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Structure Preserving Reduced-Order Modeling of Linear Periodic Time-Varying Systems

Ting Mei, Heidi Thornquist, Eric Keiter and Scott Hutchinson
{tmei, hkthorn, erkeite, sahutch}@sandia.gov
Sandia National Laboratories, Albuquerque, NM, USA

***Abstract*— Many subsystems encountered in communication systems can be modeled as linear periodic time-varyiing (LPTV) systems. In this paper, we present a novel structure preserving reduced-order modeling algorithm for LPTV systems. A key advance of our approach is that it preserves the periodic time-varying structure during the reduction process, thus resulting in reduced LPTV systems. Unlike prior LPTV model order reduction (MOR) techniques which recast the LPTV systems to artificial linear time-invariant (LTI) systems and apply LTI MOR techniques for reduction, our structure preserving algorithm uses a time-varying projection directly on the original LPTV systems. Our approach always produces a smaller system than the original system, which was not valid for previous LPTV MOR techniques. We validate the proposed technique with several circuit examples, demonstrating significant size reductions and excellent accuracy.**

I. Introduction

Linear periodic time-varying (LPTV) systems are encountered in many communication systems. Examples of LPTV systems include mixers, switched-capacitor filters, up/down-conversion circuits, etc. To verify systems hierarchically, it is essential to reduce large subsystems to much smaller macromodels with similar behaviors.

Reduced-order modeling techniques have been successfully applied for linear time-invariant (LTI) systems. There are a variety of well established LTI MOR techniques [1–5], such as PRIMA and PVL. These techniques are often based on projection of a LTI system into lower dimension subspaces. However, for LPTV systems, which normally feature frequency translations and switching behaviors, the MOR techniques are not well established.

Previous work on LPTV MOR normally first converts a LPTV system to a LTI system and then performs reduction via LTI MOR techniques. A LPTV system can be converted to an artificial LTI system by discretizing the periodic time variation of the system using a finite basis [6,7]. The artificial LTI system has a much larger size than that of the original LPTV system. LTI MOR techniques can then be applied to the artificial LTI system to generate a smaller macromodel. In [8], an operator-based MOR technique is proposed. Instead of using a fixed, a-priori discretization, the operator-based method enables flexibility of changing discretization bases dynamically during the MOR process. However, the main advantage of this method is algorithm flexibility and code modularity. Otherwise, it has the same nature as previous LPTV MOR techniques.

The previous approaches impose several limitations on the model reduction process. First, the time-varying structure of the system is not preserved. The original LPTV system is reduced to a LTI one. Time domain simulations are performed on the LTI macromodel and additional post-processing is required to recast to LPTV results. In addition, since the LPTV system is first converted to a much larger artificial LTI system and then reduced, the size of reduced LTI system can be larger than that of the original LPTV system, as shown in Section V.

In this paper, we present a novel structure preserving MOR technique for LPTV systems. The key feature of this technique is the use of a time-varying projection, thus preserving the time-varying structure during the reduction process. By properly reformulating the equations, our method is able to produce a reduced LPTV system that has the same format as the original system. Additional post-processing, necessary for previous LPTV MOR techniques, is avoided. This structure preserving feature also leads to another advantage of our method, the size of the reduced system generated from our technique is always smaller than (or equal to) that of the original LPTV system. Such a desirable property is not possible for the previous methods. Numerical results in Section V show that the macromodels generated from our method not only are smaller than the full systems, but also feature smaller sizes and better accuracy than macromodels generated from previous LPTV MOR techniques.

The remainder of the paper is organized as follows. In Section II, we review the transfer function of LPTV systems derived by using multi-time partial differential equations (MPDE). In Section III, the previous Krylov-subspace based LPTV MOR techniques, both in matrix and operator forms, are described. In Section IV, the structure preserving MOR algorithm for LPTV systems is presented. In Section V, we apply our technique to a number of examples and demonstrate the applications of our approach.

II. Transfer function for LPTV systems

Consider a system driven by a large periodic signal $b_l(t)$ and a small signal $u(t)$ to produce an output $z_y(t)$. For simplicity, we assume that both $u(t)$ and $z_y(t)$ are scalars. The system can be described by the differential algebraic equations (DAEs)

$$\begin{aligned} \frac{dq(y)}{dt} + f(y) &= b_l(t) + Bu(t), \\ z_y(t) &= d^T y(t), \end{aligned} \tag{1}$$

where $y(t)$ is a vector of circuit unknowns (node voltages and branch currents), B and d are vectors that capture the connections of the input and output to the circuit. The size of the system is n.

We assume that $y^*(t)$ is the periodic steady state solution of (1) when $u(t) = 0$. Linearizing (1) around $y^*(t)$, we obtain

$$\begin{aligned} \frac{dC(t)x(t)}{dt} + G(t)x(t) &= Bu(t), \\ z(t) &= d^T x(t). \end{aligned} \tag{2}$$

Here, $C(t) = \frac{dq(y)}{dy}|_{y^*}$ and $G(t) = \frac{df(y)}{dy}|_{y^*}$ are periodic time-varying matrices. $x(t)$ and $z(t)$ are the small signal version of $y(t)$ and $z_y(t)$, respectively.

To more conveniently derive the transfer function of LPTV systems, we first recall the MPDE [9, 10] forms of (2), which separate the input and system time scales. The MPDE form of (2) is

$$\begin{aligned} \frac{\partial C(t_2)\hat{x}(t_1,t_2)}{\partial t_1} + \frac{\partial C(t_2)\hat{x}(t_1,t_2)}{\partial t_2} + G(t_2)\hat{x}(t_1,t_2) \\ = Bu(t_1), \\ \hat{z}(t_1,t_2) = d^T\hat{x}(t_1,t_2). \end{aligned} \tag{3}$$

Here, $\hat{x}(t_1,t_2)$ and $\hat{z}(t_1,t_2)$ the bivariate form of $x(t)$ and $z(t)$ in (2).

Performing a Laplace transform with respect to t_1, we further obtain

$$\begin{aligned} sC(t_2)\hat{X}(s,t_2) + \frac{\partial C(t_2)\hat{X}(s,t_2)}{\partial t_2} + G(t_2)\hat{X}(s,t_2) \\ = BU(s), \\ \hat{Z}(s,t_2) = d^T\hat{X}(s,t_2). \end{aligned} \tag{4}$$

$\hat{X}(s,t_2)$ and $\hat{Z}(s,t_2)$ are the transformed variables.

Define the differential operator

$$\frac{D}{dt_2}[.] = \frac{\partial[C(t_2).]}{\partial t_2}. \tag{5}$$

We can now write the time-varying transfer function in operator form

$$\begin{aligned} H(s,t_2) &= \frac{\hat{Z}(s,t_2)}{U(s)} \\ &= d^T\left[sC(t_2) + \frac{D}{dt_2}[.] + G(t_2)\right]^{-1} B. \end{aligned} \tag{6}$$

III. LPTV MOR USING KRYLOV SUBSPACE METHODS

Previous LPTV MOR techniques first convert a LPTV system to a LTI system and then perform reduction via LTI MOR techniques. Compared to other LTI MOR techniques, Krylov subspace methods are computationally more efficient [8]. In this section, we will review previous LPTV MOR techniques using Krylov subspace methods, in both matrix and operator forms.

A. Matrix Form LPTV MOR Methods

A LPTV system can be converted to an artificial LTI system by discretizing the periodic time variation using a finite basis [6, 7]. For example, we can expand the t_2 dependence in (4) using a time domain Backward Euler finite difference basis. Define the following long vectors

$$\begin{aligned} \vec{X}_{TD}(s) &= [\hat{X}_0(s)^T, \hat{X}_1(s)^T, \cdots, \hat{X}_{N-1}(s)^T]^T, \\ \vec{Z}_{TD}(s) &= [\hat{Z}_0(s)^T, \hat{Z}_1(s)^T, \cdots, \hat{Z}_{N-1}(s)^T]^T, \\ \vec{B}_{TD}(s) &= [B^T, B^T, \cdots, B^T]^T, \end{aligned} \tag{7}$$

and

$$D = \begin{pmatrix} d & & & \\ & d & & \\ & & \ddots & \\ & & & d \end{pmatrix}. \tag{8}$$

Here $\hat{X}_i(s) = \hat{X}(s,t_2)|_{t_2=t_i}$ and $\hat{Z}_i(s) = \hat{Z}(s,t_2)|_{t_2=t_i}$. $t_2 \in [0, T_2]$. N is the number of sample points. We obtain time domain matrix form of (4)

$$\begin{aligned} [sC_{TD} + \mathcal{J}_{TD}]\vec{X}_{TD}(s) = \vec{B}_{TD}U(s), \\ \vec{Z}_{TD}(s) = \mathcal{D}^T\vec{X}_{TD}(s), \end{aligned} \tag{9}$$

where

$$\begin{aligned} \mathcal{J}_{TD} &= \mathcal{G}_{TD} + \Delta C_{TD} \\ C_{TD} &= \begin{pmatrix} C_0 & & & \\ & C_1 & & \\ & & \ddots & \\ & & & C_{N-1} \end{pmatrix}, \\ \mathcal{G}_{TD} &= \begin{pmatrix} G_0 & & & \\ & G_1 & & \\ & & \ddots & \\ & & & G_{N-1} \end{pmatrix}, \\ \Delta &= \begin{pmatrix} \frac{1}{\delta_0}I & & & -\frac{1}{\delta_0}I \\ -\frac{1}{\delta_1}I & \frac{1}{\delta_1}I & & \\ & & \ddots & \\ & & -\frac{1}{\delta_{N-1}}I & \frac{1}{\delta_{N-1}}I \end{pmatrix}. \end{aligned} \tag{10}$$

$C_i = C(t_2)|_{t_2=t_i}$ and $G_i = G(t_2)|_{t_2=t_i}$. $\delta_i = t_{2,i} - t_{2,i-1}$.

Define

$$\vec{H}_{TD}(s) = [H_0(s)^T, H_1(s)^T, \cdots, H_{N-1}(s)^T]^T, \tag{11}$$

where $H_i(s) = H(s,t_2)|_{t_2=t_i}$. We can write a convenient matrix representation for the time-varying transfer function in (6)

$$\vec{H}_{TD}(s) = \mathcal{D}^T\left[sC_{TD} + \mathcal{J}_{TD}\right]^{-1}\vec{B}_{TD}. \tag{12}$$

Since (9) is a LTI system, it can be reduced by LTI MOR techniques. We briefly describe the applications of block-Krylov methods [1, 4, 5, 11, 12]. The transfer function (12) can be written in the form

$$\vec{H}_{TD}(s) = \mathcal{L}^T\left[I + s\mathcal{A}\right]^{-1}\mathcal{R}, \tag{13}$$

where

$$\mathcal{L} = \mathcal{D}, \mathcal{R} = \mathcal{J}_{TD}^{-1}\vec{B}_{TD}, \mathcal{A} = \mathcal{J}_{TD}^{-1}C_{TD}. \tag{14}$$

Now (14) can be used to generate reduced-order models using block-Krylov methods. For example, by applying the block Arnoldi algorithm [1, 12] to matrices $\mathcal{A}$ and $\mathcal{R}$, we obtain an orthogonal projection matrix V_m (of size $nN \times m$) and a block Hessenberg matrix T_m (of size $m \times m$). The transfer function of the reduced-order model is given by

$$\vec{H}_m(s) = \mathcal{L}^T V_m\left[I_m + sT_m\right]^{-1} V_m^T\mathcal{R}. \tag{15}$$

The reduced LTI system has a size of m and $m \le nN$. It can be shown that $\vec{H}_m(s)$ approximates $\vec{H}_{TD}(s)$ [13].

B. Operator Form LPTV MOR Methods

The matrix based MOR methods described above rely on a fixed, a-priori discretization of the time-varying differential operators to convert a LPTV system into a LTI system. For the operator based MOR methods [8, 14, 15], the discretization basis can be changed dynamically during the model-order reduction process. This is achieved by modifying the internals of the Krylov-subspace method to use general function space operators, instead of matrices.

We use the block Arnoldi algorithm to illustrate the operator based MOR procedure. Rewrite the operator form time-varying transfer function (6) as

$$H(s,t_2) = d^T \left(I + s\mathfrak{L}[.]\right)^{-1} r(t_2). \tag{16}$$

where

$$\begin{aligned} \mathfrak{L}[.] &= \left[\frac{D}{dt_2}[.] + G(t_2)\right]^{-1} [C(t_2).], \\ r(t_2) &= \left[\frac{D}{dt_2}[.] + G(t_2)\right]^{-1} B. \end{aligned} \tag{17}$$

Define operator based Arnoldi algorithm using $\mathfrak{L}[.]$ and $r(t_2)$

$$\begin{aligned} q_0'(t_2) &= r(t_2) \\ q_0(t_2) &= \frac{q_0'(t_2)}{||q_0'(t_2)||} \\ b_1(t_2) &= \mathfrak{L}[q_0(t_2)] \\ q_1'(t_2) &= b_1(t_2) - < b_1(t_2), q_0(t_2) > q_0(t_2) \\ q_1(t_2) &= \frac{q_1'(t_2)}{||q_1'(t_2)||} \\ b_2(t_2) &= \mathfrak{L}[q_1(t_2)] \\ q_2'(t_2) &= b_2(t_2) - < b_2(t_2), q_0(t_2) > q_0(t_2) \\ &\quad - < b_2(t_2), q_1(t_2) > q_1(t_2) \\ q_2(t_2) &= \frac{q_2'(t_2)}{||q_2'(t_2)||} \\ &\vdots \end{aligned} \tag{18}$$

Here, the inner-products and norms are defined as standard L^2 inner-products and norms [8]. $r(t_2)$ and $b_{i+1}(t_2)$ can be calculated using (17),

$$\begin{aligned} \left[\frac{D}{dt_2}[.] + G(t_2)\right] r(t_2) &= B, \\ \left[\frac{D}{dt_2}[.] + G(t_2)\right] b_{i+1}(t_2) &= C(t_2) q_i(t_2). \end{aligned} \tag{19}$$

Both $C(t_2)$ and $G(t_2)$ are periodic. Therefore, $r(t_2)$, $q_i(t_2)$ and $b_{i+1}(t_2)$ are also periodic. Any periodic steady state method, such as frequent domain harmonic balance (HB), time domain finite difference or shooting method, can be applied to solve (19). The projection matrix $V(t_2)$ is defined as

$$V(t_2) = \begin{pmatrix} q_0(t_2) & q_1(t_2) & \dots & q_{m-1}(t_2) \end{pmatrix} \tag{20}$$

During the solution of (19) at each Arnoldi step, we can dynamically change the discretization basis, which is the main advantage of operator based MOR methods over matrix based MOR methods. If a fixed discretization basis is used throughout the whole Arnoldi process, then discretizing the differential operator before the Arnoldi process (the matrix based MOR methods) and discretizing the operator during the Arnoldi process (the operator based MOR methods) produce the same results. For example, let the Backward Euler finite difference basis be used in both methods to produce the m-th Krylov subspace. Let

$$V_m = \begin{pmatrix} V_0 \\ V_1 \\ \vdots \\ V_{N-1} \end{pmatrix} \tag{21}$$

be the partitioning of the projection matrix (15) corresponding to the block size n, then $V_i = V(t_2)|_{t_2=t_i}$, *i.e.*, each block in (21) corresponds to $V(t_2)$ in (20) evaluated at different time points. Therefore, both Arnoldi processes in the matrix based and the operator based MOR methods generate the same information, but the operator based approach has the flexibility of changing discretization bases dynamically during the process.

C. Limitations of Previous LPTV MOR Methods

The previous LPTV MOR techniques impose several limitations on the model reduction process. First, the time-varying structure of the system is not preserved. The original LPTV system is reduced to a LTI system. Simulations are performed on the reduced LTI system. Additional post-processing is required in order to convert the LTI solutions to LPTV solutions.

In addition, since the LPTV system is first converted to a much larger artificial LTI system and then reduced, the size of the reduced LTI system is not guaranteed to be smaller than that of the original LPTV system. The size of the artificial LTI system is nN. For example, $\vec{X}_{TD}(s)$, $\vec{Z}_{TD}(s)$ and $\vec{B}_{TD}(s)$ in (9) are vectors of size $nN \times 1$. C_{TD} and $\mathcal{J}_{TD}$ are matrices of size $nN \times nN$. The reduced LTI system can potentially have a size of up to nN, *i.e.*, $m \le nN$. Therefore, the size of the reduced LTI system can be larger than that of the original LPTV system, as shown in Section V.

IV. Structure Preserving MOR for LPTV Systems

In this section, we propose a LPTV MOR technique which preserves the time-varying structure of a system during the reduction process and produces a reduced LPTV system. We prove that the size of the reduced system generated from our technique is always smaller than (or equal to) that of the original LPTV system.

A. The Algorithm

The key feature of our method is to retain the periodic time-varying structure at every step of the reduction process. Instead of using a LTI projection matrix as in (21), our method uses the periodic time-varying format of the projection matrix and applies it directly to the original LPTV system. The information required to form the time-varying projection matrix can be obtained from either the matrix form or the operator form

MOR techniques discussed in Section III. Denote the time-varying projection matrix by $V_q(t_2)$. $V_q(t_2)$ evaluated at different values of t_2 corresponds to blocks in (21). Multiplying $V_q(t_2)^T$ to the first equation in (3) and substituting the variable $\hat{x}(t_1,t_2) = V_q(t_2)\hat{x}_q(t_1,t_2)$, we obtain

$$\begin{aligned} V_q(t_2)^T \frac{\partial C(t_2)V_q(t_2)\hat{x}_q(t_1,t_2)}{\partial t_1} + V_q(t_2)^T \frac{\partial C(t_2)V_q(t_2)\hat{x}_q(t_1,t_2)}{\partial t_2} \\ + V_q(t_2)^T G(t_2)V_q(t_2)\hat{x}_q(t_1,t_2) = V_q(t_2)^T Bu(t_1), \\ \hat{z}(t_1,t_2) = d^T V_q(t_2)\hat{x}_q(t_1,t_2). \end{aligned} \tag{22}$$

Since $V_q(t_2)^T$ does not depend on t_1, we can write

$$V_q(t_2)^T \frac{\partial C(t_2)V_q(t_2)\hat{x}_q(t_1,t_2)}{\partial t_1} = \frac{\partial V_q(t_2)^T C(t_2)V_q(t_2)\hat{x}_q(t_1,t_2)}{\partial t_1} \tag{23}$$

Applying chain rule, we obtain

$$\begin{aligned} &\frac{\partial V_q(t_2)^T C(t_2)V_q(t_2)\hat{x}_q(t_1,t_2)}{\partial t_2} = \\ &V_q(t_2)^T \frac{\partial C(t_2)V_q(t_2)\hat{x}_q(t_1,t_2)}{\partial t_2} + \frac{\partial V_q(t_2)^T}{\partial t_2} C(t_2)V_q(t_2)\hat{x}_q(t_1,t_2) \end{aligned} \tag{24}$$

Substituting (23) and (24) into (22), the reduced system can be written in the form

$$\begin{aligned} \frac{\partial \hat{C}(t_2)\hat{x}_q(t_1,t_2)}{\partial t_1} + \frac{\partial \hat{C}(t_2)\hat{x}_q(t_1,t_2)}{\partial t_2} + \hat{G}(t_2)\hat{x}_q(t_1,t_2) \\ = \hat{B}(t_2)u(t_1), \\ \hat{z}(t_1,t_2) = \hat{d}(t_2)^T \hat{x}_q(t_1,t_2). \end{aligned} \tag{25}$$

where

$$\begin{aligned} \hat{C}(t_2) &= V_q(t_2)^T C(t_2)V_q(t_2) \\ \hat{G}(t_2) &= V_q(t_2)^T G(t_2)V_q(t_2) - \frac{\partial V_q(t_2)^T}{\partial t_2} C(t_2)V_q(t_2) \\ \hat{B}(t_2) &= V_q(t_2)^T B \\ \hat{d}(t_2) &= V_q(t_2)^T d \end{aligned} \tag{26}$$

(25) is in the same form as the MPDE form of the original LPTV system (3). Therefore, we can write the DAE form of (25)

$$\begin{aligned} \frac{d\hat{C}(t)x_q(t)}{dt} + \hat{G}(t)x_q(t) = \hat{B}(t)u(t), \\ z(t) = \hat{d}(t)^T x_q(t), \end{aligned} \tag{27}$$

where $x_q(t) = \hat{x}_q(t,t)$.

The reduced system (27) is a LPTV system and is in the same form as the original system (2). Simulations can be directly performed on (27) to produce periodic time-varying results without any additional post-processing.

Following the same procedure as in Section II, we can now write the time-varying transfer function of the reduced system in operator form

$$H(s,t_2) = \hat{d}(t_2)^T \left[s\hat{C}(t_2) + \frac{D}{dt_2}[.] + \hat{G}(t_2) \right]^{-1} \hat{B}(t_2). \tag{28}$$

Define

$$W(s,t_2) = \left[s\hat{C}(t_2) + \frac{D}{dt_2}[.] + \hat{G}(t_2) \right]^{-1} \hat{B}(t_2). \tag{29}$$

Then

$$\left[s\hat{C}(t_2) + \frac{D}{dt_2}[.] + \hat{G}(t_2) \right] W(s,t_2) = \hat{B}(t_2). \tag{30}$$

$W(s,t_2)$ is periodic in t_2. At each frequency point s_i, we can calculate $W(s_i,t_2)$ by any steady state method, such as frequent domain harmonic balance (HB), time domain finite difference or shooting method. The time-varying transfer function $H(s,t_2)$ can be calculated by $H(s,t_2) = \hat{d}(t_2)^T W(s,t_2)$.

Note that [8] applies a similar time-varying projection. However, it neither leads to a reduced LPTV system to which simulations can be easily and directly applied, nor is it proved to generate a truly reduced system that always has a smaller size than the original LPTV system. The reduced system in [8] is in a multi-time format which is not in the same form as the original MPDE system since the differential operator (5) is defined differently for the two systems. Solving this multi-time format of the reduced system in [8] inevitably involves discretizing the system (enlarging the system) and mapping the solution back to the single-time format. The authors in [8] emphasize that the main utility of the operator based approach is the flexibility of changing discretization bases dynamically. In contrast, our algorithm produces a simple DAE form LPTV system which is in the same form as the original LPTV system and generates a provably smaller system, as shown in the next section.

B. Properties of the Structure Preserving LPTV MOR Method

The key advance of our method is that it preserves the periodic time-varying structure during the reduction process. This leads to a reduced LPTV system which has the same form as the original system.

This structure preserving property also results in another desirable property of our method, the size of the reduced system generated from our technique is always smaller than (or equal to) that of the original LPTV system. To prove this property, we perform a Laplace transform with respect to t_1 and expand the t_2 dependence in (22) using a time domain finite difference (for example, Backward Euler) basis. Following the same procedure as in Section III-A, we can write the time domain matrix form of (22)

$$\begin{aligned} \left[s\mathcal{V}_{TD}^T C_{TD}\mathcal{V}_{TD} + \mathcal{V}_{TD}^T \mathcal{J}_{TD}\mathcal{V}_{TD} \right] \vec{X}_{TD}(s) = \mathcal{V}_{TD}^T \vec{B}_{TD} U(s), \\ \vec{Z}_{TD}(s) = \mathcal{D}^T \mathcal{V}_{TD}\vec{X}_{TD}(s). \end{aligned} \tag{31}$$

where C_{TD}, C_{TD}, $\vec{B}_{TD}$ and $\mathcal{D}$ are defined in Section III-A and

$$\mathcal{V}_{TD} = \begin{pmatrix} V_0 & & & \\ & V_1 & & \\ & & \ddots & \\ & & & V_{N-1} \end{pmatrix}, \tag{32}$$

$V_i = V_q(t_2)|_{t_2=t_i}$. $\mathcal{V}_{TD}$ is a matrix of size $nN \times qN$. Note that the blocks V_i. in $\mathcal{V}_{TD}$ are the same as the blocks in the previous projection matrix (21). However, the blocks are arranged differently in the two matrices. It is this special structure of $\mathcal{V}_{TD}$, naturally formed from using periodic time-varying projection in our method, that preserves the periodic time-varying structure during the reduction process.

Note that [3] presents a similar structure preserving technique for LTI systems. In [3], the original LTI projection matrix is partitioned and is manually set to form a block projection matrix. In our method, however, this structure of the equivalent projection matrix $\mathcal{V}_{TD}$ is naturally formed from using periodic time-varying projection.

This structure also guarantees that the size of the reduced system generated from our technique is always smaller than (or equal to) that of the original LPTV system. When $q = n$, $\mathcal{V}_{TD}$ becomes a square matrix of size $nN \times nN$. It is straightforward to prove that the matrix form for the time-varying transfer function of the reduced system

$$\begin{aligned} &\vec{H}_{qN}(s) \\ &= \mathcal{D}^T \mathcal{V}_{TD} \left[s \mathcal{V}_{TD}^T C_{TD} \mathcal{V}_{TD} + \mathcal{V}_{TD}^T \mathcal{J}_{TD} \mathcal{V}_{TD} \right]^{-1} \mathcal{V}_{TD}^T \vec{B}_{TD} \\ &= \mathcal{D}^T \left[s C_{TD} + \mathcal{J}_{TD} \right]^{-1} \vec{B}_{TD}, \end{aligned} \tag{33}$$

i.e., the reduced system is the same as the original system. In other words, the maximum size of the reduced system is n, *i.e.*, $q \leq n$. In contrast, the projection matrix from previous techniques (21) is of size $nN \times m$. The reduced LTI system have a size of $m \leq nN$. Therefore, the size of the reduced system from previous techniques can be larger than that of the original LPTV system, as shown in Section V.

Note we write the time domain matrix form of the reduced system (31) and the transfer function (33) from our technique in order to conveniently derive the properties of the method. The matrix form is not used in actual calculation since it involves matrix operation of size qN. The reduced system and the transfer function are calculated in the periodic time-varying format (of size q) using (26) and (30). Shooting method in complex domain is implemented to calculate (30), which only involves matrix operation of size q.

V. Application and Validation

In this section, we apply the structure preserving LPTV MOR technique to three circuit examples. Note that some circuit examples, such as mixers, are nonlinear, but the RF signal path is near-linear [16]. The frequency domain responses of the reduced system generated from our algorithm are shown to match that of the original system. We verify that the LPTV macromodel generated from our technique is smaller than (or equal to) the original LPTV system, but the macromodel generated from previous techniques can be larger than the original system to match the frequency domain responses accurately.

A. A Simple Upconverter

A simple upconverter from [6] is shown in Figure 1. It consists of a low pass filter, an ideal mixer and two bandpass filters. The low pass filter has a pole at $100kHz$ and the two bandpass filters have a center frequency at $10MHz$. The two bandpass filters have bandwidths of about $10kHz$ and $30kHz$, respectively. The LO frequency of the mixer is $10MHz$. The size of the system is $n = 5$.

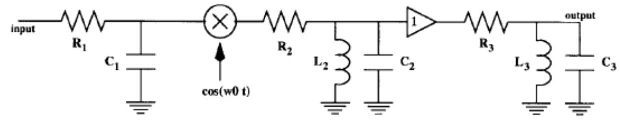

Fig. 1. A simple upconverter

For a LPTV circuit, the transfer function $H(s,t_2)$ is time-varying. We consider $H_{t_0}(s) = H(s,t_2)|_{t_2=t_0}$. Other transfer functions $H_{t_i}(s)$ can be compared in the same manner. The transfer functions $H_{t_0}(s)$ from the original LPTV system, the reduced system from previous techniques and the reduced system from our technique are compared in Figure 2.

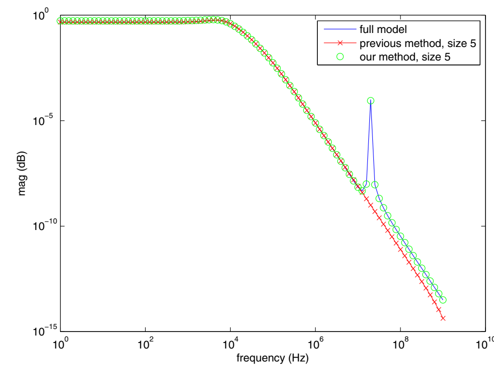

Fig. 2. Comparison of results from the original LPTV system and the reduced system.

As can be seen, the transfer function of the reduced system from our technique with a size of $q = 5$ matches that of the original system; while the transfer function from previous techniques with a size of 5 does not match. The previous techniques require at least a size of 9 to produce a good match, even though the original system is of size $n = 5$.

B. A Balanced Downconversion Mixer

A balanced direct downconversion mixer adapted from [17] is shown in Figure 3. The LO frequency of the mixer is $10kHz$. The inputs are RF signals. The size of the original system is $n = 5$.

Figure 4 shows the comparison of the transfer functions $H_{t_0}(s)$ from the full system, the reduced system from previous techniques and the reduced system from our technique. As can be seen, the transfer function from our technique with a size of $q = 2$ matches that of the original system (of size $n = 5$). However, for previous techniques, a size of $m = 5$ is not sufficient. $m = 15$ is required to produce a good match.

Time domain transfer functions $H_{t_i}(s)$ can be converted to frequency domain transfer functions using FFT.

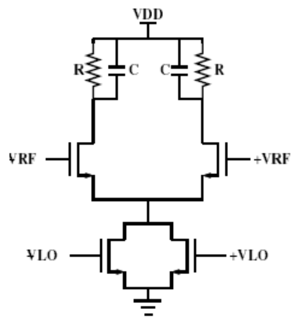

Fig. 3. A balanced direct downconversion mixer

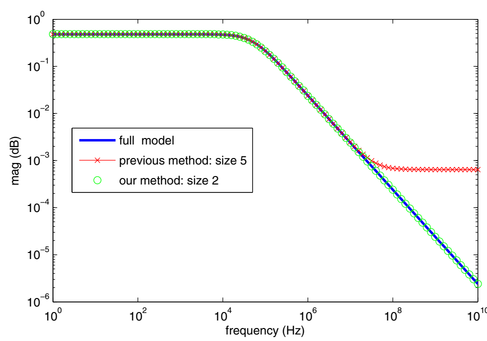

Fig. 4. Comparison of results from the original system and the reduced system.

C. PLL Phase Detector

A mixer used as the phase detector in a PLL circuit is shown in Figure 5. The mixer is followed by a low pass filter. The LO frequency of the mixer is $100MHz$. The size of the original system is $n = 12$.

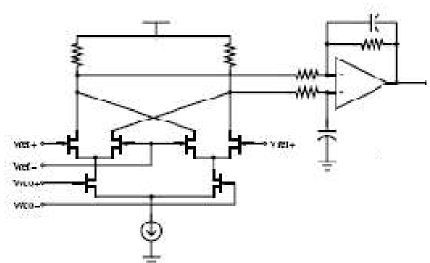

Fig. 5. A phase detector in a PLL circuit

Figure 6 shows the comparison of the transfer functions $H_{t_0}(s)$ from the full system, the reduced system from previous techniques and the reduced system from our technique. As can be seen, the transfer function from our technique with a size of $q = 4$ matches that of the original system (of size $n = 12$). However, for previous techniques, a size of $m = 12$ is not sufficient. $m = 13$ is required to produce a good match.

VI. Conclusions

We have presented a novel structure preserving reduced-order modeling algorithm for LPTV systems. Key advantages of our approach include preserving the periodic time-varying structure during the reduction process and producing significantly smaller systems than the original systems. Numerical results show that the macromodels generated from our method feature smaller sizes and better accuracy than macromodels generated from previous LPTV MOR techniques.

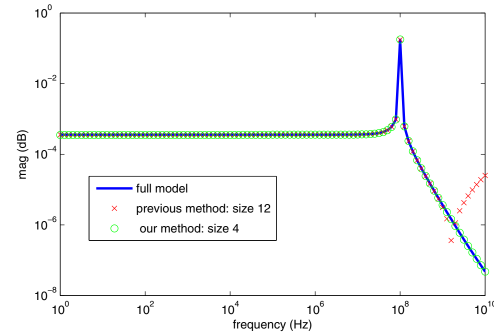

Fig. 6. Comparison of results from the original system and the reduced system.

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