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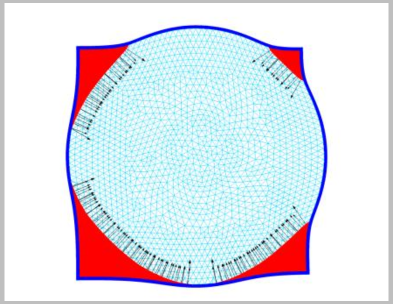

A Conformal Decomposition Finite Element Method (CDFEM) for Burning Deformable Solids

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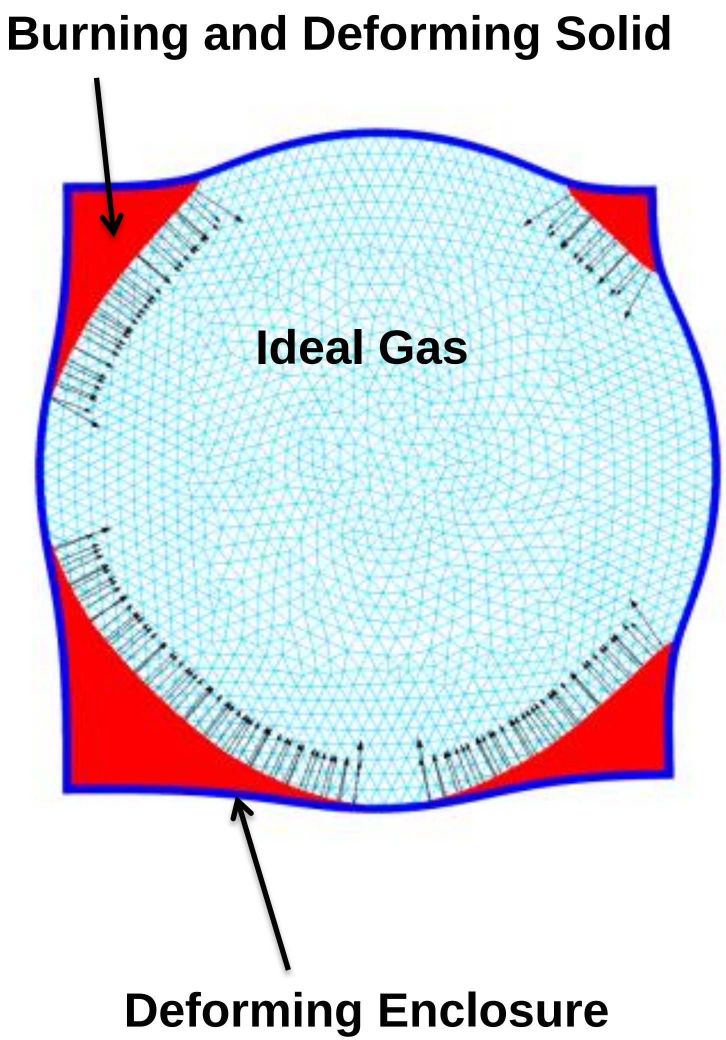
Problem of Interest



- Disparate Volumetric Physics
  - 3 Distinct materials and physics – fluid, solid, and burning solid
  - DOFs discontinuous or one-sided
- Complex Interfacial Physics
  - Momentum balance, mass balance, burn front motion
  - Fronts moving with speed other than local velocity
- Dynamic Topology
  - Precludes simple moving mesh methods
  - Interfaces created and deleted dynamically

Volume and Interface Equations

- Fluid Mechanics
  - Quasi-static, compressible ideal gas

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho u = 0 \qquad P = \frac{\rho RT}{M}$$

$$0 = -\nabla P + \nabla\cdot\mu\left(\nabla u + \nabla u^{t}\right) + \nabla\left(-\frac{2\mu}{3}\nabla\cdot u\right)$$

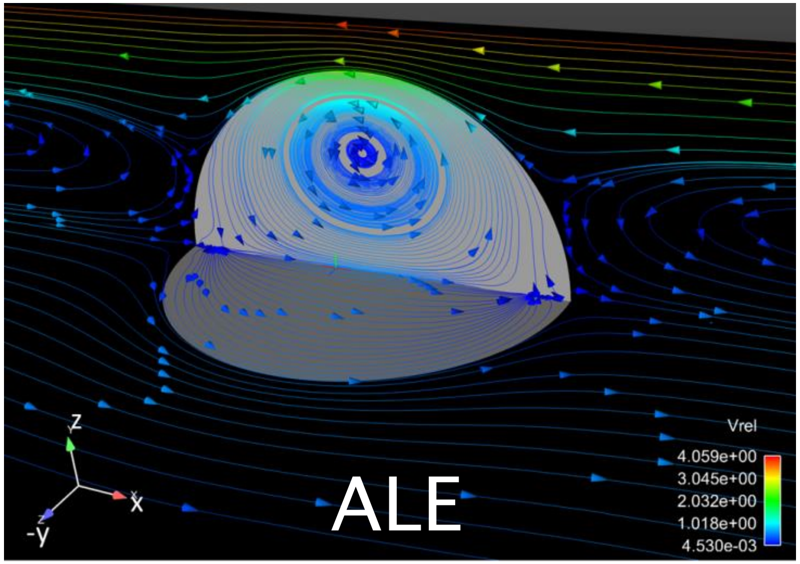
- Solid Mechanics
  - Quasi-static, linear elastic

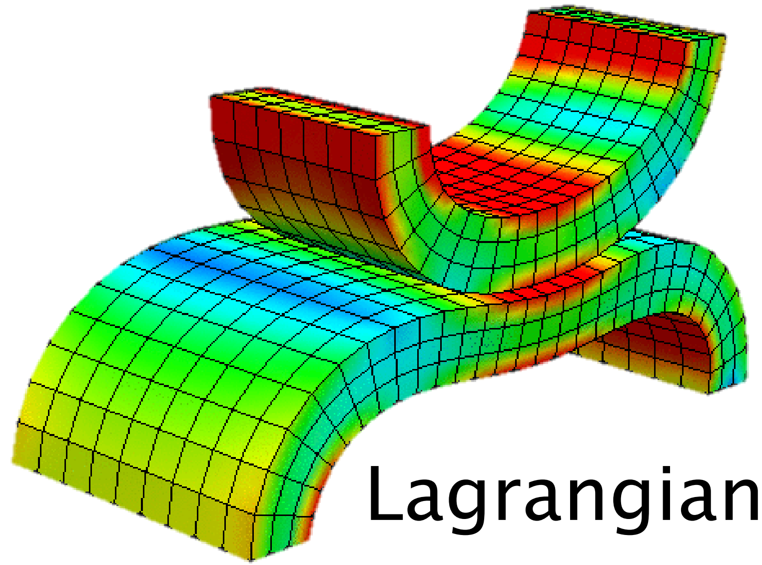
$$0 = \nabla\cdot\mu_s\left(\nabla d + \nabla d^{t}\right) + \nabla\left(\lambda_s\nabla\cdot d\right)$$

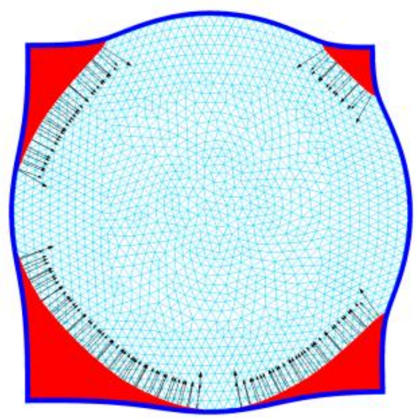
- Burn Front
  - Simple pressure-dependent speed

$$\gamma = k_0 + k\left(P - P_o\right)^{\alpha}$$

Mesh Motion Discretizations

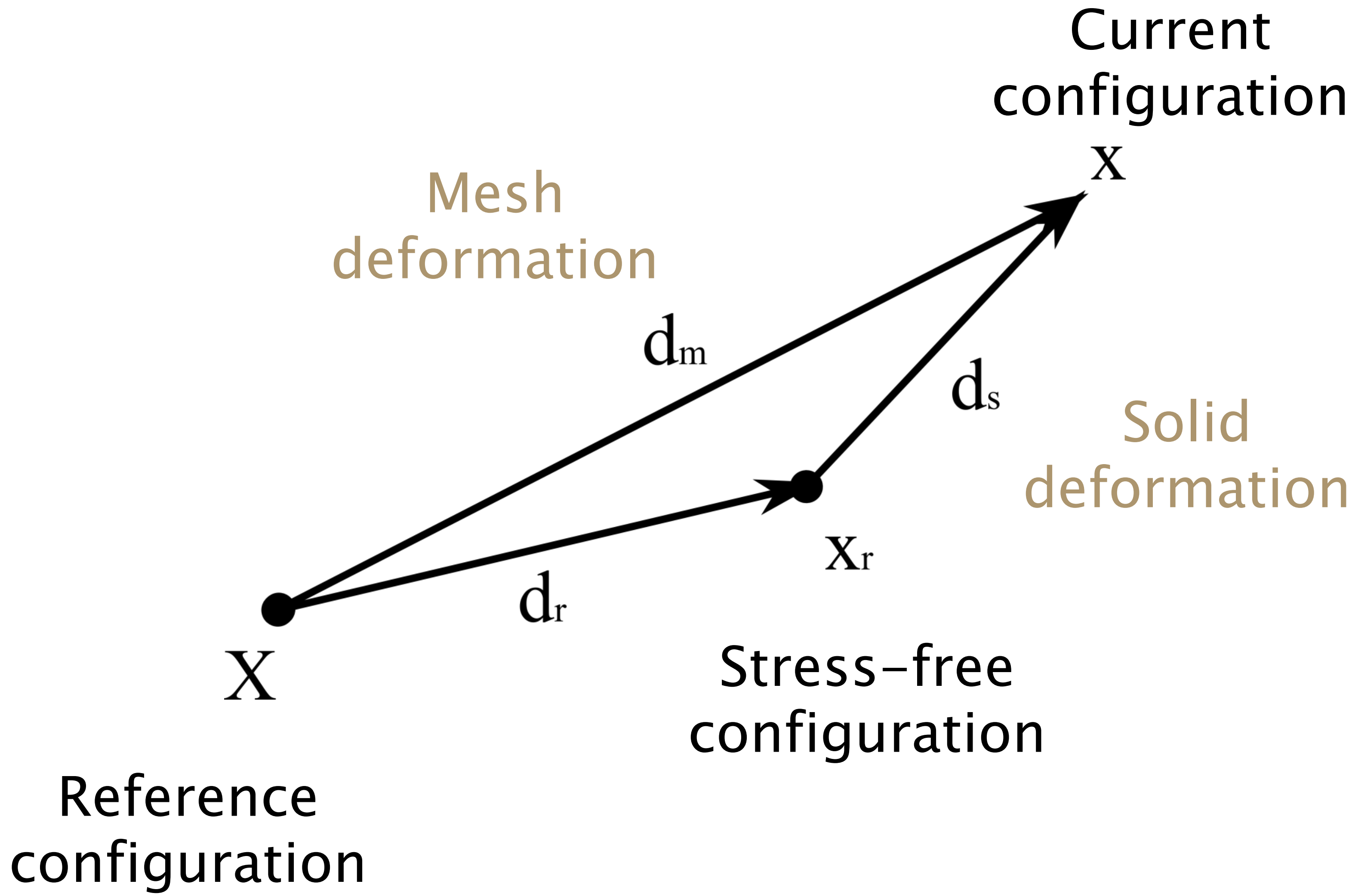






TALE: Fluid–Structural Interactions + Burning

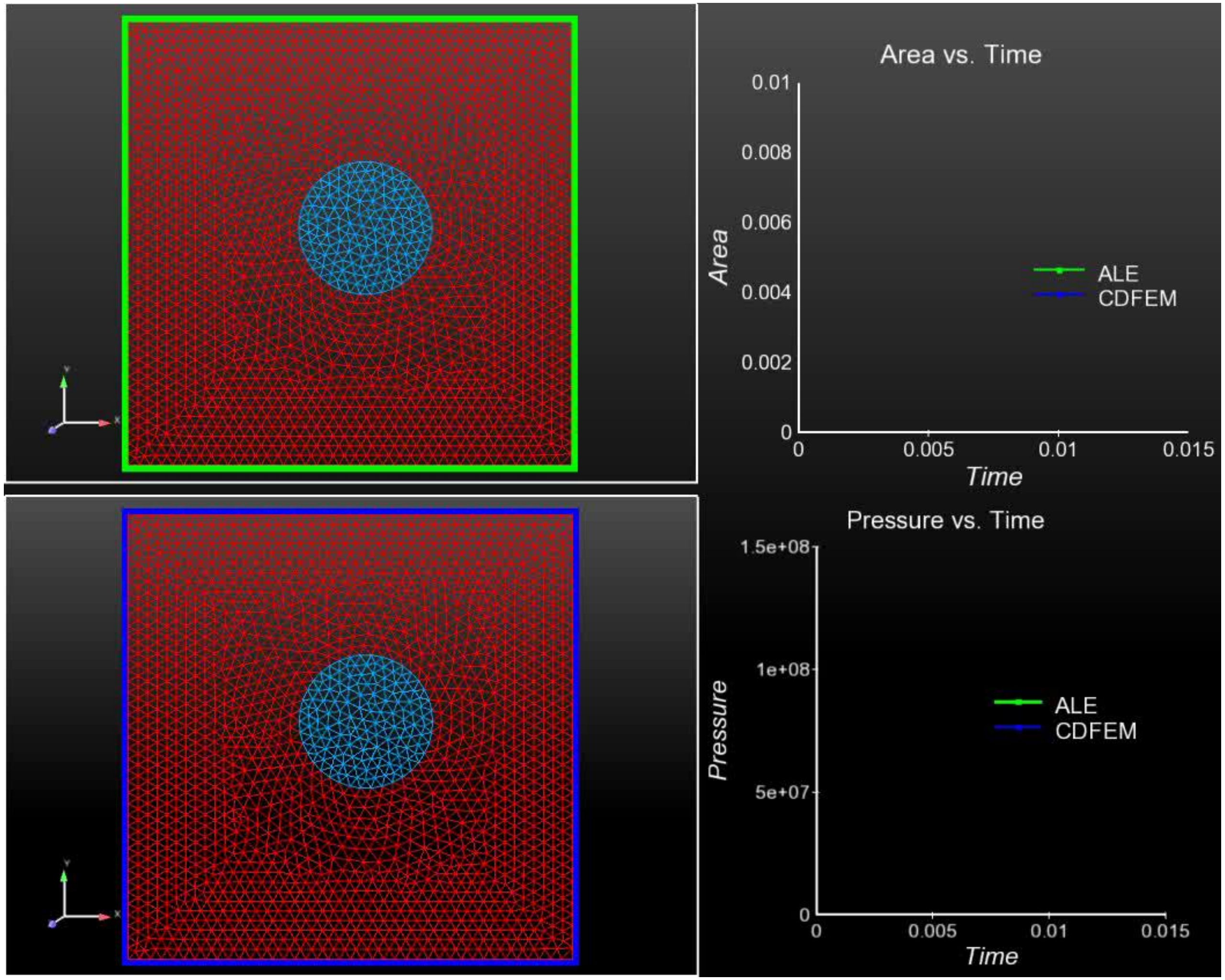
Total ALE (TALE)



TALE Boundary Conditions

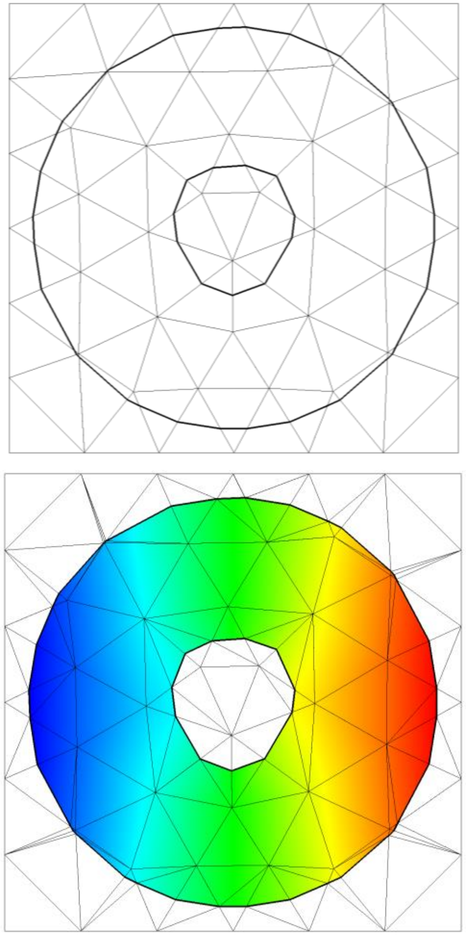
- Vector Version of Interfacial Mass Balance and No-Slip
  - Conservation of Mass Across Interface (normal direction)
    $$\mathbf{n}\cdot\left(\rho_f\mathbf{u}_f-\rho_s\dot{\mathbf{d}}_s-\left(\rho_f-\rho_s\right)\dot{\mathbf{d}}_m\right)\mathbf{n}=0$$
  - No-slip (tangent direction)
    $$\left(\mathbf{I}-\mathbf{nn}\right)\cdot\left(\mathbf{u}_f-\dot{\mathbf{d}}_s\right)=0$$
  - Result
    $$\rho_1\left(\mathbf{u}_f-\dot{\mathbf{d}}_s\right)+\mathbf{n}\left(\rho_f-\rho_s\right)\left(\mathbf{n}\cdot\dot{\mathbf{d}}_s-\mathbf{n}\cdot\dot{\mathbf{d}}_m\right)=0$$
- Vector Version of Solid Mass Balance
  - Solid Kinematic Condition (normal direction)
    $$\left(\dot{\mathbf{d}}_m-\dot{\mathbf{d}}_s\right)\cdot\mathbf{n}=\gamma$$
  - No-slip Condition (tangent direction)
    $$\left(\mathbf{I}-\mathbf{nn}\right)\cdot\left(\dot{\mathbf{d}}_m-\dot{\mathbf{d}}_s\right)=0$$
  - Result
    $$\left(\dot{\mathbf{d}}_m-\dot{\mathbf{d}}_s\right)=\left(k_0+k\left(P-P_o\right)^\alpha\right)\mathbf{n}$$

Comparison Between ALE and CDFEM Simulation of Burning, Rigid Solid



Conformal Decomposition Finite Element Method (CDFEM)

- Simple Concept
  - Use one or more level set fields to define materials or phases
  - Decompose non-conformal elements into conformal ones
  - Obtain solutions on conformal elements
- Related Work
  - Li et al. (2003) FEM on Cartesian Grid with Added Nodes
  - Ilinca and Hetu (2010) Finite Element Immersed Boundary
- Properties
  - Supports wide variety of interfacial conditions (identical to boundary fitted mesh)
  - Avoids manual generation of boundary fitted mesh
  - Supports general topological evolution (subject to mesh resolution)
- Similar to finite element adaptivity
  - Uses standard finite element assembly including data structures, interpolation, quadrature

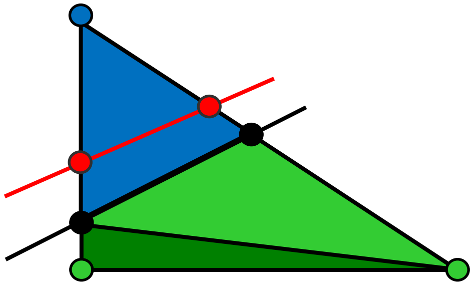


XFEM - CDFEM Requirements Comparison for Thermal/Fluids

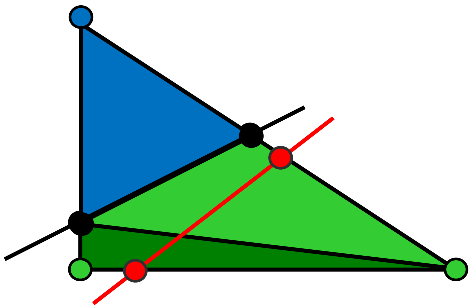
	XFEM	CDFEM
Volume Assembly	Conformal subelement integration, specialized element loops to use modified integration rules	Standard Volume Integration
Surface Flux Assembly	Specialized volume element loops with specialized quadrature	Standard Surface Integration
Phase Specific DOFs and Equations	Different variables present at different nodes of the same block	Block has homogenous dofs/equations that may differ from block to block
Dynamic DOFS and Equations	Require reinitializing linear system	Require reinitializing linear system
Various BC types on Interface	Dirichlet BCs are research area	Standard Techniques available

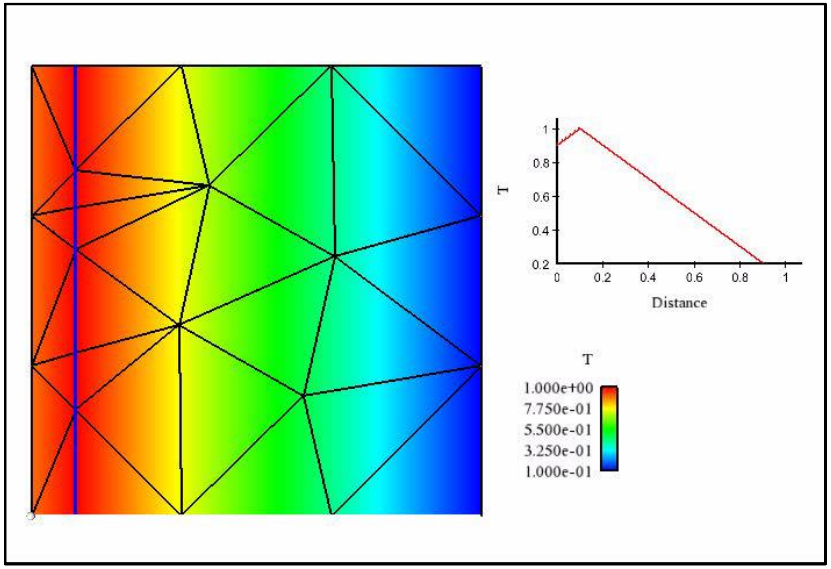
CDFEM for Moving Interfaces

- How do we handle the moving interface?



- What do we do when nodes change sign?



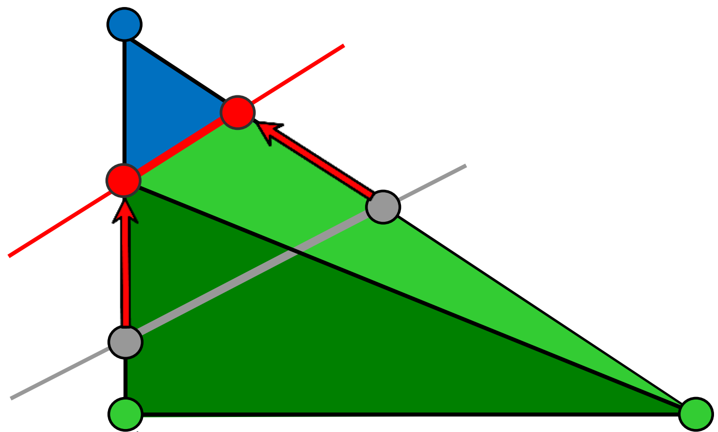


Patch Test: Exact preservation of discontinuous gradient with constant advection

Approach for Dynamic Discretizations Due to Moving Interfaces: Mesh Motion

- ALE – CDFEM Approach
  - Consider deforming domain
  - Apply chain rule to time derivative

$$\left.\frac{\partial \psi}{\partial t}\right\|_{\xi} = \left.\frac{\partial \psi}{\partial t}\right\|_{\mathbf{x}} + \left.\frac{\partial \mathbf{x}}{\partial t}\right\|_{\xi} \cdot \nabla_{\mathbf{x}} u$$



  - Constant advection – Backward Euler

$$\int_{\Omega}\left(\frac{\partial \psi}{\partial t} + u \cdot \nabla \psi\right) N_i \, d\Omega = 0 \Rightarrow R_i = \int_{\Omega}\left(\left.\frac{\partial \psi}{\partial t}\right\|_{\xi} + (\mathbf{u} - \dot{\mathbf{x}}) \cdot \nabla \psi\right) N_i \, d\Omega$$

$$R_i = \int_{\Omega_+^{n+1}}\left(\frac{\psi_+^{n+1}(\xi) - \psi_+^{n}(\xi)}{\Delta t} + u \cdot \nabla \psi_+^{n+1}\right) N_i \, d\Omega + \int_{\Omega_-^{n+1}}\left(\frac{\psi_-^{n+1}(\xi) - \psi_-^{n}(\xi)}{\Delta t} + u \cdot \nabla \psi_-^{n+1}\right) N_i \, d\Omega$$

  - Requires integration only over new decomposition
  - Requires definition of mesh velocity, $\dot{\mathbf{x}}$
    - Current algorithm: Find nearest point on old interface

Level Set Interface Motion

- Level Set Evolves with Extension Speed
  - Rather than typical advection with fluid velocity

$$\mathbf{u}_\phi = -e\frac{\nabla\phi}{|\nabla\phi|} \qquad \frac{\partial\phi}{\partial t} + \mathbf{u}_\phi\cdot\nabla\phi = 0 \rightarrow \frac{\partial\phi}{\partial t} = e$$

- Extension Speed Equation
  - Minimize gradients in extension speed along level set gradients

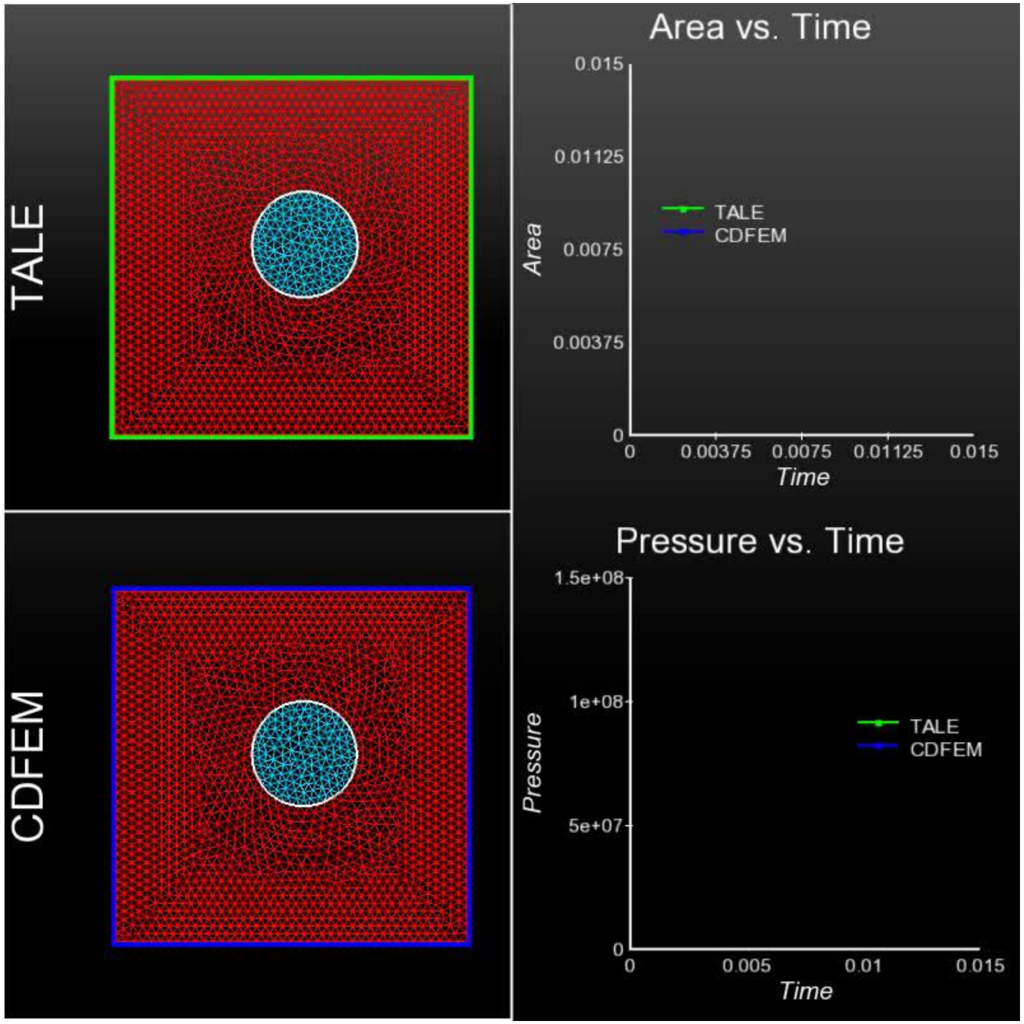
$$\min \text{wrt}\, e_i : \frac{1}{2}\frac{\partial}{\partial e_i}(\nabla e\cdot\nabla\phi)^2 \rightarrow (\nabla e\cdot\nabla\phi)(\nabla w_i\cdot\nabla\phi) = 0$$

  - Results in anisotropic diffusion equation
  - CDFEM discretization allows specification of interface speed via Dirichlet BCs

TALE-CDFEM Boundary Conditions

- Vector Version of Interfacial Mass Balance and No-Slip
  - Conservation of Mass Across Interface (normal direction)
    $$\mathbf{n}\cdot\left(\rho_f\mathbf{u}_f-\rho_s\dot{\mathbf{d}}_s-\left(\rho_f-\rho_s\right)\left(\dot{\mathbf{d}}_m-e\mathbf{n}\right)\right)\mathbf{n}=0$$
  - No-slip (tangent direction)
    $$\left(\mathbf{I}-\mathbf{nn}\right)\cdot\left(\mathbf{u}_f-\dot{\mathbf{d}}_s\right)=0$$
  - Result
    $$\rho_f\left(\mathbf{u}_f-\dot{\mathbf{d}}_s\right)+\mathbf{n}\left(\rho_f-\rho_s\right)e+\mathbf{n}\left(\rho_f-\rho_s\right)\left(\mathbf{n}\cdot\dot{\mathbf{d}}_s-\mathbf{n}\cdot\dot{\mathbf{d}}_m\right)=0$$
- Solid Mass Balance for Extension Speed (normal direction)
  $$\left(\dot{\mathbf{d}}_m-\dot{\mathbf{d}}_s\right)\cdot\mathbf{n}-e=\gamma$$
  - Mesh is free to minimize mesh stress rather than enforce kinematic condition
    - Eliminates tangling
    - Required to maintain straight element sides in cut elements

Comparison Between TALE and CDFEM Simulation of Burning, Deformable Solid



Summary

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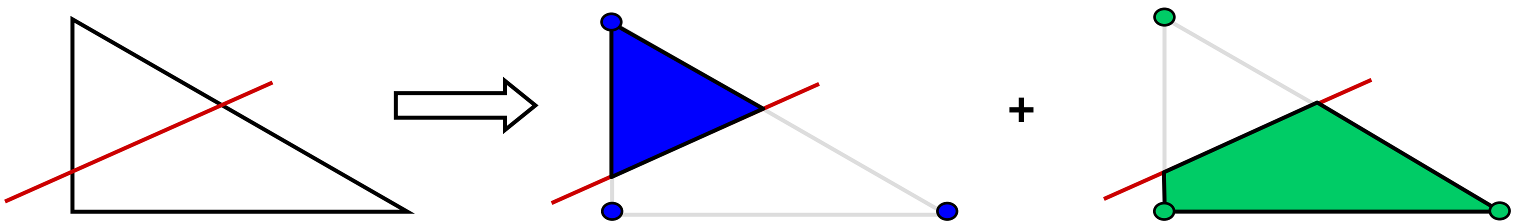
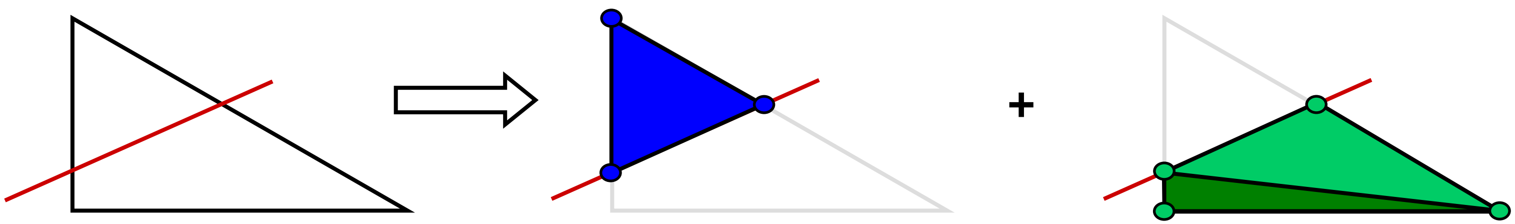
- TALE Method for Solid Mechanics and Interfacial Reactions
  - Moving mesh method with separately element blocks and sidesets for material volumes and material interfaces
  - Separate deformation field for accurately capturing solid mechanics
  - Cannot address topology change
- CDFEM-TALE Method for Solid Mechanics and Interfacial Reactions
  - Combined moving mesh and level set method with dynamically generated blocks and sidesets for material volumes and material interfaces
  - Minimizes mesh deformation
  - Captures topology change

XFEM – CDFEM Discretization Comparison

- XFEM Approximation



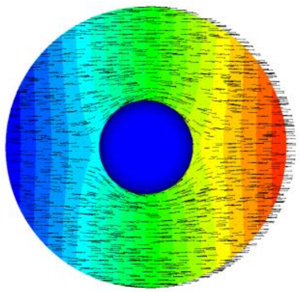
- CDFEM Approximation

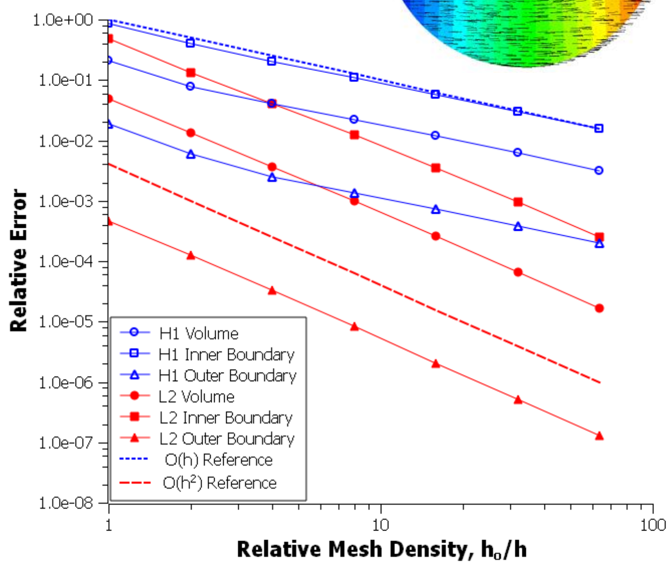


  - Identical IFF interfacial nodes in CDFEM are constrained to match XFEM values at nodal locations
  - CDFEM space contains XFEM space
    - CDFEM is no less accurate than XFEM (Li et al., 2003)
    - XFEM can be recovered from CDFEM by adding constraints

CDFEM Verification

- Steady Potential Flow about a Sphere
  - Embedded curved boundaries
  - Dirichlet BC on outer surface, Natural BC on inner surface
  - Optimal convergence rates for solution and gradient both on volume and boundaries





- Steady, Viscous Flow about a Periodic Array of Spheres
  - Embedded curved boundaries
  - Dirichlet BC on sphere surface
  - Accurate results right up to close packing limit
  - Sum of nodal residuals provides accurate/convergent measure of drag force