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A Massively Parallel Time-Domain Infinite Element Approach for Far-Field Acoustic Calculations

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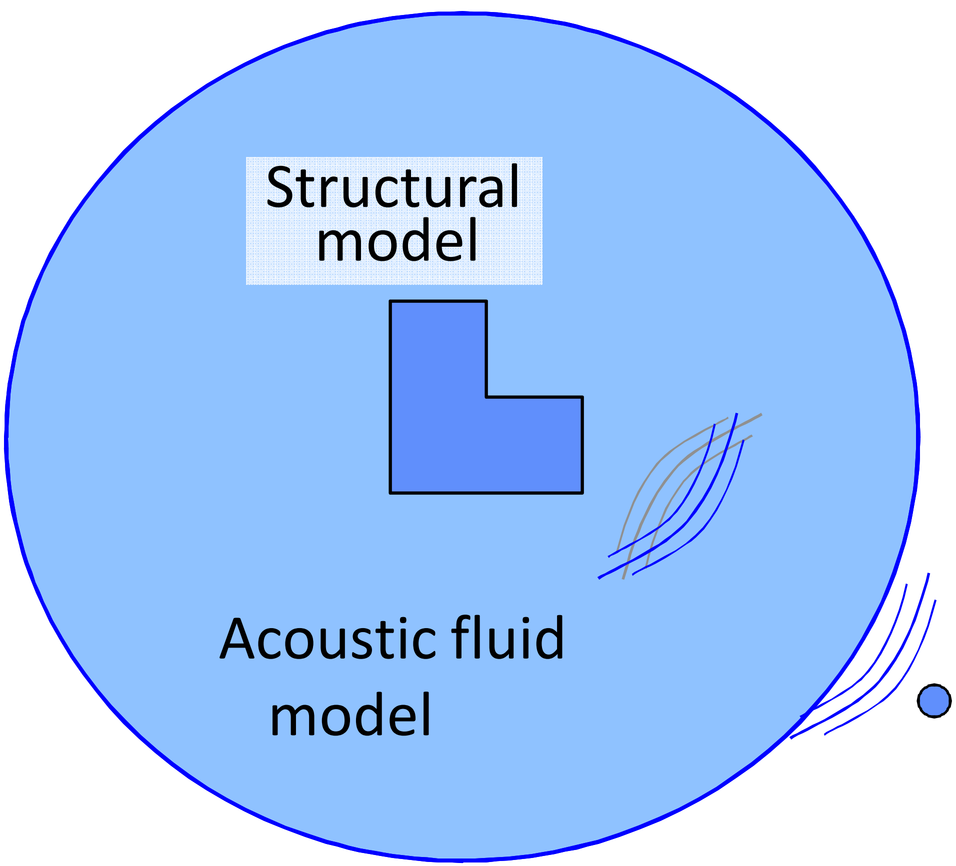
Time-Domain Far-Field Acoustics



Common Requirement:
compute acoustic pressure outside of acoustic mesh

microphone:
compute far-field response

Time-Domain Far-Field Acoustics

Two separate requirements:

1. Absorbing boundary condition on exterior acoustic surface
2. Far-field post-processor to compute response outside of acoustic mesh

Two different approaches:

- **Absorbing boundary condition (PML, high-order absorbing boundary, etc) followed by Kirchoff integral postprocessor**
- **Time-domain infinite elements?**

Comparison of Kirchoff Integral and Infinite Elements in the Time Domain

Kirchoff integral

- Large data storage requirement (entire exterior boundary for all times)
- Potential numerical instabilities (similar to time-domain boundary elements)
- Requirement for spatial and temporal derivatives of finite element solution
  - Loss of accuracy

Infinite Element

- Low (or no) data storage required
- Numerically stable provided zero-mass condition is satisfied (Astley, 2006)
- Need to identify host infinite element of far-field point of interest, and master element coordinates
  - Nonlinear problem

Brief History of Infinite Elements

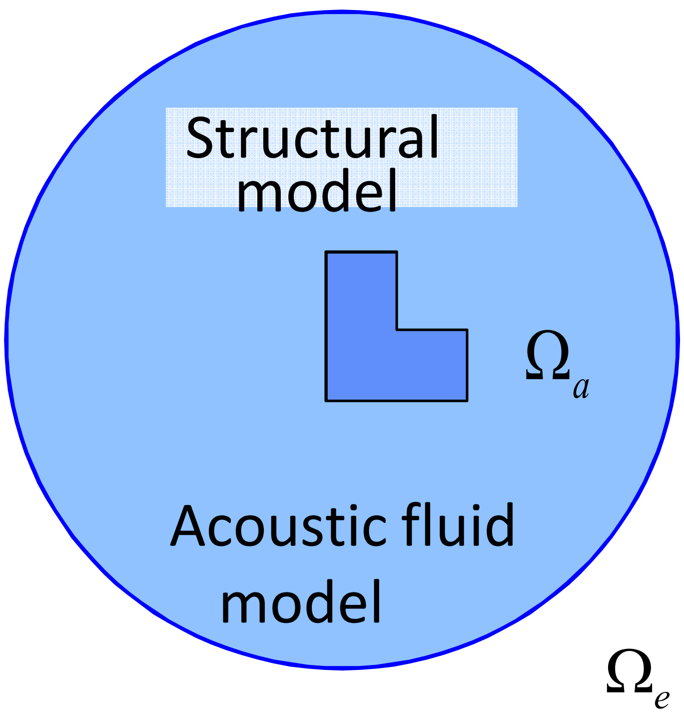
Originally developed for frequency domain calculations

- Bettess, Burnett, Astley, Demkowicz, etc

Time-domain versions originated with "mapped wave envelope" elements by Astley et al. using a conjugated formulation

- **Later extended to time-domain infinite elements**

Infinite Element Formulation



$$\Omega = \Omega_a + \Omega_e$$

Acoustic wave equation for fluid

$$\frac{1}{c^2} p_{tt} - \Delta p = 0 \quad \Omega x [0, T]$$

$$\frac{\partial p}{\partial n} = g(x,t) \qquad \Gamma x [0, T]$$

Weak formulation on exterior domain

$$\int_{\Omega} \frac{1}{c^2} \ddot{p}\, q\, dV + \int_{\Omega} \nabla p \bullet \nabla q \, dV = \int_{\Gamma} g q \, dS$$

Trial and weight functions

$$\phi(x,\omega) = P(x) e^{-ik\mu(x)} \quad q = D(x) P(x) e^{ik\mu(x)}$$

$$\left(-\omega^2 M + i\omega C + K\right) p = f$$

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Time-Domain Infinite Elements

$$P \approx \left(\frac{\alpha_1}{r} + \frac{\alpha_2}{r^2} + \ldots + \frac{\alpha_n}{r^n} \right) e^{-ikr}$$

- Trial functions derived from expansion of exact far-field solution
- Singular Jacobian maps infinitely-long elements to unit master elements
- Order defined as how many terms kept in expansion
  - Conjugated (Petrov-Galerkin) $q \approx D(x)P(x)e^{-ikr}$
  - Unconjugated (Galerkin) $q \approx D(x)P(x)e^{ikr}$
- Conjugated leads to frequency-independent K, M, C

$$\left(-\omega^2 M + i\omega C + K\right)p = f \quad \Longleftrightarrow \quad M\ddot{p} + C\dot{p} + Kp = f$$

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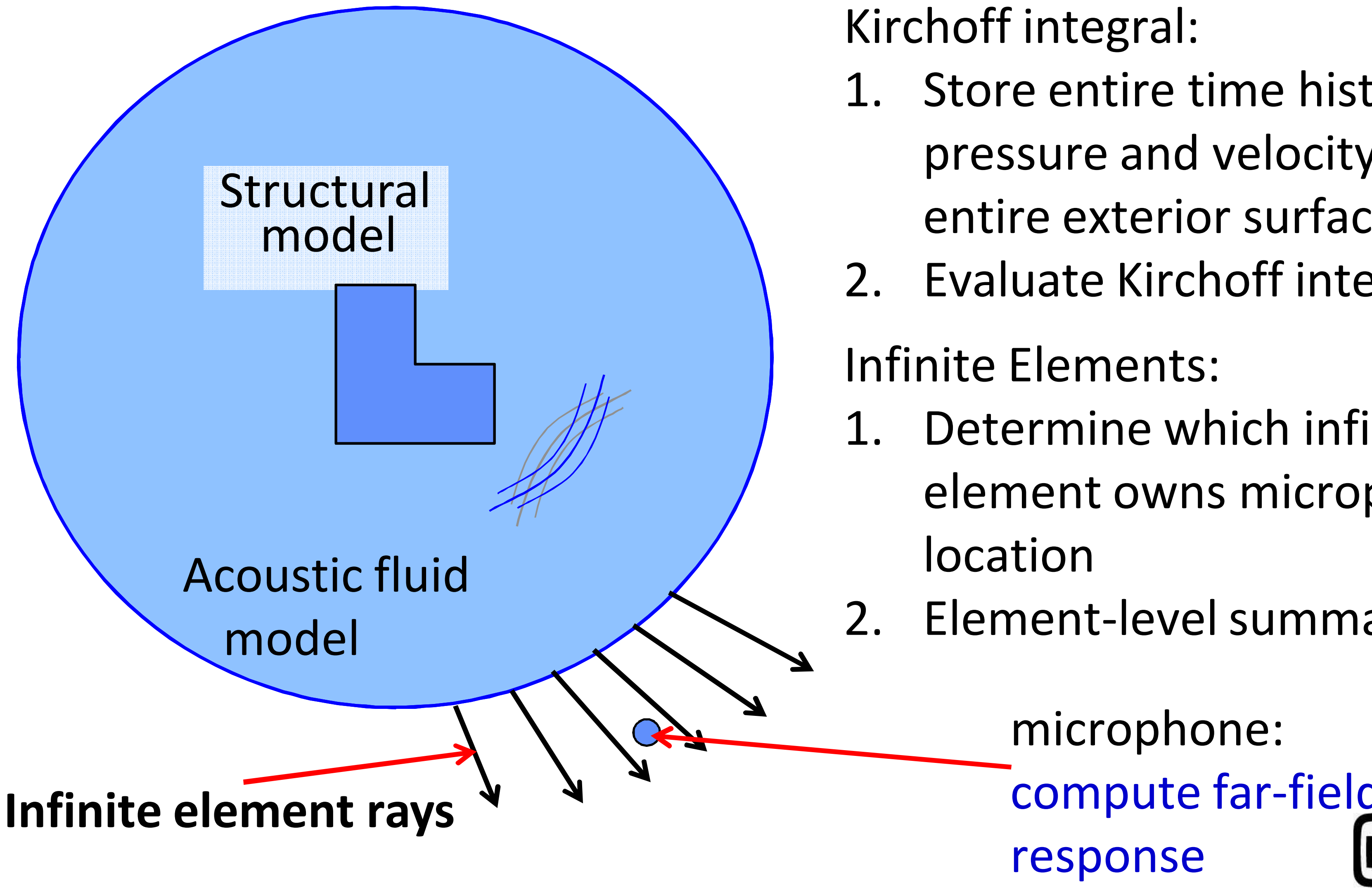
Comparison of Kirchoff Integral and Infinite Elements



Kirchoff integral:

1. Store entire time history of pressure and velocity on entire exterior surface
2. Evaluate Kirchoff integral

Infinite Elements:

1. Determine which infinite element owns microphone location
2. Element-level summation

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Infinite Elements for Far-Field

Far-Field microphone calculation using Infinite Elements:

1. Determine which infinite element owns microphone location
   - Nonlinear problem requiring Newton iterations
   - Embarrasingly parallel
2. Element-level summation
   - Similar to finite element interpolation

Infinite Element Interpolation

- **Infinite elements discretize the entire (infinite) exterior region.**
- **A singular mapping is necessary to transform finite-size element face into prism of infinite length.**

$$x = \sum_{j=1}^{N} M_j(s,t,v)\,x_j$$

Given coordinates of far-field point, need to determine master element coordinates (s,t,v)

- Nonlinear problem

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Infinite Element Interpolation

- Given an arbitrary point outside of the acoustic mesh, we need to determine which infinite element owns the point.
  - Thus far no convergence problems
- Current approach is to loop through all infinite elements (embarrasingly parallel operation) and do Newton iterations to see which one converges.
- Then, once (u,v,w) are known, acoustic pressure at far-field point computed with standard element-level interpolation

Kirchoff Integral Formulation

$$p(x,t) = \frac{\rho}{4\pi} \int_S \frac{a_n(x, t - R/c)}{R} H(t - R/c) dS$$

$$+ \frac{1}{4\pi c} \int_S e_R \cdot n_S \frac{\partial}{\partial t} \frac{p(x_S, t - R/c)}{R} H(t - R/c) dS$$

$$+ \frac{1}{4\pi c} \int_S \frac{c}{R^2} p(x_S, t - R/c) H(t - R/c) dS$$

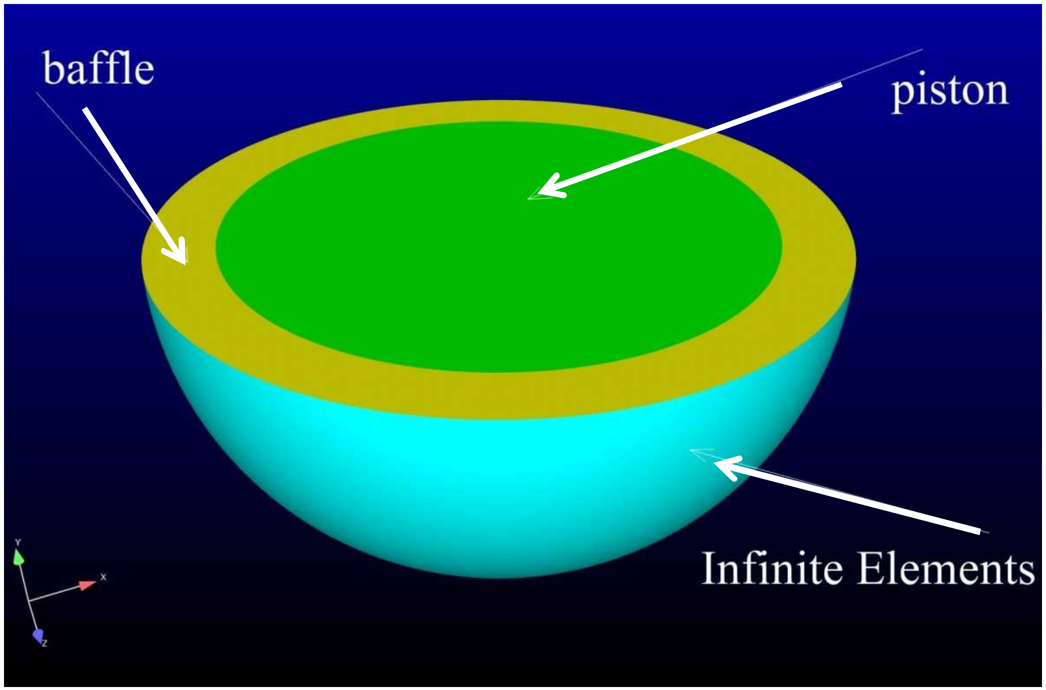
Required quantities:

$p(x,t)$	Acoustic pressure from finite element solution
$a_n(x,t)$	Spatial gradient of acoustic pressure
$\frac{\partial p(x,t)}{\partial t}$	Temporal gradient of acoustic pressure

Kirchoff Integral Formulation

- Requirement for spatial and temporal gradients of acoustic pressure (from finite element calculation)
  - These do not come directly from the finite element calculation
    - Must be computed numerically after-the-fact
  - Loss of order of accuracy

Piston on Infinite Baffle



Rigid piston, with plane harmonic wave applied

Baffle (symmetry BC)

Model synopsis:

Acoustic mesh : tets

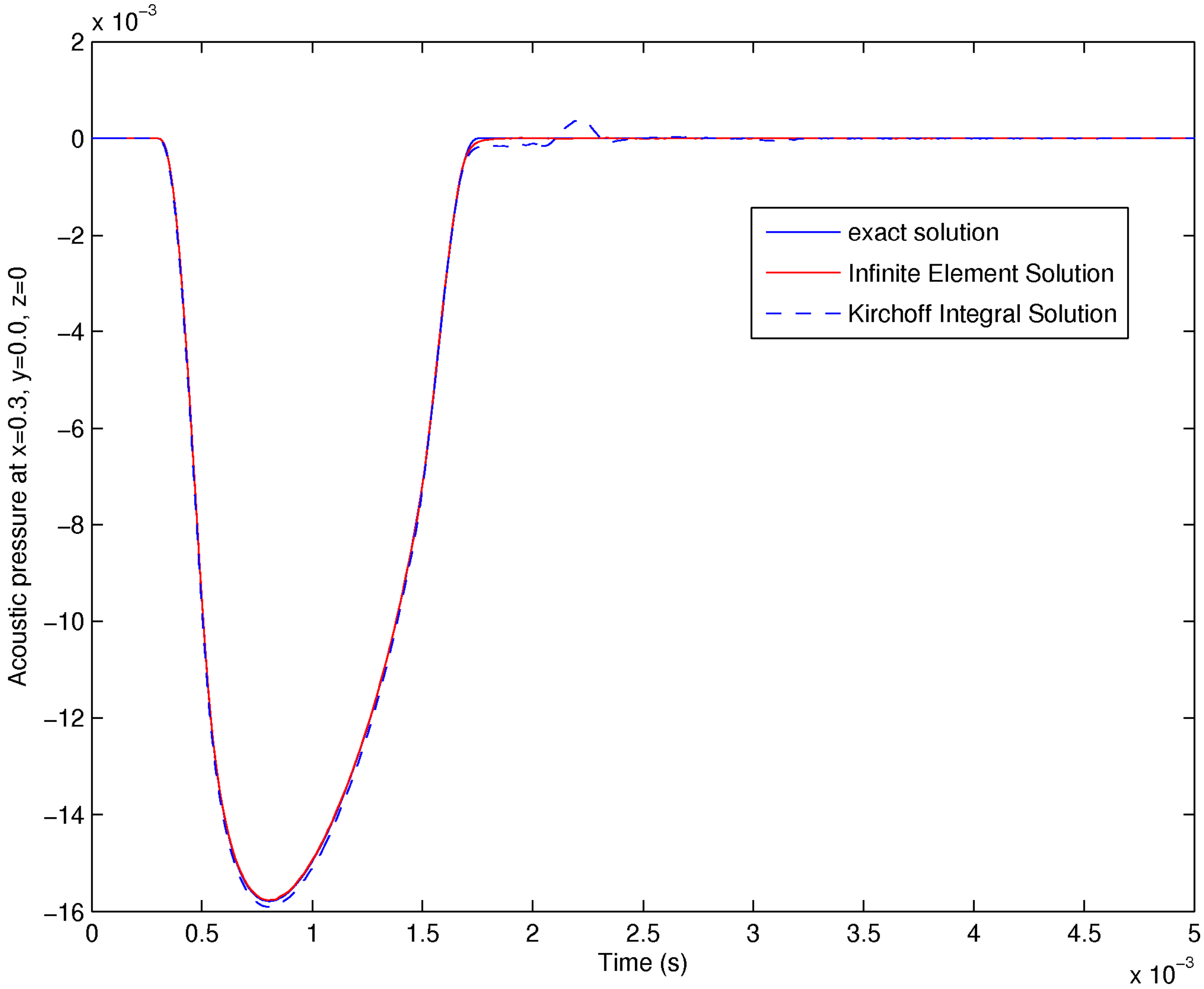
Fluid boundary verification

Analytic solution for acoustic pressure

[Ref]: Pierce, *Acoustics*, McGraw-Hill, 1981.

$$p(x,t) = \frac{\rho}{4\pi} \int_S \frac{a_n(x, t - R/c)}{R} H(t - R/c) dS$$

Piston on Infinite Baffle

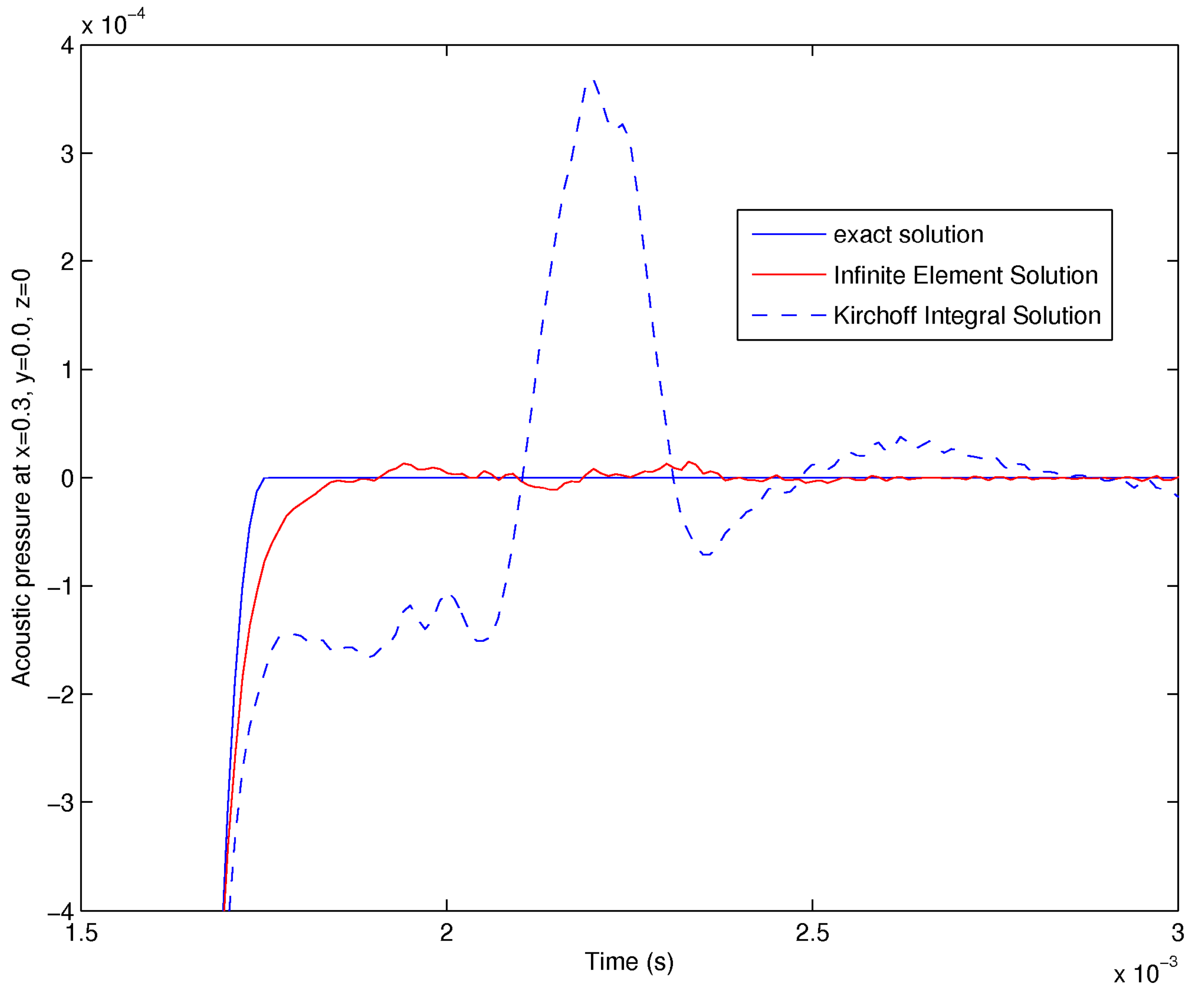


r/R = 1.1

r: radius to far-field point

R: radius of acoustic mesh

Piston on Infinite Baffle



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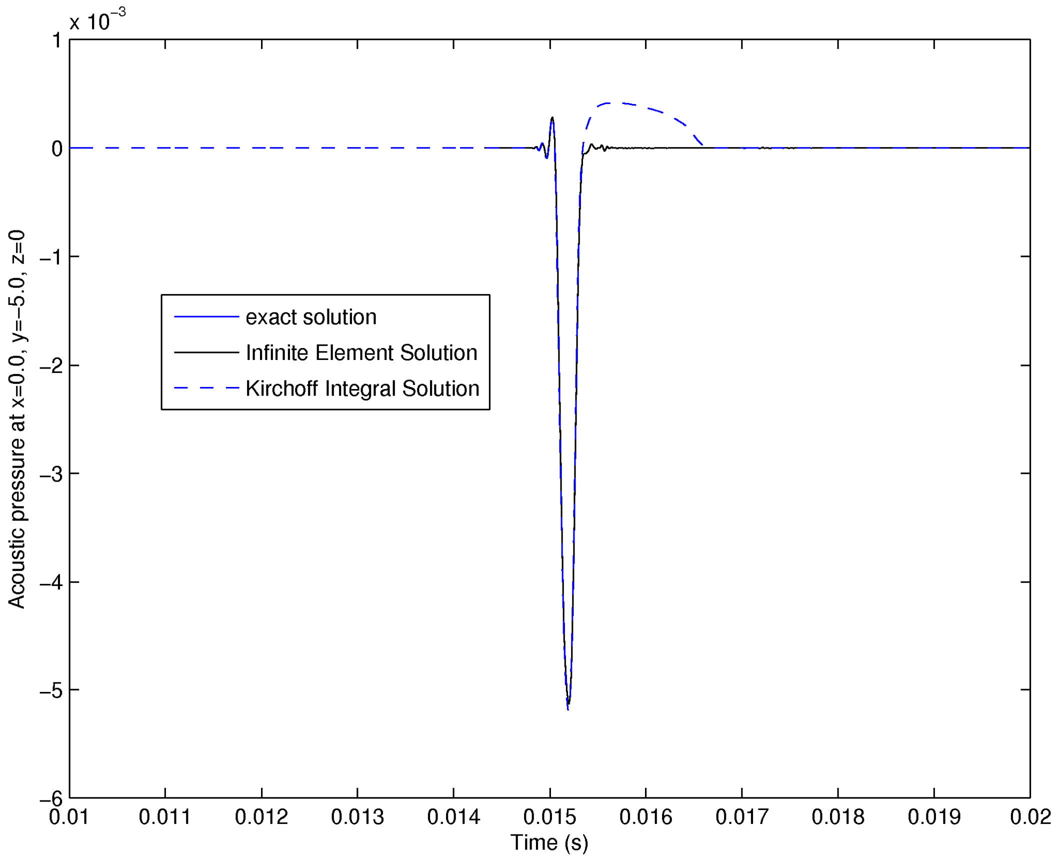
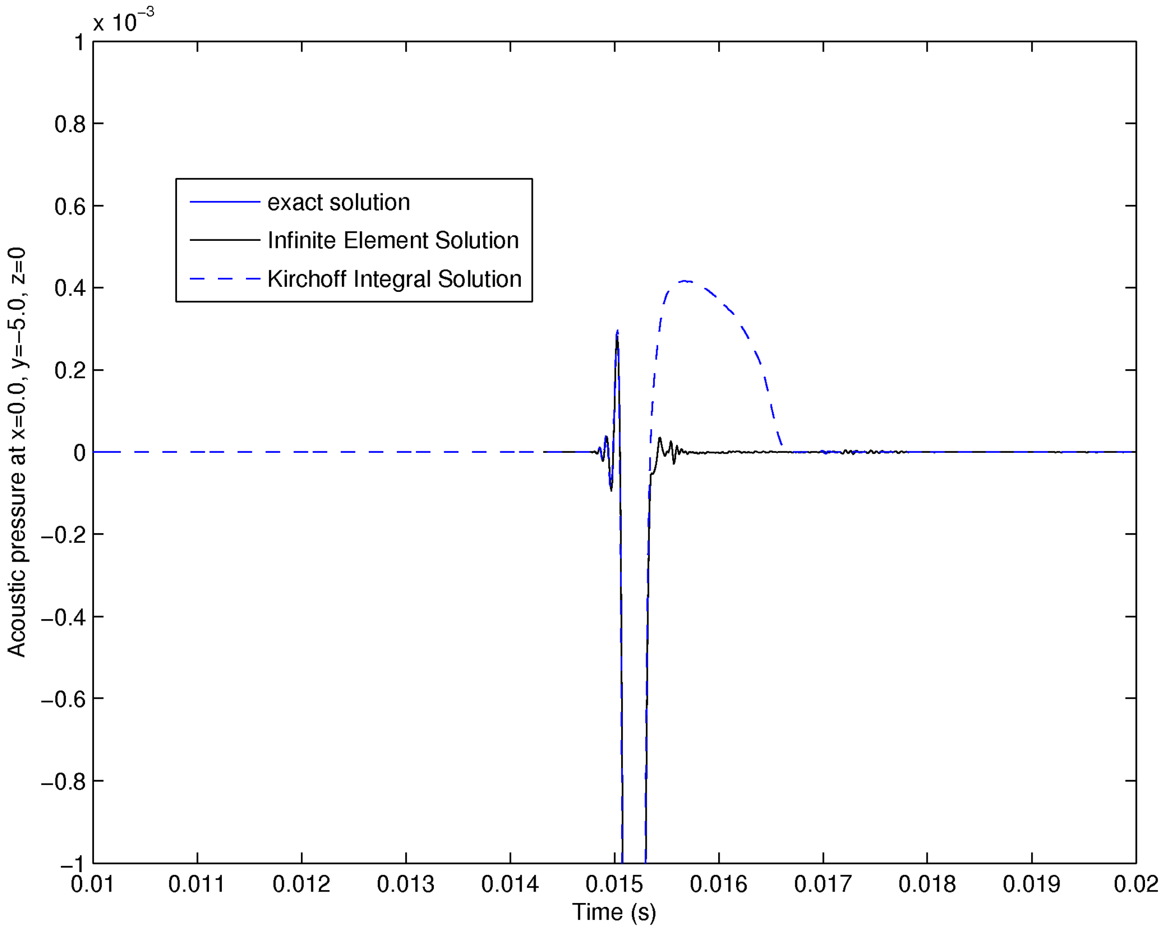


r/R = 20.0

r: radius to far-field point

R: radius of acoustic mesh

Piston on Infinite Baffle



r/R = 20.0

r: radius to far-field point

R: radius of acoustic mesh

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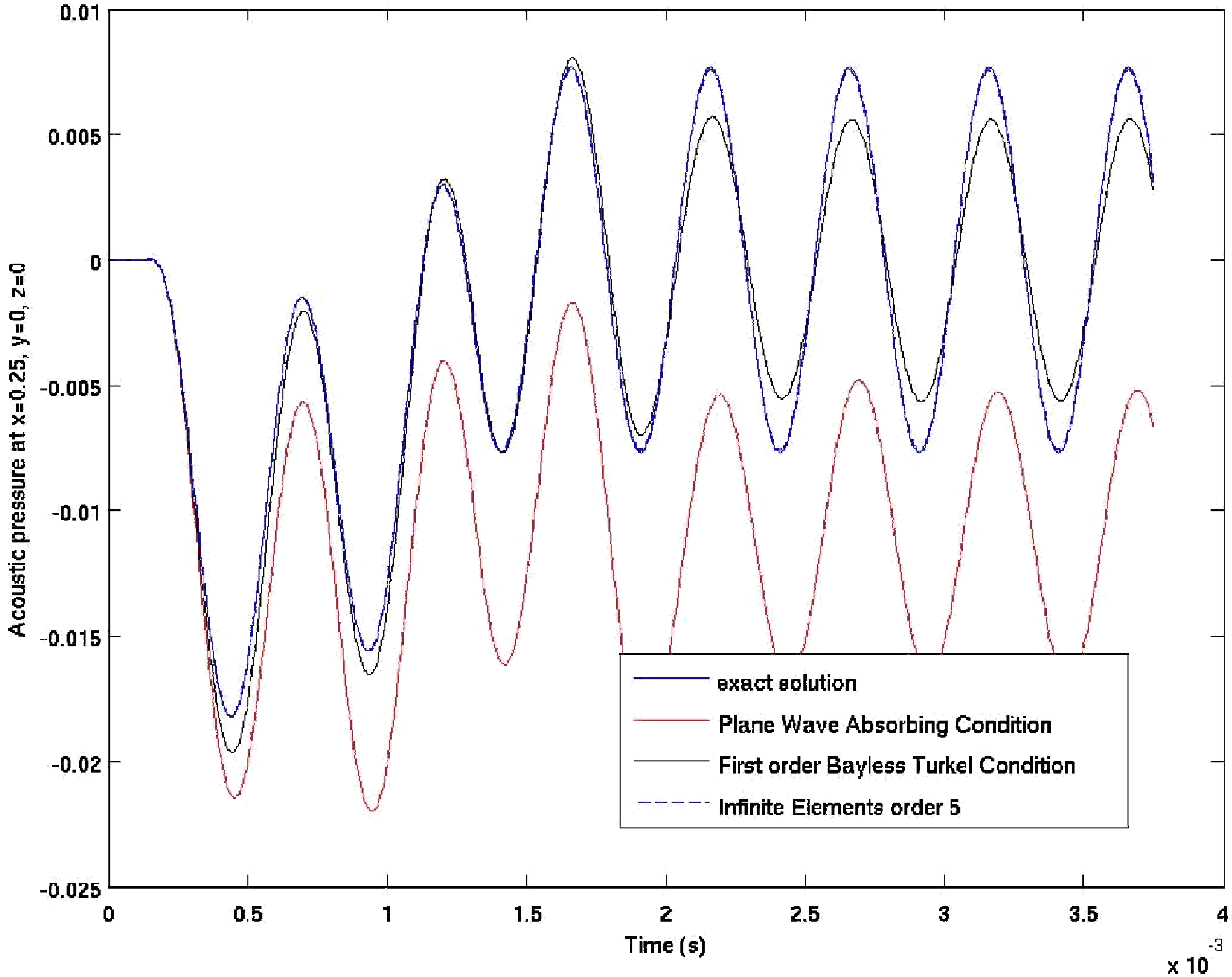
System Stability with Infinite Elements

- Anytime introduce absorbing BC, could impact stability. Contributions to C, K, and sometimes M
- In IE case, M, K, and C are all modified.
- In particular, M impacts system stabiilty.

Conclusions

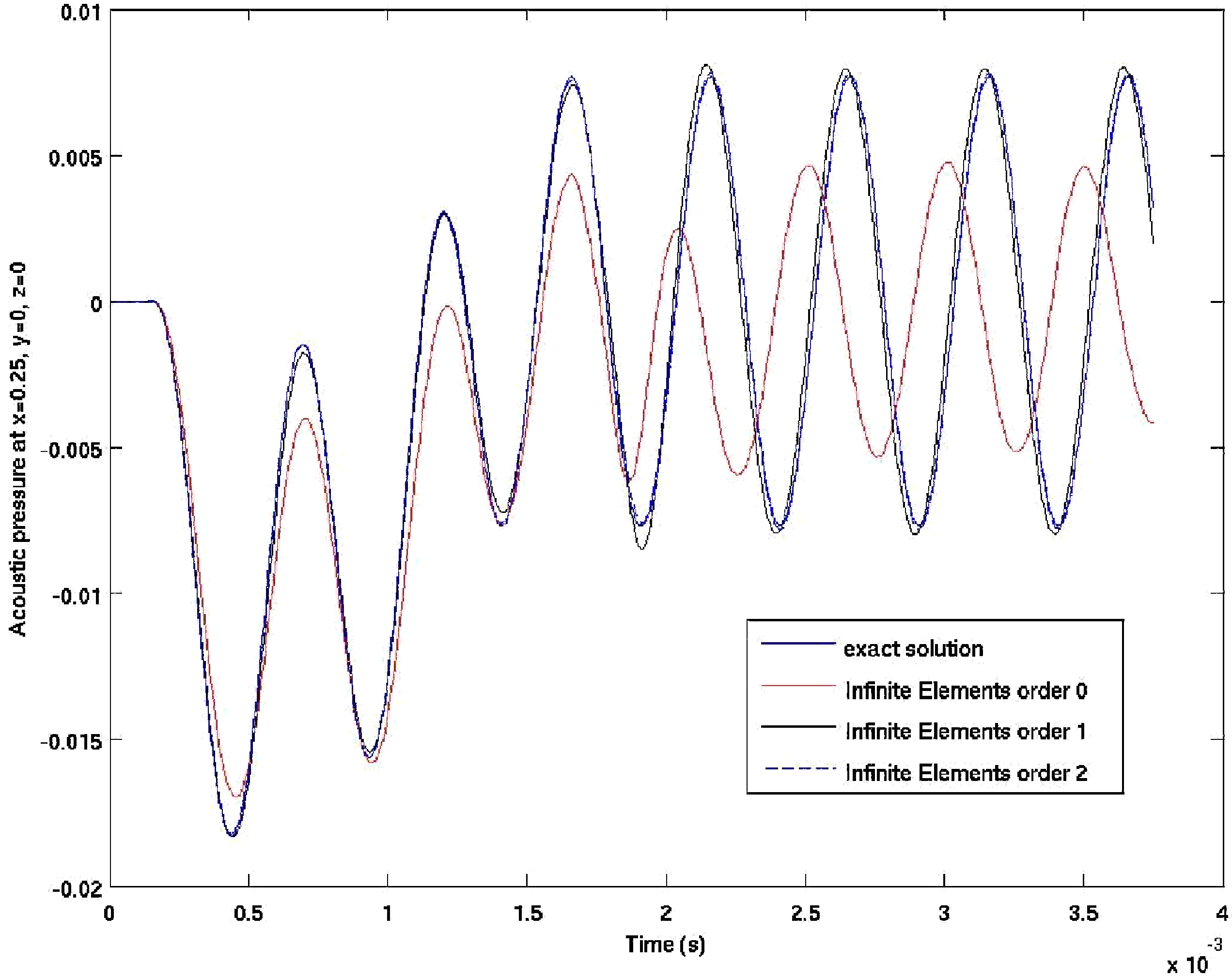
- Time-Domain infinite elements appear to give more accurate results than time-domain Kirchoff integral for far-field acoustic postprocessing
- Time-domain infinite elements also have significant advantages of less data storage than the Kirchoff approach
- Nonlinear iterations are required to locate host infinite element and parametric coordinates for a given field point
- Zero-mass condition is required for system stability.

Structural-Acoustics verification
Piston on Infinite baffle problem



Structural-Acoustics verification

Piston on Infinite baffle problem



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