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Variable Nonlocal Length Scale in a Peridynamic Body with Applications to Local-Nonlocal Coupling

ASME 2013 International Mechanical Engineering Congress and Exposition

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Peridynamic Theory of Solid Mechanics

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WHAT IS PERIDYNAMICS?

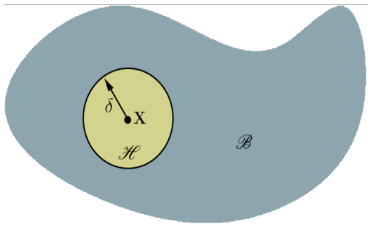
Peridynamics is a mathematical theory that unifies the mechanics of continuous media, cracks, and discrete particles

HOW DOES IT WORK?

- Peridynamics is a *nonlocal* extension of continuum mechanics
- Remains valid in presence of discontinuities, including cracks
- Balance of linear momentum is based on an *integral equation*:

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t) = \underbrace{\int_{\mathscr{B}} \left\{ \underline{\mathbf{T}}[\mathbf{x},t]\left\langle \mathbf{x}'-\mathbf{x}\right\rangle - \underline{\mathbf{T}}'[\mathbf{x}',t]\left\langle \mathbf{x}-\mathbf{x}'\right\rangle \right\} \, \mathrm{d}V_{\mathbf{x}'}}_{\text{Divergence of stress replaced with integral of nonlocal forces.}} + \mathbf{b}(\mathbf{x},t)$$

The point X interacts directly with all points within its horizon



S.A. Silling. Reformulation of elasticity theory for discontinuities and long-range forces. *Journal of the Mechanics and Physics of Solids*, 48:175-209, 2000.

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Peridynamic Theory of Solid Mechanics

CONSTITUTIVE LAWS IN PERIDYNAMICS

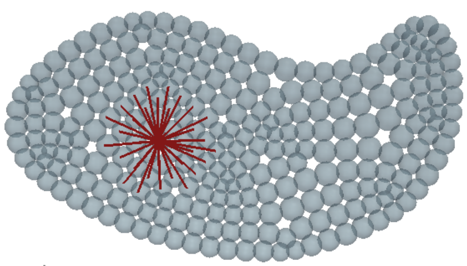
- Peridynamic *bonds* connect any two material points that interact directly
- Peridynamic forces are determined by *force states* acting on bonds

$$\underbrace{\underline{\mathbf{T}}[\mathbf{x},t]}_{\text{Force State}}\ \underbrace{\langle \mathbf{x}_i' - \mathbf{x}\rangle}_{\text{Bond}}$$

- Force states are determined by constitutive laws and are functions of the deformations of all points within a neighborhood
- *Material failure* is modeled through the breaking of peridynamic bonds
  - Example: critical stretch bond breaking law

DISCRETIZATION OF A PERIDYNAMIC BODY

Direct discretization of the strong form of the balance of linear momentum [1]



$$\rho(\mathbf{x})\ddot{\mathbf{u}}_h(\mathbf{x},t) = \sum_{i=0}^{N} \left\{ \underline{\mathbf{T}}[\mathbf{x},t]\langle \mathbf{x}_i' - \mathbf{x}\rangle - \underline{\mathbf{T}}'[\mathbf{x}_i',t]\langle \mathbf{x} - \mathbf{x}_i'\rangle \right\} \Delta V_{\mathbf{x}_i'} + \mathbf{b}(\mathbf{x},t)$$

[1] S.A. Silling and E. Askari. A meshfree method based on the peridynamic model of solid mechanics. *Computers and Structures*, 83:1526-1535, 2005.

Classical (Local) Material Models Can be Applied within the Peridynamic Framework

NON-ORDINARY STATE-BASED APPROACH [1]

1. Compute an approximate deformation gradient based on the initial and current locations of material points in nonlocal neighborhood

Approximate Deformation Gradient

$$\bar{\mathbf{F}} = \left(\sum_{i=0}^{N} \underline{\omega}_i \, \underline{\mathbf{Y}}_i \otimes \underline{\mathbf{X}}_i \, \Delta V_{\mathbf{x}_i} \right) \mathbf{K}^{-1}$$

Shape Tensor

$$\mathbf{K} = \sum_{i=0}^{N} \underline{\omega}_i \, \underline{\mathbf{X}}_i \otimes \underline{\mathbf{X}}_i \, \Delta V_{\mathbf{x}_i}$$

2. Kinematic data passed to classical material model
3. Classical material model computes stress
4. Stress converted to pairwise forces

$$\underline{\mathbf{T}} \left\langle \mathbf{x}' - \mathbf{x} \right\rangle = \underline{\omega} \, \sigma \mathbf{K}^{-1} \left\langle \mathbf{x}' - \mathbf{x} \right\rangle$$

5. Apply stabilization term to suppress low-energy modes (optional)

[1] S. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari. Peridynamic states and constitutive modeling. Journal of Elasticity, 88:151-184, 2007.

Suppression of Low-Energy Modes

Penalize deformation that deviates from regularized deformation gradient [1]

Predicted location of neighbor

$$\mathbf{x}_n'^{\star} = \mathbf{x}_n + \bar{\mathbf{F}}_n \ (\mathbf{x}_o' - \mathbf{x}_o)$$

Hourglass vector

$$\mathbf{\Gamma}_{\mathrm{hg}} = \mathbf{x}_n'^{\star} - \mathbf{x}_n'$$

Hourglass vector projected onto bond

$$\gamma_{\mathrm{hg}} = \mathbf{\Gamma}_{\mathrm{hg}} . \, (\mathbf{x}_n' - \mathbf{x}_n)$$

Stabilization force

$$\mathbf{f}_{\mathrm{hg}} = -\mathrm{C}_{\mathrm{hg}} \underbrace{\left(\frac{18k}{\pi\delta^4}\right)}_{\substack{\text{micro-}\\\text{modulus}}} \underbrace{\frac{\gamma_{\mathrm{hg}}}{\|\mathbf{x}_o' - \mathbf{x}_o\|}}_{\substack{\text{hourglass}\\\text{stretch}}} \underbrace{\frac{\mathbf{x}_n' - \mathbf{x}_n}{\|\mathbf{x}_n' - \mathbf{x}_n\|}}_{\substack{\text{bond unit}\\\text{vector}}} \Delta V_{\mathrm{x}} \, \Delta V_{\mathrm{x}'}$$

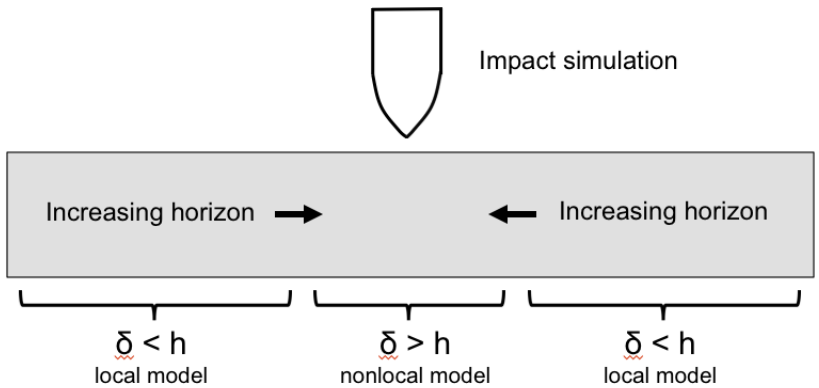
[1] D. Littlewood, K. Mish, and K. Pierson. Peridynamic simulation of damage evolution for structural health monitoring. Proceedings of ASME 2012 International Mechanical Engineering Congress and Exposition (IMECE2012), Houston, TX, November 9-15, 2012.

New Research Focus: Variable Nonlocal Length Scale

Employ a variable peridynamic horizon to better facilitate local-nonlocal coupling in combined peridynamic / classical FEM simulations

MOTIVATION

- A variable horizon provides a smooth transition from a nonlocal model to a local model



CHALLENGE

- How can we vary the peridynamic horizon without introducing artifacts?

New Research Focus: Variable Nonlocal Length Scale

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DO CURRENT PERIDYNAMIC MODELS SUPPORT A VARIABLE LENGTH SCALE?

- Standard peridynamic constitutive laws have very limited support for a variable length scale
  - A linearly varying horizon can be supported without introducing artifacts
  - Difficulties persist at transition from a constant horizon to a varying horizon

PATH FORWARD

- Goal: Develop an alternative formulation that mitigates spurious artifacts in the presence of a variable nonlocal length scale
- Target one-dimensional patch tests (expose spurious artifacts, if any)
  - Linear displacement field must be equilibrated
  - Quadratic displacement field must produce constant acceleration

Partial Stress Formulation for Peridynamic Constitutive Laws

Discovery: A novel peridynamic formulation that is guaranteed to pass the patch test

Approach is based on the peridynamic stress

$$\nu(x) := \int_0^\infty \int_0^\infty \left(\underline{T}[x-z]\langle z+w \rangle - \underline{T}[x+z]\langle -z-w \rangle \right) \, dz \, dw.$$

If $\underline{T}$ is independent of *x*, the peridynamic stress field reduces to

$$\nu = \int_{-\delta}^{\delta} \xi \underline{T}\langle \xi \rangle \, d\xi.$$

This leads to a definition of the peridynamic *partial stress*

$$\nu_0(x) := \int_{-\delta}^{\delta} \xi \underline{T}[x]\langle \xi \rangle \, d\xi.$$

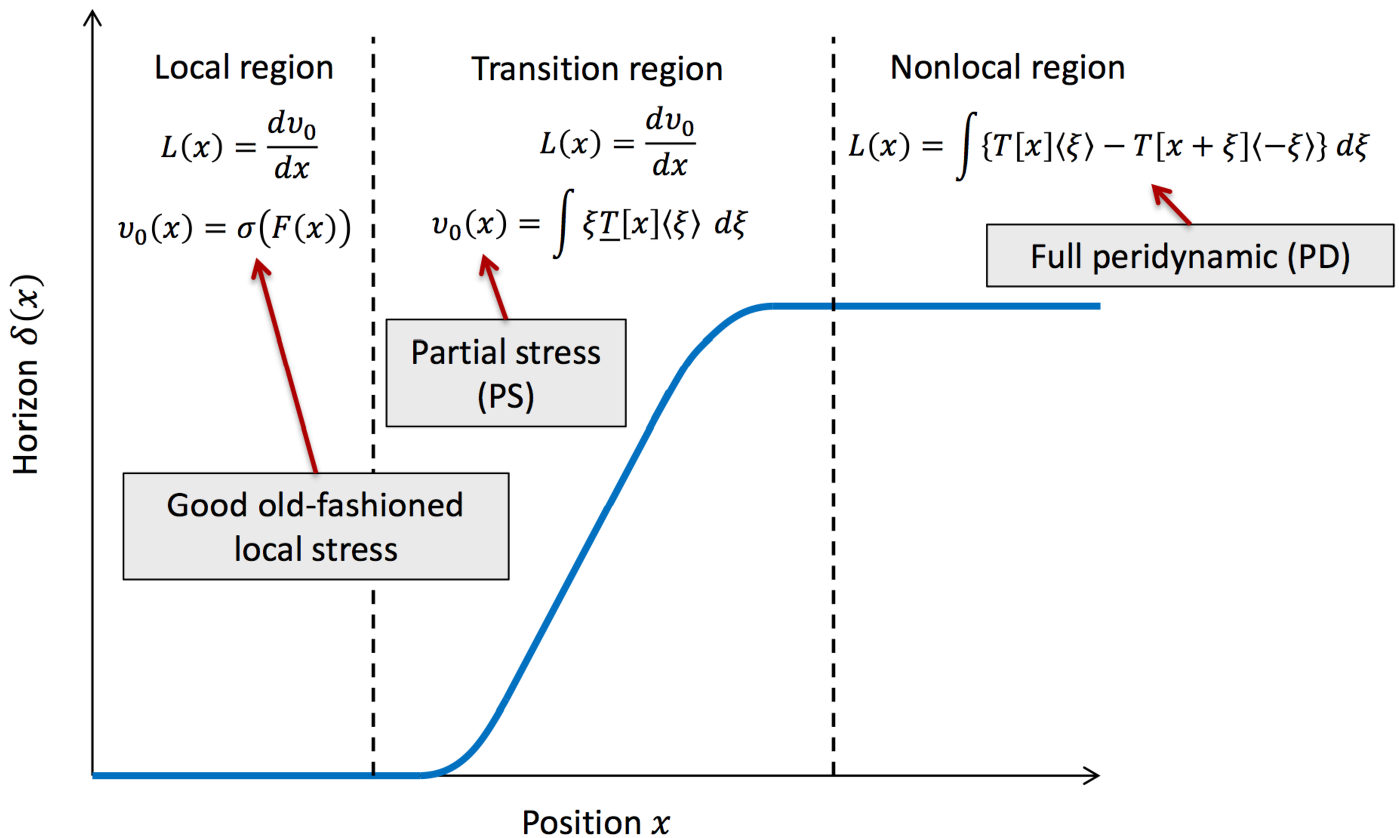
The partial stress is independent of delta in the linear patch test

➡ Utilize the partial stress within regions of variable horizon to greatly reduce numerical artifacts

Stewart Silling, Variable Length Scale in a Peridynamic Continuum, Multiscale Technical Information Exchange Meeting, May 6th, 2013, Livermore CA.

Utilize the Partial Stress Formulation in a Transition Region

ALTER THE PERIDYNAMIC HORIZON WITHIN A BODY TO APPLY NONLOCALITY ONLY WHERE NEEDED



[Stewart Silling]

Evaluation of Patch Tests for Partial Stress Formulation

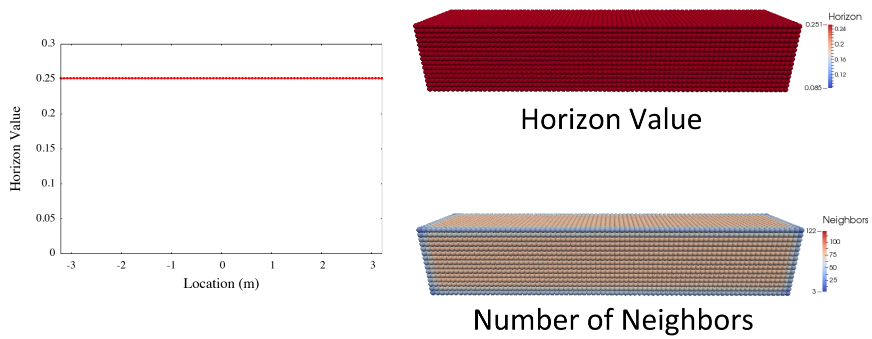
SUBJECT RECTANGULAR BAR TO PRESCRIBED DISPLACEMENT FIELDS

- Examine response under linear and quadratic displacement fields
- Investigate standard formulation with both constant and varying peridynamic horizon
- Investigate partial stress formulation with both constant and varying peridynamic horizon

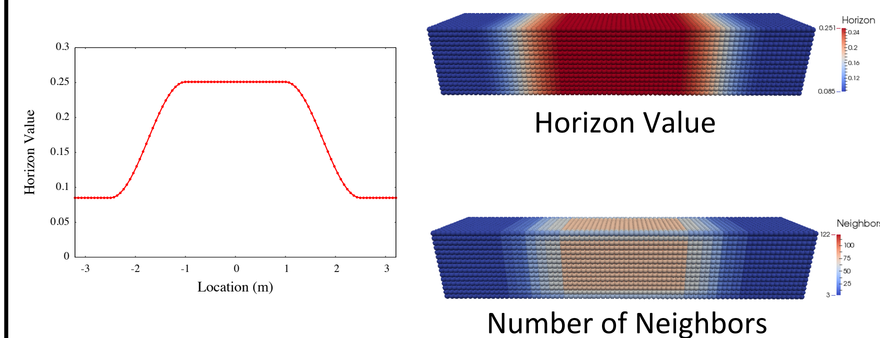
Elastic Correspondence Material Model

Density	7.8 g/cm^3
Young's Modulus	200.0 GPa
Poisson's Ratio	0.0
Stability Coefficient	0.0

Constant Horizon



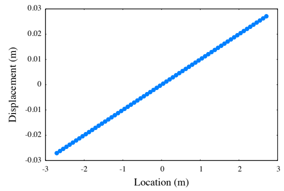
Varying Horizon



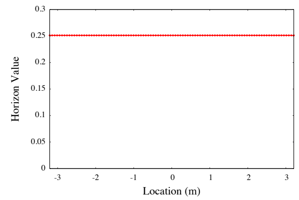
Patch Test: Prescribed Linear Displacement

Test set-up

Prescribe linear displacement field



Constant horizon throughout bar

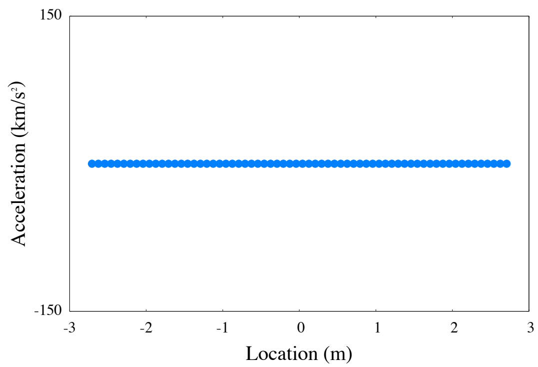


Can the standard model and the partial-stress model recover the expected zero acceleration?

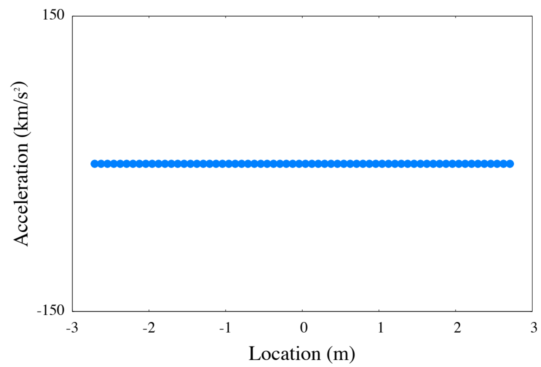
Both models produce the expected result when the horizon is **constant**

Test Results: Acceleration over the length of the bar

Standard material model



Partial-stress formulation

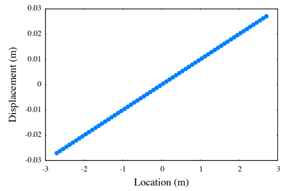


Note: nodes near ends of bar excluded from plots

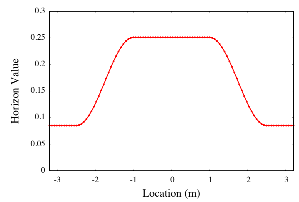
Patch Test: Prescribed Linear Displacement

Test set-up

Prescribe linear displacement field



Variable horizon

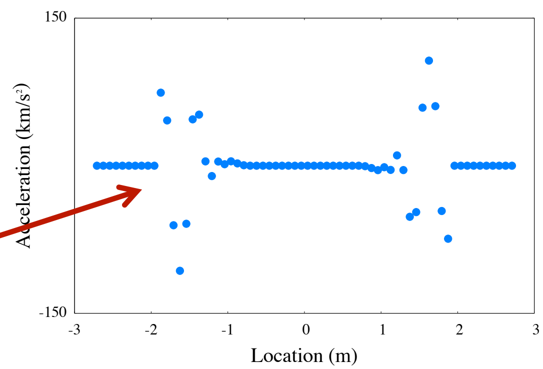


Can the standard model and the partial-stress model recover the expected zero acceleration?

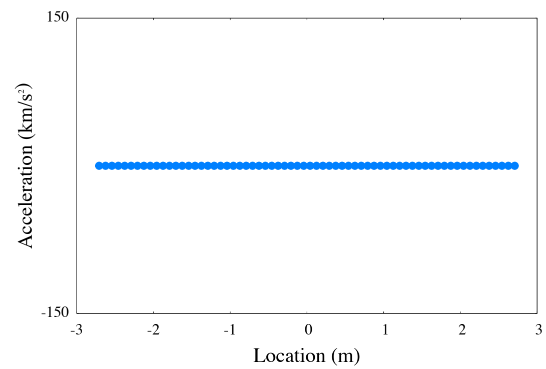
Only the **partial stress** formulation produce the expected result when the horizon is **varying**

Test Results: Acceleration over the length of the bar

Standard material model



Partial-stress formulation

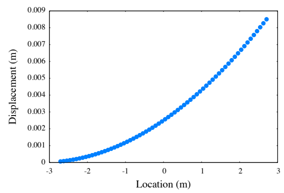


Spurious “ghost forces” present in standard formulation

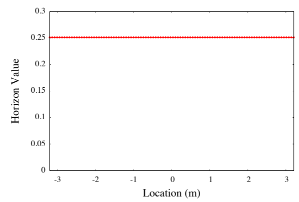
Patch Test: Prescribed Quadratic Displacement

Test set-up

Prescribe linear displacement field



Constant horizon throughout bar

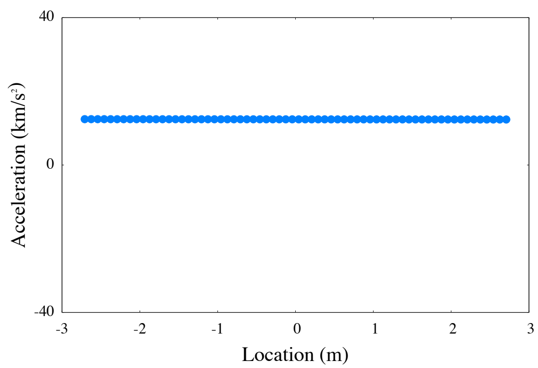


Can the standard model and the partial-stress model recover the expected constant acceleration profile?

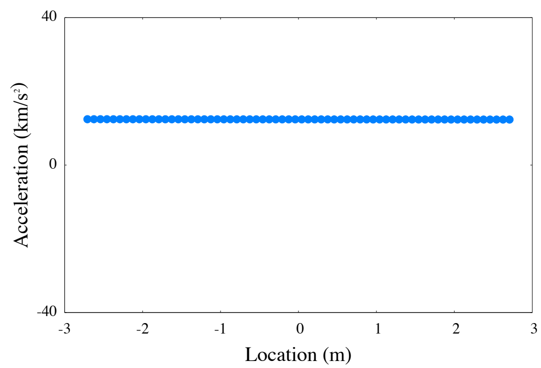
Both models produce the expected result when the horizon is **constant**

Test Results: Acceleration over the length of the bar

Standard material model



Partial-stress formulation



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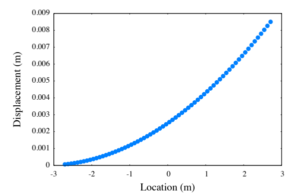
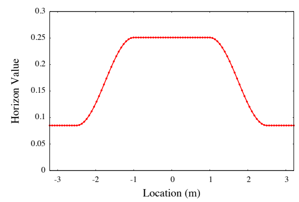


Variable horizon

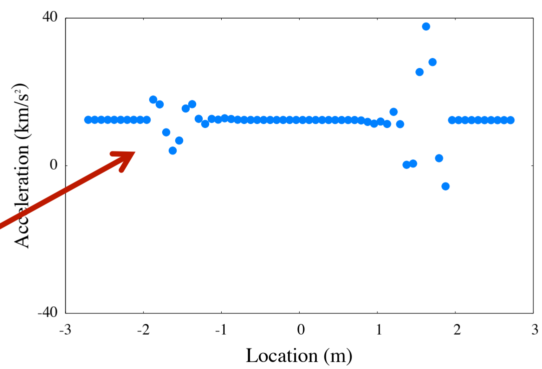


Can the standard model and the partial-stress model recover the expected constant acceleration?

Only the **partial stress** formulation produce the expected result when the horizon is **varying**

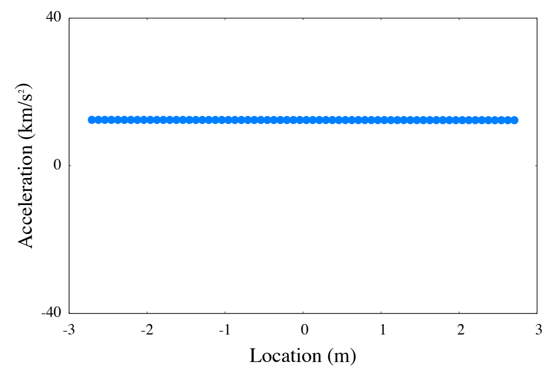
Test Results: Acceleration over the length of the bar

Standard material model



Spurious "ghost forces" present in standard formulation

Partial-stress formulation



Wave Propagation at Local-Nonlocal Interface

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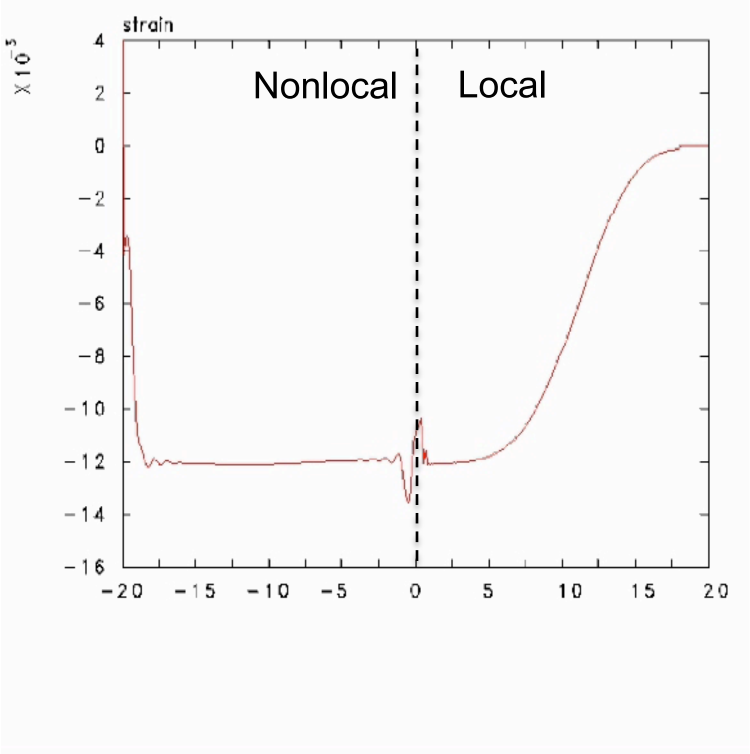
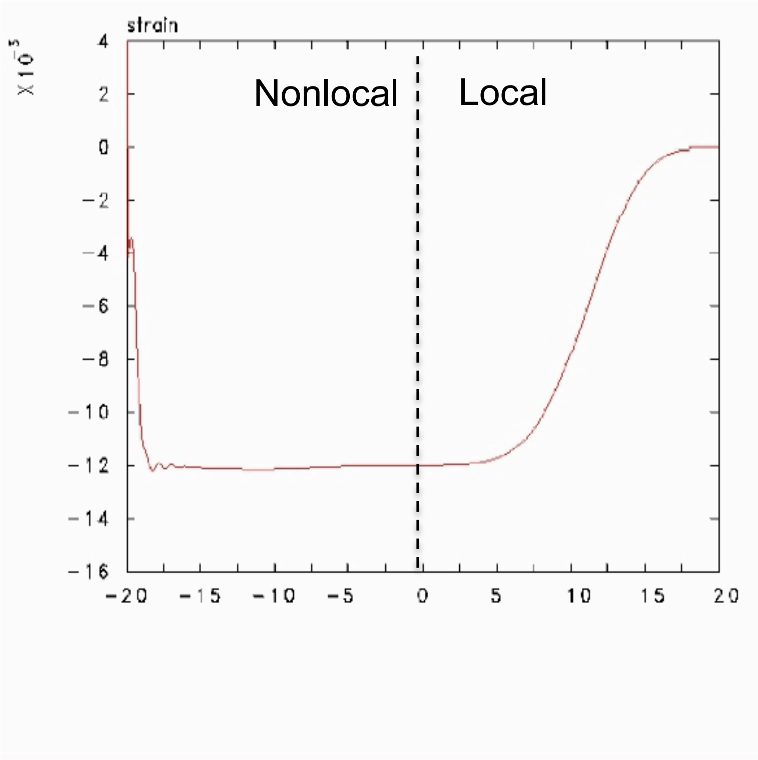
Standard peridynamic model

Numerical artifacts at local-nonlocal boundary



Partial-stress approach

Greatly reduces artifacts, enables smooth transition between local and nonlocal models



Stewart Silling, Variable Length Scale in a Peridynamic Continuum, Multiscale Technical Information Exchange Meeting, May 6th, 2013, Livermore CA.

What about Performance?

USE OF A VARIABLE HORIZON CAN REDUCE OVERALL COMPUTATIONAL EXPENSE

- Variable-horizon reduces neighborhood size
  - Less computational cost per internal force evaluation
  - Reduces number of unknowns in stiffness matrix for implicit time integration
- Variable-horizon reduces critical time step
  - Critical time step is strongly dependent on the horizon
  - Smaller time step results in more total steps to solution

Total Number of Bonds

(equal to number of nonzeros in stiffness matrix)

Constant Horizon	92.6 million
Varying Horizon	46.5 million

Stable Time Step [1]

Constant Horizon	2.03e-5 sec.
Varying Horizon	7.15e-6 sec.

Total Run Time for Patch Test

(1000 time steps with fixed time step of 1.0e-7 sec.)

Standard Formulation, Constant Horizon	185 sec.
Standard Formulation, Varying Horizon	127 sec. (bad results)
Partial Stress Formulation, Constant Horizon	271 sec.
Partial Stress Formulation, Varying Horizon	172 sec.

[1] S.A. Silling and E. Askari. A meshfree method based on the peridynamic model of solid mechanics. *Computers and Structures*, 83:1526-1535, 2005.

Ongoing Work

PARTIAL STRESS FORMULATION

- Construction of tangent stiffness matrix for quasi-statics and implicit dynamics
- Application of improved peridynamic quadrature (with Pablo Seleson)
- Extension of initial serial implementation to parallel

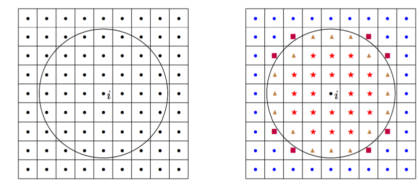
COUPLING PERIDYNAMICS WITH CLASSICAL FINITE ELEMENT ANALYSIS

- Application of partial stress approach within standard finite elements
- Coupling of *Peridigm* peridynamics code with *Albany* classical finite element code
- Introduction of additional blending scheme at local-nonlocal interface, if needed

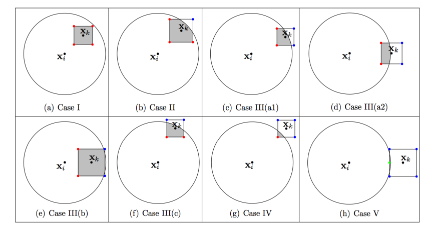
Improved Single-Point Peridynamic Quadrature

FOCUS ON NEIGHBORS THAT FALL ONLY PARTIALLY WITHIN THE HORIZON

- Current practice treats neighbors as “all in” or “all out”
- Improvements can be made by considering partial volumes
  - Alter weighting in nonlocal integral
  - Bond vector can be drawn from centroid of partial volume



(a) Neighborhood of i on a square lattice. (b) Classes of lattice points with respect to i.

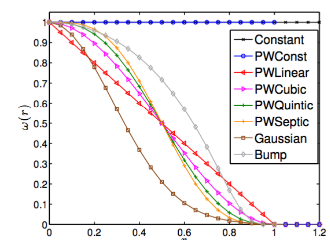


Consider different intersection cases at horizon boundary

RESULTS

- Improved scheme for handling neighbors at horizon boundaries
  - Improved accuracy
  - Small performance hit
- Identified influence functions that best mitigate errors at horizon boundaries

Consider effect of influence function on errors at horizon boundary



P. Seleson, Improved one-point quadrature algorithms for two-dimensional peridynamic models based on analytic calculations. *Submitted for publication*, 2013.

Questions?

Variable Peridynamic Horizon with Application to Local-Nonlocal Coupling

http://peridigm.sandia.gov

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Multiphysics Simulation Technologies (Org. 1444)

References

SEMINAL WORK IN PERIDYNAMICS

S.A. Silling, Reformulation of elasticity theory for discontinuities and long-range forces, *Journal of the Mechanics and Physics of Solids*, 48(1), pp. 175-209, 2000.

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