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A Stieltjes-Lanczos Procedure for Parameterized Matrices

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Follow along! http://bit.ly/chebfun_cg

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RUN FOR YOUR LIVES!

$2O_3 \longrightarrow 3O_2$

C : concentration of O_3

$\frac{dC}{dt} = -\lambda C$

Half-life $t_{1/2} = \frac{\ln(2)}{\lambda}$

Given measurements of λ, what is $\mathrm{E}[t_{1/2}]$?

Inspired by Weilandics et al. 1988

Parameterized Matrices

$$\mathbf{A}(s)\mathbf{x}(s) = \mathbf{b}(s)$$

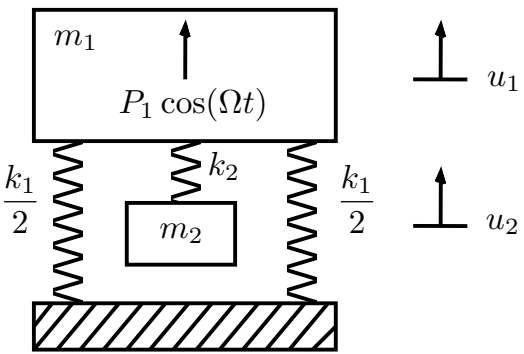
Running Lanczos with Stieltjes produces a quadrature rule for

$$\int \mathbf{b}(s)^T \mathbf{A}(s)\mathbf{b}(s)\, ds$$

WATCH YOUR HEADS!

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} P_1 \cos(\Omega t) \\ 0 \end{bmatrix}$$

R. R. Craig and A. J. Kurdila. Fundamentals of Structural Dynamics. 2nd ed. Wiley, 2006. Chapter 9.



WATCH YOUR HEADS!

Assume solution

$$u_1 = U_1 \cos(\Omega t) \qquad u_2 = U_2 \cos(\Omega t)$$

Yields the parameterized matrix problem

$$\begin{bmatrix} k_1 + k_2 - \Omega^2 m_1 & -k_2 \\ -k_2 & k_2 - \Omega^2 m_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ 0 \end{bmatrix}$$

Given uncertainty in Ω

$$\int_0^{0.5} U_1 \, d\Omega = \int_0^{0.5} e_1^T K_\Omega^{-1} e_1 \, d\Omega$$

Thanks Chebfun!

Come see more UQ and
parameterized matrices!

Minisymposium 181
Room 12 - Today! 3pm