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Fault Oblivious HPC with Dynamic Task Replication and Substitution

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Outline

- Example (motivating) application domain
- The DAS Architecture
- Trading off replication and substitution
- The Grid World application
- Simulation results

Motivating Application: Distributed Value Iteration

Markov Decision Process

- A collection of states S
- Can take an action *a* in each state *s*
- Next state s_{t+1} depends on current state s_t and and current action a_t
- Receive reward (payment) $r_t = r(s_t)$
- Discount factor δ: future rewards exponentially discounted
- Goal: collect maximum expected sum of rewards

MDP + Value Iteration

- Let ***V*(s)*** be the maximum expected sum of rewards starting at state ***s***
- **Bellman optimality:**

$$V^*(s) = r(s) + \delta \max_a \sum_{s'} P^a_{ss'} V^*(s')$$

- $P^a_{ss'}$ is probability of going to state ***s'*** from ***s*** if action ***a*** is taken
- **Value iteration (starting at $V_0(s) = r(s)$):**

$$V_{n+1}(s) \leftarrow r(s) + \delta \max_a \sum_{s'} P^a_{ss'} V_n(s')$$

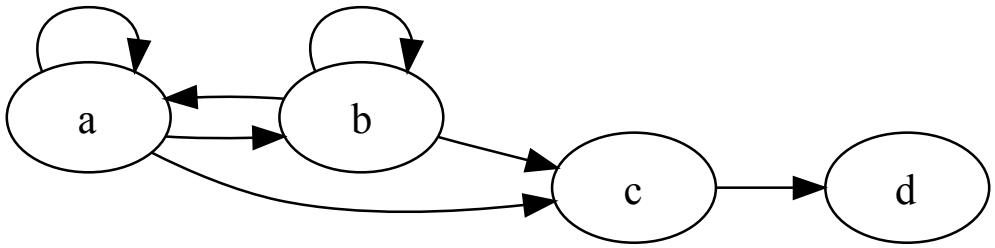
Value Iteration

- Provably converges to V^*
- Standard implementation updates values synchronously; as long as values for all states keep being updated, asynchronous variant also converges
- Requires broadcast transmission of the entire vector V_n in every step
- Requires storing the entire table of transition probabilities, etc, of total size |S||A|
- Cannot break this up into independent subproblems: no independent subsets of values

Dependency Graph

Definition

- Each node *i* represents a computational task
  - Computing $V^*(s)$ (and intermediate $V_s(s)$)
- A directed edge from *i* to *j* means that *i* depends on data generated by *j*
  - A non-zero probability of going from state *s* to *s'* means that *V(s)* and *V(s')* depend on each other
  - A task can depend on itself



Using Dependency Graph

- Can be specified/change dynamically
- Don’t need to broadcast the entire value vector: nodes can query current value from each neighbor
- Much smaller storage requirements for each node (only information about itself and neighbors)

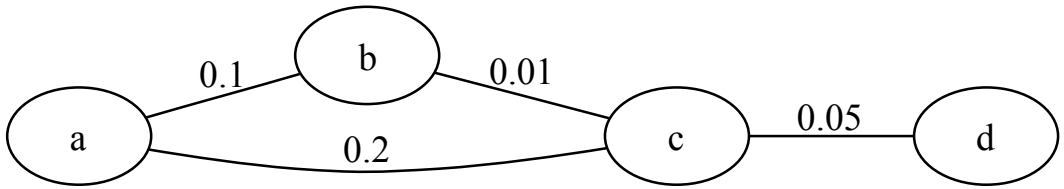
Using Dependency Graph

- Fault tolerance via replication
  - If node *i* detects that its neighbor has failed, send request to the system to restart that computation
- Checkpointing:
  - Intermediate results can be multicast to nodes that depend on task *i* (this could be the data required by these neighbors anyway)
  - If *i* fails, it can be hot-started based on intermediate results
  - Checkpointing can be (potentially) less frequent

Substitution Graph

Substitution Graph

- Edge between *i* and *j* means that computation performed at node *i* can be substituted by that at node *j* (and vice versa)
- Assume substitution relationships are reciprocal (undirected edges); not really necessary, but convenient
- Substitution can be imperfect: an edge has a weight representing an upper bound on error introduced by substitution
- Can be dynamic: graph can change "on the fly"



Transitivity of Substitution

- Note that substitution relationships need not be transitive:
  - If we only include edges with error below some threshold e, can have error(a,b) < e, error(b,c) < e, but error(a,c) > e

Fault Obliviousness via Substitution

- Suppose task *i* fails and another task *j* depends on its computation. Instead of replicating *i*, can substitute by *k*, incurring some error
- In general, substitution error depends on who uses the computation, so in general could have a different graph for each task
- Note that we can store it in a distributed way
  - each task stores only substitutions for tasks it depends on and corresponding weights

Trading Off Replication and Substitution

Notation

- Suppose errors are additive (e.g., using l_1 norm)
- w_{ijk}: error incurred by task k if computation it requires performed by i is substituted by j
- I: the set of failed tasks
- D_i: set of tasks that depend on I
- v_i = 1 iff i is substituted for (rather than replicated)
- z_{ijk} = 1 iff j substitues for i for a dependent task k (repaces k's dependence on i with j)

Mixed Integer Program Formulation

Objective: max # of substitutions (minimize # of replications)

$$\max_{v,z} \sum_{i \in I} v_i$$

Can account for the fact that subsets of tasks are allocated to different processors minimizing inter-processor communication by adding a constraint on the number of substitutions that cross processor boundaries

Only a single substitute for *i,k*

$$\sum_{j \notin I} z_{ijk} = v_i \; \forall i \in I, k \in D_i$$

Don't substitute with failed tasks

$$z_{ijk} = 0 \; \forall i, j \in I, k \in D_i$$

(unnecessary in practice)

Binary variables

$$v_i, z_{ijk} \in \{0,1\}$$

Simplified Optimization

- Previous MIP is NP-Hard to solve, probably not something you can do in real time
- Let's simplify:
  - N_i: number of tasks that depend on i
  - $w_{ij} = \max_k w_{ijk}$
  - Focus on a best substitute for a failed i, with

$$w_i = N_i \min_{i \notin I} w_{ij}$$

Simplified MIP

Objective: max # of substitutions
(minimize # of replications)

$$\max_{v} \sum_{i \in I} v_i$$

Subject to:

Error budget

$$\sum_{i \in I} w_i v_i \leq \varepsilon$$

Binary variables

$$v_i \in \{0, 1\}$$

This is just a Knapsack problem; can be approximated by a simple, fast greedy heuristic:

add tasks in increasing order of w_i until error budget is saturated

Obtaining Substitution Error Weights

- Upper bound on errors can be specified for the specific problem instance based on domain knowledge
- Empirical: observe data streams produced at different nodes and compare based on some relevant distance metric; if similar enough, add a substitution edge (and specify weight)
- Combined: specify edges a priori, but use empirical weights

Simulation Results: Value Iteration on the Grid World

Grid World

- An agent moves around the grid world
- Each cell represents a state and yields some reward *r*
- Agent can move up, down, left, right
- Resulting location is a probabilistic function of the move (agent is drunk)
  - Agent actually goes in the direction of his intended action with probability *p = 0.8*
  - Moves to any other adjacent location with equal probability
  - Grid boundaries are “walls” (no wrap around)

Example grid world

Value Iteration, Substitution Errors

- **Set discount factor to 0.95**
- **Stopping criterion at 0.001 (stop once values are not changing by more than this)**
- **Values of neighboring states *s* and *s'* are close substitutes for one another; can bound error of such substitution (w_{ij}):**

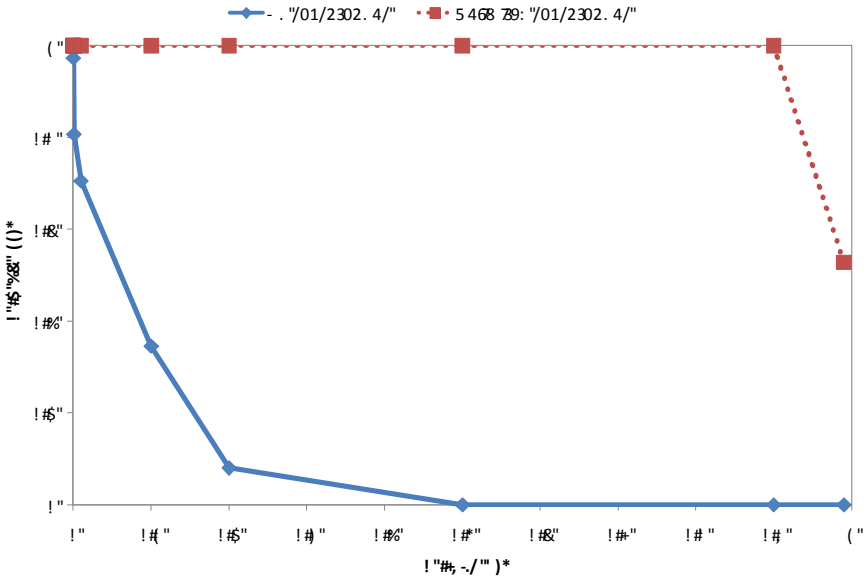
$$|V(s) - V(s')| \leq \frac{1 - \delta p}{1 - \delta}$$

Simulation Setup

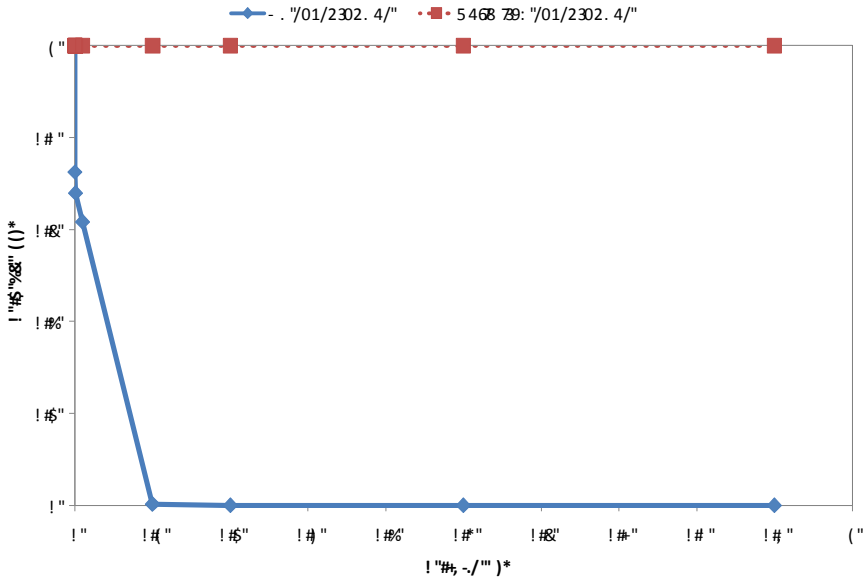
- Simulate a cluster; allocate a task per cluster node; run 100 iterations
- Nodes can fail and are fixed probabilistically (independently); ½ of all processors are functional on average; initialized to steady state
- Number of tasks is ½ of total number of nodes (saturate expected number of functioning processors)
- Compare pure replication (always replicate failed tasks) to pure substitution (always substitute)

Resilience to Failures

10x10 grid



50x50 grid



Advantage of substitution greater on a larger problem

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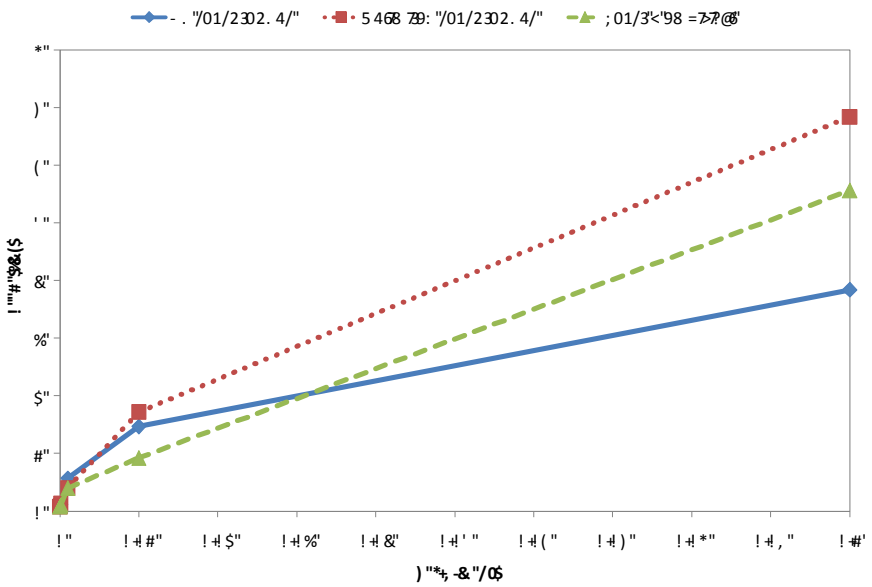
Computation Error

10x10 grid



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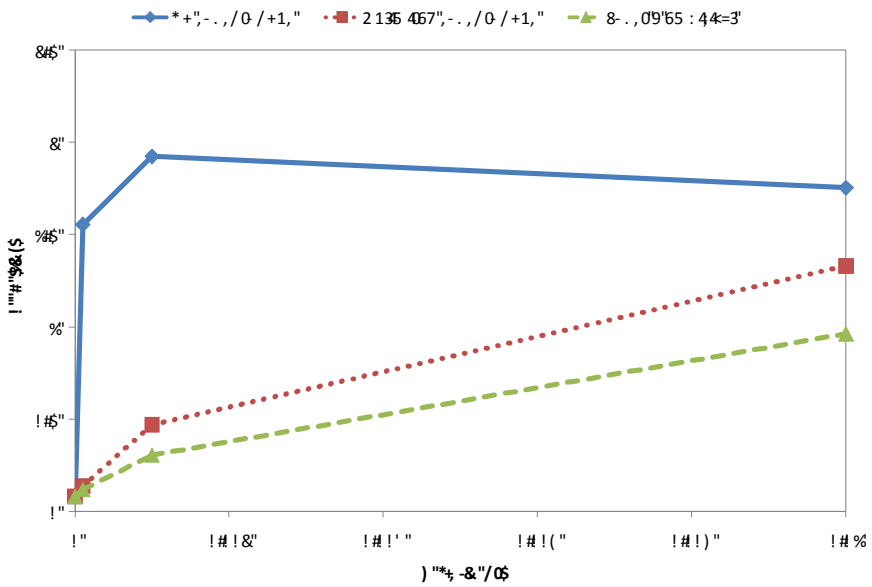

Empirical substitution is best when individual node failure probability is low; advantage greater on the larger problem

Conclusions

- Novel architecture for fault tolerance/obliviousness using a combination of task dependency and substitution graphs
- Optimization problem formulates tradeoffs between replication and substitution; can approximately solve a simplified problem in real time
- Simulation results suggest that ability to substitute tasks is advantageous for resilience and keeps errors down