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Three-dimensional Extended Finite Element Method based on Nodal Tetrahedra

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Overview

- Extended Finite Element Method (XFEM)[1] is now widely accepted for mesh-independent fracture
- 3D XFEM is challenging due to topology issues
- Tetrahedra have advantages over hexahedra for XFEM
  - Fewer possibilities of edges cut by a plane
  - Element cut planes are quadrilaterals or triangles
- Tetrahedra suffer from volumetric locking
- Nodal tetrahedra proposed as alternative to overcome that problem

[1]Moes, Dolbow, and Belytschko, IJNME, 1999

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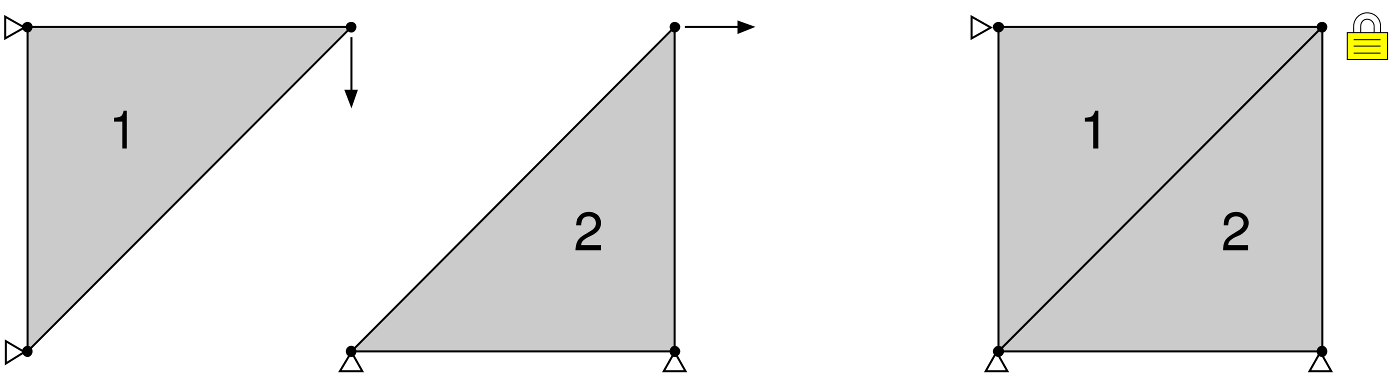
Pitfalls of Tetrahedral Elements

- Classic volumetric locking
  - Incompressible materials
  - Free nodes of elements 1 & 2 can only move in indicated directions
  - Combining elements 1 & 2 “locks” that node
  - Plastically deforming materials are nearly incompressible and exhibit this behavior



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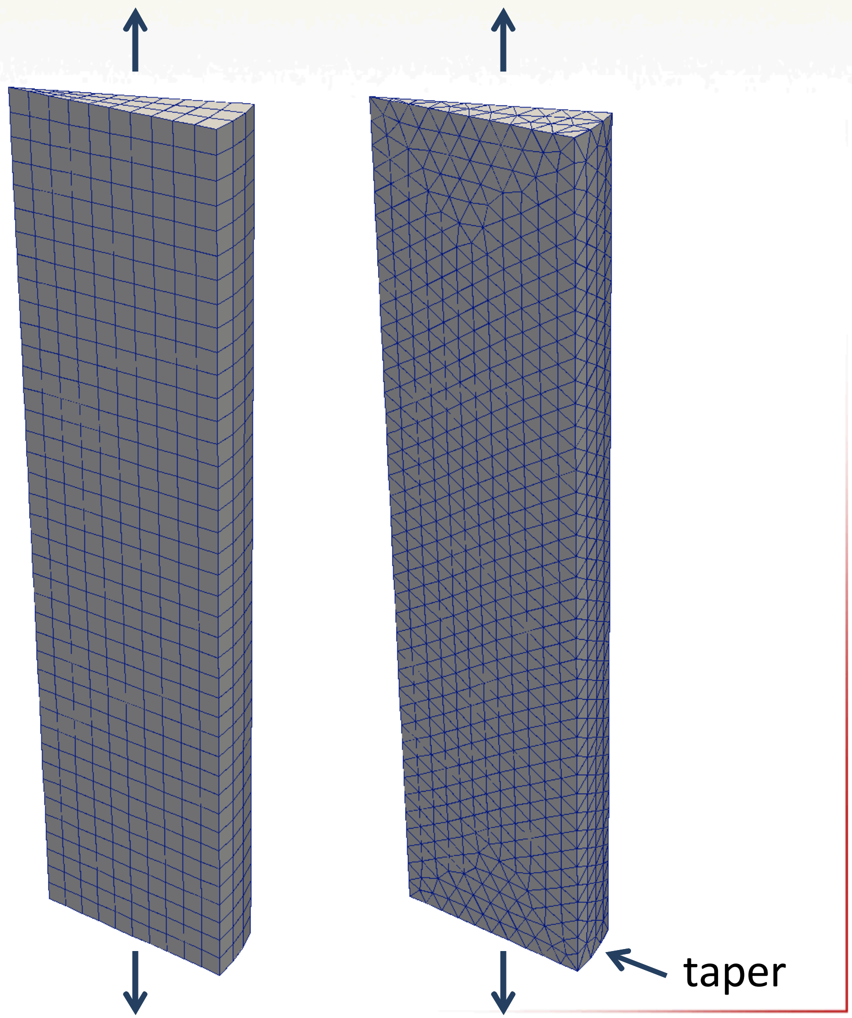
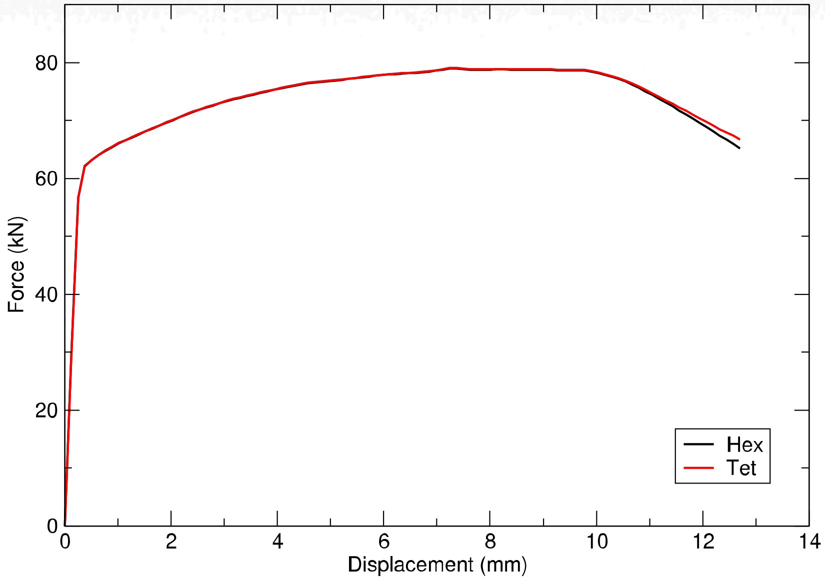
Locking Demonstration: Bar Pull

- Compare Hex & Tet mesh of slice of bar
- 2024-T3 Aluminum
- 51mm long, 6.4mm radius
- 15° wedge
- Symmetry plane on bottom
- Equivalent mesh sizes
- Slight taper at bottom symmetry plane to induce necking in that region
- No failure modeled

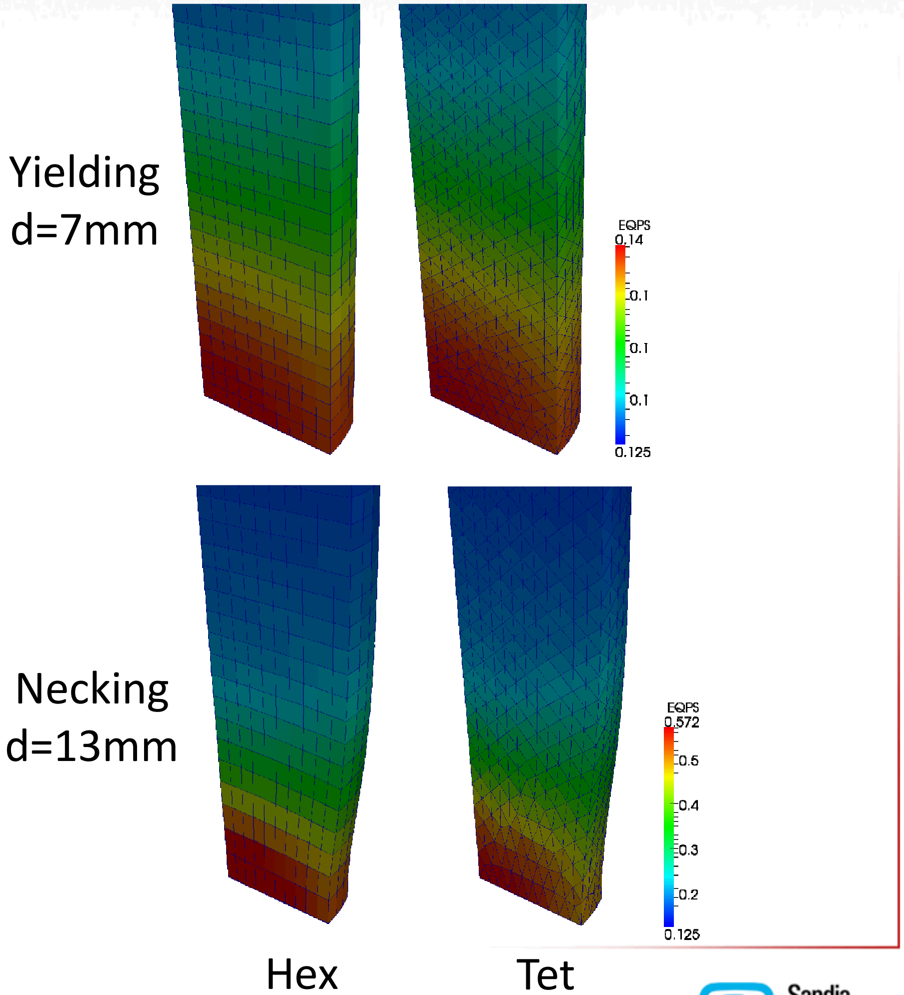


Bar Pull Hex vs Tet: Global Response & EQPS

Force/Displacement



Equivalent Plastic Strain



- Force/displacement response very similar
- Tet slightly stiffer in necking
- Equivalent plastic strain almost identical

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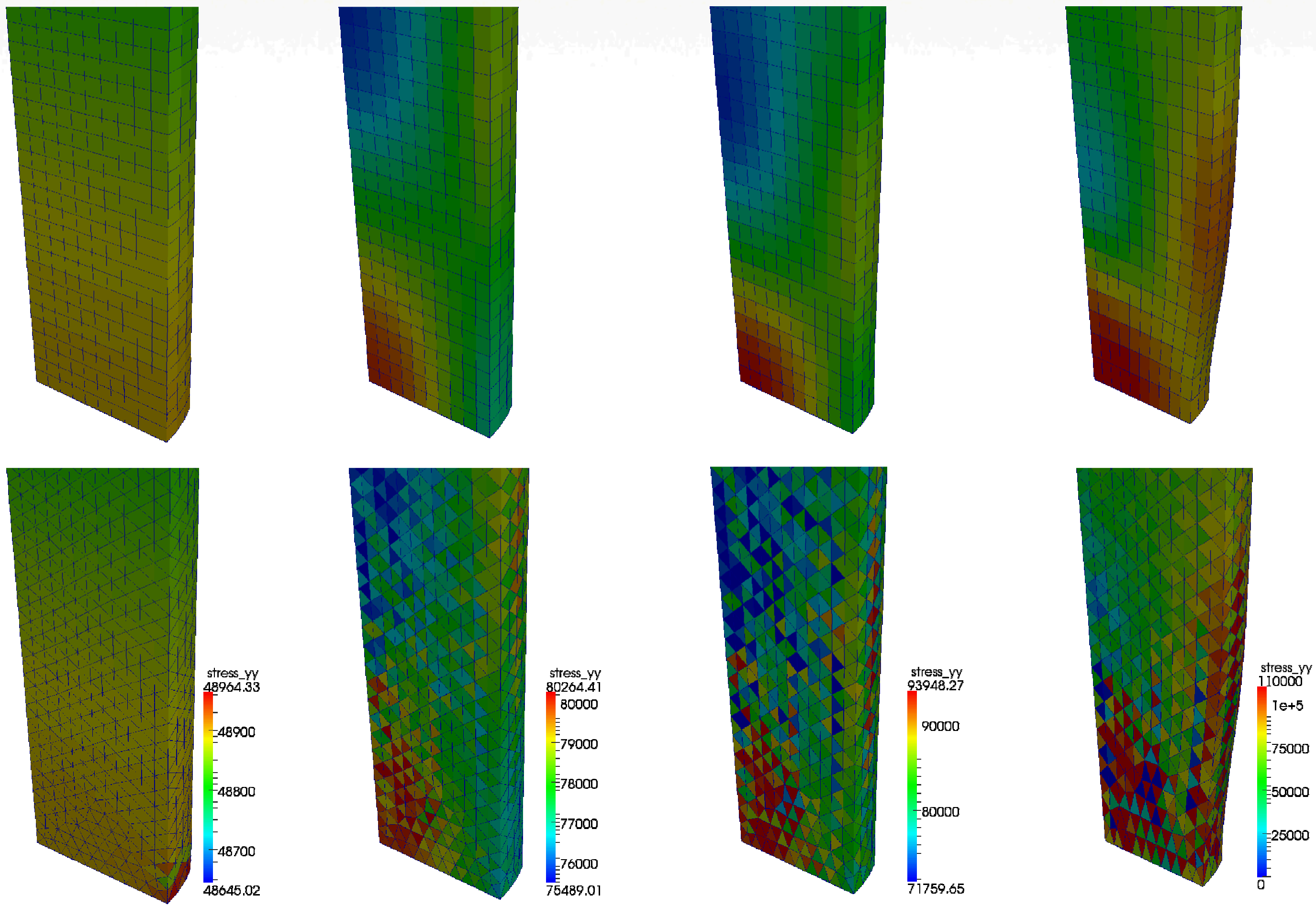
Bar Pull Hex vs Tet: stress(yy)

Hex

Tet



Elastic d=0.2mm

Yielding d=7mm

Failure d=10mm

Necking d=13mm

Effect of Locking on Tearing Parameter

- Tearing parameter used as crack growth criterion in this work[1]:

$$TP = \int_0^{\varepsilon_f} \left\langle \frac{2\sigma_T}{3(\sigma_T - \sigma_m)} \right\rangle^4 d\varepsilon_p$$

  - The Macaulay brackets indicate that failure only occurs in tension. No healing occurs.
  - The power of 4 on the stress state is empirical and is required to match notched tensile tests.
  - The critical value of the tearing parameter can be determined from a tensile test.
- Highly influenced by errors in the mean stress caused by volumetric locking in tets.

[1]Brozzo, DeLuca, and Rendina, 1972

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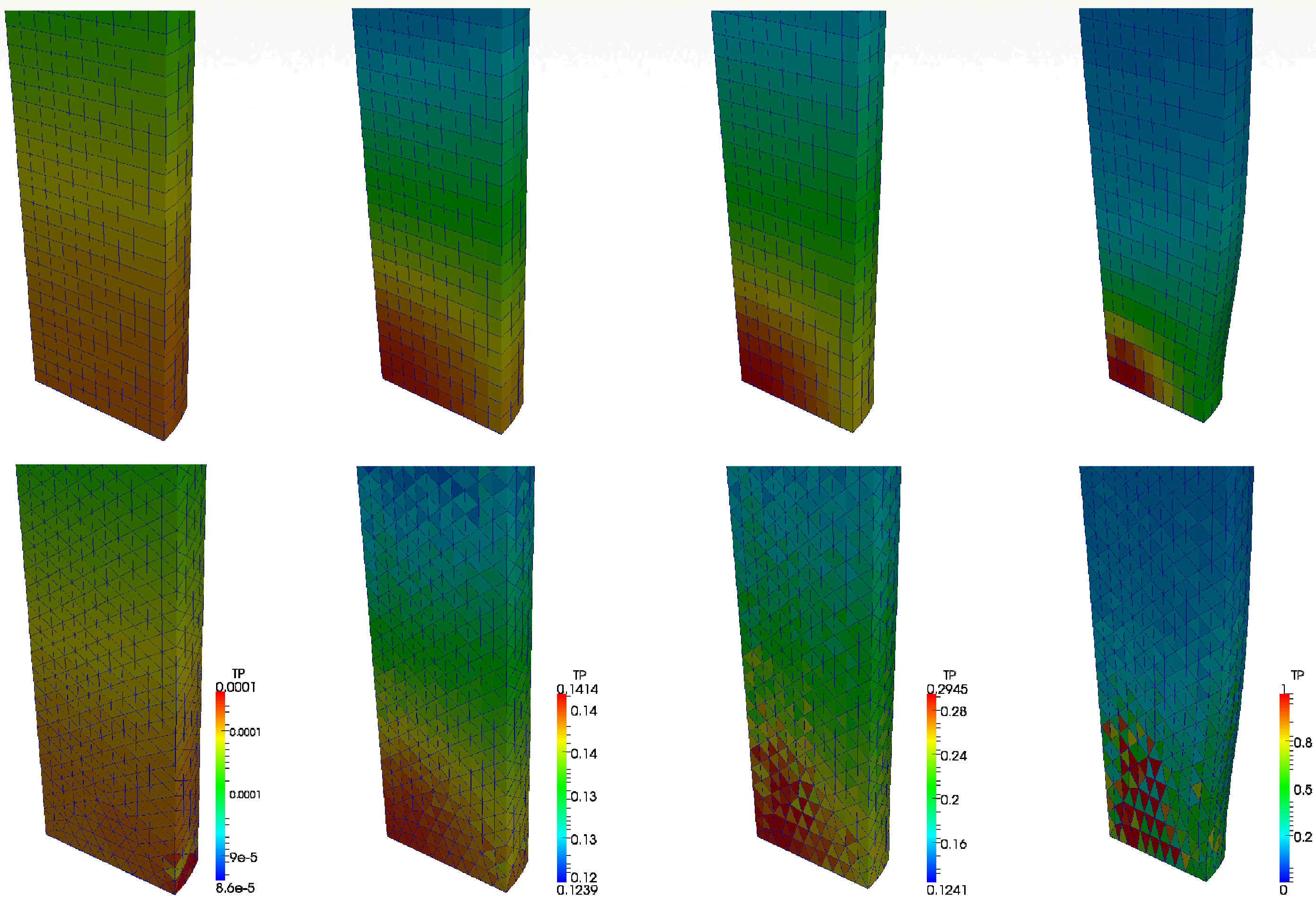
Bar Pull Hex vs Tet: Tearing Parameter

Hex

Tet



Elastic	Yielding	Failure	Necking
d=0.2mm	d=7mm	d=10mm	d=13mm

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Node-based Tetrahedral Element

- Tetrahedral element using nodal integration proposed[1] to reduce constraints and overcome volumetric locking.
  1. Calculate standard element-centric gradient operator ***B*** for each element
  2. Compute tributary volume of each node
  3. Compute volume-averaged nodal ***B*** for each node
  4. Evaluate nodal stress
  5. Compute stress divergence and assemble to internal force

[1]Dohrmann, Heinstein, Jung, Key, Witkowski, IJNME, 2000

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Node-based Tet "Hourglass Control"

- Node-based tet has low energy modes similar to hourglass modes.
- Add small contribution from standard linear tet to stiffen those modes:

$$\acute{\sigma}^{ij} = (1 - \beta)\, \acute{\sigma}^{ij}_{(np)} + \beta\, \acute{\sigma}^{ij}_{(el)}$$

- β controls amount of contribution from linear tet, can range from 0 to 1.

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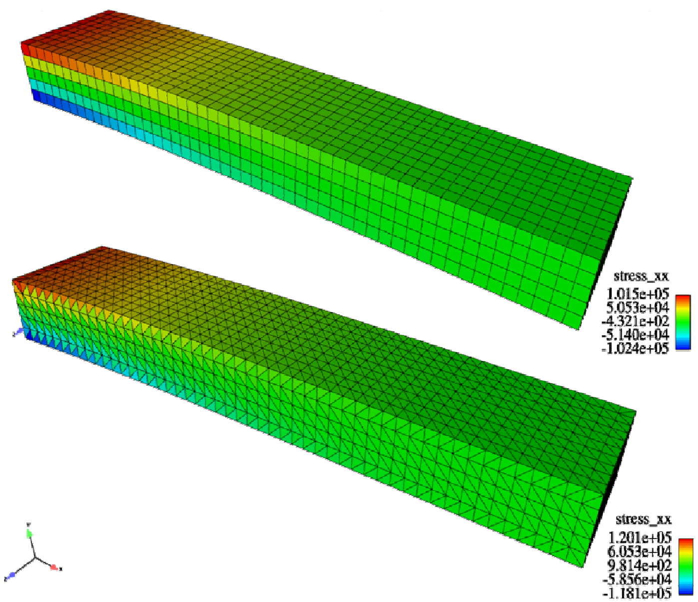
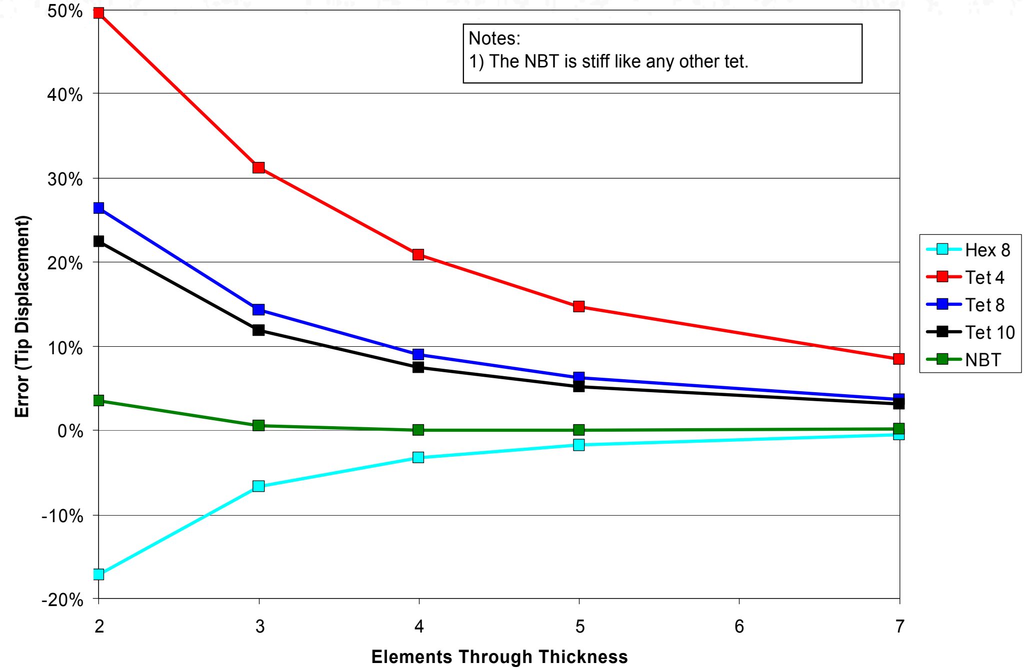
Bending Performance of Node-based Tets

Elastic Beam Bending Problem





- Nodal tet has lower error than more refined hex mesh
- Computational cost is roughly equivalent for similar accuracy

[1]Gruda, 2002 (internal communication)

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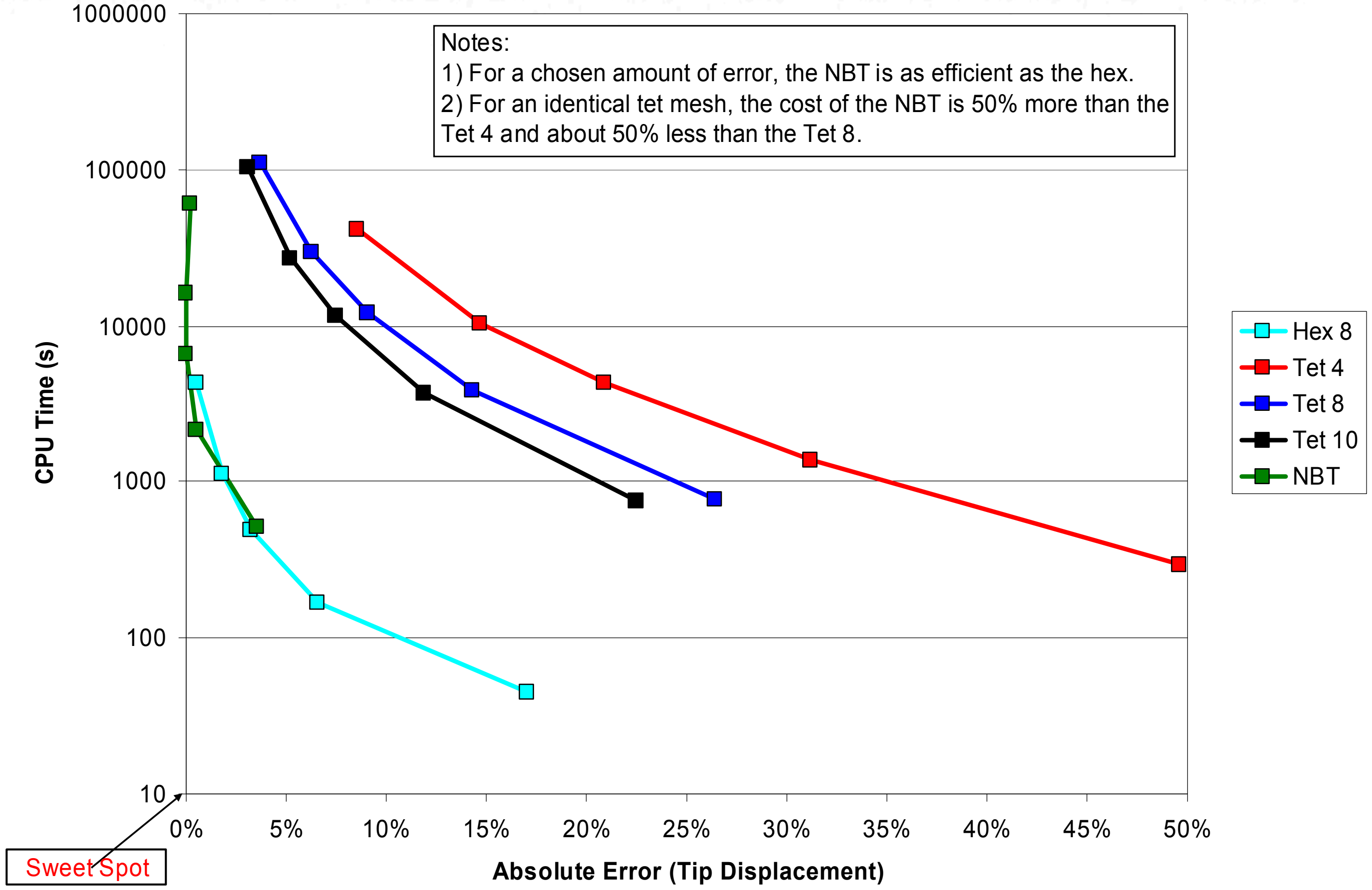
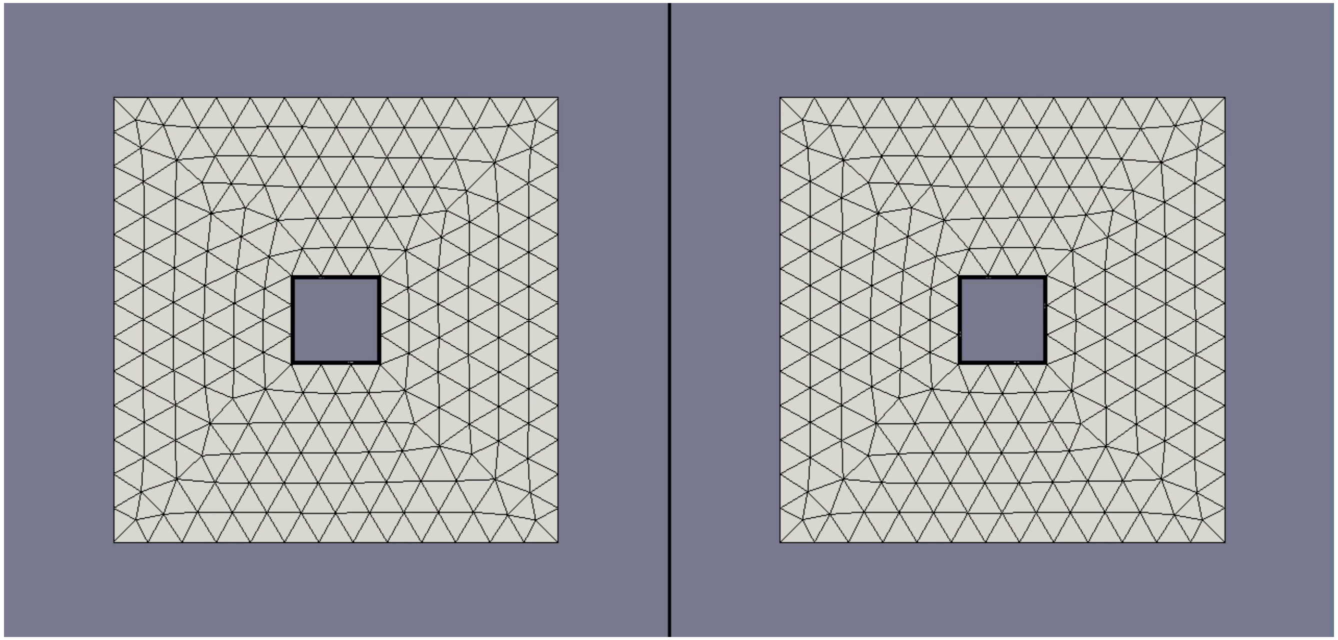
CPU Time vs Error



Nodal Tet & Adaptive Remeshing

- Because material state is maintained on nodes, element connectivity can be changed while maintaining nodal data



No Remeshing

Remeshing

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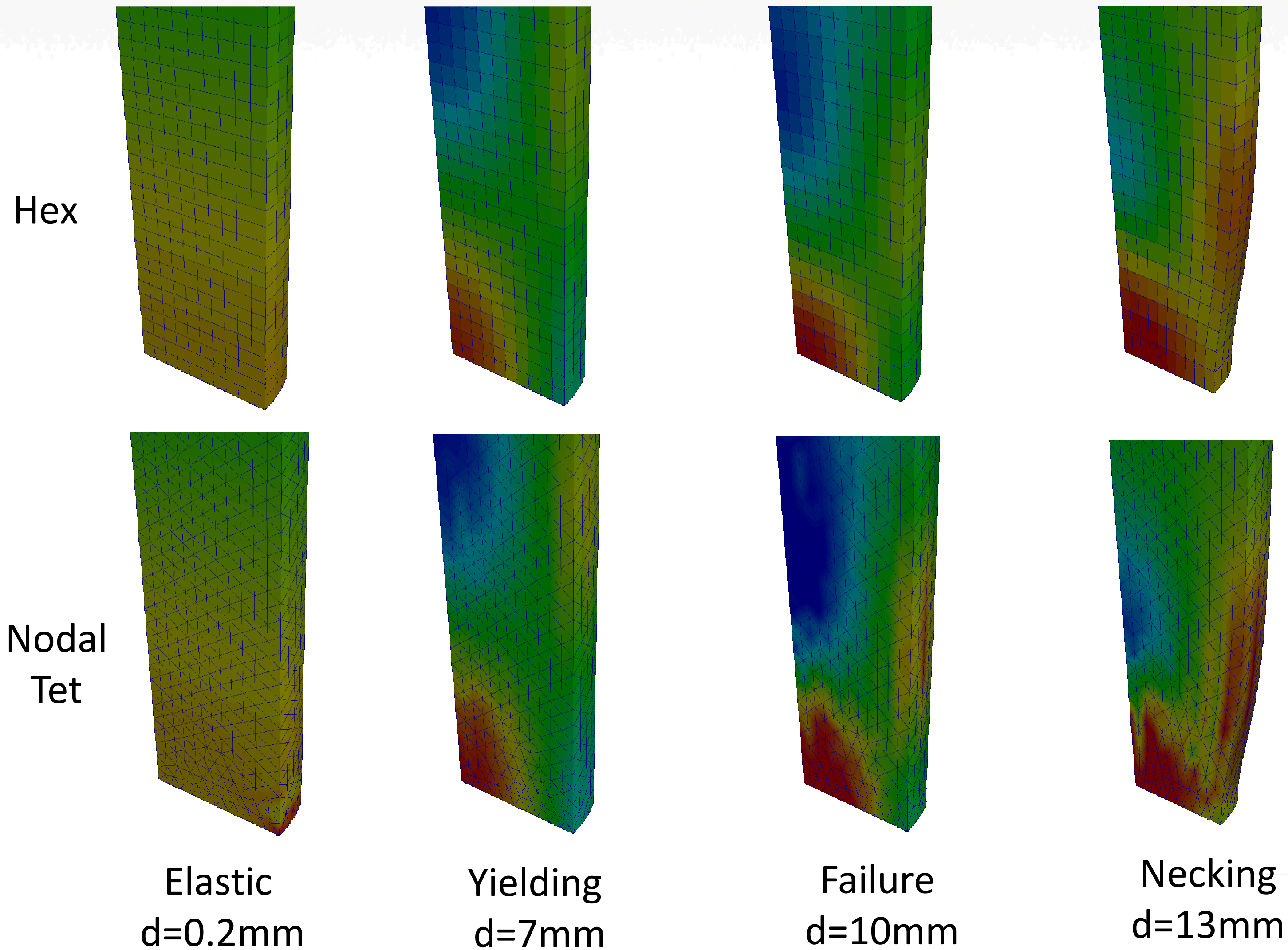
Bar Pull Hex vs Nodal Tet: stress(yy)



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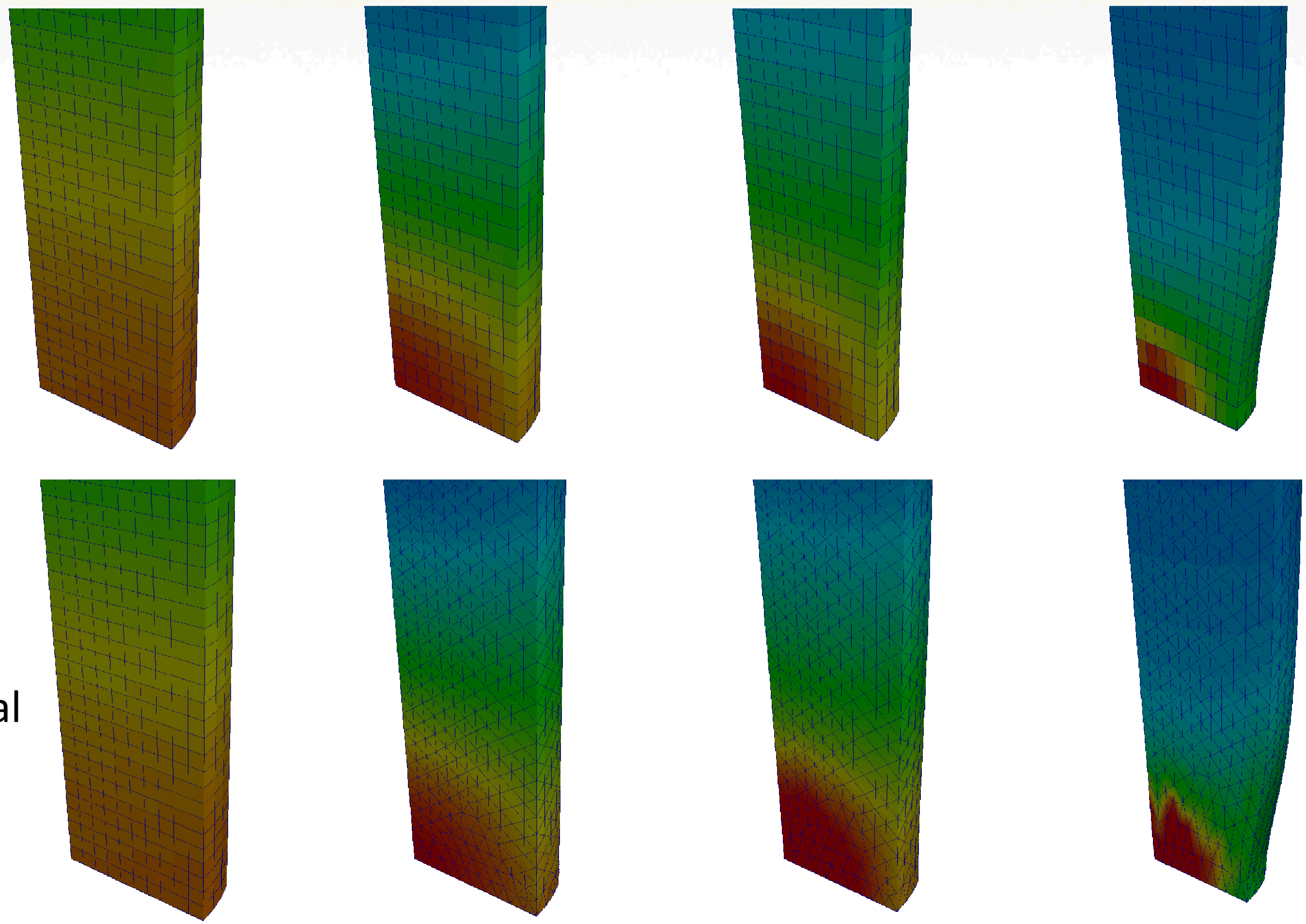
Bar Pull Hex vs Nodal Tet: Tearing Parameter

Hex

Nodal Tet



Elastic
d=0.2mm

Yielding
d=7mm

Failure
d=10mm

Necking
d=13mm

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XFEM Heaviside Enrichment

- Standard shape functions:

$$\mathbf{u}^h(\mathbf{x}, t) = \sum_{I=1}^{3} N_I(\mathbf{x})\mathbf{u}_I(t)$$

  - Continuous displacement field interpolated from nodal displacements **u**

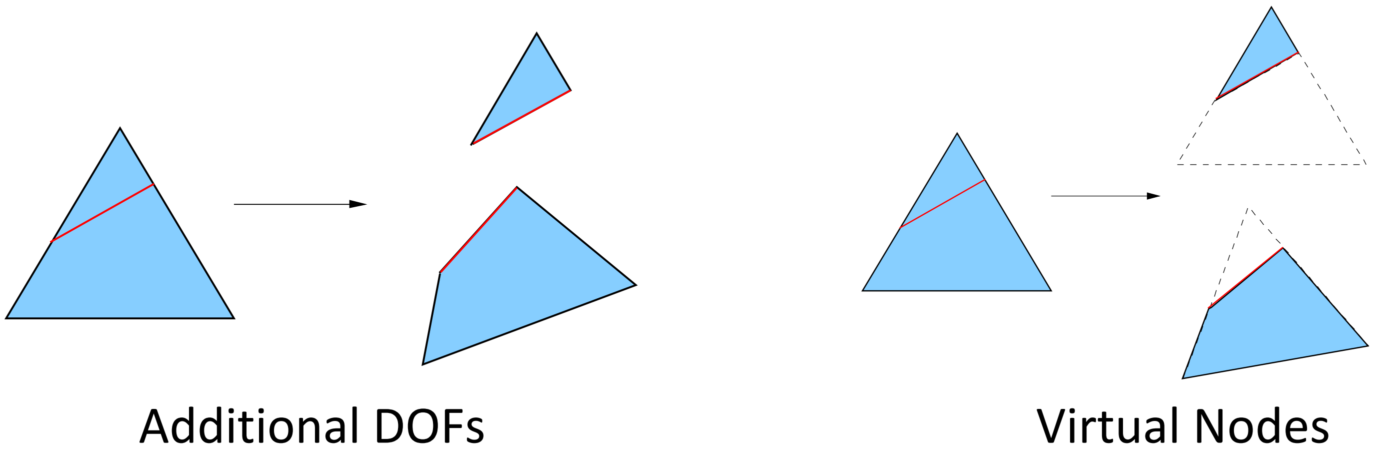
- XFEM with Heaviside-enrichment:

$$\mathbf{u}^h(\mathbf{x}, t) = \sum_{I=1}^{3} N_I(\mathbf{x})\left(\mathbf{u}_I(t) + H(\mathbf{x})\mathbf{e}_I(t)\right)$$

  - Continuous displacement field enriched by Heaviside function to account for discontinuities within element
  - Additional degrees of freedom **e** added at nodes for compatibility of discontinuous fields across elements

Virtual Node Method

- Virtual node method[1] enriches displacement field elements by duplicating cut elements



- Results in same number of degrees of freedom as Heaviside-enriched XFEM
- Two approaches shown to be equivalent[2]

[1]Hansbo & Hansbo, CMAME, 2004
[2]Areias & Belytschko, CMAME, 2006

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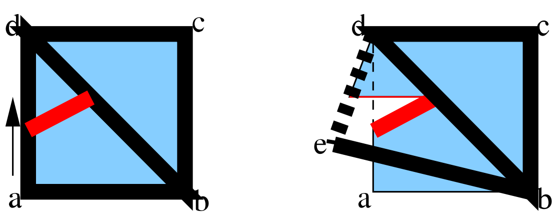
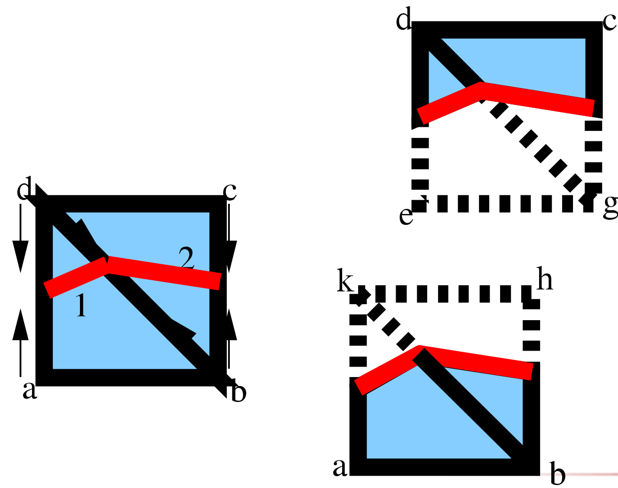
Nodal One-Ring Algorithm

- One-ring algorithm[1] used for bookkeeping

- Node donates itself to neighbors when its one-ring is cut



Cutting First Element



Cutting Second Element

[1]Molino, Bao & Fedkiw, SIGGRAPH, 2004

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XFEM Implementation in Sierra

- Sierra is Sandia's suite of massively parallel coupled physics codes
  - Provides separate modules for different physics (solid, fluid, thermal)
  - Sierra framework provides mesh data base, supports dynamic mesh modification
- Heaviside-enriched XFEM implemented through virtual node method
- Linear or Node-based Tets
- Support for contact and cohesive zones on cut planes through multi-point constraints

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Fracture Application

- Compact tension specimen with grooves along side
- 32mm X 32mm X 6 mm
- Initial fatigue crack
- 2024-T3 Aluminum
- Subjected to cyclic loading
- Periodic loading prescribed in terms of gauge displacement

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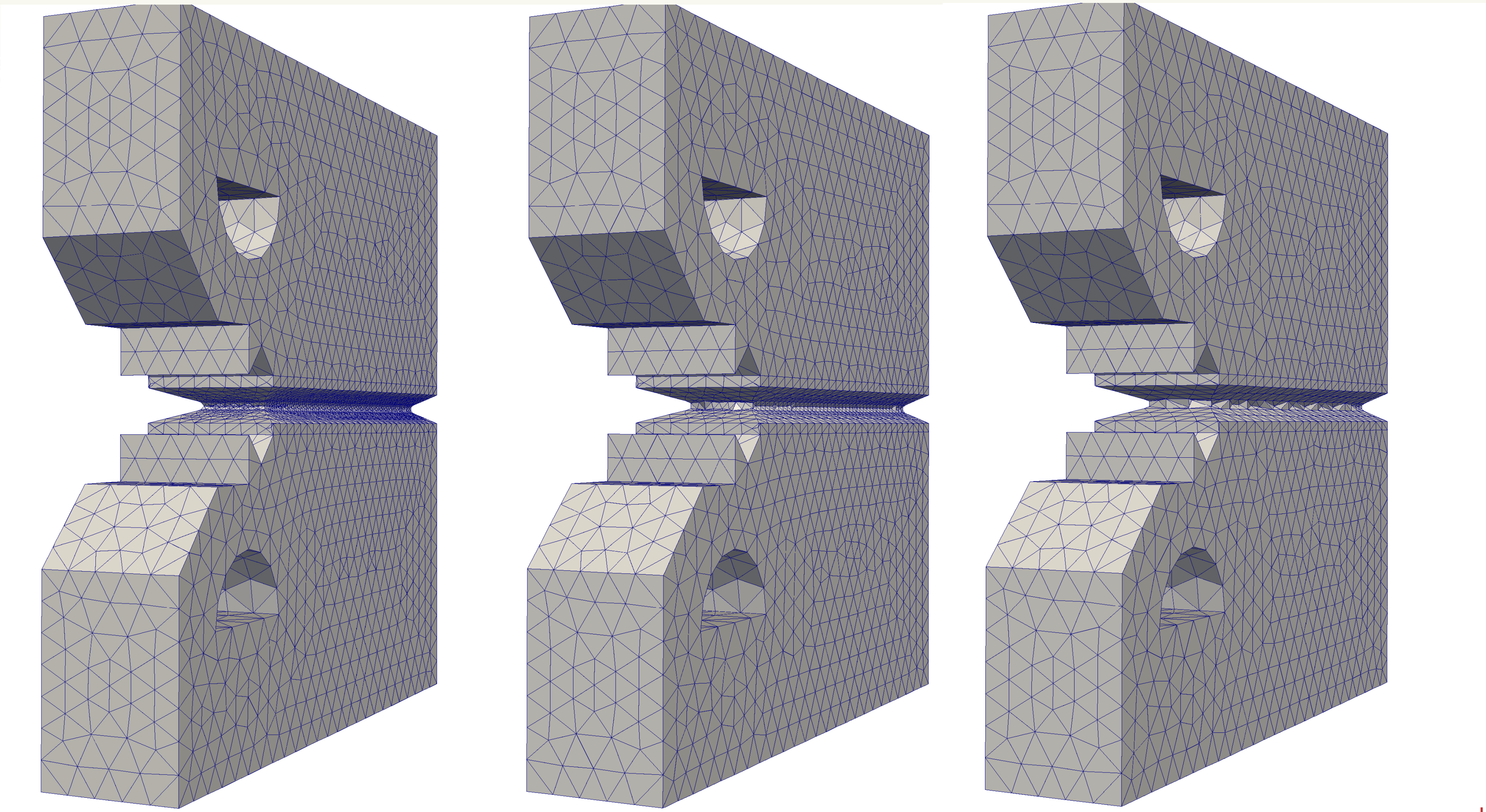
Meshes Used in This Study



Fine (0.25 mm) Medium (0.5mm) Coarse (1.0mm)

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Modeling Approach

- Initial fatigue crack presribed as pre-existing XFEM cut plane
- Crack allowed to propagate based on tearing parameter
- Cohesive zones inserted on newly created cut planes
  - Not inserted on initial fatigue crack cut plane
  - Tvergaard-Hutchinson interface constitutive model
- Crack growth permitted only in cut plane for this study
  - In general, direction can be driven by mechanics

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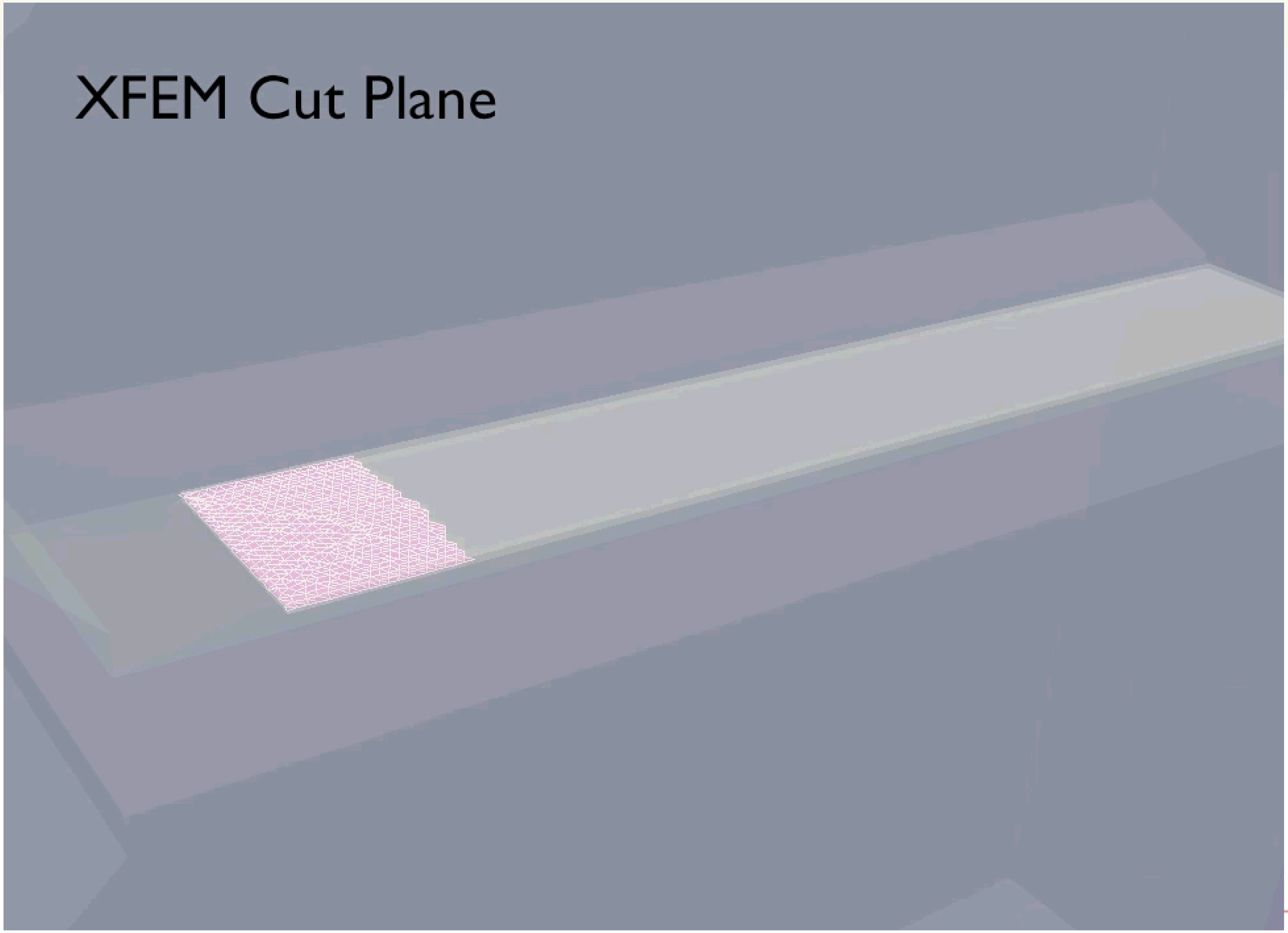
Fine Node-Based Tet Model



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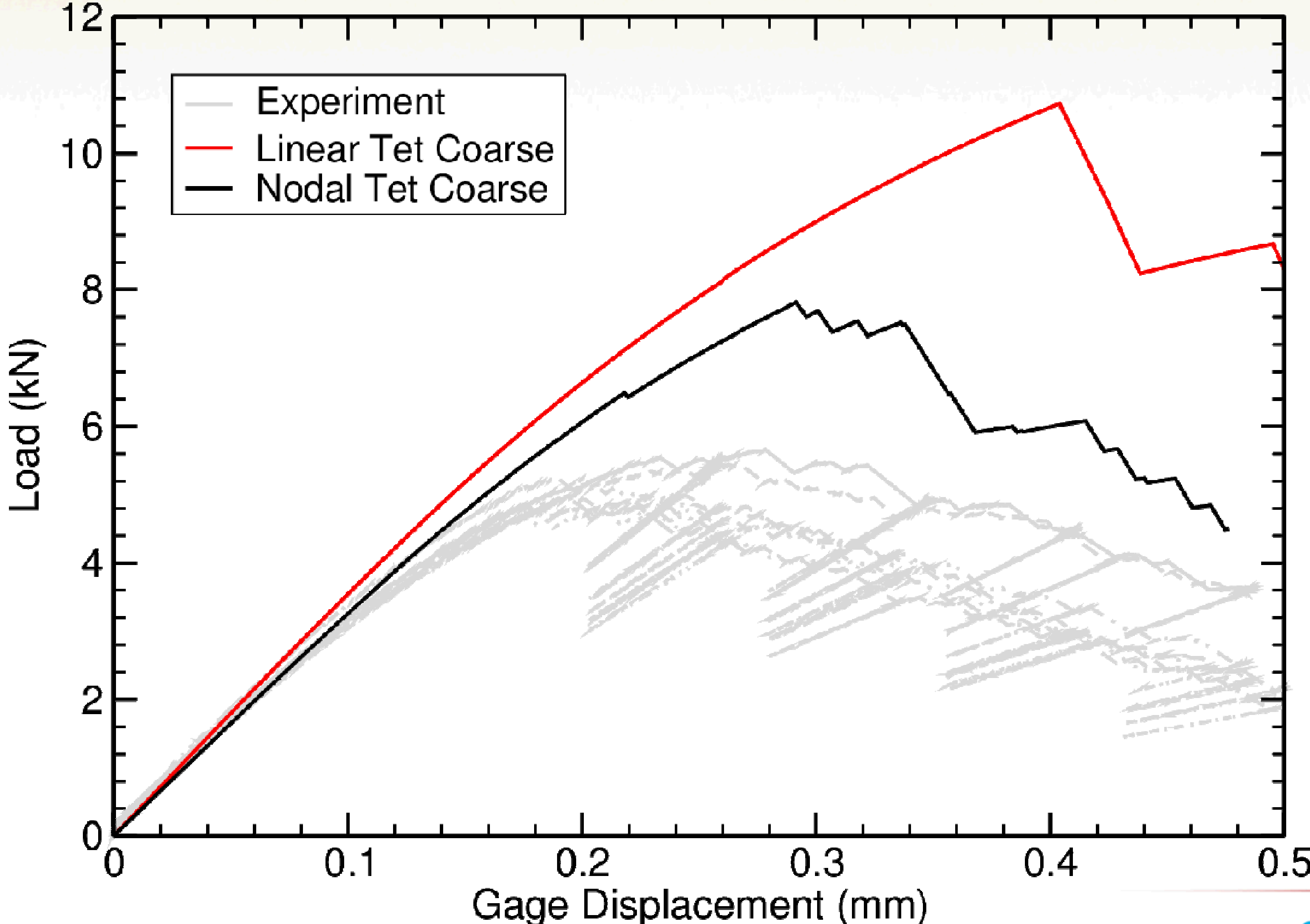
Linear Tet vs Nodal Tet: Coarse Mesh



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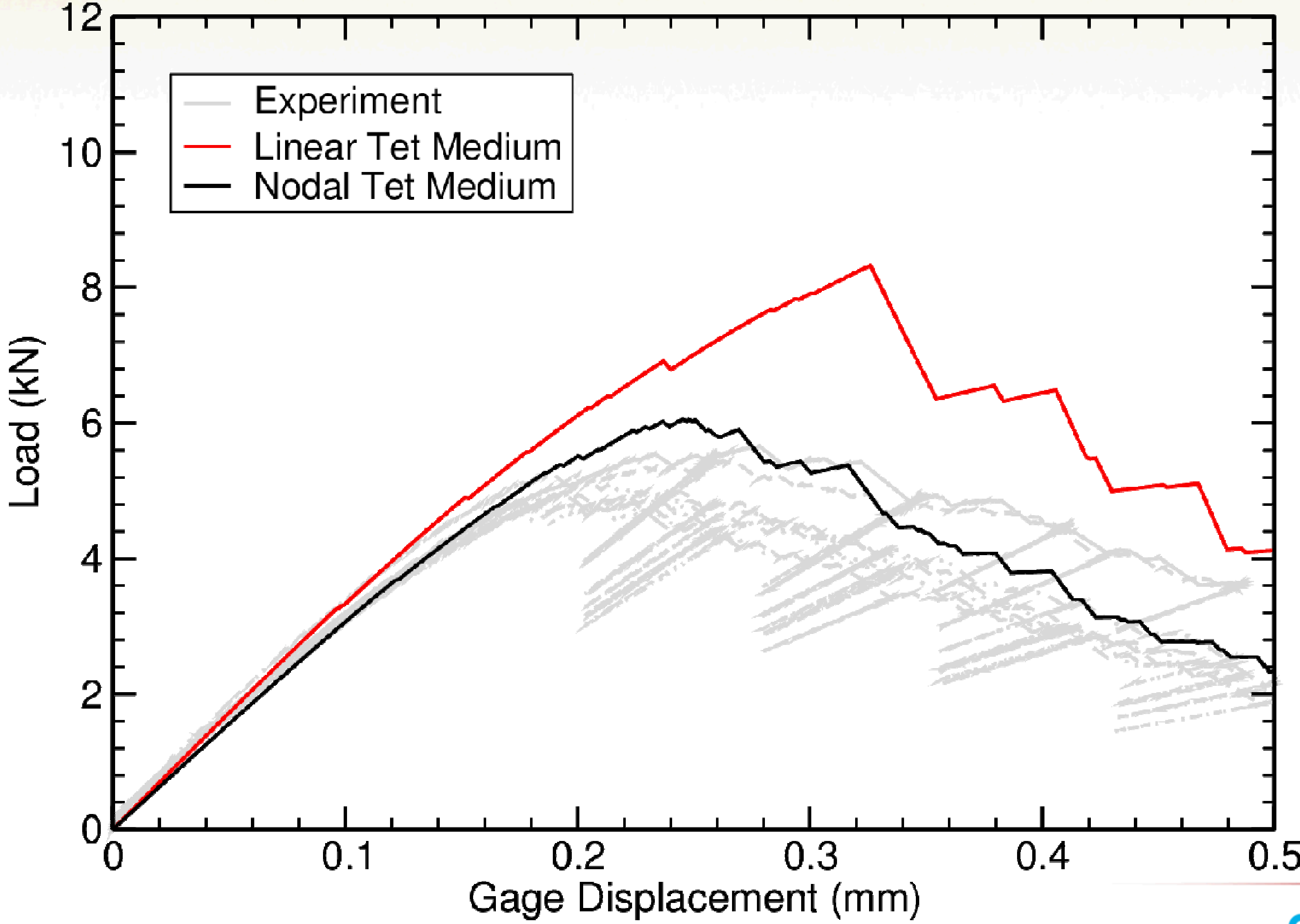
Linear Tet vs Nodal Tet: Medium Mesh



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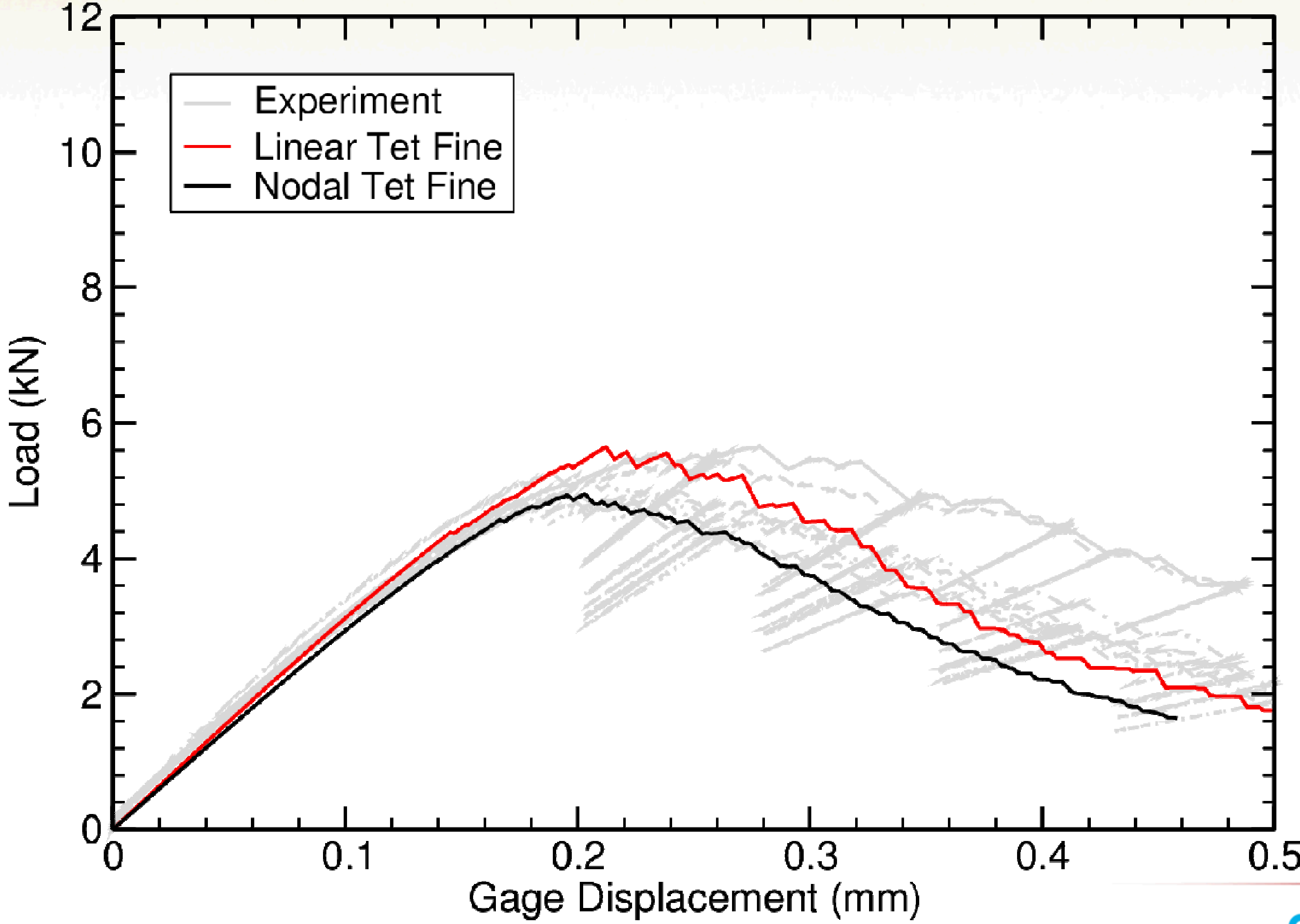
Linear Tet vs Nodal Tet: Fine Mesh



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Conclusions

- Nodal tetrahedra are demonstrated to overcome volumetric locking seen during yielding
- Particularly beneficial for ductile fracture problems
- Performance on fracture test
  - Softer response prior to fracture
  - Smoother stress fields give more reliable growth criterion
  - Appears to be more quickly convergent than linear tet
    - More mesh refinement needed to show this conclusively
- Virtual node method demonstrated to work well on single crack problem
  - Paving the way for more complex pervasive failure problems with many cracks