

Fundamental Limits of Force and Transductions in Nano-Optomechanics. SAND2011-3703C

Phononics 2011

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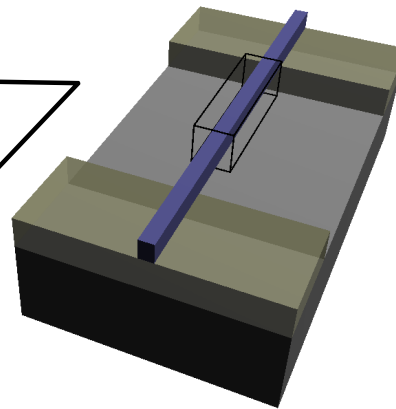
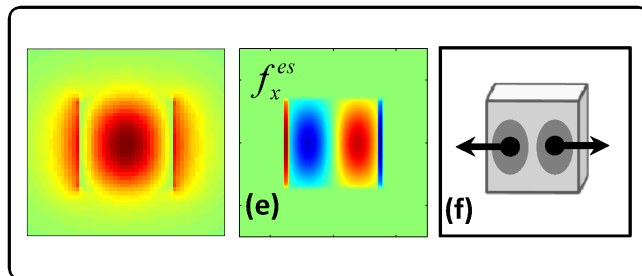


Outline of Presentation:

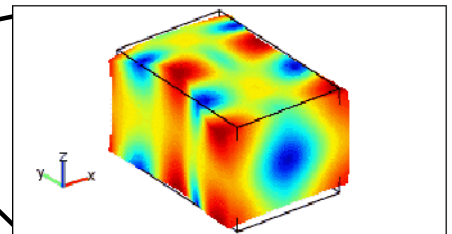
Outline of Presentation:

1. Overview of photon-phonon coupling at micro- and nanoscales.
2. Examine origins of optical forces within nanoscale materials and geometries.
3. Explore scaling of stimulated photon-phonon coupling via new framework.

Material induced optical forces



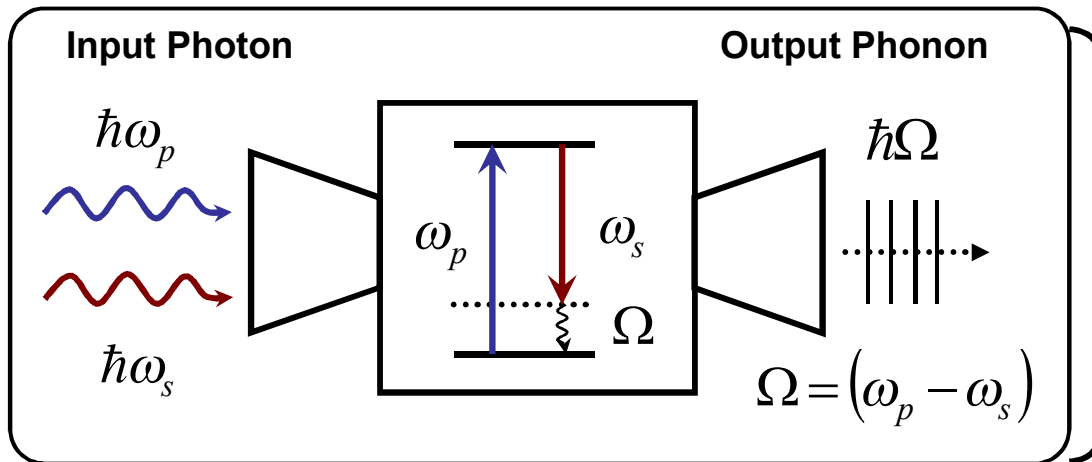
Nanoscale photon-phonon coupling



Result: Giant Enhancement of Stimulated Brillouin Scattering at Nanoscales

How Does Stimulated Brillouin Scattering Work?

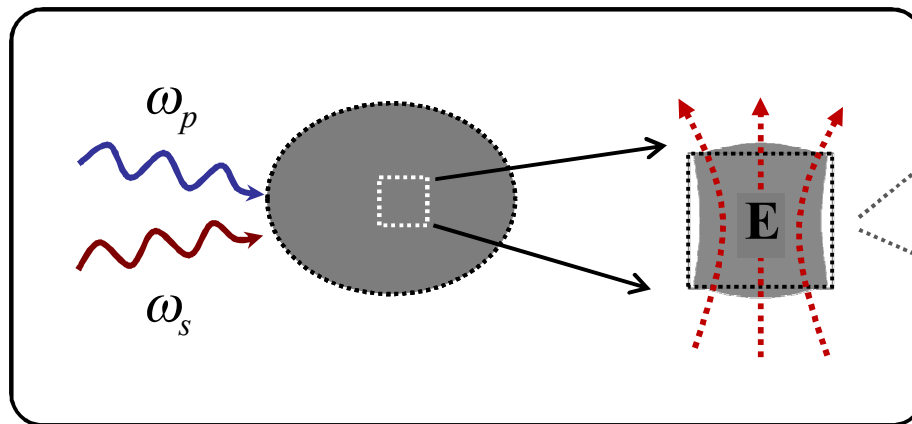
Physics of stimulated phonon generation:



Stimulated Brillouin Scattering (SBS)

- Yields coherent phonon generation.
- Mediated by optical forces.

Micro-scale origins of parametric process:



Interference yields intensity "Beat Note".

$$|E(t)|^2 = 2E_p E_s \cdot \cos(\Omega \cdot t) + C$$

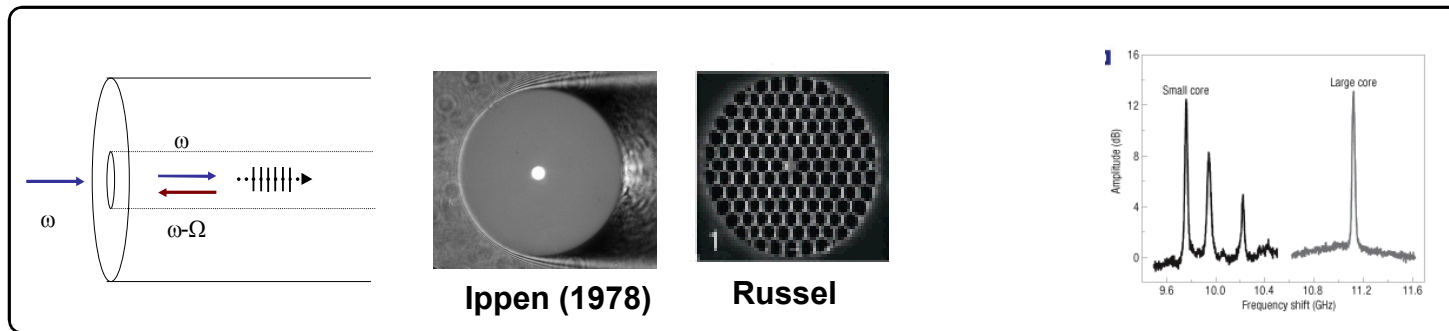
Optical Force: Proportional to Intensity.

$$F(t) \approx \alpha \cdot \sqrt{P_p \cdot P_s} \cdot \cos(\Omega \cdot t)$$

Time Varying Forces Transduce Phonon.

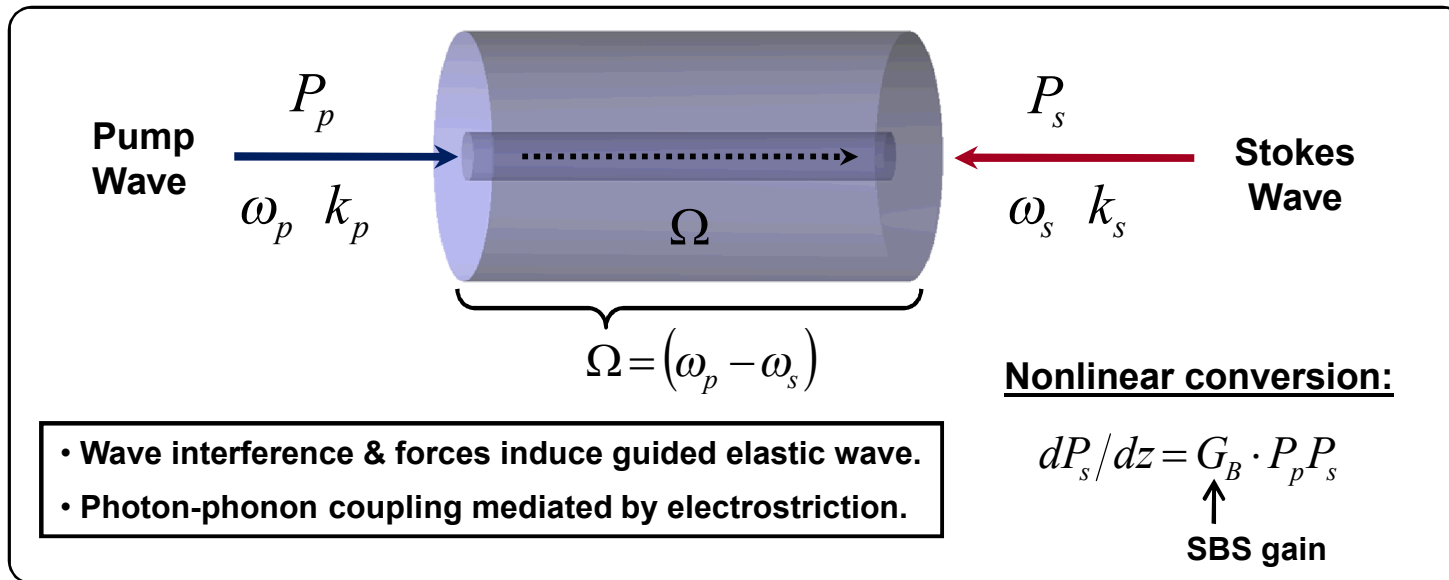
Guided-Wave Stimulated Brillouin Scattering:

Microscale Guided-Wave Stimulated Brillouin Scattering :



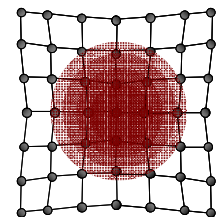
Lineage:
Nonlinear optics
Spectroscopy

How microscale stimulated Brillouin scattering works:



SBS: Mediated by
electrostriction.

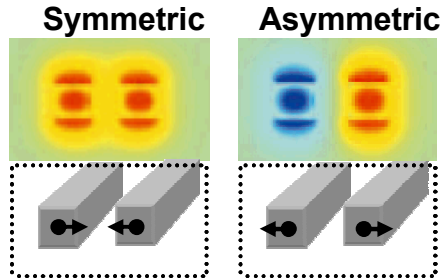
Electrostriction:



From dynamic
material response.

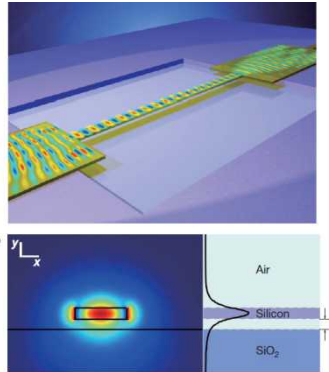
Nanoscales: light-matter interactions change in a fundamental way....

Enhanced Radiation Pressure at Micro and Nanoscales



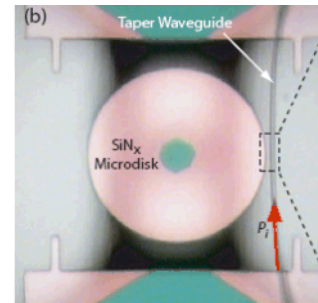
Attractive force Repulsive force

Povinelli, et. al. Opt. Lett. 30,3042 (2005).

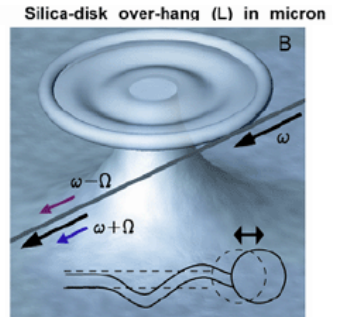


Mo Li et al. Nature 1, 480–484 (2008).

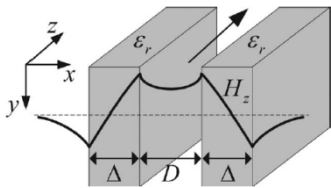
Optical microdisk



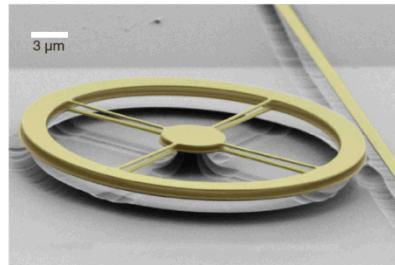
Eichenfield, M. et al. Nat. Photon. 1, 416–422 (2007).



T. Carmon, H. Rokhsari, L. Yang, T. Kippenberg, and K. Vahala, Phys. Rev. Lett. 94, 223902 (2005).

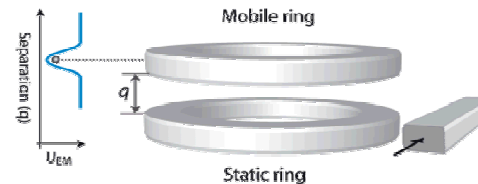


A. Mizrahi & L. Schächter Opt. Lett. 32, (2007), p.692-694

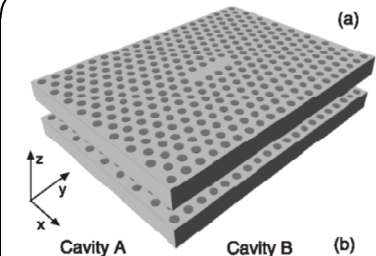


G. S. Wiederhecker, L. Chen et al Nature 462 2009, p.633

Dual microring geometry:



Rakich, Popovic et al. Nat. Photon. 1, 685-669 (2007).

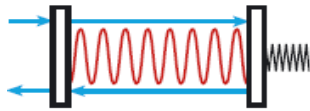


Notomi, M. and Taniyama, H. Phys. Rev. Lett. 94, p23903 (2006)

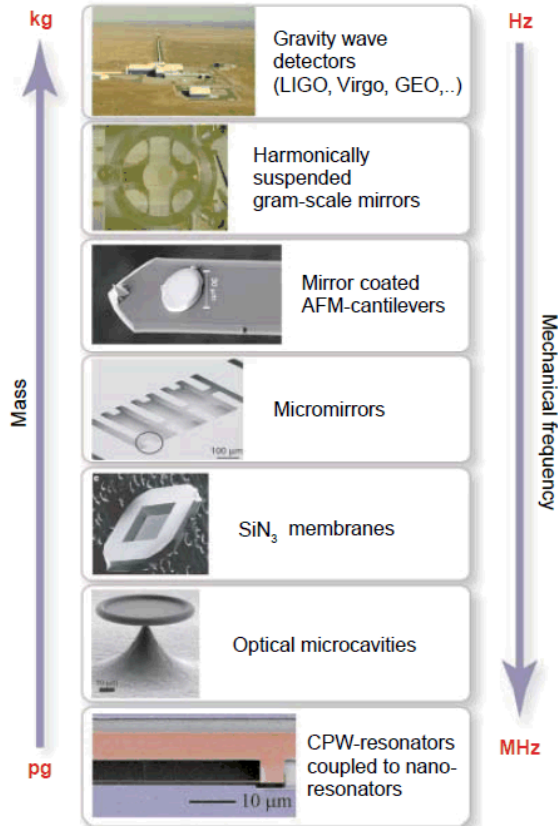
- Most of studies have focused on forces between discrete bodies.
- We will see that optical forces within bodies become very important to consider as well...

Nano Optomechanics: Stimulated Scattering

Cavity Optomechanics

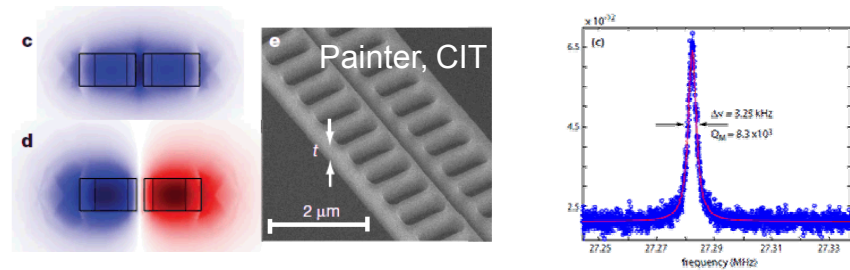


Walther 1983



Kippenberg, Vahala, Science 2008

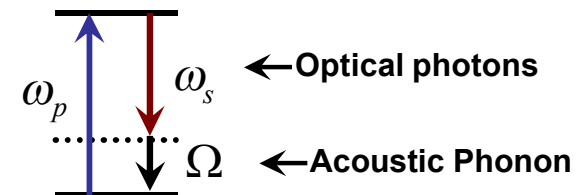
Optomechanical Parametric Oscillation



Radiation Pressure

Eichenfield, *et. al.* Nature, 459, 550 (2009).

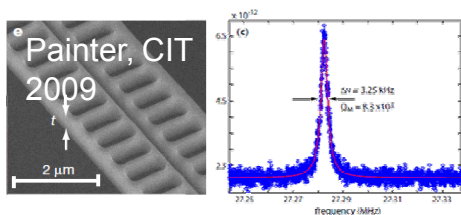
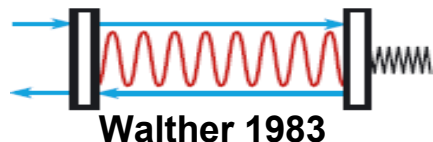
Radiation pressure mediates photon-phonon coupling.



- Formally equivalent to Brillouin process.
- A key distinction: mediated by radiation pressure.

Traveling-wave SBS at nanoscales:

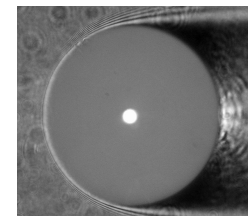
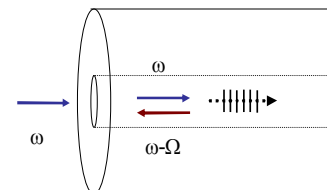
Cavity Optomechanics



Painter (2009)

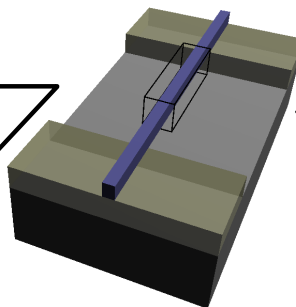
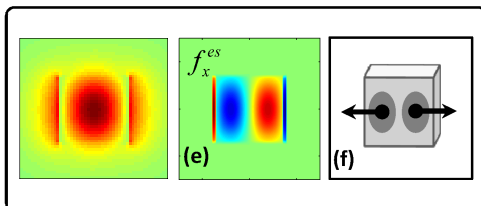
Nanoscales:
Radiation pressure
Mediated Stimulated
Brillouin processes

Microscales:
Electrostrictively
Mediated Stimulated
Brillouin processes

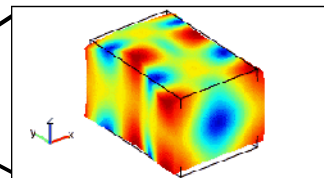


Result: Giant Enhancement of Stimulated Brillouin Scattering at Nanoscales

Electrostriction + Radiation Pressure



10-40 GHz Phonons

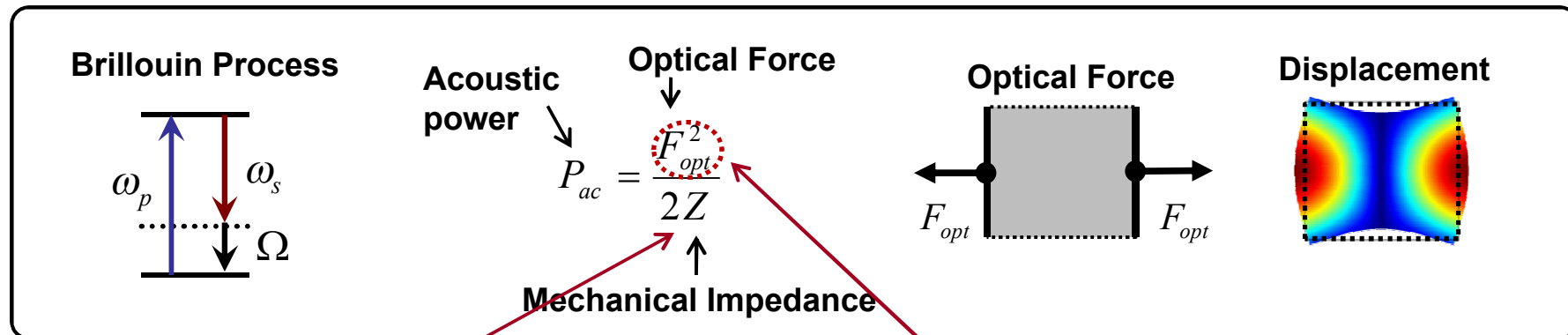


Radically enhanced SBS.

Electrostriction
& Radiation
Pressure
mediate SBS

How to Formulate SBS Gain in Nanoscale Systems?

For a time-harmonic force, $F(t) = F_{opt} \cdot \cos(\Omega \cdot t)$, produced by modal beating:



“Mechanical Transduciton in Periodic Media,”
R. Camacho, (Paper 0130), Next.

“Analysis of Forces in Nanophotonic
Waveguides,” C. Reinke. (Paper 0170),
2:40-3:00 PM

$$dP_s/dz = G_B \cdot P_p P_s$$

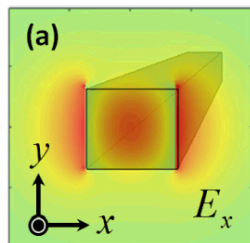
SBS gain

Exact solution: $G_B = \left(\frac{\omega_s}{\Omega} \right) \cdot \frac{1}{P_s P_p} \frac{F_{opt}^2}{Z} \alpha$ ← Geometric scale-factor

Strong photon-phonon coupling requires: (1) large optical force, (2) small mechanical impedance.

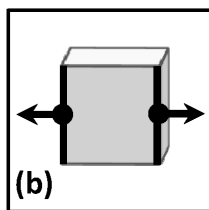
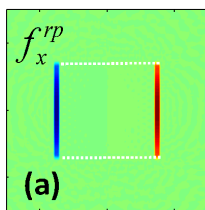
Optical Forces at Nano-scales

Silicon Waveguide.

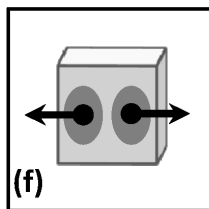
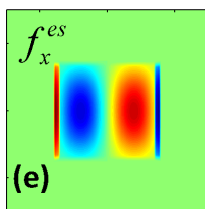


300nm x 300nm

Radiation Pressure



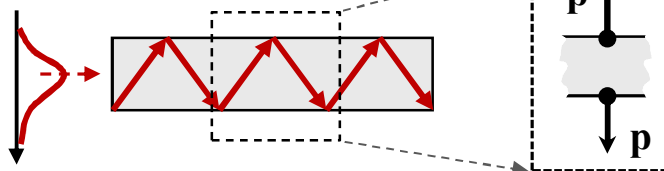
Electrostrictive Forces



Radiation Pressure.

Radiation Pressure: Scattering from boundaries.

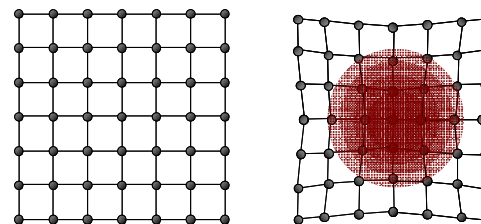
Light within a waveguide:



Entirely depends on geometry.

Electrostrictive Forces

Electrostriction: dynamic response of media to light.



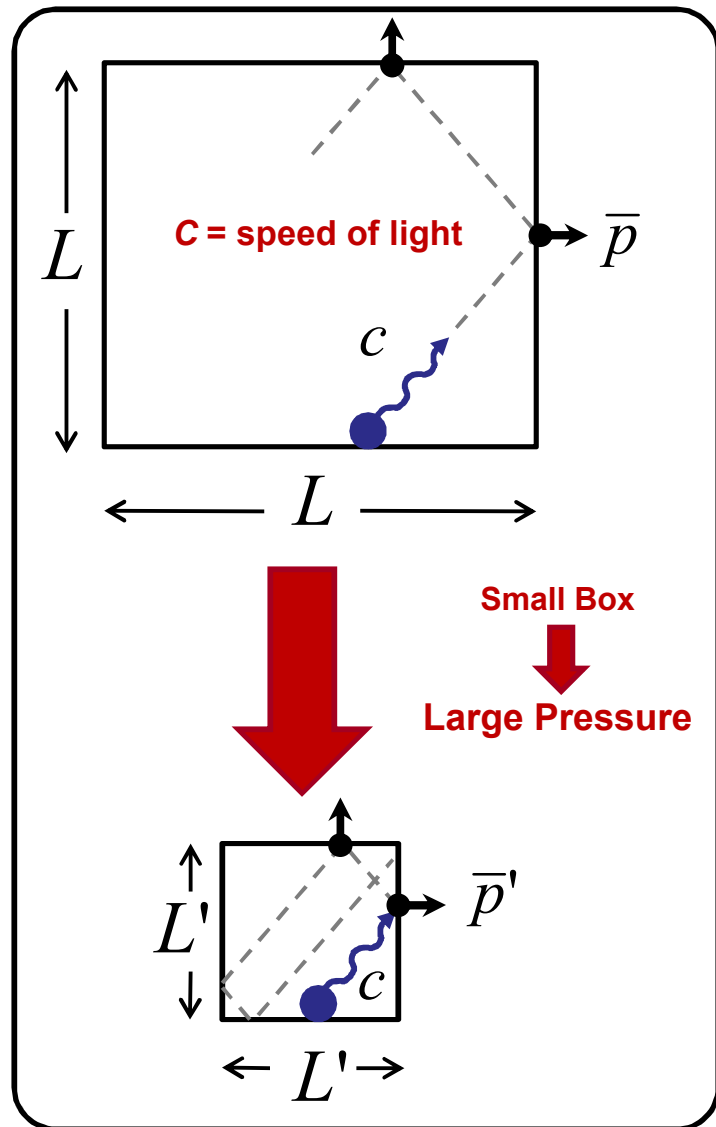
Depends primarily on material properties.

Both: Radically enhanced at nanoscales.

How to understand origin and enhancement?

Why is Radiation Pressure Enhanced at Nano-scales?

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Equation of State for an Ideal Photon Gas:

$$\text{Pressure} \rightarrow \bar{p}V = \frac{1}{3} \cdot N \cdot \hbar\omega \leftarrow \text{Photon Energy}$$

Volume # of photons

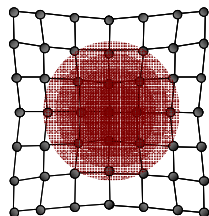
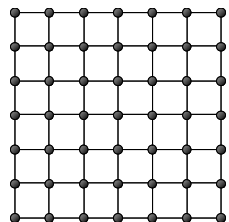
Pressure Per Photon:

$$\bar{p} = \frac{1}{3} \cdot \frac{\hbar\omega}{V} = \frac{1}{3} \cdot \frac{\hbar\omega}{L^3}$$

For $L/L' = 1000$, $p'/p = 1,000,000,000$.

Each photon packs more punch at nanoscales!

Optical Forces Within Dielectric Media



$$S = \alpha \cdot E_i + \beta \cdot E_i E_j$$

Strain
Piezo Coeff.
Electrostrictive Coeff.

Electrostriction = Material induced optical forces.

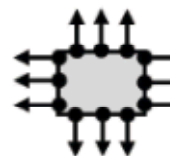
- All dielectrics exhibit electrostriction (not piezo electricity).
- Sign and magnitude are tailorable by choice of material



Germanium



Silicon

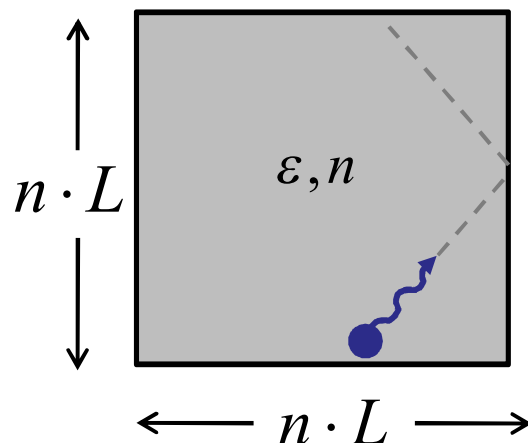
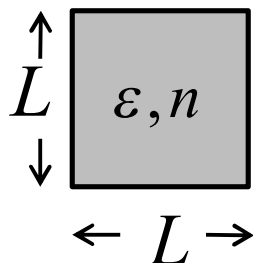


GaAs

What's going on?

Box: Photon's Perspective

Box: Real Space



From Photon's Perspective:

- Space is now quite different.

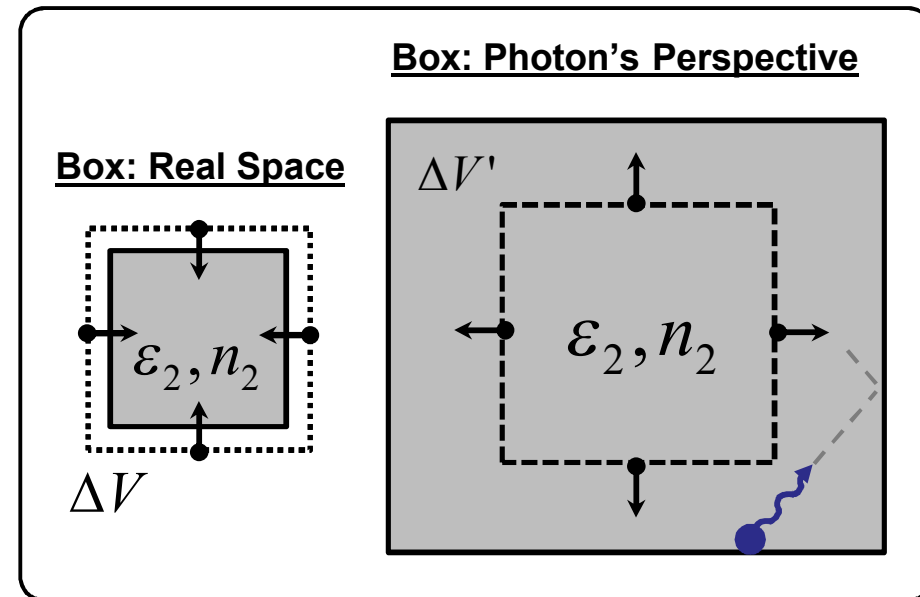
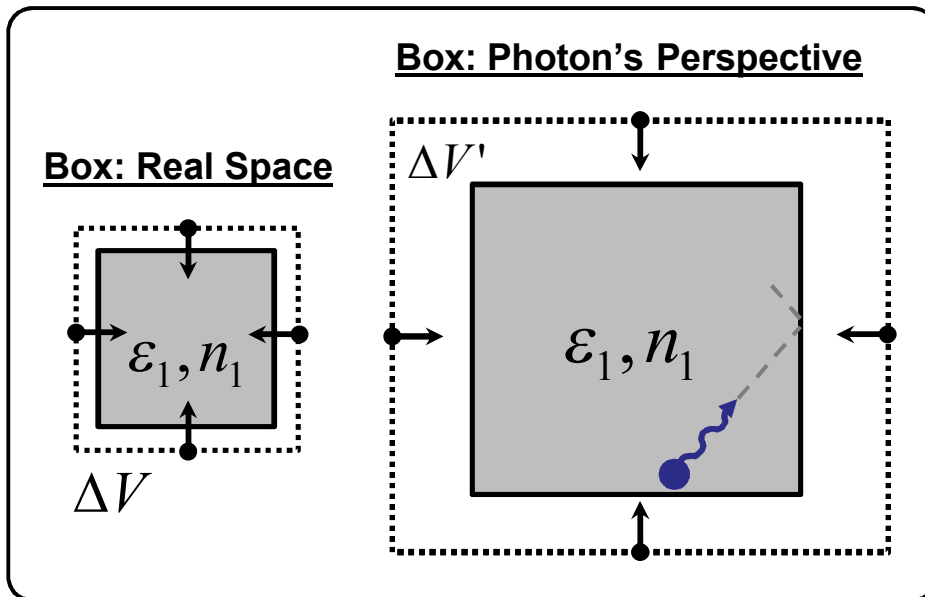
$$V \Rightarrow V \cdot n^3$$

Box seems much bigger.

Oddities don't end here...

Electrostriction: Material Induced Forces

Since $\partial n / \partial V \neq 0$, distortion of the box is perceived quite differently by a photon:



Photon gas within a dielectric:

$$\Delta V' \Rightarrow n^3 \Delta V + 3n^2 V \cdot \left(\frac{\partial n}{\partial V} \right) \cdot \Delta V$$

Correction due to dynamic response

Photon gas within a dielectric:

$$\bar{p} = \frac{1}{3} \cdot \frac{N \cdot \hbar \omega}{V} \left[1 + \frac{3}{n} \left(\frac{\partial n}{\partial V} \right) \right]$$

Electrostriction

Alternative: Electrostrictive Stress & Force.

Dependence on material properties:

$$\left\{ \begin{array}{l} \epsilon_{kl} \quad (\text{Dielectric Tensor}) \\ p_{jkmn} \quad (\text{Elasto-optic Tensor}) \end{array} \right\}$$

Electrostrictive Stress in Cubic Crystal (Si):

$$\sigma_{kl}^{es} = -\frac{1}{2} \epsilon_o \cdot n^4 \cdot p_{ijkl} \cdot E_i E_j$$

Force Density:

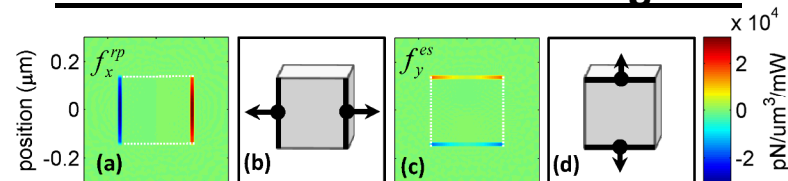
$$\mathcal{F}_j = -\partial_i \sigma_{ij}$$

Important Properties of Stress/Force

1. Increases as n^4 .
2. Proportional to p_{ijkl} .
3. Sign & magnitude depends on p_{ijkl}

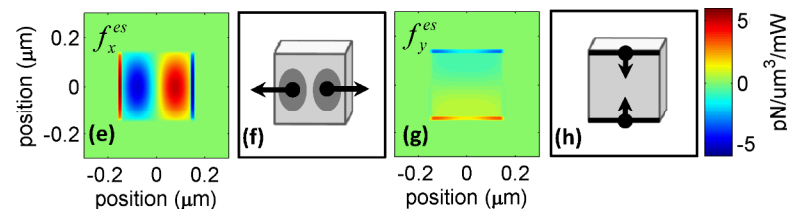
Example: Silicon waveguide.

Radiation Pressure: Si waveguide



1. Force localized to boundary.
2. Directed outward.

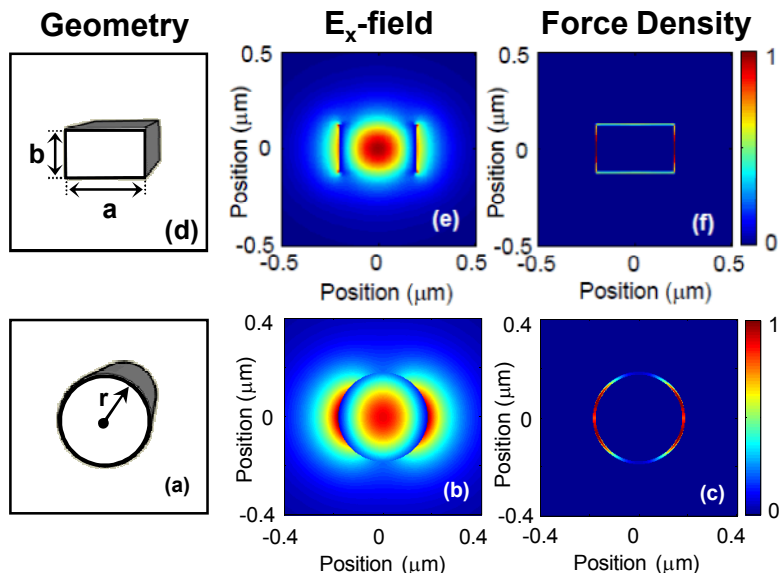
Electrostriction: Si waveguide



1. Force distributed within volume.
2. Directed outward or inward.

[1] P. T. Rakich, P. Davids, and Z. Wang, "Tailoring Optical Forces in Waveguides Through Radiation Pressure and Electrostriction," Opt. Express 18, 14439-14453 (2010)

Key to Radiation Pressure: Dispersion.



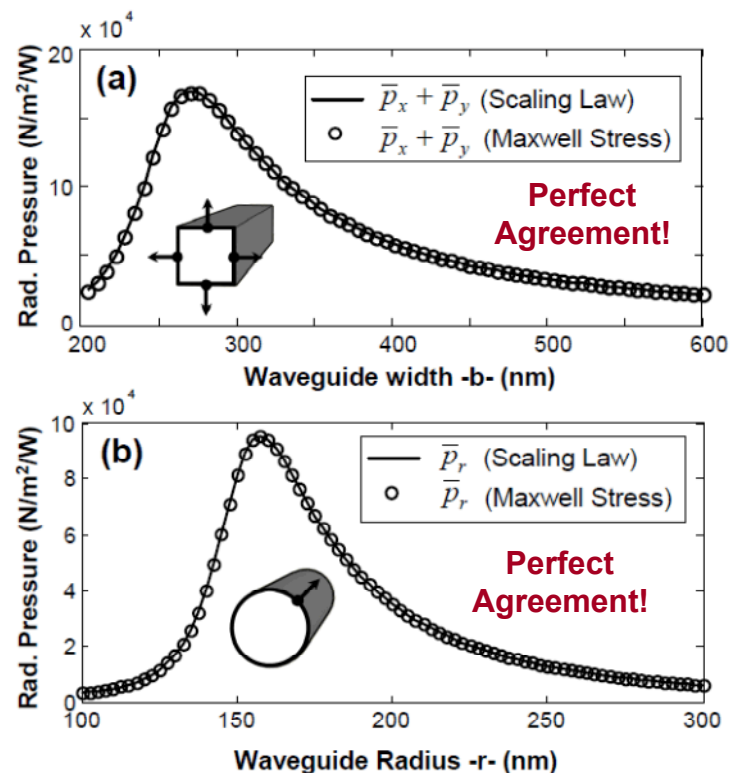
Radiation Pressure:

$$\bar{p}^{rp} = \frac{P_{opt}}{c \cdot A} \cdot (n_g - n_p) \leq \frac{P_{opt}}{c \cdot A} \cdot (2\Delta n)$$

Relation Holds for Any Dielectric Waveguide and Any Guided Mode!

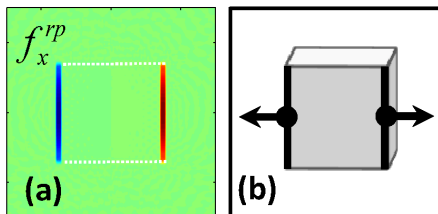
[1] P. T. Rakich, Z. Wang, and P. Davids "Scaling of Optical Forces in Dielectric Waveguides: Rigorous Connection Between Dispersion and Radiation Pressure," *Optics Letters*.

Comparison of Maxwell Stress & Scaling Law



Nanoscales: The Neglected Optical Force Dominates!

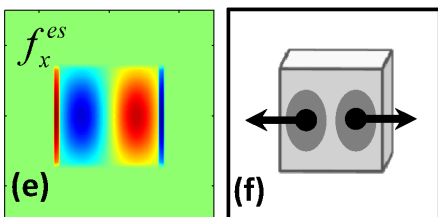
Radiation Pressure



Rad. Pressure: $\bar{p}^{rp} = \frac{P_{opt}}{c \cdot A} \cdot (n_g - n_p) = \frac{P_{opt}}{c \cdot A} \cdot \alpha^{rp}$

Electrostriction: $\bar{p}^{es} = \frac{P_{opt}}{c \cdot A} \cdot n_g n^2 (p_{11} + 2p_{12}) / 2 = \frac{P_{opt}}{c \cdot A} \cdot \alpha^{es}$

Electrostrictive forces



Material	Symmetry	p_{11}	p_{12}	$p_{11} + 2p_{12}$	n	α^{rp}	α^{es}
Si	cubic	-0.09	+0.017	-0.056	3.5	-5	-1.7
Ge	cubic	0.27	0.235	0.74	4.2	-6.4	+40
GaAs	cubic	-0.165	-0.14	-0.445	3.4	-4.8	-12
Silica	amorphous	0.121	0.27	0.661	1.45	-0.89	+1.0
As ₂ S ₃	amorphous	0.25	0.24	0.73	2.4	-2.8	+6.5
As ₂ Se ₃	amorphous	-	-	-	2.8	-3.6	-

How Large are Forces?

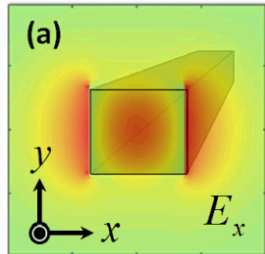
Material	Pressure (Pwr = 100mW)
Si	$\sim 5 \times 10^4 \text{ N/m}^2$
Ge	$\sim 10^6 \text{ N/m}^2$

**5-50 People
standing on
manhole
cover**

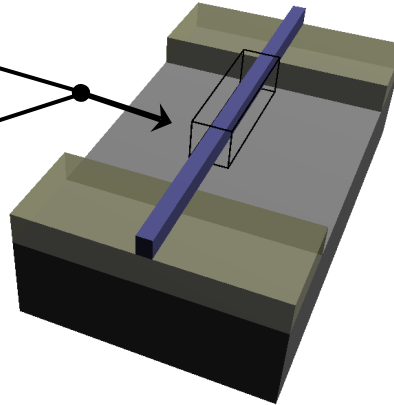
Material	Pressure (Pwr = 1kW)
Si	$\sim 5 \times 10^8 \text{ N/m}^2$
Ge	$\sim 10^{10} \text{ N/m}^2$

**Stresses
Approach
Material Yield
Strength**

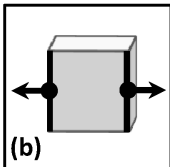
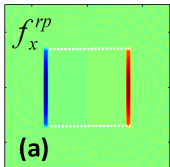
Next: Examine photon-phonon coupling.



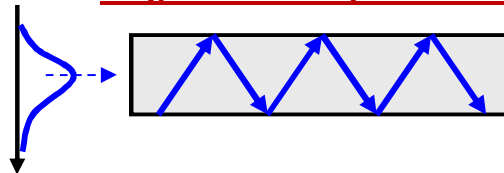
Guided mode within suspended dielectric waveguide. (300x300nm)



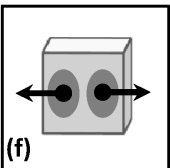
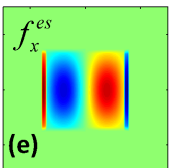
Radiation Pressure



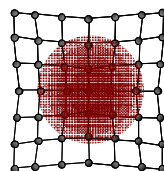
Origin: Boundary Scattering



Electrostrictive forces



Origin: Dynamic Material Response

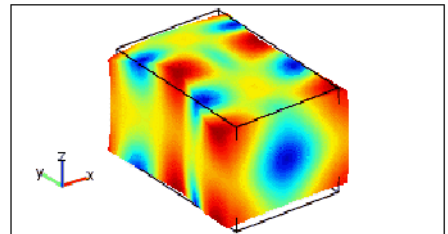


Akin to piezoelectricity

$$S = \alpha \cdot E_i + \beta \cdot E_i E_j$$

Strain Piezo Coeff. Electrostrictive Coeff.

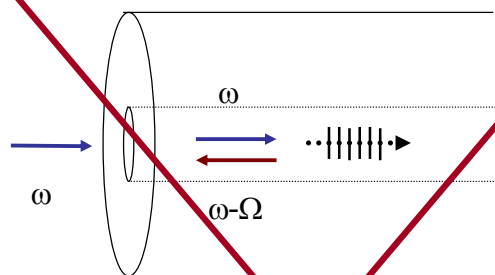
Generalized theory of photon-phonon coupling:



Reveals powerful new photon-phonon coupling processes at nanoscales.

How to Formulate SBS Gain in Nanoscale Systems?

Microscale SBS Theory:



$$g_B = \frac{2\pi^7 p_{12}^2}{e\lambda_p^2 \rho v_a \Delta v_B}$$

- Neglects radiation pressure.
- Simplified elastic-wave mode.

Not valid at nanoscales.

Unified Treatment of SBS: Valid at any Length-Scale

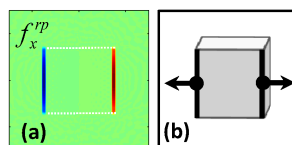
Canonical relation for SBS gain:

$$dP_s/dz = G_B \cdot P_p P_s$$

SBS gain

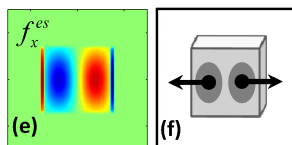
$$G_B(\Omega) = \frac{1}{\delta z} \frac{\omega_s}{\Omega} \cdot \frac{1}{P_p \cdot P_s} \int_{\delta V} \langle f_{\Omega}(\mathbf{r}, t) \cdot \dot{\mathbf{u}}(\mathbf{r}, t) \rangle \cdot dV,$$

Radiation Pressure:



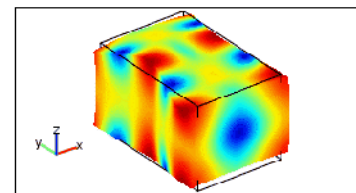
+

Electrostriction:



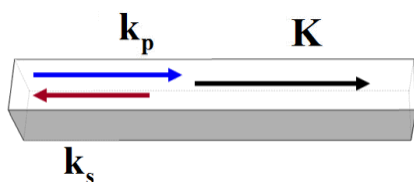
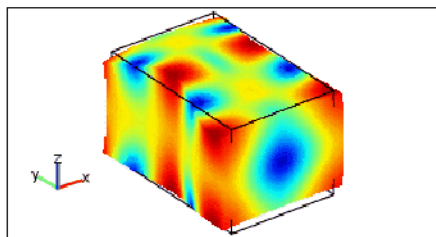
Time harmonic force

Velocity distribution

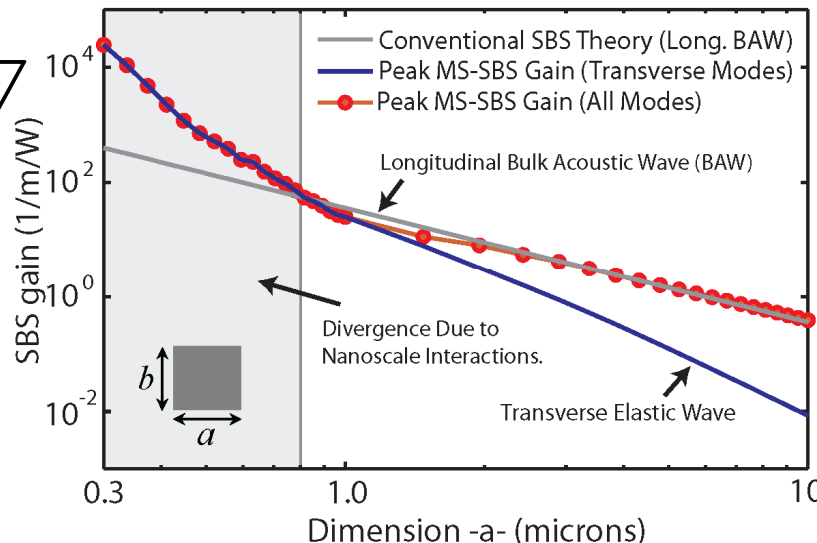


SBS Radically Enhanced at Nanoscales:

Nanoscale Stimulated Brillouin Scattering (SBS)



Photon-phonon coupling vs dimension.



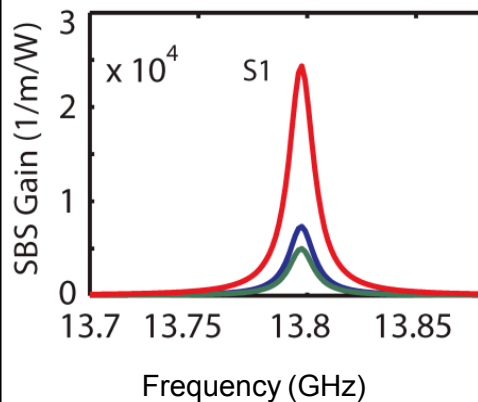
Conventional Treatment of SBS.

$$g_B = \frac{2\pi n^7 p_{12}^2}{c \lambda_p^2 \rho v_a \Delta \nu_B}$$

Conventional theory:

1. Silicon material properties used.
2. Perfect agreement from 2-10 microns.

- Electrostriction
- Radiation Pressure
- Electrostriction & Radiation Pressure

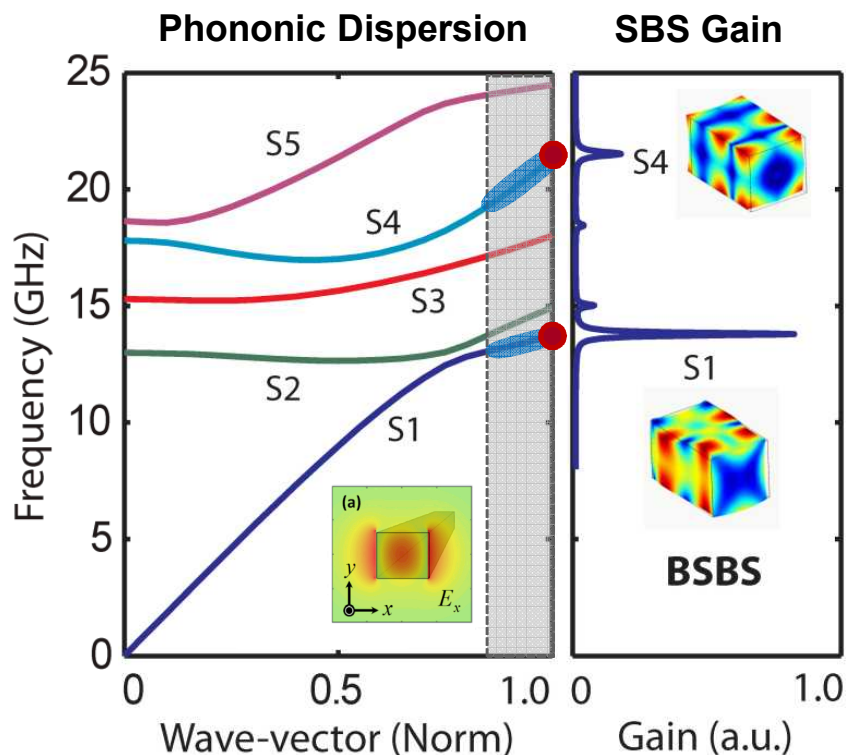


Nanoscale SBS:

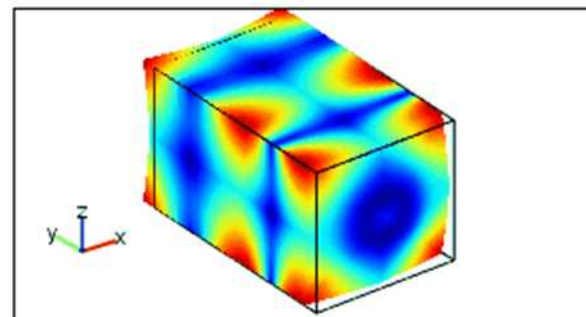
1. For dimensions < 800nm, conventional SBS theory breaks down.
2. Rigorous model reveals 100x enhanced SBS at small dimensions.

Coherent combination of electrostriction and radiation pressure produce further enhancement.

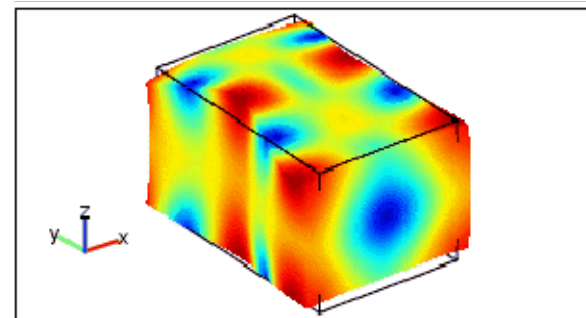
Nanoscale Backward-SBS: Tunable Phonon Emission.



Excitation: 21.6 GHz Phonons

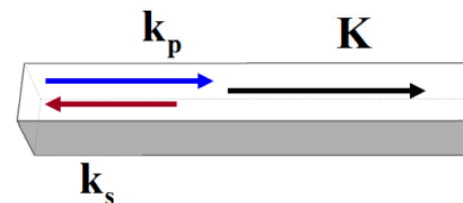


Excitation: 13.8 GHz Phonons

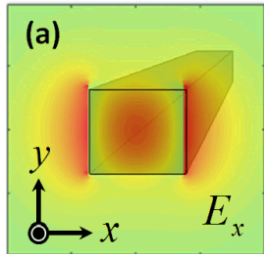


Nano-optomechanical backward-SBS:

1. Gain is 10^6 x Larger than in Fibers.
2. 20% frequency tunable phonon emission.



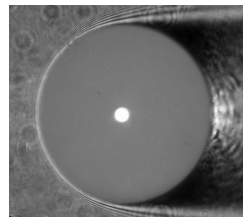
How Strong is SBS:



Guided mode within
suspended dielectric
waveguide. (300x300nm)

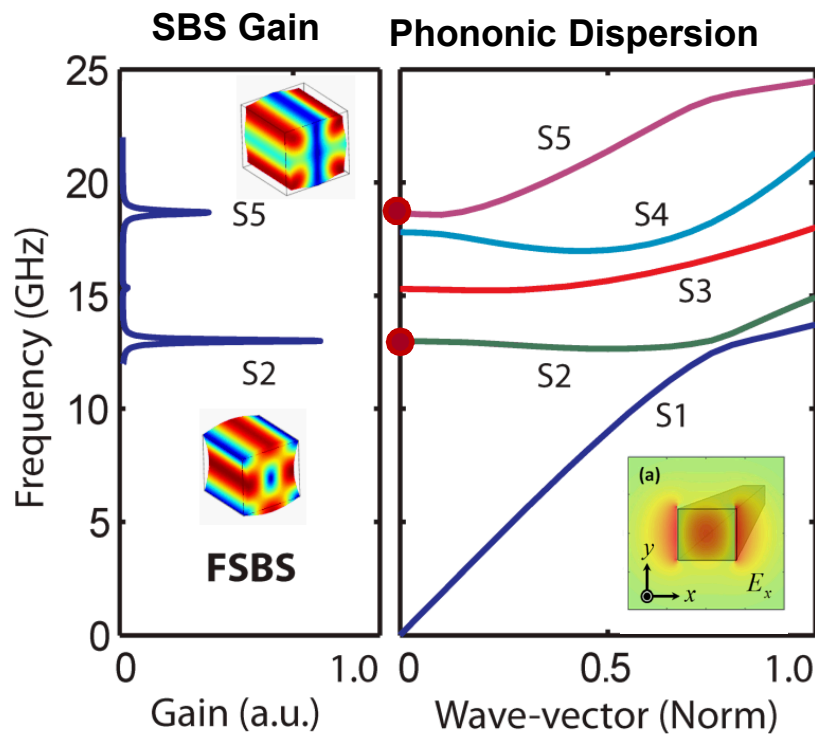
Suspended waveguide: $L = 100$ microns

= SBS nonlinearity
of 10-100 meters of fiber

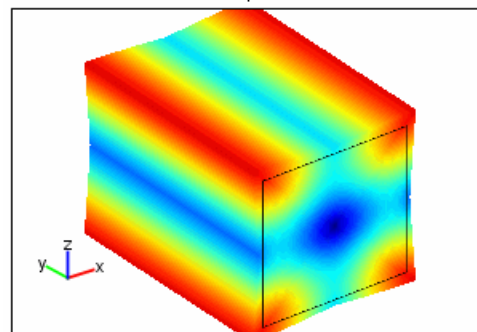


Fiber optic: $L = 10$ -100 meters

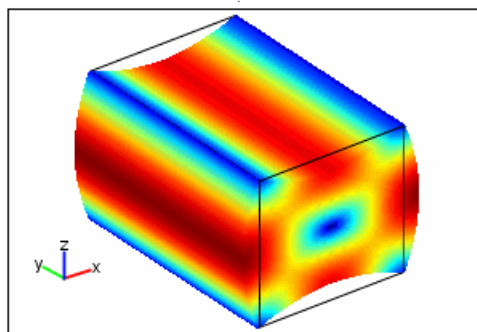
Forward-SBS: Fixed Frequency Resonances.



Excitation: 18.6 GHz Phonons

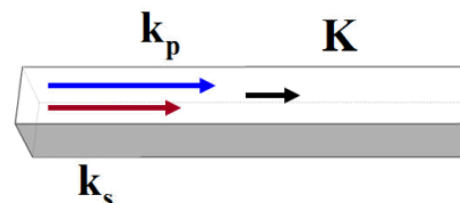


Excitation: 13.0 GHz Phonons



Nano-optomechanical Forward-SBS:

1. Generally forbidden in guided wave-systems.
2. Ultra-low threshold parametric oscillation possible.



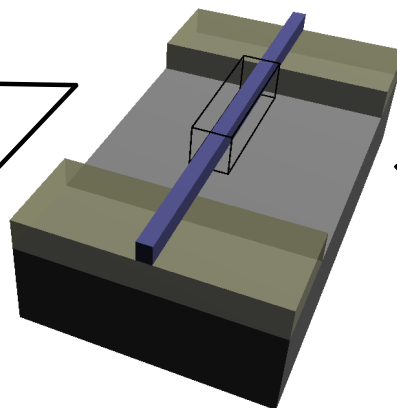
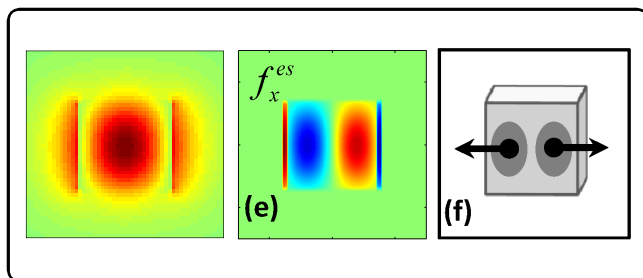
Conclusions:

Developed unified treatment of SBS at nanoscales:

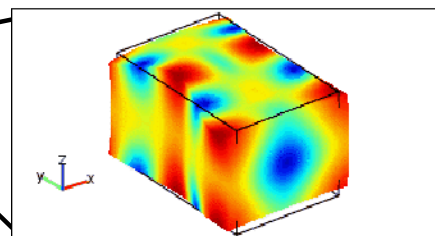
1. Valid at any lengthscale (micro- to nano-scales).
2. Radically enhanced SBS processes found
→ Resulting from electrostriction & radiation pressure.
3. Forward SBS: Excitation of ultra-high frequency modes.
4. Backward SBS 1E6 x stronger than in fiber.
5. Important step towards highly tailorable chip-scale SBS.

“Giant Enhancement of Stimulated Brillouin Scattering at Nanoscales,” P. Rakich, C. Reinke, R. Camacho, P. Davids, Z. Wang, (submission to PRL).

Material induced optical forces



10-40 GHz Phonons





Acknowledgments:

Special thanks:

Ihab El-kady, Troy Olsson, Milos Popovic

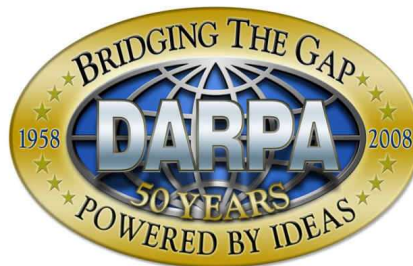
Funding Agencies:

DARPA—MTO (PMs: Mike Haney, Scott Rodgers, Jeff Rogers).

DOE—Laboratory Directed Research and Development funding.

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

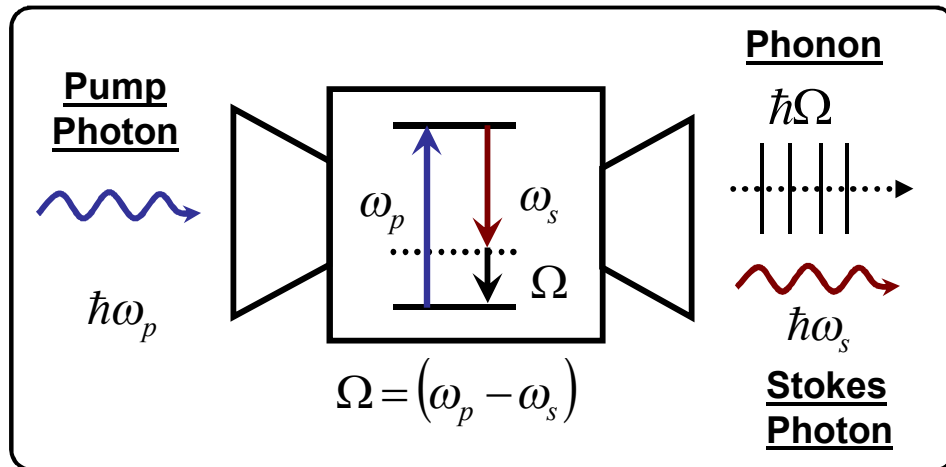
This work was supported in part by the office of the Director of Defense Research and Engineering under Air Force contract FA8721-05-C-0002.



Backup Slides Follow:

What is a Brillouin Process?

Brillouin Scattering Energy Level Diagram:

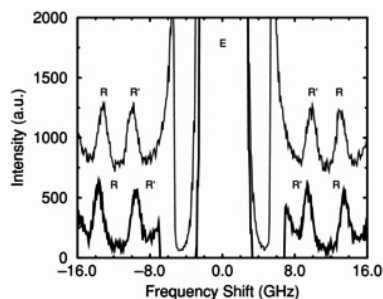


Brillouin Scattering:

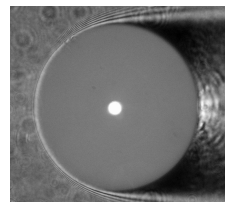
3rd order Parametric process by which photons couple to acoustic phonons.

———— Bulk Media ————— Micro-Scales ————— Nano-Scales —————→

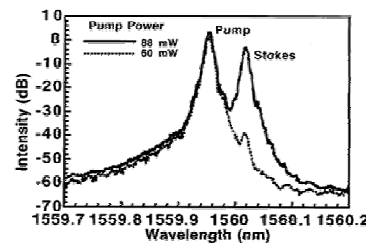
BLS in Bulk Media



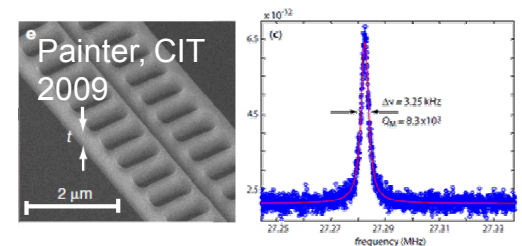
SBS bulk media and optical fibers



Ippen (1978)

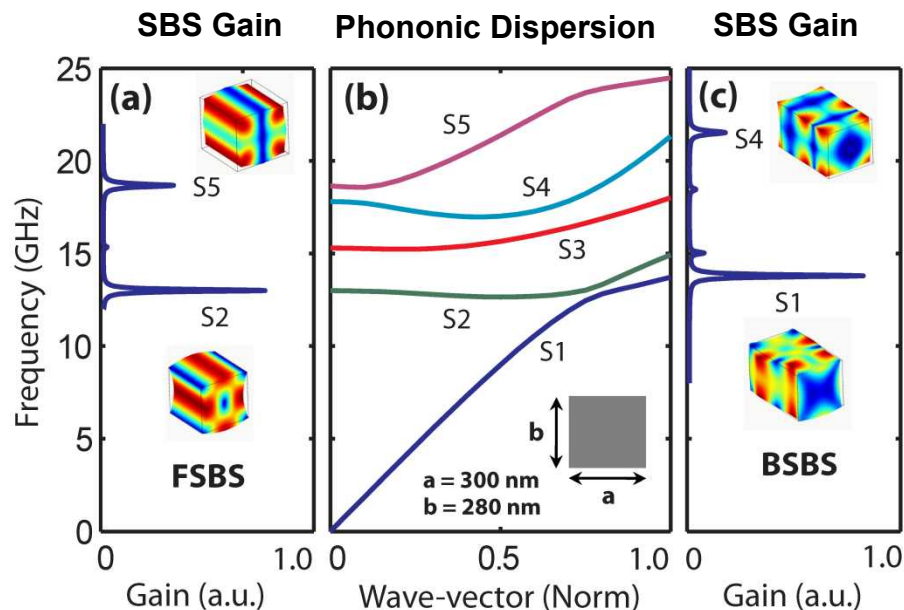


Nano-scale SBS and BLS

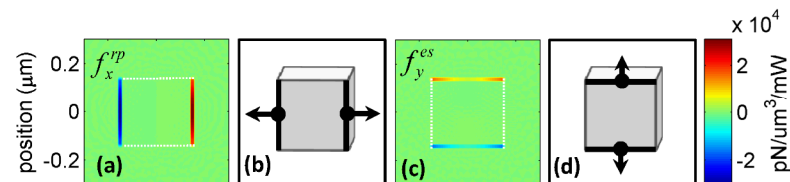


Painter (2009)

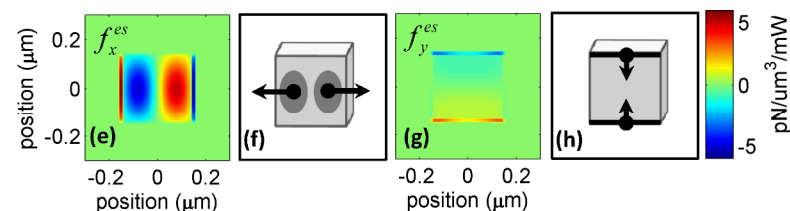
Contributions of Different Forces at Nanoscales:



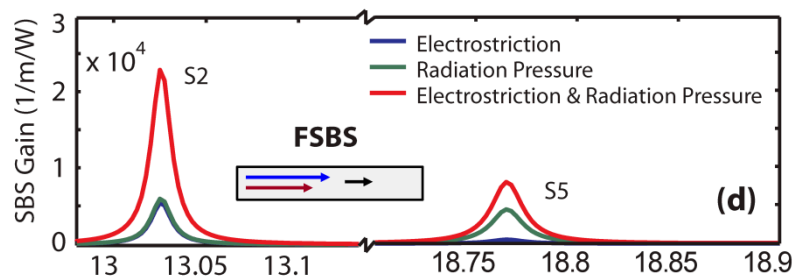
Radiation Pressure:



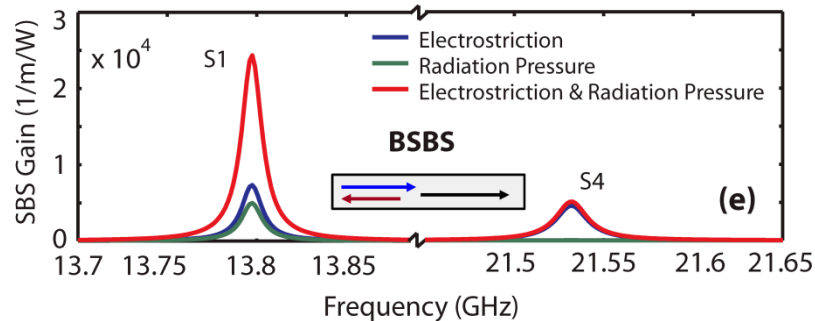
Electrostriction:



Forward SBS (FSBS):

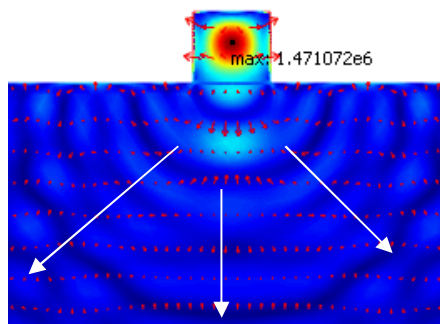


Backward SBS (BSBS):



SBS in Silicon Photonics?

Transduced Waves:

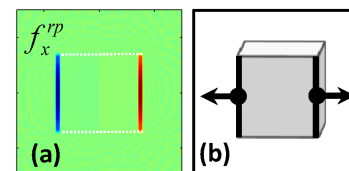


Canonical Treatment of SBS is not valid for Si waveguides!

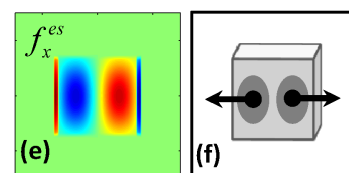
What's Different?

- Elastic Wave Leakage:
→ Makes SBS extremely weak.
- Must treat combination of:
 1. Radiation Pressure
 2. Electrostriction

Radiation Pressure



Electrostrictive forces

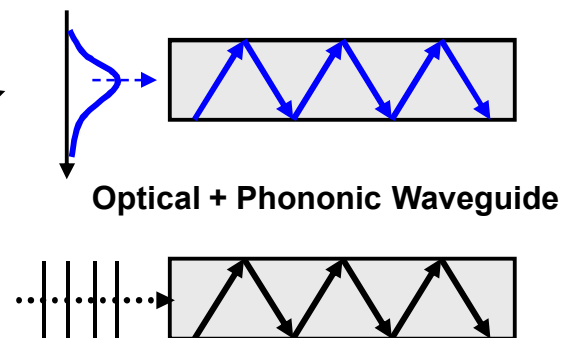
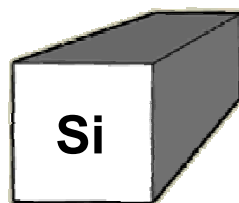


Simplest Way to Control Photons and Phonons for Enhanced SBS:

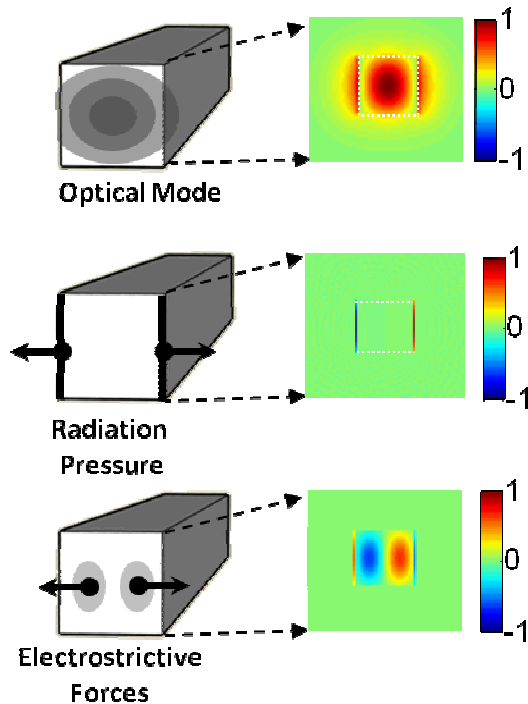
Nano-wire Supports:

- Guided *Optical Wave*
- Guided *Elastic Wave*

Suspended Waveguide



Nanoscales: The Neglected Optical Force Dominates!



$$\text{Rad. Pressure: } \bar{p}^{rp} = \frac{P_{opt}}{c \cdot A} \cdot (n_g - n_p) = \frac{P_{opt}}{c \cdot A} \cdot \alpha^{rp}$$

$$\text{Electrostriction: } \bar{p}^{es} = \frac{P_{opt}}{c \cdot A} \cdot n_g n^2 (p_{11} + 2p_{12}) / 2 = \frac{P_{opt}}{c \cdot A} \cdot \alpha^{es}$$

Material	Symmetry	p_{11}	p_{12}	$p_{11} + 2p_{12}$	n	α^{rp}	α^{es}
Si	cubic	-0.09	+0.017	-0.056	3.5	-5	-1.7
Ge	cubic	0.27	0.235	0.74	4.2	-6.4	+40
GaAs	cubic	-0.165	-0.14	-0.445	3.4	-4.8	-12
Silica	amorphous	0.121	0.27	0.661	1.45	-0.89	+1.0
As ₂ S ₃	amorphous	0.25	0.24	0.73	2.4	-2.8	+6.5
As ₂ Se ₃	amorphous	-	-	-	2.8	-3.6	-

How Large are Forces?

Material	Pressure (Pwr = 100mW)
Si	$\sim 5 \times 10^4 \text{ N/m}^2$
Ge	$\sim 10^6 \text{ N/m}^2$

5-50 People standing on man-hole cover

Material	Pressure (Pwr = 1kW)
Si	$\sim 5 \times 10^8 \text{ N/m}^2$
Ge	$\sim 10^{10} \text{ N/m}^2$

Stresses Approach Material Yield Strength

Significance of LDRD: Big Picture & Impact.

Physical Optics & Optomechanics:

1. Origins of nanoscale optical forces.
 - a. Effect of geometry and material.
 - b. Effective medium properties.
2. Radically new stimulated phonon emission mechanisms at nanoscales.

Heat Transport & Ultrafast Science:

1. Stimulated Mach-wave emission enables:
 - a. Phononic pump probe experiments
 - b. Phonon transport & lifetimes.
 - c. Phonon dispersion & bandgaps meas.
 - d. Mechanical properties of media.

LDRD: Goals

1. Advance science of nanoscale optical forces & transduction.
2. Explore technological impact & potential benefits to DOE mission.

Theory & Modeling:

1. Multi-scale models of photon-phonon coupling (micro- and nano-scales).
2. Powerful new formalisms for optical forces.
3. Novel scaling laws for optical forces with nanoscale geometry and material

Novel Devices & Applications:

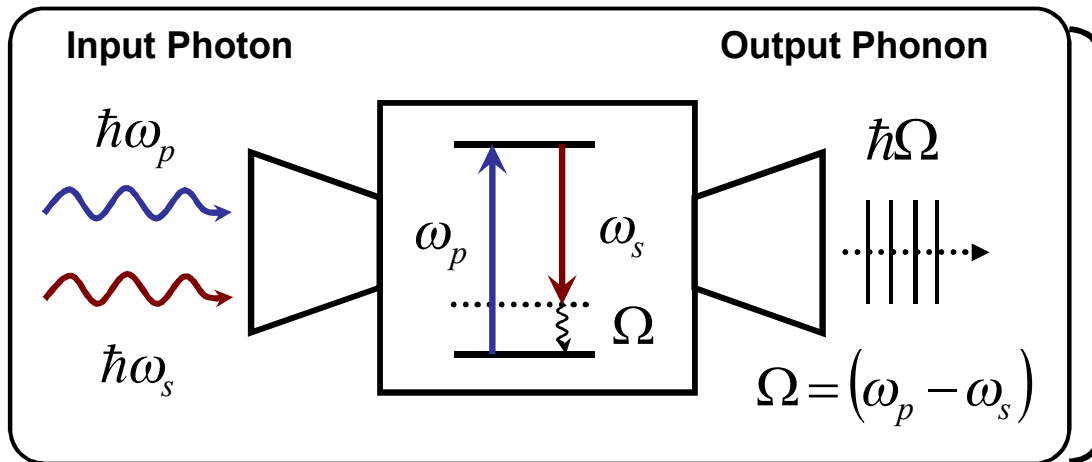
1. Ultra-broadband chip-scale delays.
2. Pulsed phonon lasers & frequency combs.
3. Tunable ultrahigh frequency oscillators.
4. Sensors and microphones.
5. Phonon amplifiers and modulators.
6. Ultra-broadband acousto-optics... etc.

Fabrication & Metrology:

1. Unique and general experimental platform for study of optical forces & photon-phonon coupling.
2. First-in-class methods for ultrafast pump probe studies of phononic effective media.
3. First-in-class Brillouin scattering measurements.

How are Phonons (acoustic waves) Generated by Light?

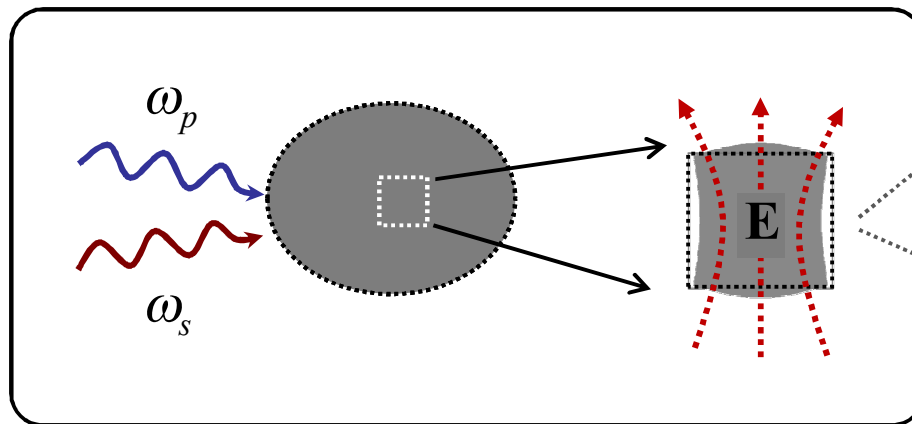
Physics of stimulated phonon generation:



Stimulated Brillouin Scattering (SBS)

Describes all optomechanical photon-phonon coupling processes (including cavity optomechanics).

Micro-scale origins of parametric process:



Interference yields intensity “Beat Note”.

$$|E(t)|^2 = 2E_p E_s \cdot \cos(\Omega \cdot t) + C$$

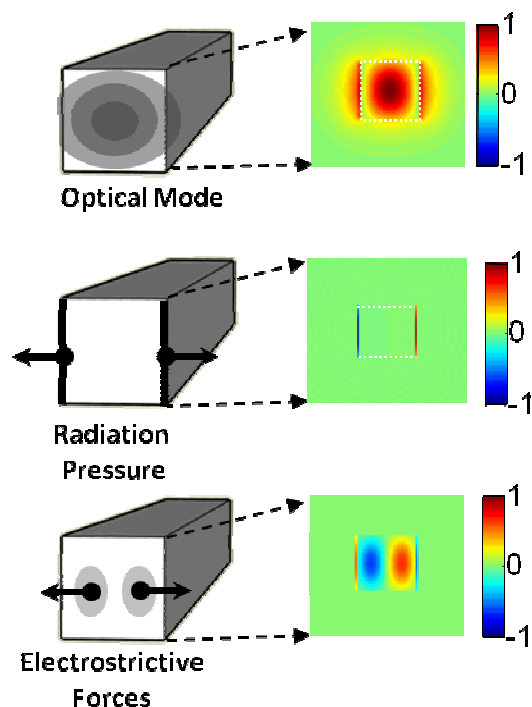
Optical Force: Proportional to Intensity.

$$F(t) \approx \alpha \cdot \sqrt{P_p \cdot P_s} \cdot \cos(\Omega \cdot t)$$

Time Varying Forces Transduce Phonon.

30

Optical Forces at Nano-scales

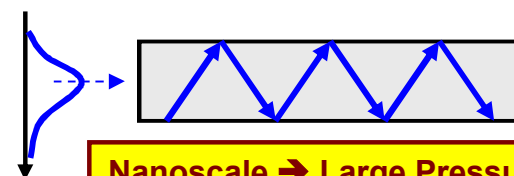


Radiation Pressure Generally VERY Small: Not Any More.

Photon momentum

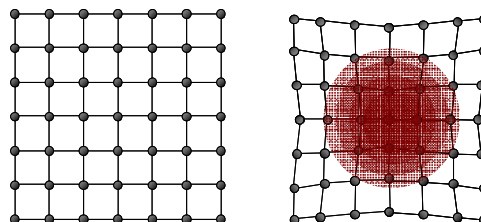
$$p = \hbar k$$

Radiation Pressure



Pressure increases to 10^4 N/m² in nanoscale waveguides.

Electrostrictive forces → Material induced forces



Akin to piezoelectricity

$$S = \alpha \cdot E_i + \beta \cdot E_i E_j$$

Strain ↑ Piezo Coeff. ↑ Electrostrictive Coeff. ↑

Electrostriction = Material induced optical forces.

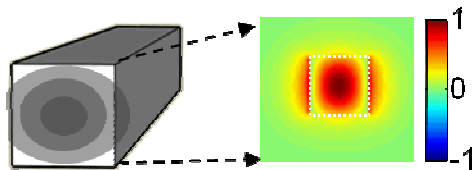
- All dielectrics exhibit electrostriction (not piezo electricity).
- Sign and magnitude are tailorable by choice of material.
- *We are the first to treat electrostriction in nano-scale systems.*

Forces produced by two Effects:

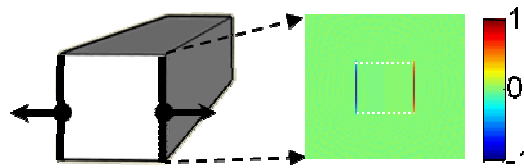
1. Radiation Pressure
2. Electrostriction

Both Scale to large values with nanometer-scale optical confinement (e.g. 10^4 N/m²)

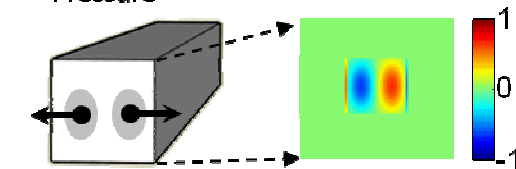
Nanoscales: The Neglected Optical Force Dominates!



Optical Mode



Radiation Pressure



Electrostrictive Forces

$$\text{Rad. Pressure: } \bar{p}^{rp} = \frac{P_{opt}}{c \cdot A} \cdot (n_g - n_p) = \frac{P_{opt}}{c \cdot A} \cdot \alpha^{rp}$$

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5-50 People standing on man-hole cover

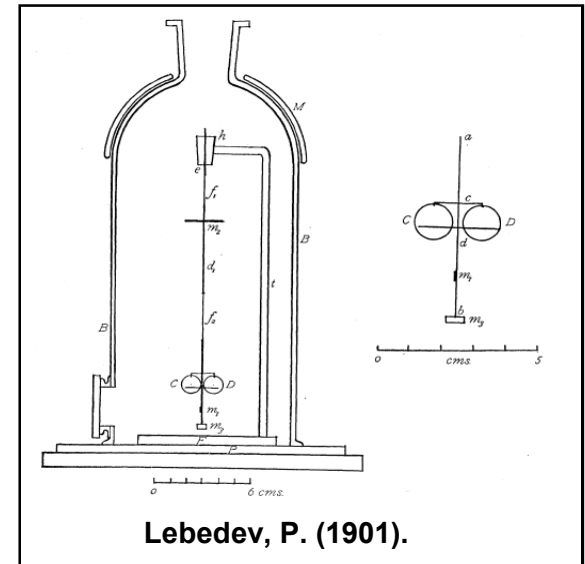
Material	Pressure (Pwr = 1kW)
Si	$\sim 5 \times 10^8 \text{ N/m}^2$
Ge	$\sim 10^{10} \text{ N/m}^2$

Stresses Approach Material Yield Strength

Historical Introduction to Optical Forces:

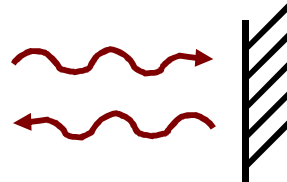
The concept of light induced motion has a very rich history:

- Timeline for Radiation Pressure:
 - (1619) Kepler: Speculated solar repulsion.
 - (1873) Maxwell: Theoretical basis for pressure.
 - (1901) Lebedew, Nichols: Experimental evidence.
- Optical Forces: *Very difficult to observe.*
(60 Watt lamp \rightarrow Force $\approx 400\text{nN}$)
- First observation: Thermal lamp & torsion balance



Photon momentum:

$$|p| = \hbar k$$



Imparted momentum:

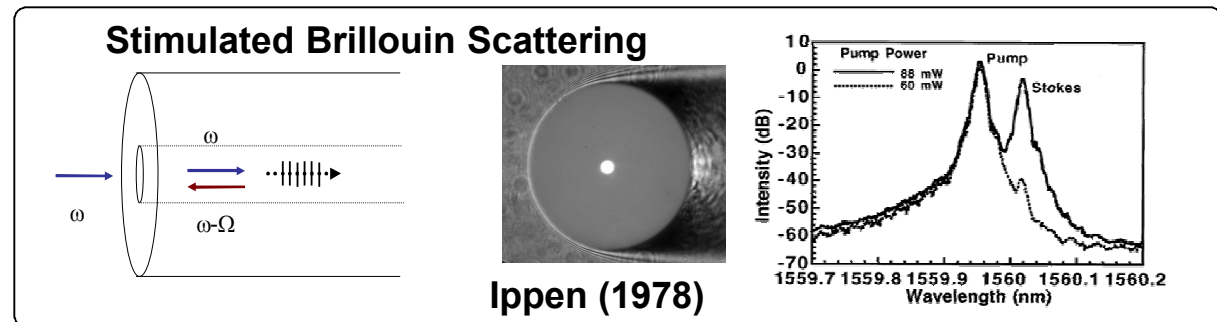
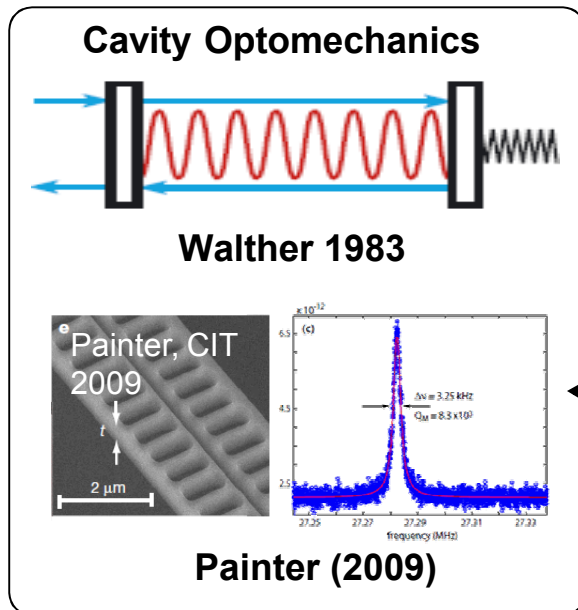
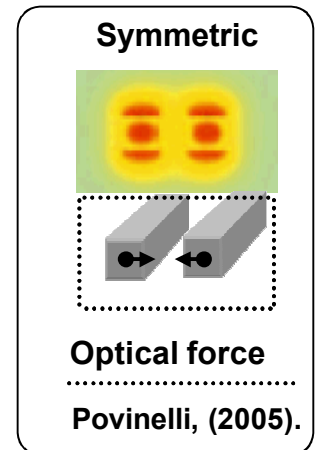
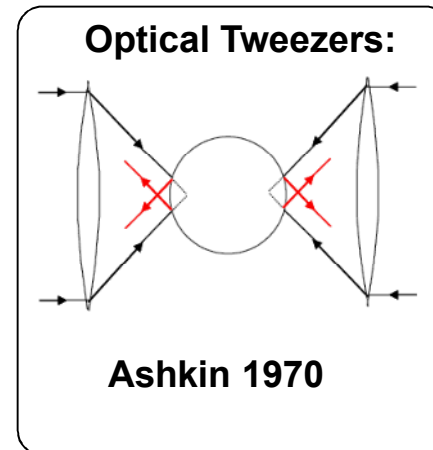
$$|\Delta p| = 2\hbar \cdot k$$

Radiation Pressure: Produced by photon recoil.

1. Nichols, E. F. & Hull, G. F. *Phys. Rev.* 13, 307–320 (1901).
2. Maxwell, J. C. *A Treatise on Electricity and Magnetism* (1873).
3. Lebedew, P. *Ann. Phys. (Leipz.)* 6, 433–458 (1901).

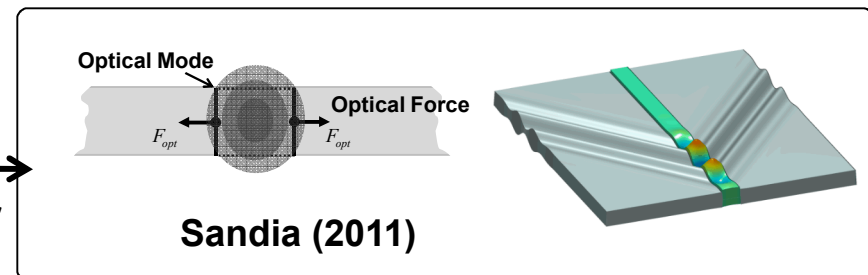
More Recent Work Involving Optical Forces:

- (1970s) Optical tweezers: Trapping small of particles (Power = 50 mW, Force \approx 50 pN)
- (1970s) Coherent phonon generation in fiber.
- (1980s) Free-space Interferometers: Optical Bistability (Power = 100 mW, Force \approx 650 pN)
- (1990s) Laser Trapping and Cooling: Atoms
- (2000s) Nano-scale actuation with light and optically driven parametric oscillation.



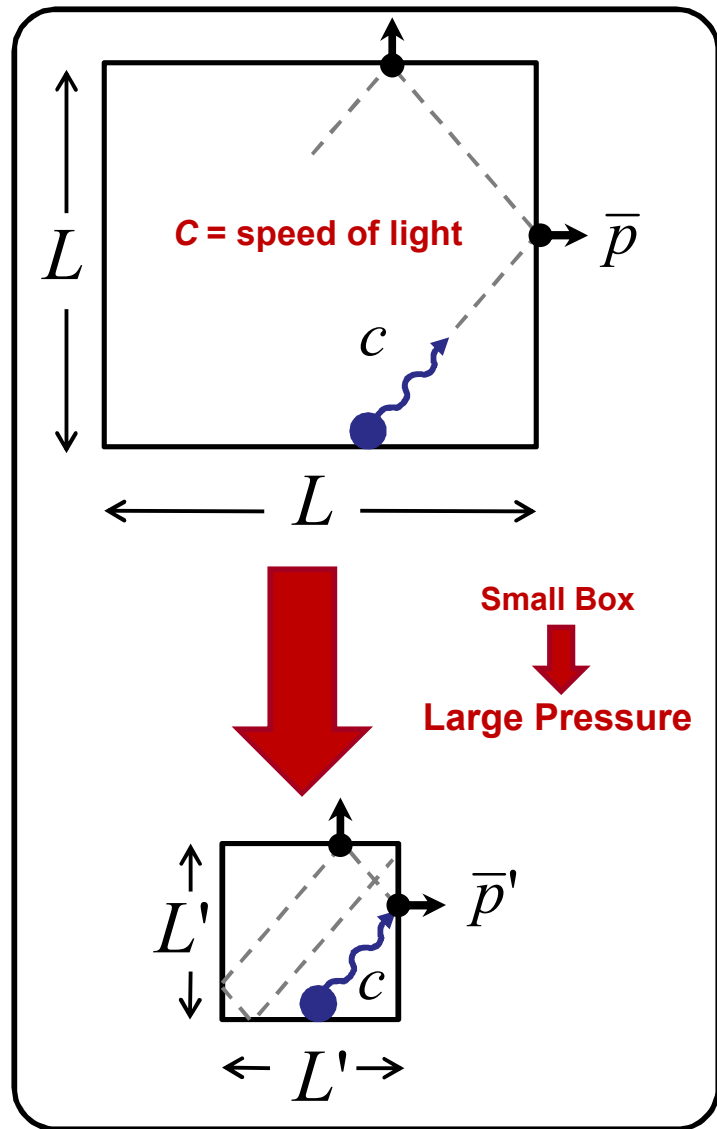
← **High fidelity
“tuning-fork”**

**Broadband
phonon emitter** →



Radiation Pressure at Nano-scales

The image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been corrupted. Restart your computer, and then open the file again. If the red x still appears, you may have to delete the image and then insert it again.



Atomic Gas:

Volume Energy

$$\bar{p}V = N \cdot k_B T$$

pressure # of atoms

Photon Gas:

Photon Energy

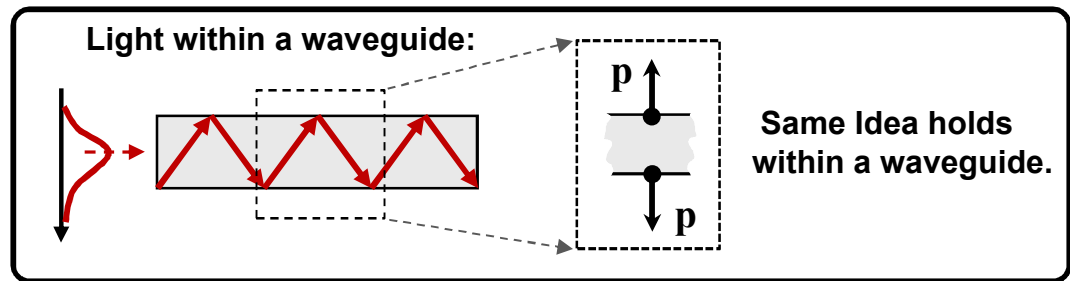
$$\bar{p}V = \frac{1}{3} \cdot N \cdot \hbar\omega$$

of photons

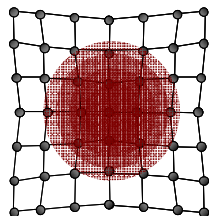
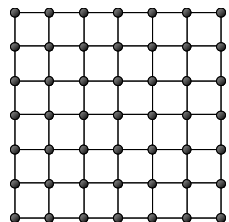
Pressure Per Photon:

$$\bar{p} = \frac{1}{3} \cdot \frac{\hbar\omega}{V} = \frac{1}{3} \cdot \frac{\hbar\omega}{L^3}$$

For $L/L' = 1000$, $p'/p = 1,000,000,000$.



Optical Forces Within Dielectric Media

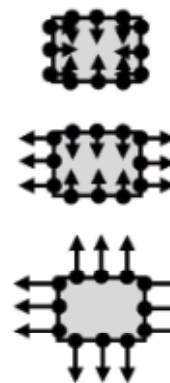


$$S = \alpha \cdot E_i + \beta \cdot E_i E_j$$

Strain
Piezo Coeff.
Electrostrictive Coeff.

Electrostriction = Material induced optical forces.

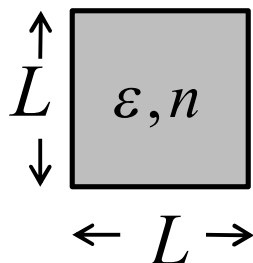
- All dielectrics exhibit electrostriction (not piezo electricity).
- Sign and magnitude are tailorable by choice of material



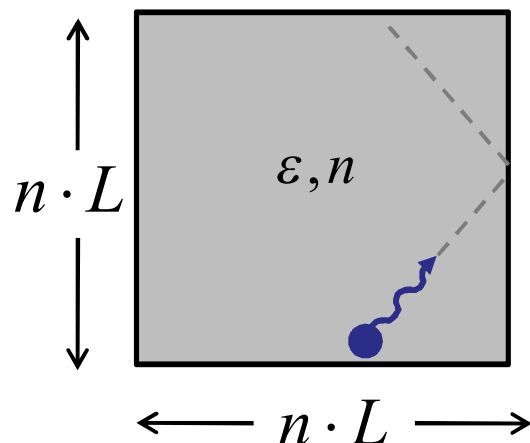
Electrostriction:
possible force
distributions.

What's going on?

Box: Real Space



Box: Photon's Perspective



From Photon's Perspective:

- Space is now quite different.

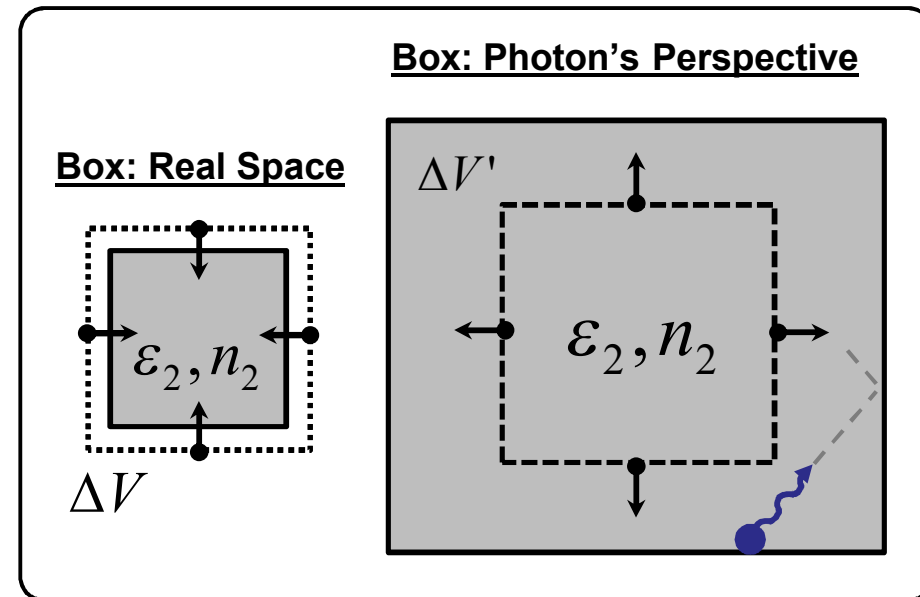
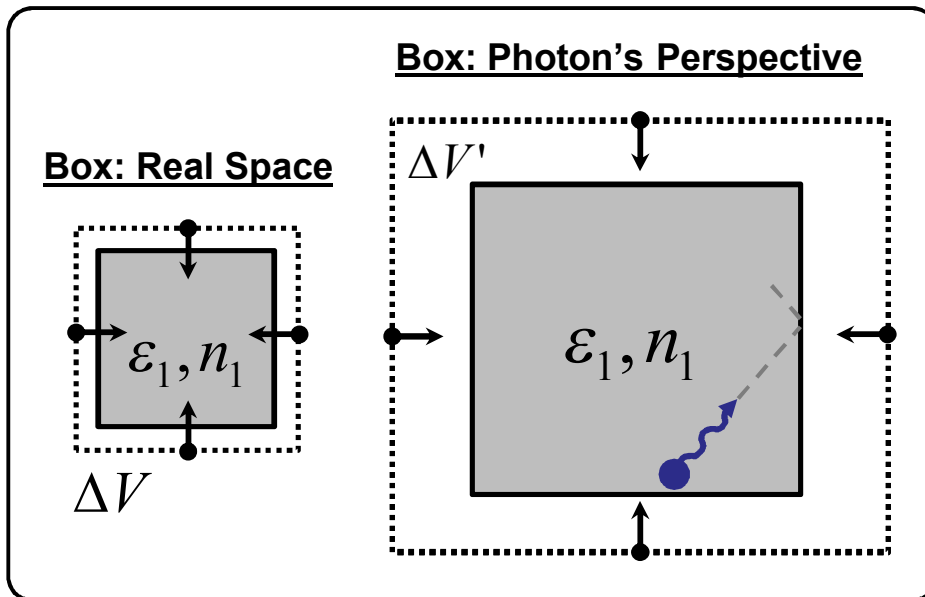
$$V \Rightarrow V \cdot n^3$$

Box seems much bigger.

Oddities don't end here...

Electrostriction: Material Induced Forces

Since $\partial n / \partial V \neq 0$, distortion of the box is perceived quite differently by a photon:



Photon gas within a dielectric:

$$\Delta V' \Rightarrow n^3 \Delta V + 3n^2 V \cdot \left(\frac{\partial n}{\partial V} \right) \cdot \Delta V$$

Correction due to dynamic response

Photon gas within a dielectric:

$$\bar{p} = \frac{1}{3} \cdot \frac{N \cdot \hbar \omega}{V} \left[1 + \frac{3}{n} \left(\frac{\partial n}{\partial V} \right) \right]$$

Electrostriction

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