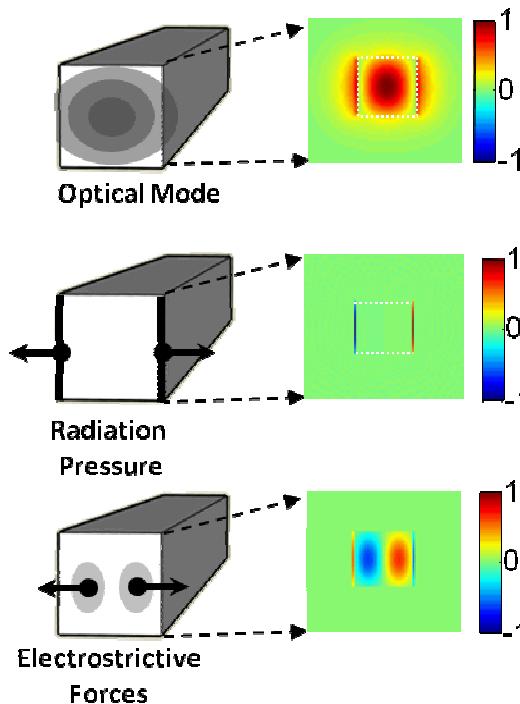
$$|E(t)|^2 = 2E_p E_s \cdot \cos(\Omega \cdot t) + C$$

Optical Force: Proportional to Intensity.

$$F(t) \approx \alpha \cdot \sqrt{P_p \cdot P_s} \cdot \cos(\Omega \cdot t)$$

Time Varying Forces Transduce Phonon.

# Optical Forces at Nano-scales

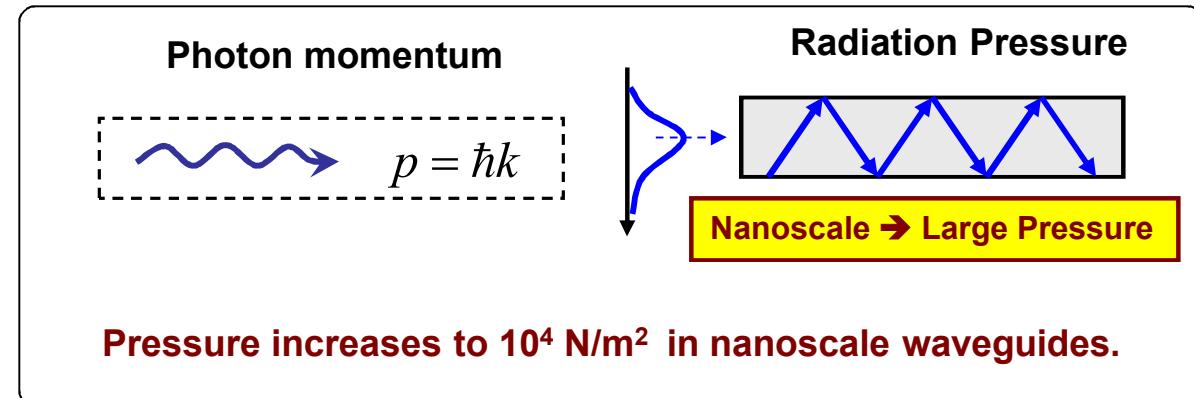


Forces produced by two Effects:

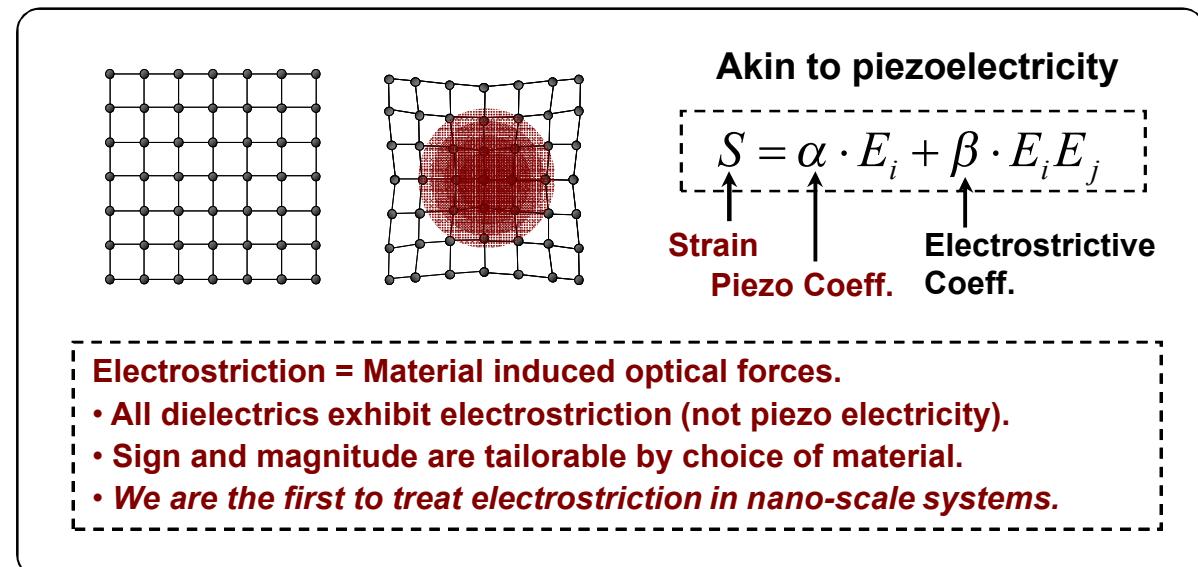
1. Radiation Pressure
2. Electrostriction

Both Scale to large values with nanometer-scale optical confinement (e.g.  $10^4 \text{ N/m}^2$ )

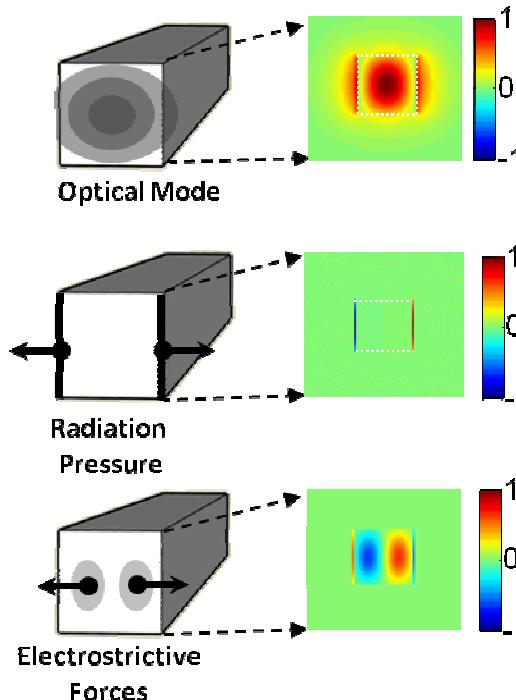
Radiation Pressure Generally VERY Small: Not Any More.



Electrostrictive forces → Material induced forces



# Nanoscales: The Neglected Optical Force Dominates!



**Rad. Pressure:**  $\bar{p}^{rp} = \frac{P_{opt}}{c \cdot A} \cdot (n_g - n_p) = \frac{P_{opt}}{c \cdot A} \cdot \alpha^{rp}$

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## How Large are Forces?

Material	Pressure (Pwr = 100mW)
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Ge	$\sim 10^6 \text{ N/m}^2$

5-50 People standing on man-hole cover

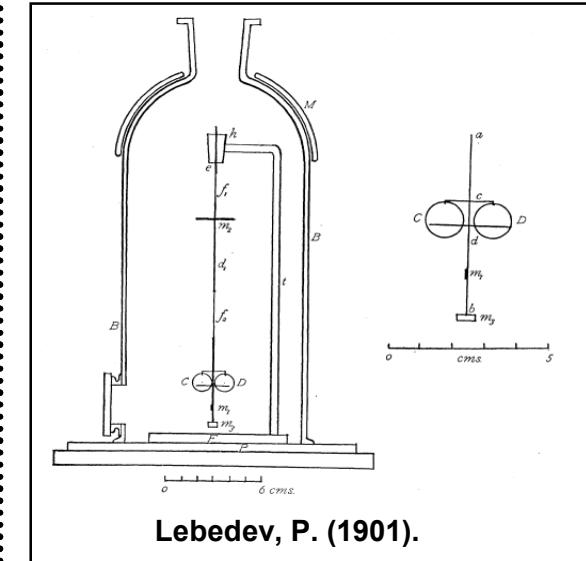
Material	Pressure (Pwr = 1kW)
Si	$\sim 5 \times 10^8 \text{ N/m}^2$
Ge	$\sim 10^{10} \text{ N/m}^2$

Stresses Approach Material Yield Strength

# Historical Introduction to Optical Forces:

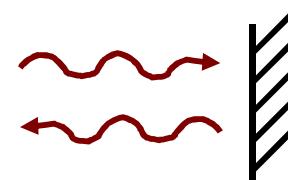
The concept of light induced motion has a very rich history:

- Timeline for Radiation Pressure:
  - (1619) Kepler: Speculated solar repulsion.
  - (1873) Maxwell: Theoretical basis for pressure.
  - (1901) Lebedew, Nichols: Experimental evidence.
- Optical Forces: Very difficult to observe.  
(60 Watt lamp  $\rightarrow$  Force  $\approx 400\text{nN}$ )
- First observation: Thermal lamp & torsion balance



Photon momentum:

$$|p| = \hbar k$$



Imparted momentum:

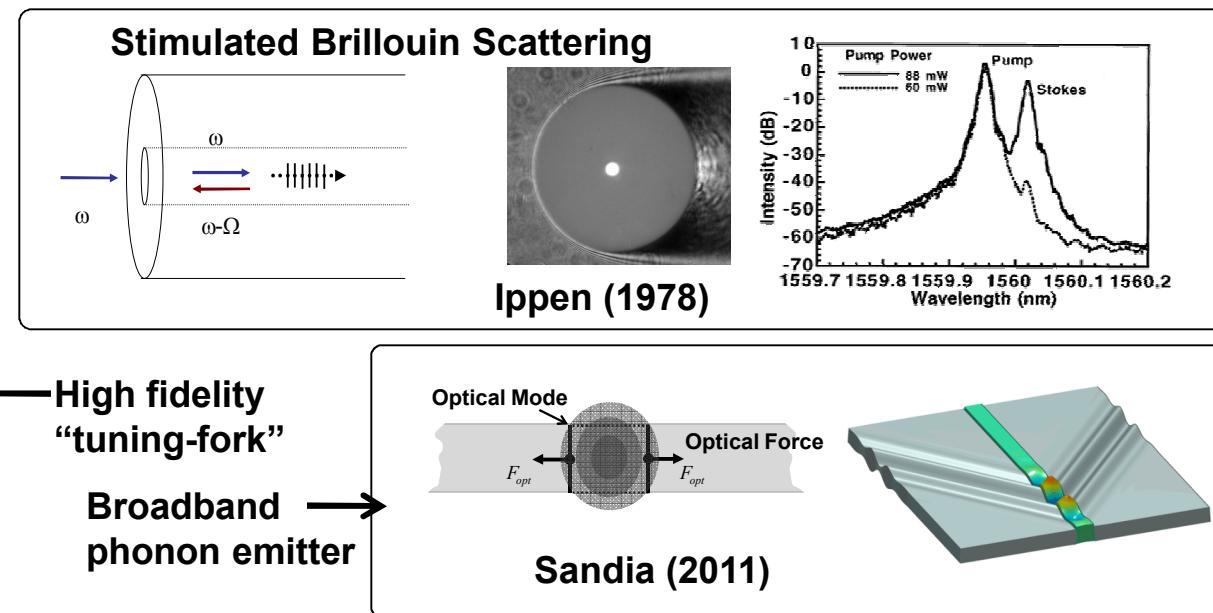
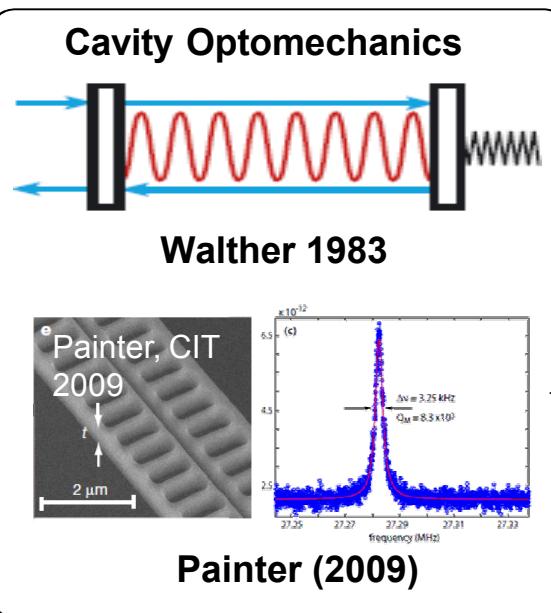
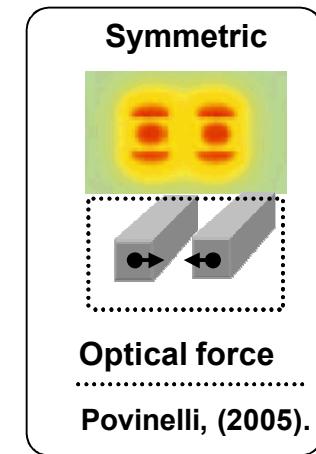
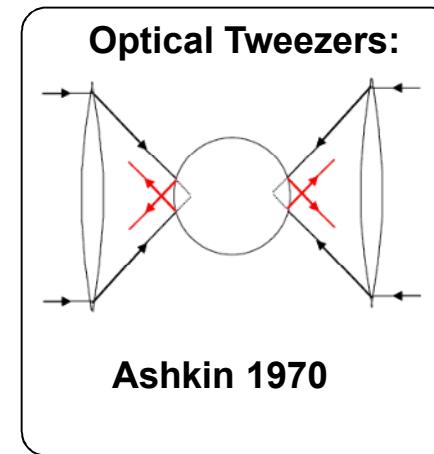
$$|\Delta p| = 2\hbar \cdot k$$

Radiation Pressure: Produced by photon recoil.

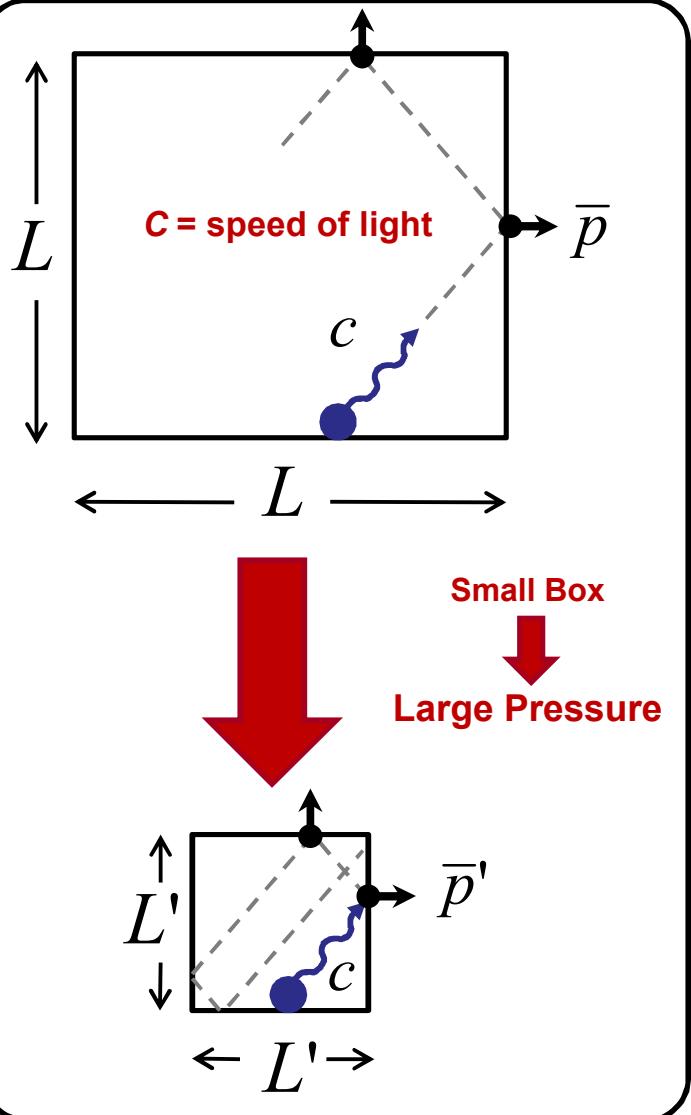
1. Nichols, E. F. & Hull, G. F. *Phys. Rev.* 13, 307–320 (1901).  
2. Maxwell, J. C. *A Treatise on Electricity and Magnetism* (1873).  
3. Lebedev, P. *Ann. Phys. (Leipzig)* 6, 433–458 (1901).

# More Recent Work Involving Optical Forces:

- (1970s) Optical tweezers: Trapping small of particles (Power = 50 mW, Force  $\approx$  50 pN)
- (1970s) Coherent phonon generation in fiber.
- (1980s) Free-space Interferometers: Optical Bistability (Power = 100 mW, Force  $\approx$  650 pN)
- (1990s) Laser Trapping and Cooling: Atoms
- (2000s) Nano-scale actuation with light and optically driven parametric oscillation.



# Radiation Pressure at Nano-scales



## Atomic Gas:

$$\bar{p}V = N \cdot \underbrace{k_B T}_{\substack{\# \text{ of atoms} \\ \text{pressure}}}$$

## Photon Gas:

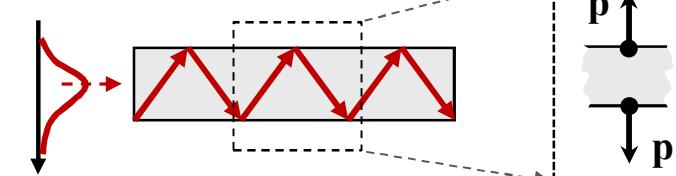
$$\bar{p}V = \frac{1}{3} \cdot N \cdot \underbrace{\hbar\omega}_{\substack{\# \text{ of photons}}}$$

## Pressure Per Photon:

$$\bar{p} = \frac{1}{3} \cdot \frac{\hbar\omega}{V} = \frac{1}{3} \cdot \frac{\hbar\omega}{L^3}$$

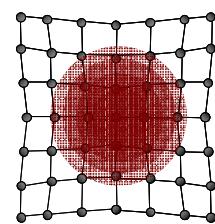
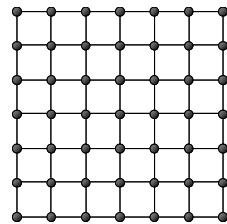
For  $L/L' = 1000$ ,  $p'/p = 1,000,000,000$ .

## Light within a waveguide:



Same Idea holds within a waveguide.

# Optical Forces Within Dielectric Media



$$S = \alpha \cdot E_i + \beta \cdot E_i E_j$$

Strain  
Piezo Coeff.  
Electrostrictive Coeff.

**Electrostriction = Material induced optical forces.**

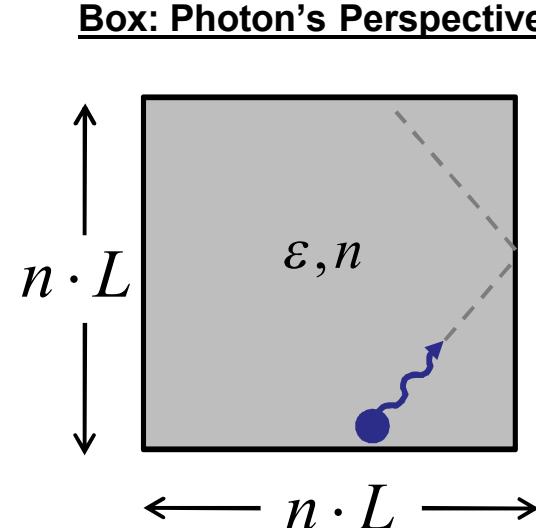
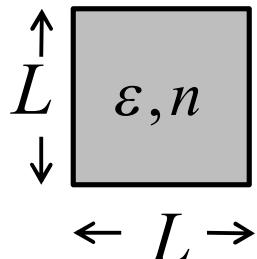
- All dielectrics exhibit electrostriction (not piezo electricity).
- Sign and magnitude are tailorable by choice of material



**Electrostrictiton:**  
possible force distributions.

What's going on?

Box: Real Space



From Photon's Perspective:

• Space is now quite different.

$$V \Rightarrow V \cdot n^3$$

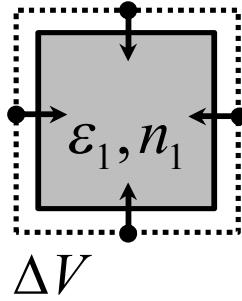
Box seems much bigger.

Oddities don't end here...

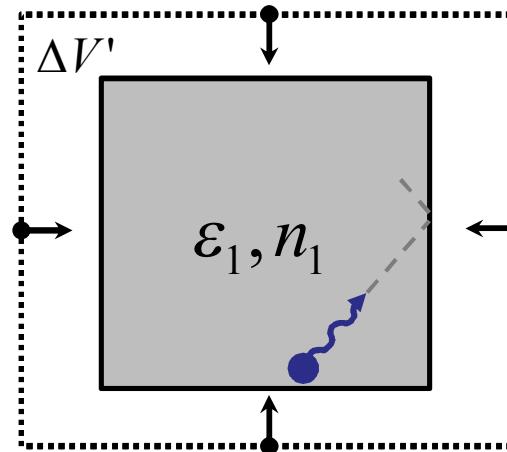
# Electrostriction: Material Induced Forces

Since  $\partial n / \partial V \neq 0$ , distortion of the box is perceived quite differently by a photon:

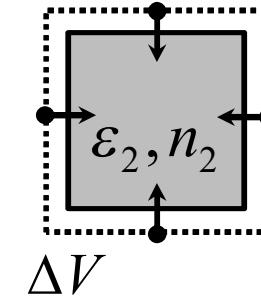
Box: Real Space



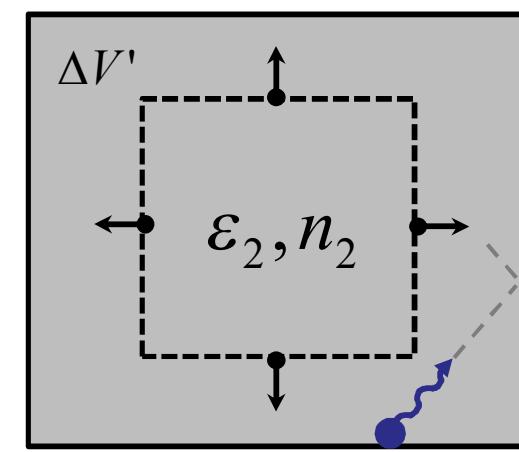
Box: Photon's Perspective



Box: Real Space



Box: Photon's Perspective



Photon gas within a dielectric:

$$\Delta V' \Rightarrow n^3 \Delta V + 3n^2 V \cdot \left( \frac{\partial n}{\partial V} \right) \cdot \Delta V$$

**Correction due to dynamic response**

Photon gas within a dielectric:

$$\bar{p} = \frac{1}{3} \cdot \frac{N \cdot \hbar \omega}{V} \left[ 1 + \frac{3 \frac{\partial n}{\partial V}}{n} \right]$$

**Electrostriction**