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Peridynamic Theory: An Approach to Computational Mechanics without Spatial Derivatives

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Outline of Presentation

- What is Peridynamic Theory?
- Why Use Peridynamics?
- The Fundamental Equations of Peridynamic Theory
- Material Modeling in Peridynamics
- Numerical Method for Solving the Fundamental Equation of Peridynamics
- Implementation of Peridynamic Theory in Kraken
- High-Impulse Impact Loading of Structures
- Detonation Modeling in Peridynamic Theory
- High-Impulse Explosive Loading
- Some Geomechanics Examples
- Concluding Remarks

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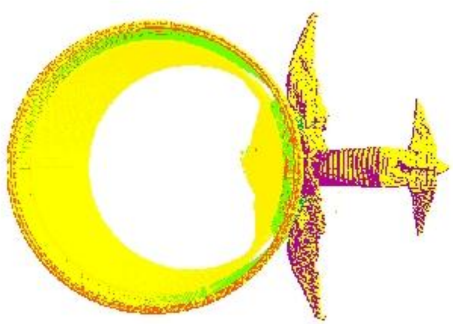
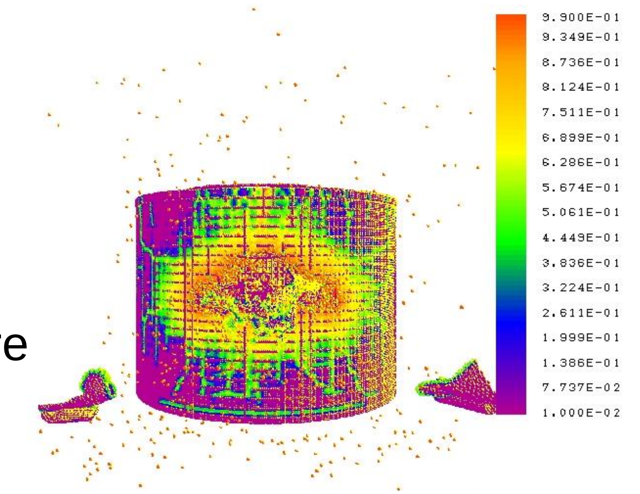
What is Peridynamic Theory (PD)?

- ***Peridynamic theory*** **is a theory of continuum mechanics that uses integro-differential equations without spatial derivatives rather than partial differential equations.**
  - **Bond-Based Peridynamics[1]**
  - **State-Based Peridynamics[2]**



Peridynamic means "near force". It is a reformulation of continuum mechanics that applies everywhere regardless of discontinuities.



[1]Silling, "Reformulation of elasticity theory for discontinuities and long-range forces", in *Journal of the Mechanics and Physics of Solids*, 48 (2000) , pp. 175-209. (Silling 2000)
[2] S.A. Silling et al. "Peridynamic States and Constitutive Modeling", in *J Elasticity*, 88 (2007), pp. 151–184. (Silling 2007)

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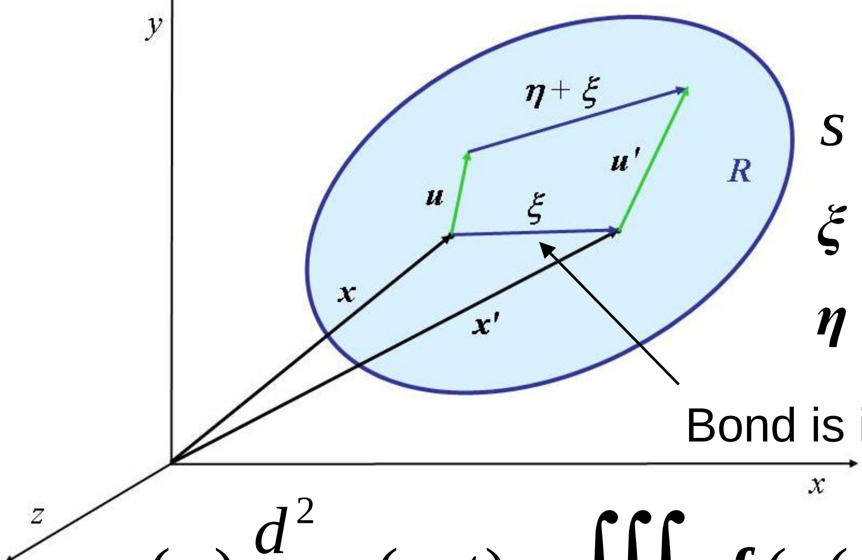
Why use Peridynamics?

- The fundamental partial differential equations used in conventional finite element or particle codes do not apply at discontinuities.
- Peridynamics can solve problems that are difficult to solve using other methods.
- With peridynamics, cracks initiate and grow spontaneously as a consequence of the governing equation and constitutive model, and there is no need for externally supplied laws or locations of cracks.

With peridynamics, cracks are part of the solution, not part of the problem.

What is Bond-Based Peridynamics?

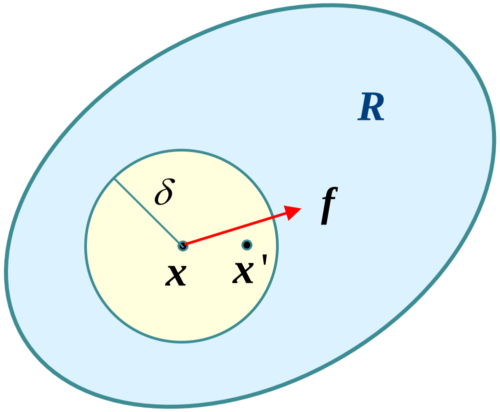
- In bond-based peridynamics the force state at a point is given by a functional over the pairwise interactions with all other points in the continuum.



$$s = (|\eta + \xi| - |\xi|)/|\xi|$$

$$\xi = x' - x$$

$$\eta = u(x', t) - u(x, t)$$



Bond is interaction between ***x*** and ***x'***.

$$\rho(x)\frac{d^2}{dt^2}u(x,t) = \iiint_R f(u(x',t) - u(x,t), x' - x)\,dV' + b(x,t)$$

where

ρ is the density at x, x is the position vector,

t is the time, u is the displacement vector,

R is the computational domain, f is the pairwise force function,

b is the body force, δ is the horizon.

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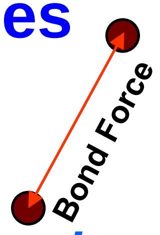
Material Modeling in Bond-Based Peridynamics



- The force per unit volume squared between particles located a two points is given by the *pairwise force function* (*PFF*) f .
  - Peridynamic interaction between two points is called a *bond*.
  - Presently, for solids the PFF magnitude is a piecewise linear function (elastic, perfectly plastic).
- Constitutive properties of materials are given by specifying the *PFF*.
  - Thus, material response, damage, and failure are determined at the bond level.
  - A bond fails when its stretch exceeds an input called the critical stretch.
- Bond properties are derivable from measured material properties including:
  - elastic modulus, yield properties, and fracture toughness.

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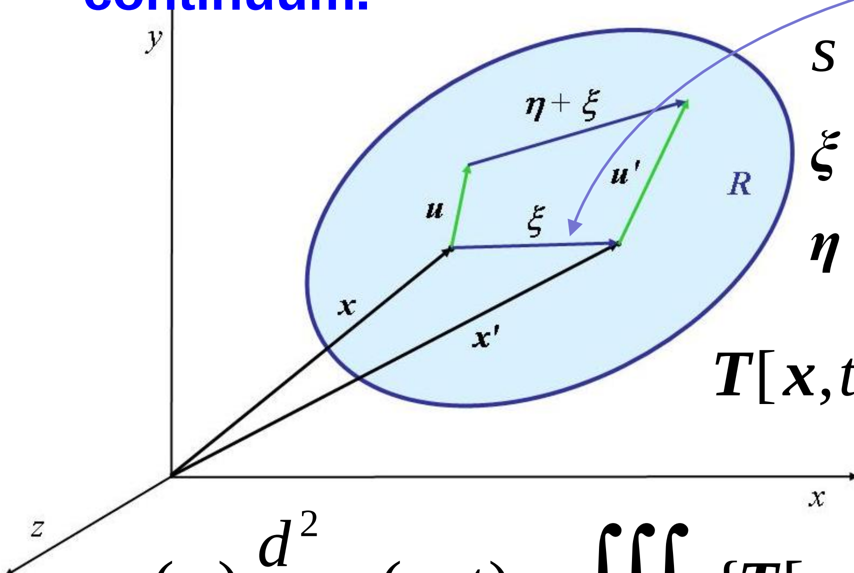
What is State-Based Peridynamics?

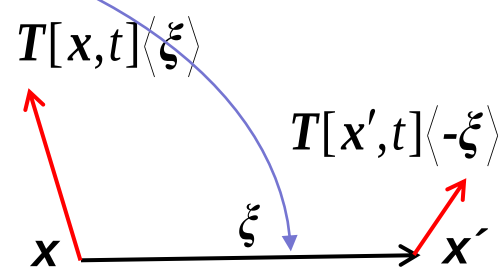
- **In state-based peridynamic theory the force state at a point is given by a functional over the relative displacements of all other points in the continuum.**



$$s = \left(\left|\boldsymbol{\eta}+\boldsymbol{\xi}\right| - \left|\boldsymbol{\xi}\right|\right) / \left|\boldsymbol{\xi}\right|$$

$$\boldsymbol{\xi} = \boldsymbol{x}' - \boldsymbol{x}$$

$$\boldsymbol{\eta} = \boldsymbol{u}(\boldsymbol{x}',t) - \boldsymbol{u}(\boldsymbol{x},t)$$



$$\boldsymbol{T}[\boldsymbol{x},t]\langle\boldsymbol{\xi}\rangle = \hat{\boldsymbol{T}}(\boldsymbol{\eta}+\boldsymbol{\xi}, \Lambda)$$

$$\rho(\boldsymbol{x})\frac{d^2}{dt^2}\boldsymbol{u}(\boldsymbol{x},t) = \iiint_R \{\boldsymbol{T}[\boldsymbol{x},t]\langle\boldsymbol{\xi}\rangle - \boldsymbol{T}[\boldsymbol{x}',t]\langle -\boldsymbol{\xi}\rangle\} dV' + \boldsymbol{b}(\boldsymbol{x},t)$$

where

ρ is the density at $\boldsymbol{x}$,	$\boldsymbol{x}$ is the position vector,
t is the time,	$\boldsymbol{u}$ is the displacement vector,
R is the computational domain,	$\boldsymbol{T}[\boldsymbol{x},t]\langle\boldsymbol{\xi}\rangle$ is the force state at $\boldsymbol{x}$,
$\boldsymbol{b}$ is the body force,	Λ are internal state variables.

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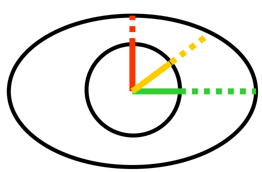
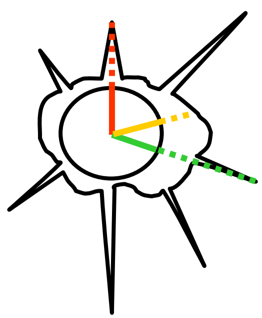
Why do we want to implement state-based peridynamics?

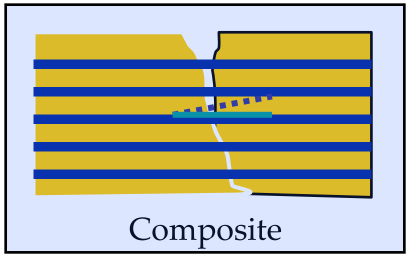
- **State-based version has the potential to**
  - utilize material models that were developed for continuum mechanics, including models developed for finite-element methods
  - model composite materials.
- **Force states provide for infinite "degrees of freedom"**
  - Classical theory only provides six "degrees of freedom



$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}$$

  - Force State provides infinite "degrees of freedom"





- Fibers can be modeled separately from matrix.
- Fibers in any direction can be modeled separately from the others.

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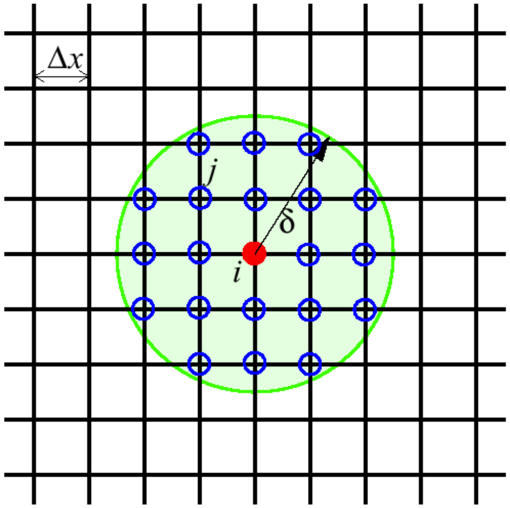
Numerical Method



- **The computational region is discretized into nodes with a known volume in the reference configuration, forming a *grid of nodes*.**
- **The fundamental equation is replaced by a finite sum, which at time t_n is**

$$\rho_i \frac{d^2}{dt^2} \boldsymbol{u}_i^n = \sum_p \boldsymbol{f}(\boldsymbol{u}_p^n - \boldsymbol{u}_i^n, \boldsymbol{x}_p - \boldsymbol{x}_i) V_p + \boldsymbol{b}_i^n, \quad \boldsymbol{u}_i^n = \boldsymbol{u}(\boldsymbol{x}_i, t_n)$$

- **For each node, the peridynamic interaction is assumed to be zero outside a distance δ called the *horizon*.**

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Implementation in Kraken Computer Code

- Peridynamic theory is implemented in the Kraken computer code.
- Kraken is
  - *mesh free* (no elements, just generate a grid of nodes),
  - *Lagrangian* (each node represents a fixed amount of material),
  - *explicit* (simple, reliable time-integration method),
  - *parallel* (executes on multiple processors).
- To enhance usability, we are developing KFragS (Kraken Fragmentation Analysis System), a graphical user interface (GUI).

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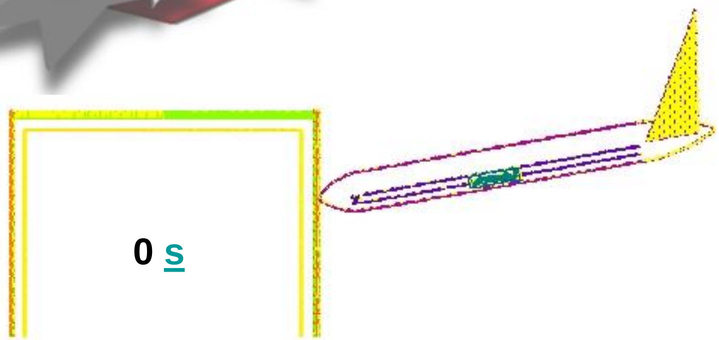
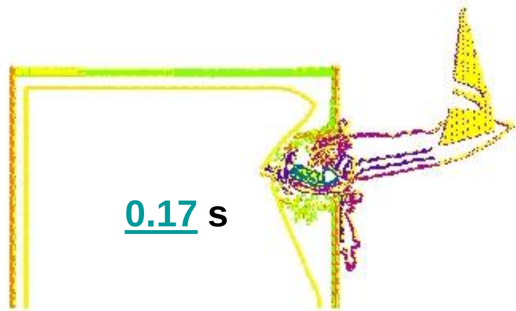
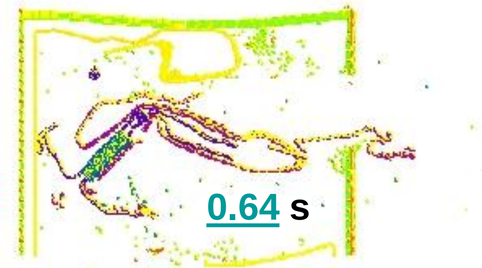
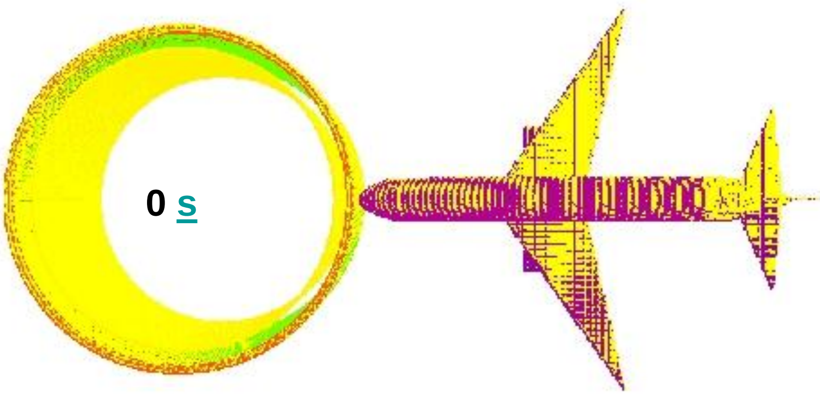
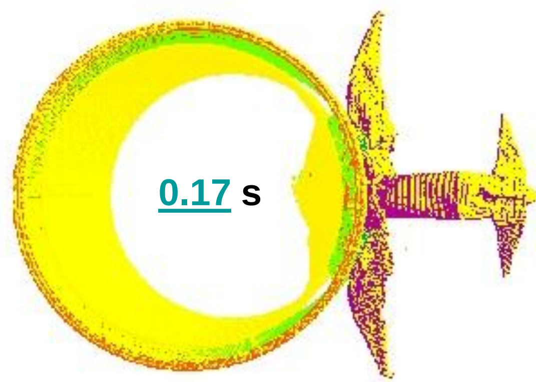
High-Impulse Impact Loading of a Structure

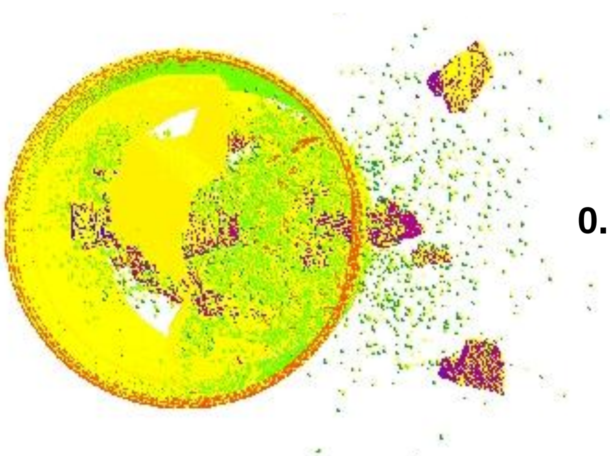




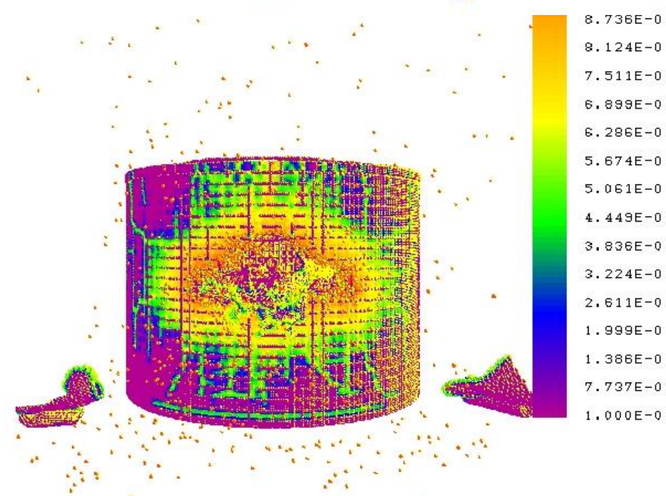






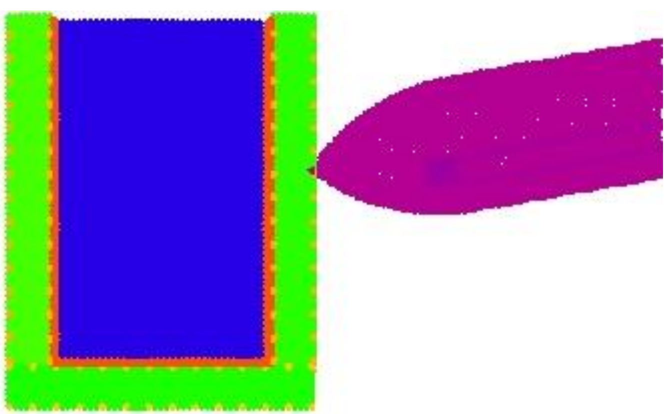


0.64 s

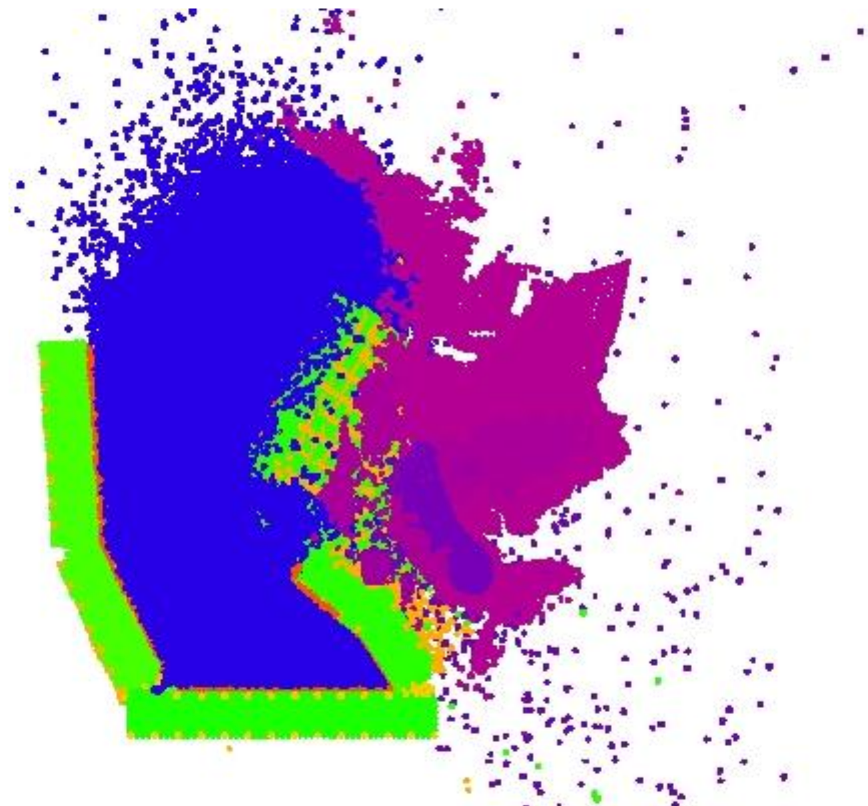


Damage at 0.64 s

High-Impulse Impact Loading of a Structure Filled with Water



0.0 s



0.50 s

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Peridynamic Detonation Model

$(x_{det}, y_{det}, z_{det}, t_{det})$, $\rho_{unreacted}$, v_{det}

- **Detonation model inputs:**
  - Location of detonation point(s) and initial detonation time(s), density of unreacted explosive, and detonation speed.
  - Parameters for equation of state (ideal gas or JWL).
- **Program burn model for detonation times.**
  - Detonation times computed prior to time advancement using Huygen’s construction.
  - Detonations can propagate around obstacles.
- **Upon detonation:**
  - Reaction products are treated as ideal or Jones-Wilkins-Lee gas undergoing an adiabatic expansion.
  - PFF for reaction products developed from equating the work in expanding a bond to the change in internal energy.
  - Energy conserved using volume-burn algorithm.

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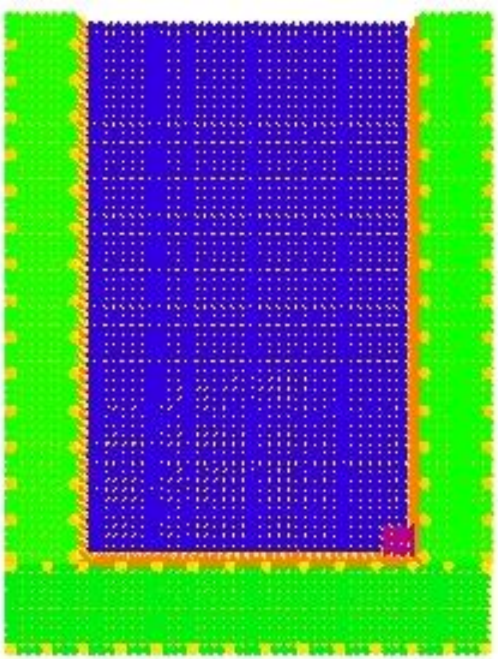
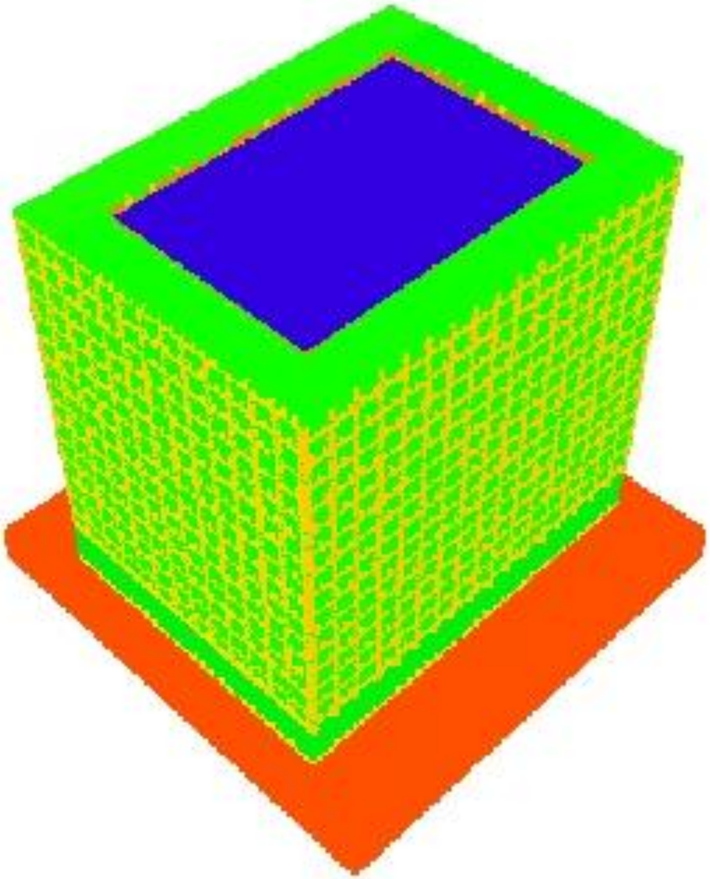
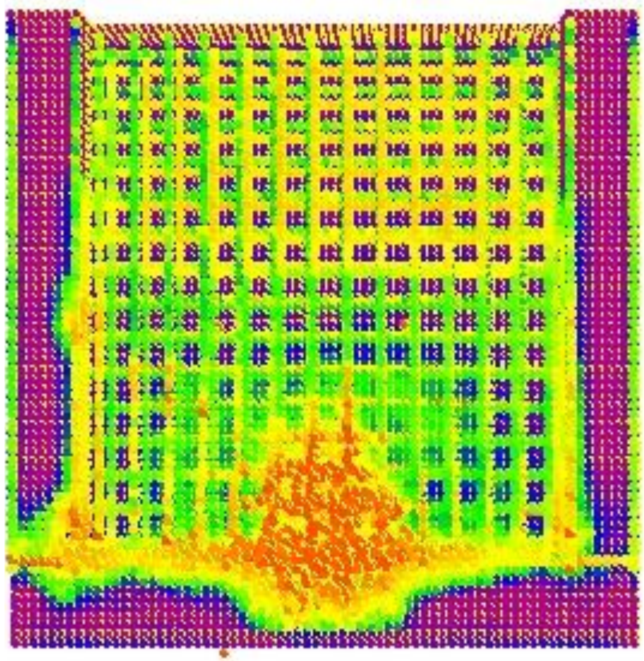
Blast Loading of a Structure

Structure has 6-ft thick walls and floor slab. The floor slab is 40 ft by 52 ft. The walls are 45 ft above the floor. All concrete is reinforced with #18 rebar at 12-in spacing. A cubic yard of explosive with unreacted density 1785 kg/m^3 and detonation speed 8747 m/s is placed on the floor at the center of the wall and detonated at time zero.



Materials





Damage

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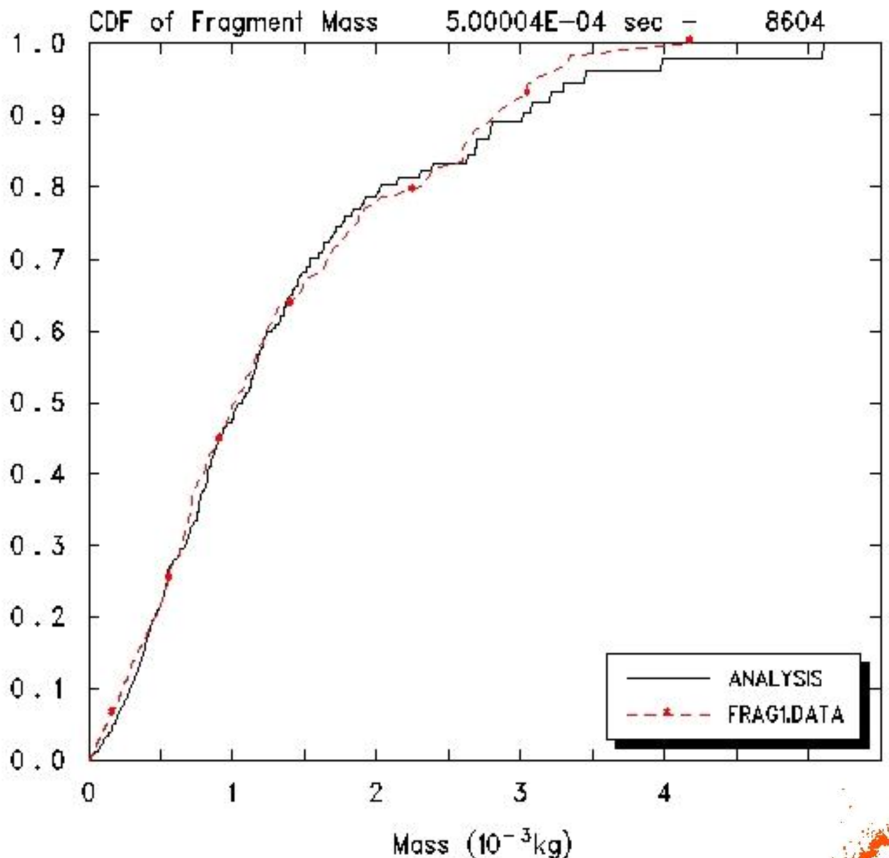
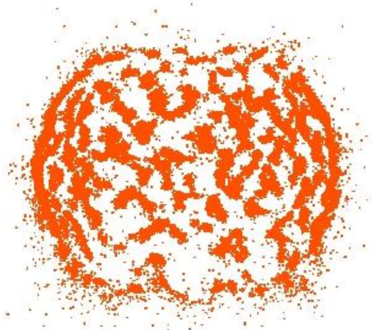
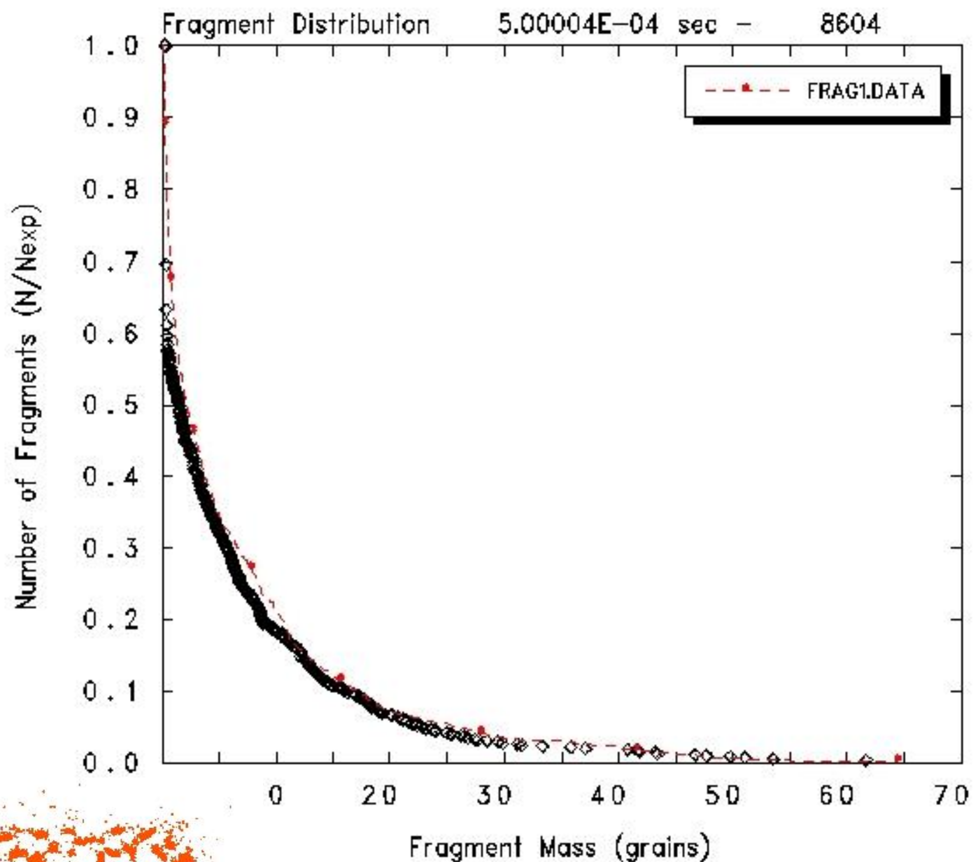
Mass Distribution for Fragmentation Test with 4340 Steel and TNT



Cumulative Mass Distribution (CMD) at 500 μs



80 μs



Fragment Distribution at 500 μs

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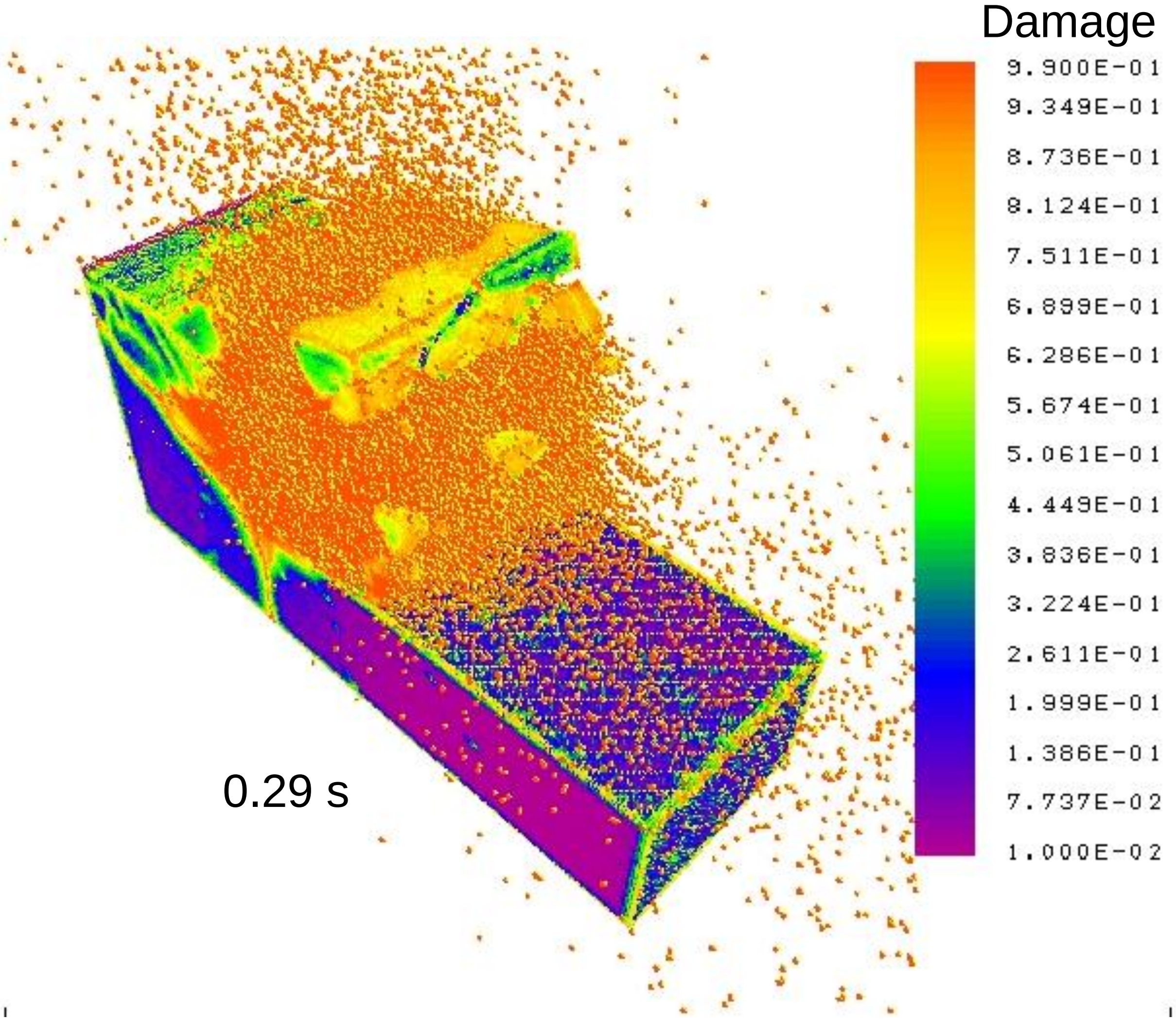
Tunnel-Wall Stability

- **Objectives**
  - To obtain a better understanding of the fundamental physical phenomena governing tunnel wall stability under shock loading including identification of key parameters and their relative importance for tunnel wall stability during shock loading.
  - To develop advanced numerical models that relate ground motion parameters, rock strength properties, tunnel geometry, features, and reinforcement such as rock bolts.
- **General Approach**
  - Develop a stochastic peridynamics theory that combines peridynamic theory with random or fractal material characterization of geomaterials.
  - Develop peridynamic shock-loading model and study shock propagation in random or fractal media with joints and faults.

Bench Blasting



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Concluding Remarks

- Peridynamic theory is a physically reasonable and viable approach to high-impulse loading and modeling fracture and fragmentation phenomena.
- An iterative verification and validation process is improving KRAKEN and increasing confidence in its predictive capability for its intended applications.
- We are expanding our interests to geomechanics problems with the tunnel-wall stability and bench blasting work.
- We are interested in combining peridynamics and the discrete element method to solve geomechanics problems.
- I will be happy to discuss common interests with you.
- I want to thank you for the opportunity to share with you our work in peridynamics and the KRAKEN code.

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