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ANALYTIC MODELS FOR PULSED X-RAY IMPULSE COUPLING

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Abstract. High-energy pulsed X-ray momentum coupling is a promising technology for early deflection of NEOs (Near Earth Objects) that might impact Earth. Analytic models for the radiation interactions can often preclude the need for large hydrocode analyses, and offer the advantage of many point calculations that reveal important features of the nonlinear phenomena, *e.g.*, thresholds, peak coupling, and high-energy scaling limits. However, model validation is an important element. One such model is used to analyze relevant experiments conducted on the Sandia Z-pinch machine. Samples were exposed to X-ray pulses approximating a 200-eV blackbody at fluences of ~1 kJ/cm^2. Target momenta were measured. Model calculations give impulse couplings somewhat greater than the data, but a more appropriate value for the one uncertain model parameter (the effective target decomposition energy), can account for this discrepancy. The analytic model is thus appropriate for system-level parameter studies that will be important constituents of all NEO mitigation investigations.

Keywords: Pulsed X rays, impulse generation, analytic models, NEO mitigation.
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INTRODUCTION AND COMPUTATIONAL APPROACH

One relatively new application for shock-physics-like phenomena and analysis is NEO (Near Earth Object) mitigation [1]. The impact on Earth by an NEO such as a large asteroid or comet could have catastrophic consequences. Altering its trajectory at a time and location early enough to preclude such a collision is probably our first line of defense. NEO mitigation technologies that have been proposed include tractoring, magnetic deflection, and engagement with a nuclear explosive. The latter could involve direct comminution, or dynamic radiation loading by either neutrons or X rays generated by stand-off detonations. Because a generic nuclear explosive releases about three-quarters of its total energy as a short pulse of low-energy X rays, this last approach is probably the most promising one for study.

There are many, but solvable issues with this mitigation technology. They include: 1) a full understanding the nonlinear phenomenology that controls how pulsed radiation generates momentum in targets; 2) how to design and size the requisite nuclear devices, including spectral optimization and total output; and 3) how to plan and execute the launch and transport requirements for such devices to achieve the needed NEO trajectory deflection. With current capabilities and expertise the overall technology to successfully address all these questions is achievable, but it is the first of these three that we will consider here.

For the initial parameter studies and scoping analyses there are two major tools that can be employed. The first, large single- and multi-dimensional hydrocodes, can provide time dependent and much greater detailed solutions. But because of their complexity, they require extensive setup involving material equations of state and constitutive relations, as well as elaborate initial and boundary conditions. They can thus be expensive to run, both in terms of time and effort. The number of cases examined may therefore be limited.

In contrast, simple closed-form analytic models based fundamentally on conservation of momentum and energy, offer the attributes of easy setup and limited input requirements. Hence they allow for many point calculations that can clarify most of the important trends in the impulse generation process. The so-called MBBAY model represents this latter approach, and will be investigated here.

A SIMPLE ANALYTIC MODEL

The Modified Bethe, Bade, Averell, Yos, or MBBAY model, was developed in the 1960s and 1970s to study the impulse-driven structural response of reentry vehicles to nuclear weapon effects [2]. In its basic form it can be expressed as

$$I = \alpha\sqrt{2}\left[\int_0^{z_0}\left\{E(z) - E_0\left(1 + \ln\frac{E(z)}{E_0}\right)\right\}\rho^2\, z\, dz\right]^{1/2},$$

where I is the impulse, $E(z)$ is the deposited energy as a function of the target depth z, E_0 is the target decomposition energy, $E(z_0) = E_0$, ρ is the target density, and $1 \le \alpha \le 2^{1/2}$. This expression may need to be numerically integrated over the appropriate photon energies to account for the major variations in target absorption coefficients for the incident X-ray spectra. However, if the photon-energy-dependent absorption coefficients, $\mu(h\nu)$, can be approximated by a single *effective* value, μ_{eff}, then the basic model can be integrated directly to give

$$I^* = \alpha\sqrt{2}\left\{\Phi_0^* - \left[1 + \ln\Phi_0^* + \frac{1}{2}(\ln\Phi_0^*)^2 + \frac{1}{6}(\ln\Phi_0^*)^3\right]\right\}^{1/2}.$$

Asterisks represent nondimensional variables, with $I^* = \mu_{eff}\, I/E_0^{1/2}$ for the impulse, and for the fluence, $\Phi_0^* = \mu_{eff}\,\Phi_0/E_0$. With μ_{eff}, the energy deposition is

$$E(z) = \mu_{eff}\,\Phi_0\, \exp(-\mu_{eff}\,\rho\, z)\ .$$

At high fluencies ($\Phi_0^* >> 1$) the impulse takes on a particularly simple form, $I^* = \alpha(2\Phi_0^*)^{1/2}$. The impulse coupling coefficient, C_M, *i.e.*, the impulse divided by the fluence, is then given as $C_M^* = \alpha(2/\Phi_0^*)^{1/2}$. Here both I^* and C_M^* are in their nondimensional forms.

An important consequence of this model is that there is a threshold fluence for impulse generation at $\Phi_0^* = 1$. Also, because of the high-fluence scaling, the efficiency for generating impulse—the impulse coupling coefficient C_M—has a maximum value at a fluence of $\Phi_0^* \approx 10$.

Another key aspect of this model is that, even when the photon-energy-dependent absorption coefficients are fully incorporated in the calculations to account for broad continuous X-ray spectra, they are well known. Thus the only required input parameter with significant uncertainty is the target decomposition energy E_0. Effective values, to account for, *e.g.*, dynamic processes or various mixtures of elemental materials, can be used to calibrate the model when only little or uncertain data are available. This will not alter basic features of the phenomenology being considered. The MBBAY model is incorporated in an X-ray deposition utility code [3], which was used for the present calculations.

VALIDATION

As with any theoretical or computational study V and V (verification and validation) are important issues. Verification (*i.e.*, that the numerical techniques correctly solve the originally posed problem) will not be addressed here. Validation will be accomplished through comparison of model predictions with experimental results. The experiments were obtained through momentum or impulse measurements on relevant target materials exposed to high-fluence X-ray loads in the Sandia Z-pinch machine. Details, including the target material properties, are provided elsewhere [4, 5], but summarizing, the radiation environment in Z can be characterized as a low-energy thermal component (~0.2 keV blackbody) superimposed with a small line spectrum characteristic of the wire array producing the pinch. These data are shown in Fig. 1. The fluence incident on the target materials was ~1 kJ/cm^2. Source spectra for each of the different wire arrays were calculated, even though they were very similar. Specific test materials included Allende, Dunite, Odessa, and for control purposes, iron and aluminum. The decomposition energies, E_0, used for these samples were those generally accepted by the space-science community, and thus might not be fully appropriate for the dynamic radiation loading conditions being examined here.

The results of these validation tests are shown in Fig. 2, where the generic model calculations are represented with only one spectrum and sample combination. Note that the figure indicates that the peak impulse coupling coefficient, $(C_M)_{max}$, occurs at a fluence of $\Phi_0 \approx 1$ J/cm^2, and has a value of ~4 x 10^{-4} s/m. At higher fluences C_M decays as $\Phi_0^{-1/2}$. Although varied, the agreement of the data with the model does point the way for system-level parametric studies for NEO mitigation investigations. The only input adjustment that might be required would involve modification of E_0 to account for different interaction phenomena, as based on available experimental test results.

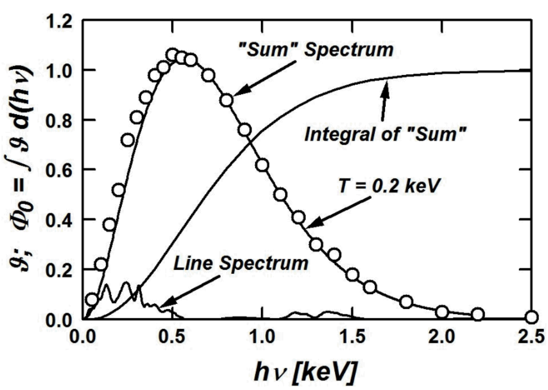

Figure 1. X-ray spectra for Z experiments as functions of the photon energy, $h\nu$. The intensities, ϑ, include the blackbody curve for $T = 0.2$ keV, the line spectrum from a copper wire array, and the "sum" of those two spectra. The additional curve is the total fluence, Φ_0, *i.e.*, the integral of the intensity "sum." The ordinate is normalized to a total fluence of one.

PARAMETER STUDIES

To show the utility of this simple approach, we have used the model, as incorporated in the X-ray deposition code [3], to calculate the radiation-generated impulse results for a series of operationally realistic pulsed radiation environments. Although somewhat more energetic than the X-ray spectra available in the Z machine, they can be easily characterized with a single variable, the Planckian or blackbody temperature T. Here we have used 0.25 keV $\leq T \leq$ 4 keV, values that should be operationally achievable. Note that continuous blackbody spectra are characterized by peaks at $h\nu = 2.82\ T$ and that 99% of the total energy is below $h\nu = 10\ T$ (*cf.* Fig.1).

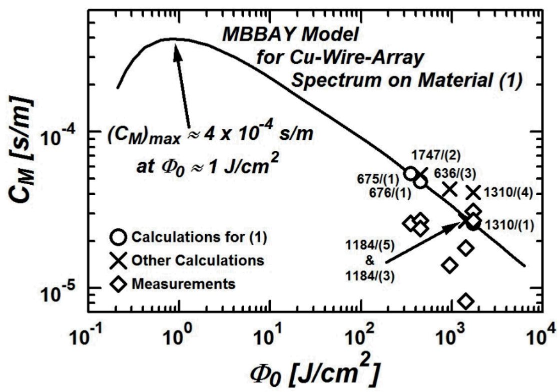

Figure 2. Model validation results showing the impulse coupling coefficients, C_M, as a function of the incident fluence, Φ_0, for the samples exposed in Z. The solid line and circles are for a copper wire array source incident on Allende samples. The Xs are for other conditions, with the specific shot numbers and targets as indicated [4]. The materials are (1) Allende, (2) Dunite, (3) Odessa, (4) aluminum, and (5) iron. The diamonds represent the experimental measurements from Z as they align with the fluences from the calculated points.

In these new calculations the single target material was iron, which is a major elemental constituent in most typical meteorite and planetary materials, or other NEOs, which might pose a threat to Earth. Although calculations for other targets will need absorption coefficients weighted by their elemental compositions, which are readily available, iron will emphasize the effects of $\mu(h\nu)$ on both I and C_M. Note that in the photon energy range of interest, $\mu(h\nu)$ scales as $(h\nu)^{-3}$, with photoelectric absorption edges that jump by roughly an order of magnitude (at photon energies of 0.7 and 7.1 keV for iron), and so need to be explicitly incorporated in the calculations.

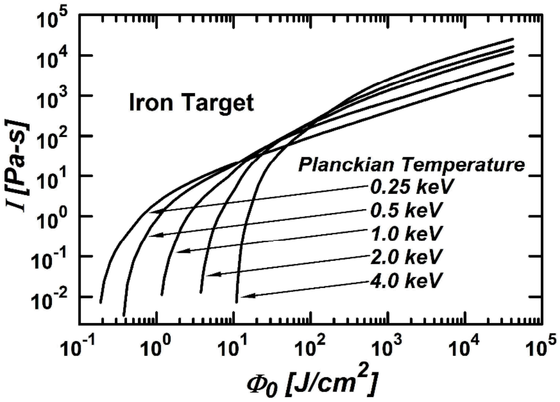

Figure 3. Impulse, I, as a function of incident fluence, Φ_0. The target is iron, and the Planckian or blackbody spectral temperatures span potentially operational values. Note the threshold fluences for impulse generation, and the high-fluence scaling of I with $\Phi_0^{1/2}$. Note that for impulse, 1 Pa-s = 0.1 dyne-s/cm^2.

Figures 3 and 4 show the results, where all the main features are exhibited. The most important of these are the nonlinear phenomena, including the impulse-generation thresholds, the peak coupling efficiencies, and how they both vary with the X-ray source spectra. The latter will be important for source device optimization. However, the high-fluence scaling of both the impulse and the impulse coupling coefficients may be the most significant property for system-level parameter studies.

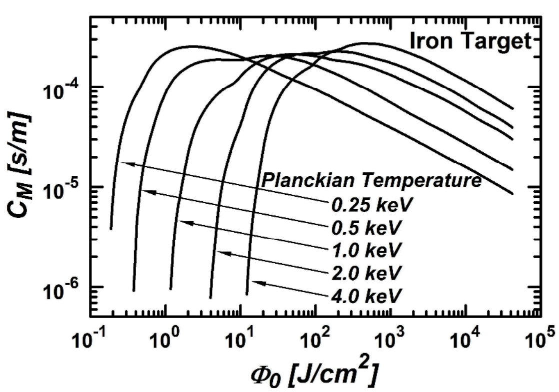

Figure 4. Impulse coupling coefficient, C_M, as a function of incident fluence, Φ_0. The blackbody temperatures span the regime of operationally achievable values. The slight irregularities in the curves near the peaks result from the photoelectric absorption edges for iron. The threshold fluences are evident, as is the high-fluence C_M scaling with $\Phi_0^{-1/2}$. Note that for coupling coefficients, 1 s/m = 1 x 10^{-5} (dyne-s/cm^2)/(J/cm^2).

CONCLUSIONS

We have suggested that deflection of NEOs by pulsed X-ray loading from nuclear devices is probably the most efficient approach for NEO mitigation. Mainly, it can be accomplished with existing capabilities and technology. Simple analytic models (*e.g.*, the MBBAY model) provide the ideal tools for the many parametric surveys that will be required for the initiation of such programs. NEO engagement studies as well as more complex hydrocode analyses will also be needed. However, it is these simple analytic models, with their limited input requirements, that will set the overall boundaries on the parameters for the optimization of the total mitigation system design and implementation.

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