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Towards Certificates for Integer Programming Computations

Higher-Confidence Integer Programming

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Abstract

Mixed-integer linear programming (MILP) is a general technology for solving combinatorial optimization problems exactly. MILP is the optimization of a linear function subject to linear and integrality constraints. MILPs naturally represent resource allocation problems. Thus they are a workhorse technology for decision support, such as scheduling, logistics, manufacturing optimization, and sensor placement, or for the study of natural systems such as bioinformatics. Recently we have been using MILP computations for scheduling quantum error correction codes for quantum computers, for planning the evolution of the US weapons stockpile, and for selecting sensor locations within large municipal water networks.

Although integer programming is an NP-complete problem, one can frequently compute (near) optimal solutions to specific instances using intelligent search. If the solution will be used to support a high-consequence, expensive, and irrevocable decision, we would like provable confidence in the result of a specific computation. A MILP solution could be incorrect due to a program error or due to properties of inexact arithmetic during computations.

We would like to compute a *certificate* for an integer programming computation. This is information that allows an independent program to check that the output is correct, preferably far faster than the time required to compute the solution. The canonical example, which we will explain, is the certificate for a linear program (LP). The certificate is the solution to the dual problem. Although solving an LP can take a large amount of time, checking a certificate requires one matrix vector multiplication and one dot product.

A "brute force" certificate for an integer program branch-and-bound computation must prove that each branching operation (subdivision of the feasible region), added cut (constraint), and fathoming operation (removable of a region) is correct. We will discuss what this entails in the context of PICO, our massively parallel integer programming solver. We will give a certificate for general Gomory cuts and discuss ways to prove the correctness of a general cut, which is equivalent to proving a region has no integer solutions. The latter problem is NP-hard in general, but may be easier for the polytopes "cut off" during a integer-programming computation. We will discuss solver tolerances and exact arithmetic for linear programs. Time permitting, we will discuss methods of computational algebra, notably the Gröbner basis test-set, which does not involve direct validation of search steps.

This presentation will report on specific avenues we have explored and current results, including some that appear to be dead-ends.

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