

# $\text{Li}^+$ transport in organic solvents: a Green-Kubo study

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# Motivation: $\text{Li}^+$ Batteries

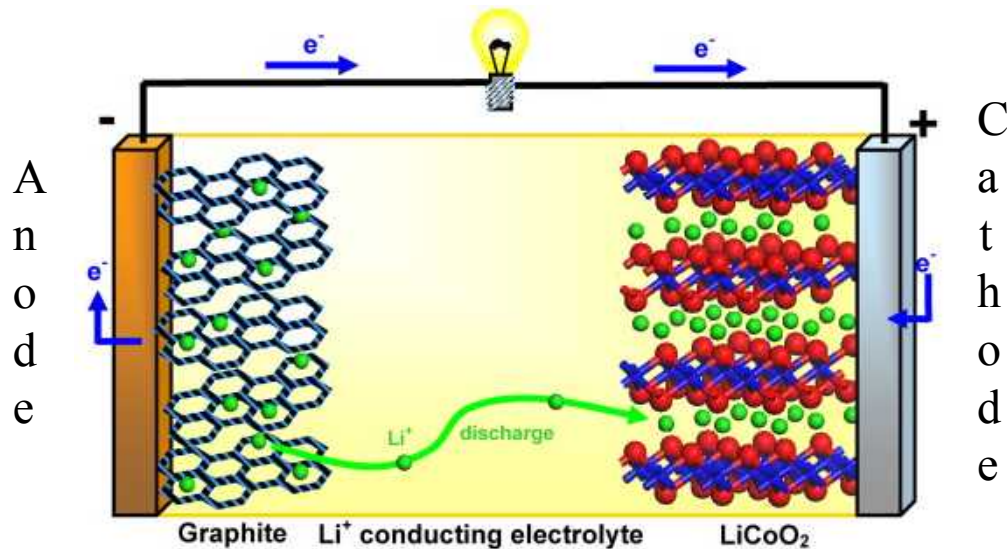


Fig. 1. Schematic representation of a lithium-ion battery. Negative electrode – graphite, positive electrode –  $\text{LiCoO}_2$ , separated by a non-aqueous liquid electrolyte  $\text{LiPF}_6$  ethylene carbonate/dimethyl carbonate. P.G. Bruce, Solid State Ionics, 179 (2008), pg. 752

- $\text{Li}^+$  batteries have become industry standard in personal electronics
- New applications for vehicles and higher energy demand fields
- Electrolytes generally consist of Lithium Salt ( $\text{LiBF}_4$ ) and organic solvent (carbonates)
- Transport between anode and cathode can be tuned by electrolyte
- Molecular dynamic (MD) methods can sample a large number of configurations



# Transport properties via Green-Kubo

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- Technique for calculating macroscopic properties from microscopic properties
- Correlation functions are determined from atom velocities

**Fick's law gives**

$$J = -D\nabla c$$

**Continuity requires**

$$\frac{\partial c}{\partial t} + \nabla \cdot J = 0$$

**Substitution gives**

$$\frac{\partial c(r, t)}{\partial t} = D\nabla^2 c(r, t)$$

**Einstein's Derivation (do some math:  $\int r^2$  and integrate)**

$$\frac{\partial}{\partial t} \int d\vec{r} r^2 c(r, t) = 2dD$$

# Transport properties via Green-Kubo

$$\frac{\partial}{\partial t} \int d\vec{r} r^2 c(r, t) = 2dD$$

**LHS is average of  $r^2$**

**Or for atomistics**

$$\frac{\partial}{\partial t} \langle r^2(t) \rangle = 2dD \quad \longrightarrow \quad \frac{\partial}{\partial t} \langle \Delta x(t)^2 \rangle = 2dD$$

**Displacements as velocities**

$$\Delta x(t)^2 = \int_0^t dt' v_x(t')$$

**Make substitutions and adjust time integrals (left for you!)**

$$D = \int_0^\infty d\tau \langle v_x(\tau) v_x(0) \rangle$$



# Transport properties via Green-Kubo

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## Viscosity

$$\eta = \frac{1}{Vk_B T} \int_0^{\infty} dt \langle \sigma_{xy}(t) \sigma_{xy}(0) \rangle$$

## Thermal Conductivity

$$\lambda_T = \frac{1}{Vk_B T} \int_0^{\infty} dt \langle j_z^e(t) j_z^e(0) \rangle$$

## Electrical Conductivity

$$\sigma_e = \frac{1}{Vk_B T} \int_0^{\infty} dt \langle j_x^{el}(t) j_x^{el}(0) \rangle$$

# CHARMM Potential

$$V = E_b + E_a + E_d + E_i + E_{nb}$$

$$E_b = \sum_{bonds} k_b (b - b_0)^2$$

$$E_a = \sum_{angles} k_\theta (\theta - \theta_0)^2$$

$$E_d = \sum_{dihedrals} k_\phi [1 + \cos(n\phi - \delta)]$$

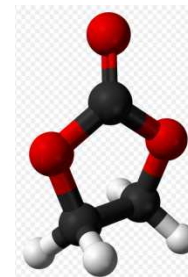
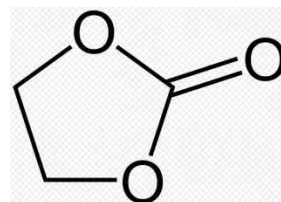
$$E_i = \sum_{impropers} k_\omega (\omega - \omega_0)^2$$

$$E_{nb} = \sum_{nonbonded} \varepsilon_1 \left[ \left( \frac{R_{min\ ij}}{r_{ij}} \right)^{12} - \left( \frac{R_{min\ ij}}{r_{ij}} \right)^6 \right] + \frac{q_i q_j}{\varepsilon_2 r_{ij}}$$

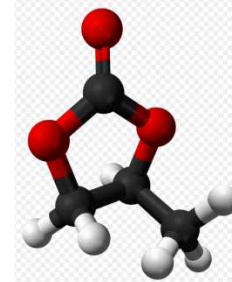
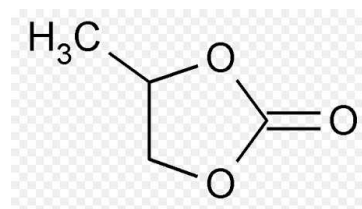
2 cases

- a) Flexible carbonates
- b) Rigid carbonates

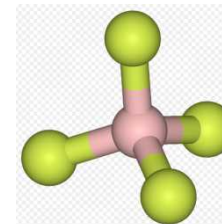
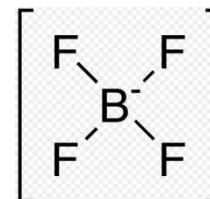
## Ethylene Carbonate (EC)



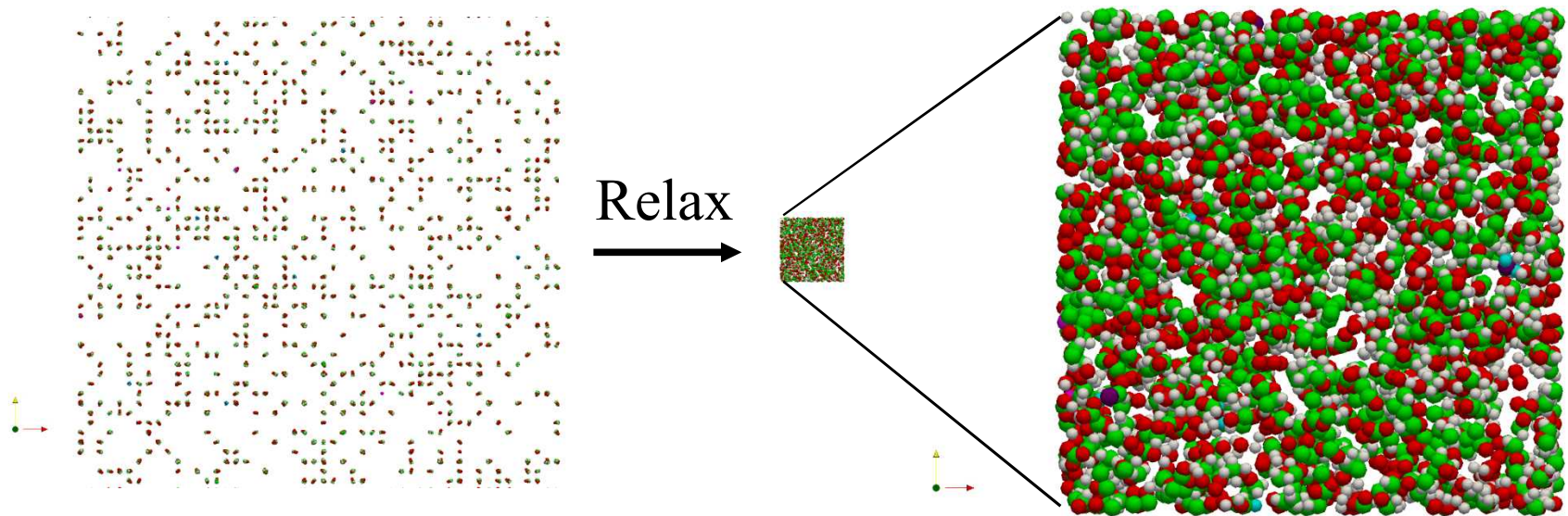
## Propylene Carbonate (PC)



## Tetrafluoraborate (BF<sub>4</sub>)



# Initial configuration



- **3 step initialization process**
  - Create sparse distribution of Carbs and separated salts
  - NVE/limit to ensure no atom ejections
  - Relax to desired temperature/pressure for  $\sim 0.5$  ns





## Simulations

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- **All configurations: ~0.1 M (10 Li<sup>+</sup>/1000 carbonates)**
- **3 Different carbonate compositions**
  - 100% EC
  - 100% PC
  - 50/50 EC/PC (by mass)
- **Range of temperatures from 300-500K**
- **Rigid/flexible molecules**
- **High pressure and low pressure**





## fix ave/correlate command

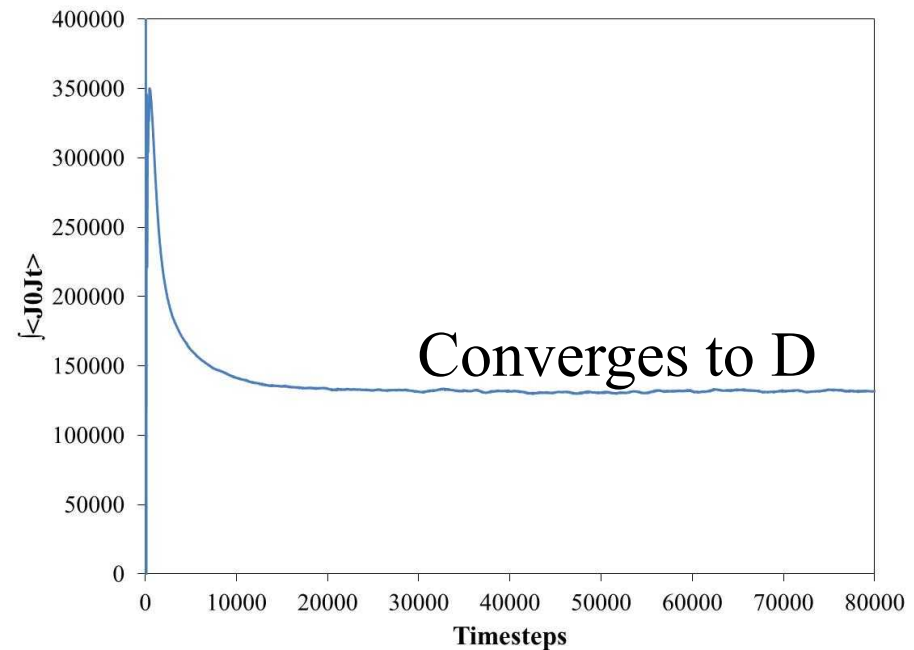
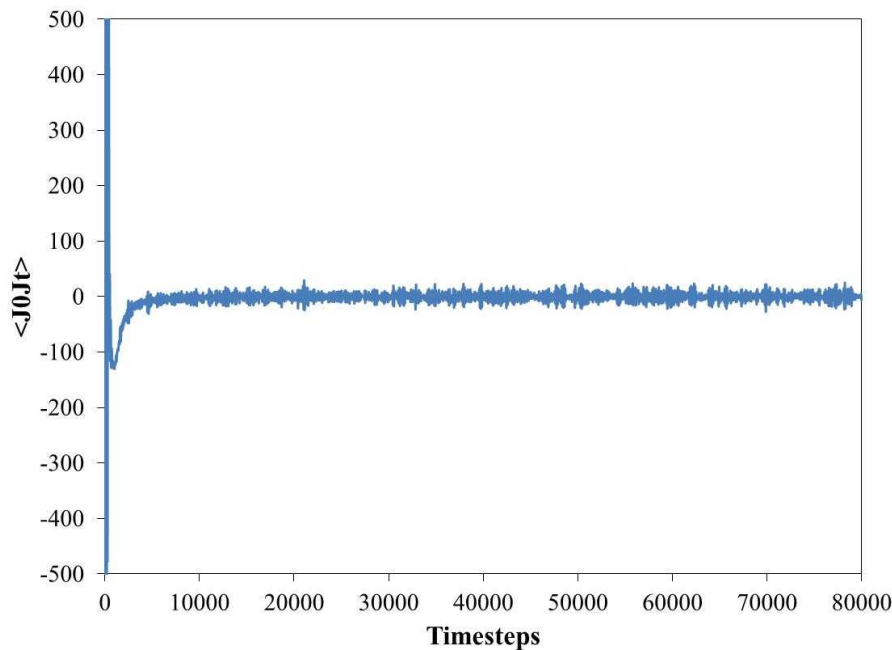
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Fix ID group-ID ave/correlate Nevery Nrepeat Nfreq value1 value2 ... Keyword args ...

- ID, group-ID are documented in fix command
- ave/correlate = style name of this fix command
- Nevery = use input values every this many timestep
- Nrepeat = # of correlation time windows to accumulate
- Nfreq = calculate time window averages every this many timesteps
- One or more input values can be listed
- Value = c\_ID, c\_ID[N], f\_ID, f\_ID[N], v\_name
- Zero or more keywords/arg pairs may be appended
- Keyword = *type* or *ave* or *start* or *prefactor* or *file* or *overwrite* or *title1* or *title2* or *title3*

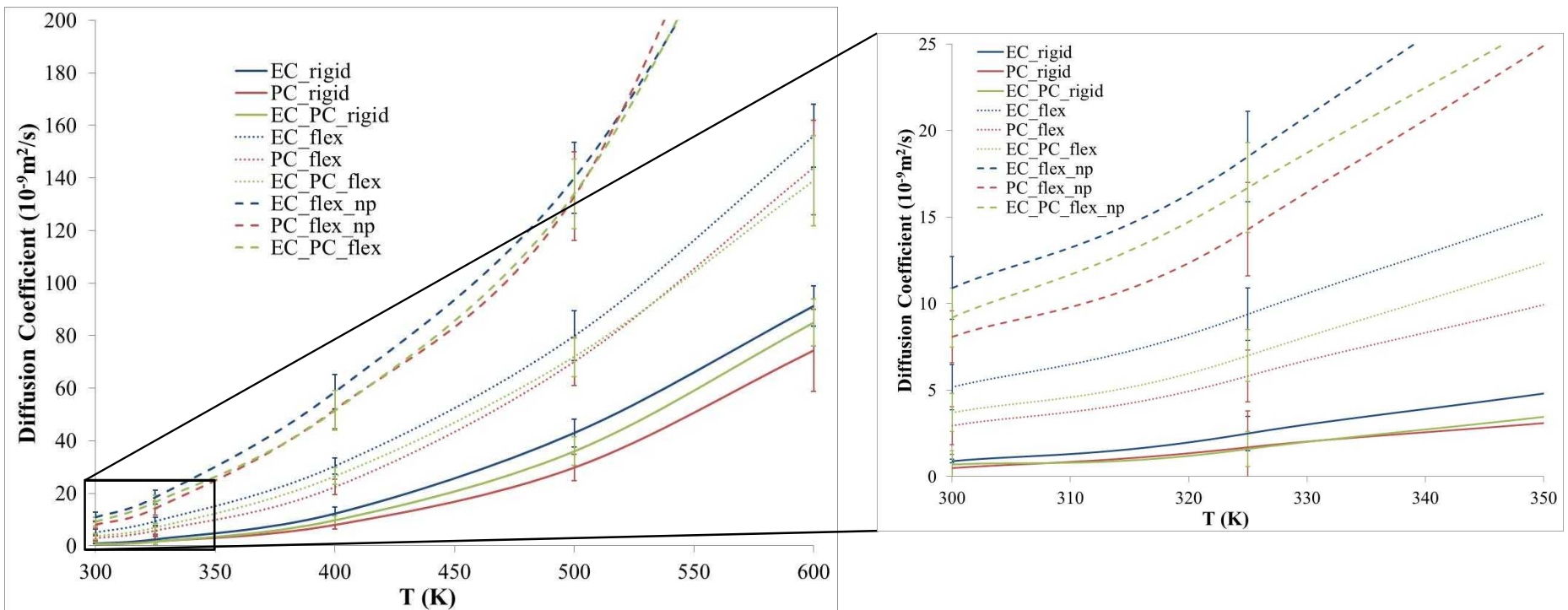
fix J0Jt\_Li Li ave/correlate 10 8000 10000000 c\_J\_Li[1] type auto/upper file J0Jt\_Li.dat ave running

# Correlation Functions



- Each correlation is the average of 10 different initial configurations
- Also averaged over the 3-dimensions due to homogeneity

# Diffusion Coefficients



- Large discrepancies between potential times
- EC tends to have higher diffusion coefficient
- Problems around  $T_g$  of potentials



## Model vs. Experiment

	Rigid	Flex	Flex (np)	Exp.
<b>EC</b> Density (g/cm <sup>3</sup> )	1.36	1.28	1.23	1.32
T <sub>g</sub> (K)	321	323	371	310
Li <sup>+</sup> diff @ 300K (10 <sup>-10</sup> m <sup>2</sup> /s)	0.9	5.2	10.9	NA
<b>PC</b> Density (g/cm <sup>3</sup> )	1.27	1.22	1.15	1.20
T <sub>g</sub> (K)	332	324	352	225
Li <sup>+</sup> diff @ 300K(10 <sup>-10</sup> m <sup>2</sup> /s)	0.5	3.0	8.1	0.2

- None of the potentials studied capture both EC and PC properties.
- Rigid potential does a good job representing EC but misses the very important component of differing T<sub>g</sub>.

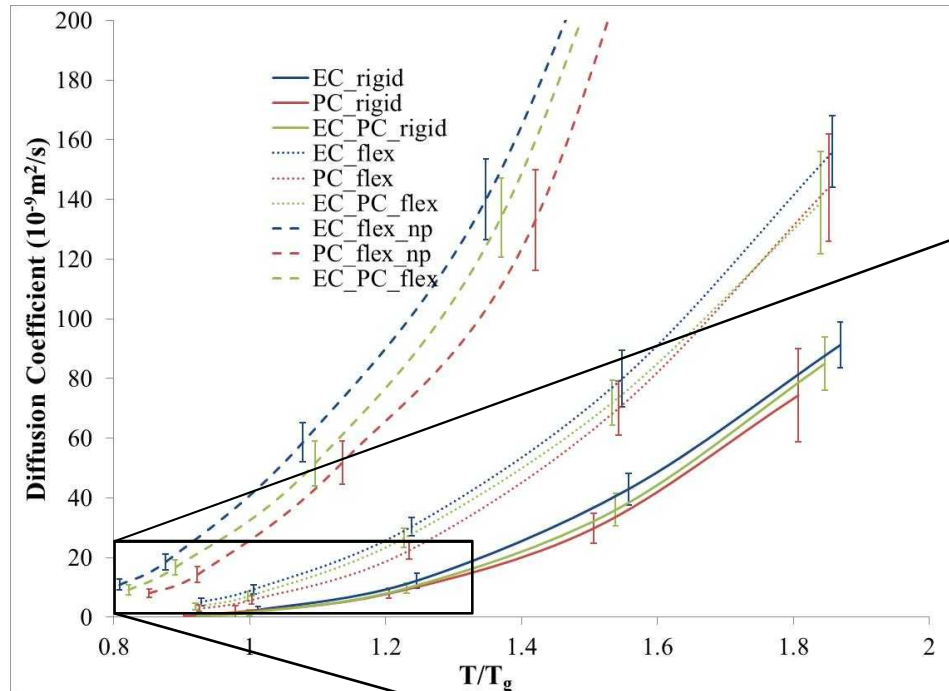


## Summary

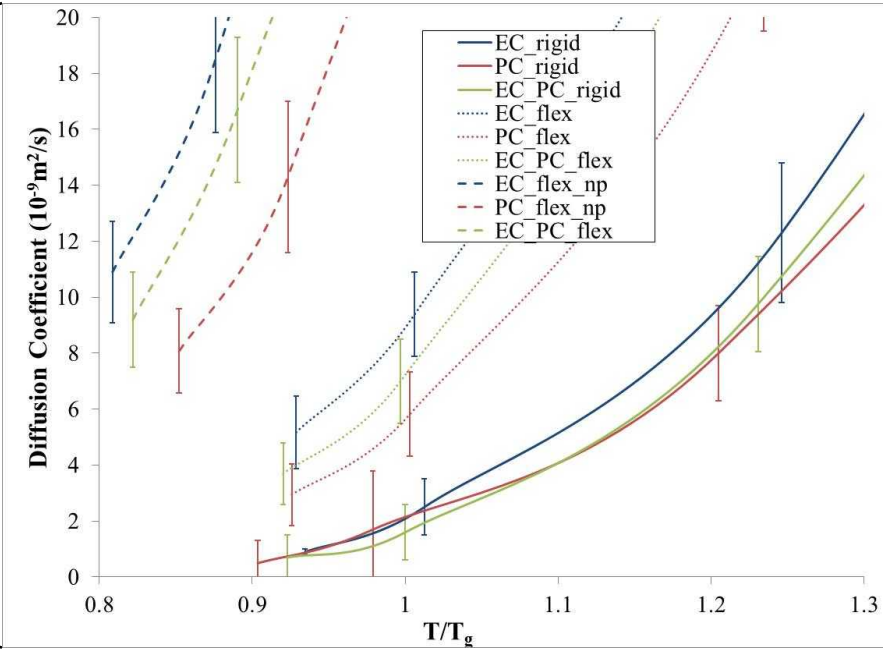
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- Green-Kubo techniques are a method for extracting macroscopic transport properties from microscopic quantities.
- “fix ave/correlate” can be used to determine the correlation functions necessary for Green-Kubo calculations
- Performed simulations on different configurations of EC/PC electrolytes with  $\text{LiBF}_4$ .
- Potentials examined do not sufficiently distinguish between EC and PC.
- Rigid potential does a reasonable job of predicting properties.

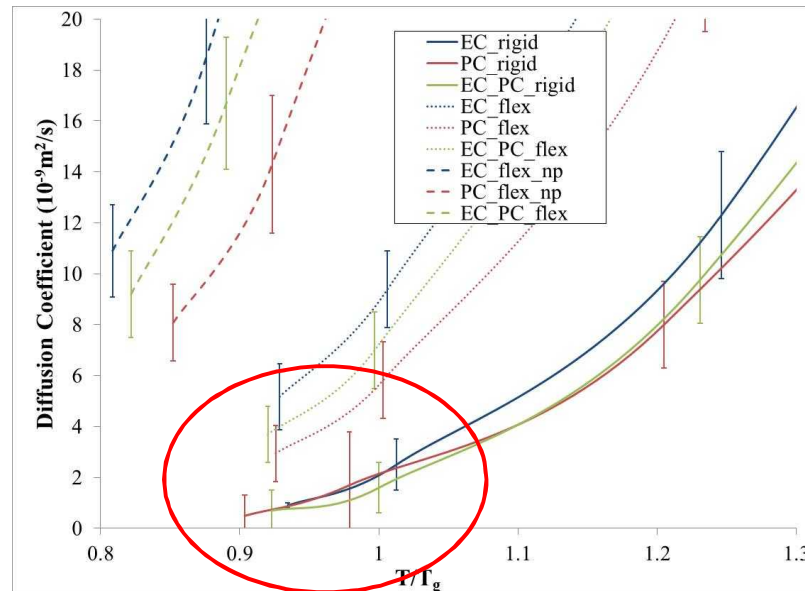
# Diffusion Coefficients



EC solvents have higher diffusion coefficients



Large discrepancies between potential times



- Generally EC has higher  $D$  than other compositions
- Rigid molecules actually have  $D$  nearest to