SAND2011-4204C

Modeling Systems of Interacting Specialists

Walter E. Beyeler, Robert J. Glass, Patrick D. Finley, Theresa J. Brown, Matthew D. Norton, Mark Bauer, Michael Mitchell, and Jacob A. Hobbs
Sandia National Laboratories
Albuquerque, New Mexico, USA
Complex Adaptive Systems of Systems (CASoS) Engineering Initiative
webeyel@sandia.gov

Many natural and artificial systems can be described abstractly as a collection of interacting entities that must meet two requirements for survival. First, entities must maintain their individual viability through some homeostatic process that consumes resources obtained from other entities in their environment. Additionally, the system must produce a pattern of resource flows among entities that creates suitable conditions for their mutual viability. We define a hybrid model that focuses on these general features. It is an abstract representation designed to capture the resource production and exchange processes that are essential to such diverse arrangements as firms interacting to form an economy, species interacting to form an ecosystem, and multi-cellular organisms. These diverse systems can be studied by setting terms and parameters of the generic model; thus, insights obtained from the study of one system can be transferred to systems that are superficially dissimilar but share common core dynamics. The model uses a set of coupled non-linear first-order differential equations to describe the dynamics of individual entities. Entities are coupled through discrete exchange events in which they seek to satisfy their resource requirements given the environmental conditions they experience. We configure the model to analyze the behavior of three disparate systems. One examines how differential production efficiencies can lead to competitive exclusion in a system of entities having complementary resource requirements. A second explores how a population of entities adapts to balance requirements for both robustness and efficiency under different exogenous stress regimes. The third considers a hierarchical system of embedded entities to investigate the effects of different patterns of system-level exchange on the internal properties of compound entities.

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

1 Introduction

The CASoS Engineering program at Sandia National Laboratories helps form policies affecting complex adaptive systems of systems. This work is an outgrowth and distillation of several years' experience applying insights from complexity science and complex systems to problems of national and international scope. Many initial problems focused on critical infrastructures such as electric power systems and banking networks [Brown et al. 2004, Glass et al. 2004]. The effects of infrastructure disruptions on economic activity and on other patterns of human interaction are of foremost concern to policy makers. Our experience with diverse technological, economic, and social systems has led us to formulate a simple abstract model that we argue captures processes essential to many complex systems.

Such systems can be described abstractly as a collection of interacting entities that must meet two requirements for stability: entities must maintain their individual viability through some homeostatic process involving the consumption of resources obtained from the environment; and the system must foster a pattern of resource flows among entities to create suitable conditions for their mutual viability.

An abstract model that focuses on these general features is useful for studying the behavior of real systems for at least two reasons. It creates a set of terms and parameters that can be used to interrelate systems in very different areas, allowing insights obtained from the study of one system to be reflected onto systems that are superficially dissimilar. A systematic study of the model itself can potentially yield insights about the behavior of many real systems arising from the basic constraints and processes in the model. We define such a model in this paper, and analyze its behavior in three simple configurations.

2 Model Summary

The model comprises a set of Entities, a set of Resources that can be stored, consumed and produced by the Entities, and a set of Markets that mediate resource exchanges between entities. The primary state variables and processes that define entities, including both the internal consumption and production processes and the interactions with other entities through exchange processes, are summarized in the causal loop diagram in Figure 1.

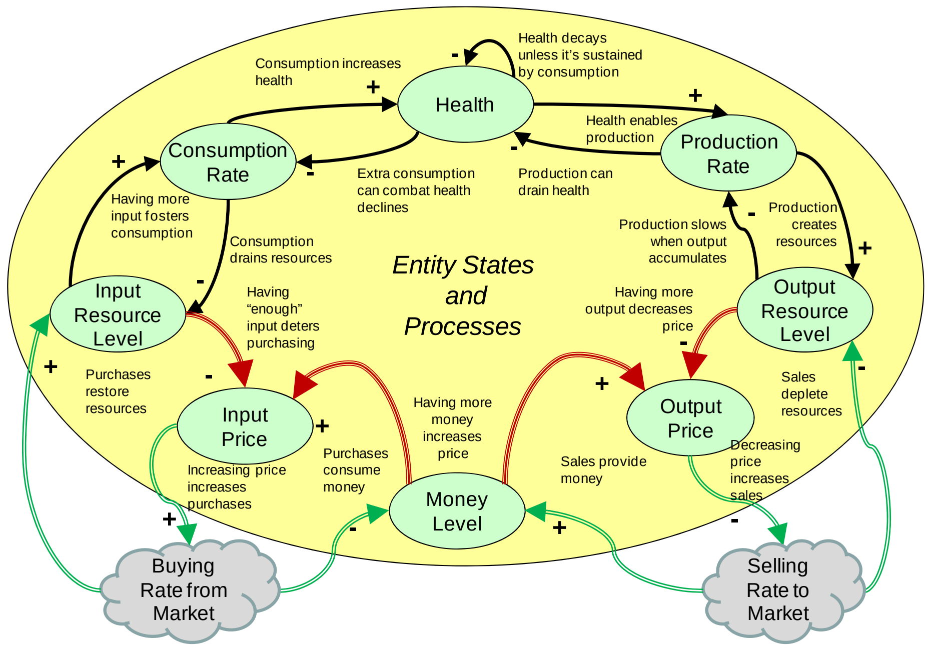

Figure 1: Causal diagram of the primary variables and interactions describing an entity. Green arrows denote interactions with the environment; red arrows denote functions that model adaptive reactions.

Entities have some internal structure that accomplishes production and that is maintained by consumption of input resources. The model does not represent this structure explicitly because its details (which vary greatly across systems) are not essential for understanding the system-level behavior arising from interactions among entities having distinctive resource requirements and production potentials. We use a scalar health variable h(t) as an index of the current state of the entity's internal structure. The meaning of this abstract variable comes from its influence on production and consumption. Parameters of the functions relating consumption, production, and health can be tailored to reflect the operational characteristics the specific internal structures that characterize real systems. Entities control their resource levels through interactions with one another via markets. Markets manage resource exchanges among the subset of entities they serve. Entities send exchange proposals to markets, and markets arrange compatible exchanges among entity pairs when possible. A common "money" resource is used in all exchanges. The terminology of markets, prices, buying and selling should be understood metaphorically and not as limiting applications to economics and commerce. The "price" an entity sets for a resource can be understood as a threshold energy level that they are willing to expend, or commit to extract, in exchange for the resource. The entity extracts energy from the environment in exchange for the resources it produces, and uses that energy to obtain the resources it needs. Entity behavior is governed by a set of simple control processes and is not assumed to be the result of optimal decision-making or other constructs commonly used in artificial economies [Testafatsion 2006] that might limit application to cognitive agents.

Entities can submit proposals at any time, and might seek to control resource levels by adjusting the size of the proposed transaction, the frequency with which they transact, or some combination of these factors. In practice Entities are configured to make proposals with some specified frequency, and to propose exchange amounts that, together with this frequency, would allow them to either obtain or dispose of resources at a rate much larger than their nominal consumption or production rates. Entities then control the actual rate of transaction by adjusting their prices, as illustrated in Figure 1. The logic for setting resource prices is therefore an essential determiner of the Entity’s behavior. Price adjustments are the primary means of managing interactions with their environment.

Entities with little need to transact signal this by proposing very low prices (as buyers) or very high prices (as sellers), rather than by adjusting their proposal frequencies and amounts. This approach has two important advantages: it greatly increases the amount of information available to the market as compared to withholding proposals, and it concentrates the Entity’s control action on the single parameter of price.

2.1 Process Definition and Stability Analysis

The equations governing the state variables shown in Figure 1, along with other details of the model formulation are given in [Beyeler et al. 2010]. The functions that specify each of the causal links in Figure 1 were designed to have qualitative properties assumed to characterize a broad class of entities, and to have few parameters with natural interpretations.

An example relevant to configurations studied here is the function defining the effect of health on potential production. This function, $p_{h_i}^*$, is defined so that production can increase, up to some limit, when health exceeds its nominal value h_0. Production decreases monotonically with health. A sigmoid function fits these criteria.

$$p_{h_i}^*\left(h/h_0 \right) = \frac{p_{sat}}{1 + (p_{sat} - 1)\left(h(t)/h_0 \right)^{-e_p}} \qquad (1)$$

where $p_{sat} > 1$ is the maximum relative production that can be achieved if health becomes large, and e_p is an elasticity parameter that describes how abruptly production changes with health. The elasticity parameter is derived from the more intuitive parameter $h_{midh_i}^*$ the relative health level required for a relative production rate of 0.5:

$$e_p = \frac{\ln\left(\frac{p_{sat} - 1}{2p_{sat_i} - 1}\right)}{\ln(h_{midh_i}{}^*)} > 0 \tag{2}$$

The existence and stability of the equilibrium solutions for certain simple configurations are determined by the parameters of $p_{h_i}{}^*$ [Beyeler et al. 2010]. For an entity that consumes and produces the same single resource, and that has no need to interact with other entities, its equilibrium condition in terms of normalized health $h^* = h(t)/h_0$ becomes:

$$p_r - h^* = \frac{p_{sat} - 1}{1 + p_{sat}\eta_T} h^{*1-e_p}; \qquad p_r \equiv p_{sat} \frac{p_0}{c_0} \frac{1 + \eta_T}{1 + p_{sat}\eta_T} \tag{3}$$

Where p_0 and c_0 are the nominal rates of resource production and consumption, and η_T is a parameter controlling the strength of the burden that production places on health.

Equation (3) cannot be solved in general in closed form, however it can be used to understand the existence and stability of fixed points. Figure 2 sketches the left and right sides of Equation (3) as functions of normalized health for the range of values that e_p can assume.

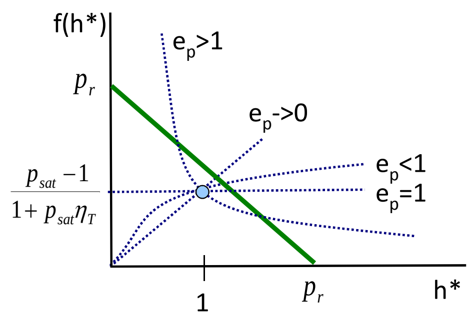

Figure. 2: Sketch of the stability conditions for an isolated entity for various ranges of e_p

The system has a stable equilibrium for $e_p < 1$, and for $e_p = 1$ provided $\frac{p_{sat} - 1}{1 + \eta_T} < p_r$. For $e_p > 1$ the system can have no solution (when the blue curve is outside the green line) or two solutions (when it is inside) with a transition between these regimes. In the case of two solutions only the lower equilibrium is stable: states between the two equilibria are driven to the upper solution, and states below cause the entity's health to collapse.

The stability analysis has interesting implications for an entity's response to certain kinds of disruption. A loss of production capacity (equivalently episodic random losses of the produced resource) corresponds to a reduction in p_0 and therefore p_r, shifting the green line toward the origin. When $e_p < 1$, the entity will be able to find a stable but reduced health value. When $e_p > 1$ however, the entity is liable to undergo catastrophic collapse as the reduced production narrows and ultimately eliminates the attractor basin for stable solutions. Larger values of e_p result from smaller values for the surplus production capacity p_{sat} or larger values for the health required to maintain production h_{mid}. Both changes represent a more "efficient" production process in the sense of being tuned to the entity's nominal operating point and having little ability to respond to stresses by either increasing production or continuing production in the event of degraded health. The third example configuration, discussed below in Section 3.3, uses this interpretation of $p_{h_i}^*$ to show how competitive pressures for efficiency might tend to push entities to the edge of stability.

2.2 Exchanges and Markets

Entities exchange resources with one another via proposals to buy or sell resources. They communicate these proposals to Markets which match buyers and sellers when their proposals are compatible. Each Market manages exchanges of one specific resource. The system can include many markets for the same resource (perhaps serving different entities) and entities can transact in multiple markets for the same resource.

Many matching algorithms might be used; however, we currently use the continuous double auction because of its simplicity and potential to settle exchanges immediately. The purpose of the Market is not to capture the operations of a specific exchange process, even in applications where economic transactions are intended, but to convey information about the relative scarcity of resources.

Proposals to markets are defined by the role of the entity (either buyer or seller), the amount of the proposed transaction, the proposed transaction price, and the length of time for which the proposal is valid. When a proposal is matched by the market, resource amounts and money are exchanged between the matched Entities. Markets may impose a levy on the money or resource involved in the exchange.

3 Example Configurations

We use the model to study the behavior of three systems. The first was designed as part of a study of simple patterns of resource interdependencies that entities might have. Exploration of parameter sensitivities led to a surprising result of competitive exclusion. The second considers two economic regions with different production characteristics in their component sectors. The effect of introducing inter-regional trade in selected resources on the flows of resources within these regions is explored. The third considers how environmental perturbations influence the outcome of a competition among entities that must trade off productive efficiency and tolerance of shocks, illustrating the potential for highly optimized tolerance [Carlson and Doyle 1999] to develop in systems of this kind.

3.1 Competitive Exclusion

This configuration was designed as part of a study of basic interaction patterns among entities rather than as a model of a real system. It contains four resources, arbitrarily labeled A, B, C, and D. Each entity in the system consumes two of the resources and produces both of the resources it does not consume. There are six basic entity types corresponding to the unique partitionings of the four resources into unordered pairs:

Table 1: Definition of the Six Entity Types by their Input and Output Resources

Entity Type	Resources Consumed	Resources Produced
CDMaker	A,B	C,D
BDMaker	A,C	B,D
BCMaker	A,D	B,C
ADMaker	B,C	A,D
ACMaker	B,D	A,C
ABMaker	C,D	A,B

The system is closed, and has redundancy at two levels. Each entity can substitute between its inputs to some degree and can shift production between its outputs. At the system level, each resource is produced by three entity types and is used by three types. This redundancy confers some resiliency against disruptions to individual entity types and to the availability of individual resources.

We used the model to explore the consequences of one entity type dominating production of a particular resource or resources by increasing the CDMaker's baseline production rate parameter p_0 for both output resources above its nominal value of 1. Our naïve expectation was that the entity having increased potential production would use this potential to increase its health at the expense of other entities. The result was surprising but easy to understand in retrospect:

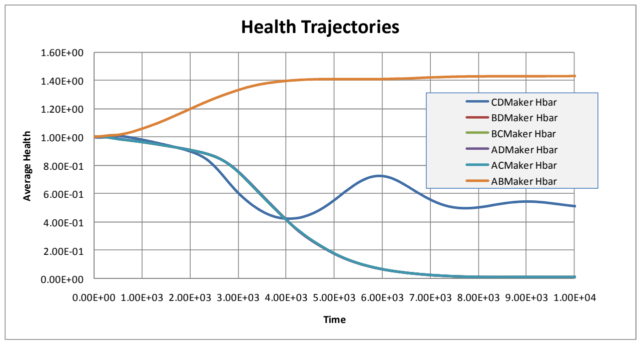

Figure 3: Average health values for each entity type when nominal production rate of CDMakers is increased

The entities that benefits from the enhanced production capacity of CDMakers are not CDMakers but ABMakers because they consumes both of the relatively abundant resources (C and D) and produces both of the relatively scarce resources (A and B). Other entity types consume one of the scarce resources and produce one of the abundant resources, but their production efficiency is lower than that of CDMakers. These entity types are unable to capture enough money (or energy) from production to obtain the input they require and are therefore driven to extinction. The elimination of competing producers allows CDMakers to “negotiate” terms of exchange that are somewhat more favorable, allowing a slight increase in health.

The exploitation of some advantage (here surplus production capacity) to eliminate competitors would not be surprising if the entity’s decision-making included a strategic picture of their environment, but it has no model of the environment at all. It is simply adjusting price levels (or energy thresholds) in response to changes in internal resource levels. This particular behavior is also of interest because the initial redundancies with which the system was endowed have been eliminated: its final configuration consists of two mutually interlocked types. Whether episodic shocks to the system would divert this trajectory and preserve a richer mixture of types is a question for further study.

3.2 Effects of Inter-Regional Trade

This configuration includes two compound entities each representing an economic region. Each region has six component entity types representing sectors of the regional economy: households, mining, manufacturing, water provision, agriculture, and energy production. These sectors exchange six kinds of resource: Labor, Food, Water, Energy, Raw Materials, and Goods. Each is produced by a specific sector using inputs from other sectors.

The two regions differ in the relative efficiency with which resources can be produced. Table 2 lists the nominal input and output coefficients for each sector entity type in the two regions. In Region 1 the processes tend to consume more energy and less labor than in Region 2, and household consumption of all resources is larger in Region 1 than Region 2.

Table 2: Nominal Input and Output Rates for Economic Sector Entities

Region 1 – More Energy Intensive, Higher Consumption by Households

	Consumption Rate for Each Resource						*Produced Resource and Rate*	
Sector Entity Type	*Labor*	*Food*	*Water*	*Energy*	*Raw Materials*	*Goods*	*Production Rate`*	*Resource*
Household		3	3	3		3	0.8	Labor
Mining	0.1		0.5	0.5			1	Raw Materials
Farming	0.1		3	1			4	Food
Water Supply	0.1			0.5			8	Water
Manufacturing	0.4			3	1		3	Goods
Energy Production	0.1		0.5				8	Energy
Total	0.8	3	7	8	1	3		

Region 2 – More Labor Intensive, Less Consumption by Households

	Consumption Rate for Each Resource						*Produced Resource and Rate*	
Sector	*Labor*	*Food*	*Water*	*Energy*	*Raw Materials*	*Goods*	*Production Rate`*	Resource
Household		1	1	1		0.5	1.2	Labor
Mining	0.2		0.5	0.2			1	Raw Materials
Farming	0.5		3	0.3			1	Food
Water Supply	0.2			0.2			7	Water
Manufacturing	0.5			1	1		0.7	Goods
Energy Production	0.1		0.5				2.7	Energy
Total	1.5	1	5	2.7	1	0.5		

In each of the two regions, we create three instances of each economic sector using the input and output coefficients in Table 2. Each region includes one market for each of the six resources, which connects all producers and consumers of the resource within the region. The nominal resource flow rates through these markets would be roughly three times the totals in Table 2 if each entity's demands could be feasibly satisfied. We first consider the resource flow rates and sector health levels in the two regions without interregional communication. Figure 4 shows the health trajectories in the two regions; resource flows rates are listed in the first column of Table 3. In Region 1 the Farming and Water Supply sectors have production rates somewhat in excess of the total nominal demand from the other sectors. The health of these sectors is therefore somewhat depressed relative to the health of other sectors, as the top half of Figure 4 shows. Mining and manufacturing are relatively depressed in Region 2 owing to the spare capacity that they enjoy.

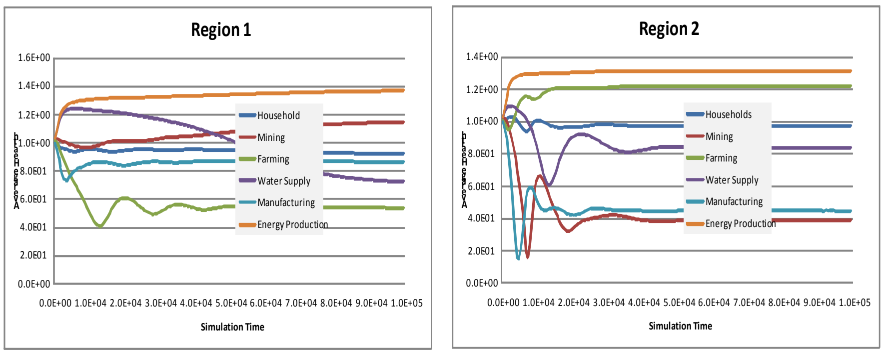

Figure 4: Average health values for each sector without inter-regional exchange

Table 3: Total Resource Flow Rates through Regional and Inter-Regional Markets for Three Cases of Inter-Regional Connection

	Inter-Regional Markets							
Resource	None		Goods			Goods and Raw Materials		
	Region 1	Region 2	Region 1	Region 2	Inter Regional	Region 1	Region 2	Inter Regional
Labor	2.03	3.14	2.04	2.69		1.91	3.14	
Food	8.53	2.70	8.28	2.50		8.96	2.71	
Water	18.45	17.25	17.89	16.78		19.02	18.11	
Energy	21.32	7.24	21.48	6.11		20.50	6.41	
Raw Materials	2.60	1.89	2.65	0.00		0.00	0.00	2.18
Goods	7.48	1.43	6.56	0.00	1.07	4.80	0.00	2.42

We next add an inter-regional market for Goods, which enables Households in both regions to buy Goods from Manufacturers in either region. No tariffs or transportation costs were imposed on inter-regional exchange (although the model allows them). Figure 5 shows the trajectories of health, and the resource flow rates are given in the central columns of Table 3. The manufacturing sector in Region 2 is extinguished, and because there is no other consumer of Raw Materials the Mining sector collapses as well. All goods are now produced in Region 1, and although the total flow of Goods from Region 1 is somewhat larger than in the case of no inter-regional exchange (6.56+1.07 vs. 7.48) this total flow is *smaller* than in the case of no inter-regional exchange. Region 1 shows a slight increase in labor use, while Region 2 sees a comparatively large decline. This decline in labor underlies the decline in total goods consumed.

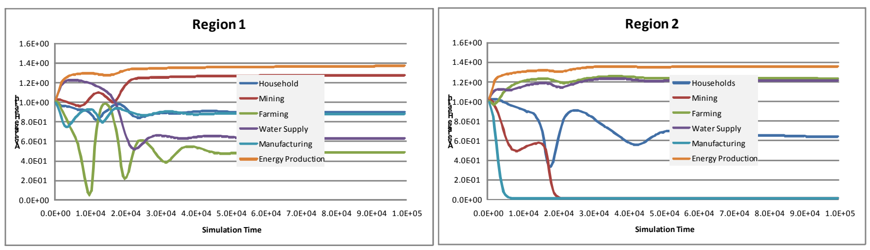

Figure 5: Average health values for each sector with inter-regional exchange of Goods

Finally, we include an inter-regional market for Raw Materials as well as Goods. Figure 6 shows the trajectories of health in the two Regions: the last columns of Table 3 list resource flow rates at the end of the simulation period.

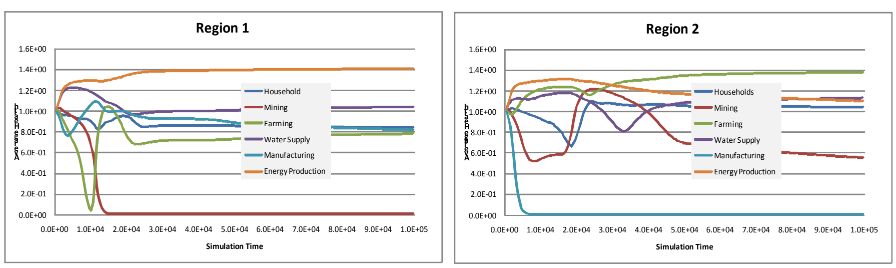

Figure 6: Average health values for each sector with inter-regional exchange of Goods and Raw Materials

Here we see a different pattern of specialization in which all Raw Materials are being produced in Region 2 and sold to the Region 1 Manufacturing sector, which is still the exclusive producer of Goods for both regions. The total production rate of Goods is again lower than in the case without inter-regional exchange, and the flow of Raw Materials is substantially lower. This is last reduction is largely due to the diversion of Raw Materials into the more-efficient Manufacturing sector in Region 1.

The definitions of economic sectors, and the coefficients used to describe them, were arbitrarily chosen for illustration. Models composed of hierarchical entities managing populations of specialized producers can clearly give insights about possible consequences of international trade patterns; this configuration is a start toward such applications.

3.3 Balancing Robustness and Efficiency under Disruption

In the first two configurations, the system adapts by changing the operating condition of component entities in response to interactions. Adaptation can also change the *composition* of a population of interacting entities through selection. The third configuration uses the model to study how trade-offs between entities' efficiency and stability are shaped by the environment to which they become adapted.

The stability analysis discussed in Section 1 helps pose the problem. There the parameters of the function $p_{h_i}^*$ leading to greater stability were seen to produce functions that maintain production as health is diminished and that can enhance production if health becomes elevated (Figure 7). These features suggest that the production process, however it is implemented by the entity, has some redundancy and surplus capacity that is lacking in entities characterized by steeper and shorter $p_{h_i}^*$ functions. Maintaining this redundancy and surplus is presumably more costly than maintaining a less robust process in that it requires the consumption of more input resources. We therefore use the parameters of $p_{h_i}^*$, as Figure 7 shows, to define a cost factor for the entity, which is used to adjust its nominal consumption rate.

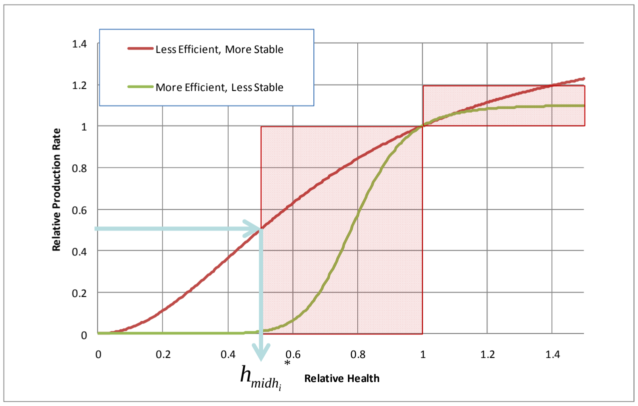

Figure 7: Production/health functions corresponding to different efficiency/robustness choices indicating the cost associated with robustness.

Intuitively the area outside the maximally efficient production function – a step function at h* = 1, represents robustness that requires additional consumption to support. If an entity's baseline consumption rate is c_{b0}, its nominal consumption rate is increased to reflect the cost of robustness:

$$c_0 = c_{b0}\left[\left(1 - h_{midh_i}{}^*\right) + \frac{1}{2}\left(\frac{p_{sat}}{1 + (p_{sat} - 1)2^{-e_p}} - 1\right)\right]^{\alpha} \tag{4}$$

Where α is a parameter that can be used to adjust the weight given to robustness costs.

We use a simple mutualistic design to study the influence of adaptive environment on the trade-offs entities make between more stable, more costly production functions and less stable, less costly versions. There are two entity types: one produces resource X by consuming resource Y, and the second produces Y by consuming X. There are 100 instances of each type. Y-producers all have identical parameters, while X-producers each have different production functions defined by random samples of the defining parameters $h_{midh_i}{}^*$ and $p_{sath_i}{}^*$ reflecting differing "choices" regarding cost and stability. X-producers and Y-producers interact through markets for these resources. These interactions are one component of the entities' environment. Exogenous shocks are a second component. Individual X-producers can be subjected to random removal of some fraction of their current inventory of resource Y.

Differences in parameter values among X-producers can lead to different health trajectories, with some ultimately dying. The populations of both X-producers (and Y-producers if needed) is periodically refreshed by replacing dead entities with new instances. This process allows us to see the selective effect of the environment on the composition of the population as ill-suited parameter combinations tend to be filtered out. Replacement by sampling the population of survivors in some way, rather than the initial distributions, would add the second half of an evolutionary dynamic, however focusing on the filtering characteristics of the environment can give important initial insights before adding new instabilities and complications to the model.

Environmental stresses are specified by two parameters: the ratio of the total nominal production rate of Y in the system to the total nominal consumption rate, and the presence or absence of random shocks to the X producers. In addition, the common production parameters $h_{midh_i}{}^*$ and $p_{sath_i}{}^*$ for the Y producers were varied from simulation to simulation.

We anticipated that increasing stress on the X producers, by decreasing abundance of the Y resource, would encourage efficiency in the X producers and filter out more robust producers (those with smaller e_p values) from the final population. Introducing random shocks to the system by episodically removing Y resource from randomly-chosen X producers was expected to favor X producers that incur the cost of robustness and filter out the more efficient (larger e_p) X producers. These expectations were substantially borne out by the results. Figure 8 shows the average value of e_p over the final population of 100 X producers for 500 sampled values of Y abundance and Y production parameters, with and without exogenous disruptions.

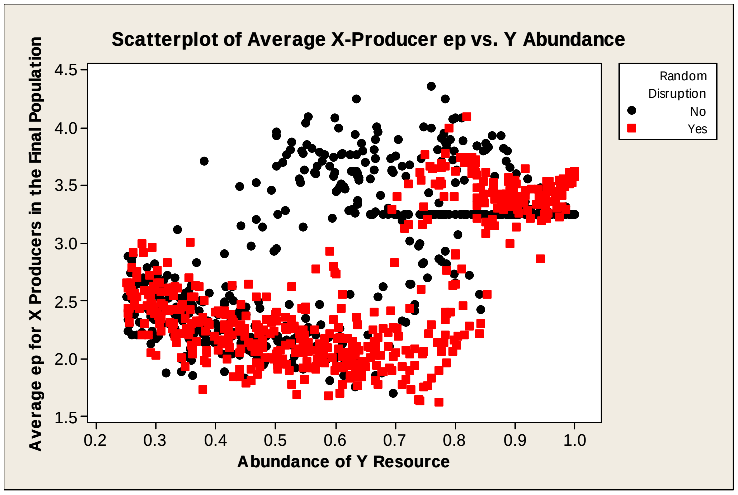

Figure 8: Average e_p in the final population of X producers vs. the relative abundance of Y in the system

There is a general trend toward more efficient X producers (larger average e_p values) as the Y resource becomes less abundant, but this trend is seen in two distinct clusters of results: one consisting of relatively efficient producers at moderate to high levels of abundance and a second of relatively robust producers at low to moderate levels of abundance. The system response can be understood by first considering the cases with high abundance and no disruption. In many cases, all entities in the initial population of X producers survive until the end of the simulation, so that the population average of $e_p \approx 3.25$ is unchanged from its value over the initial set of samples. These cases create the line of results extending down to an abundance of approximately 0.65. In other cases some X producers do expire and are replaced, allowing some selection on the basis of e_p. For high abundance levels above 0.9 adding exogenous disruption can evidently foster *efficiency* by pushing marginal entities into extinction.

As abundance declines from 0.7 to 0.5, the system develops distinct stable states: one with relatively high resource flows and health, and the second with roughly half the resource flows. In an environment with no disruption the system can often occupy the "higher" state and entities can continue to compete on efficiency; adding disruption in this range invariably filters for more robust configurations by eliminating entities with larger e_p from the population. For the lower cluster of runs there is effectively no distinction between disrupted and undisrupted runs. This implies that the random removal of input inventory from individual X producers is a small stress relative to the general shortage of Y in the system.

3 Summary

Many complex systems at many scales can be modeled as specialized entities that interact to exchange resource in a way that satisfies requirements of the component entities and that maintains the system as a whole, in the sense of a stabilized pattern of interactions among entities. We have defined a simple model that focuses on these processes, abstracting over the details of the internal structure of the entities. Illustrative applications to three problems suggest that it can be used to gain insights into diverse systems.

The model is currently being applied to represent supply networks, economic interactions, and international relationships. Features of the model not emphasized in this paper, such as dynamic formation of composite entities and creation of persistent "contractual" exchanges among entities are also being studied in simple configurations.

Bibliography

[1] Beyeler, W.E, Glass, R.J., Finley, P.D., Brown, T.J., Norton, M.D., Bauer, M., and Mitchell, M., A General Model of Resource Production and Exchange by Interacting Specialists, forthcoming

[2] Brown, T.J., Beyeler, W.E. and Barton, D., 2004, Assessing infrastructure interdependencies: the challenge of risk analysis for complex adaptive systems in International Journal of Critical Infrastructures, Vol 1 No 1.

[3] Carlson, J. M. and Doyle, J., 1999, Highly optimized tolerance: A mechanism for power laws in designed systems in Physical Review E Vol 60 No 2.

[4] Devine, P. and Paton, R. C., 1997, Herby: An Evolutionary Artificial Ecology in IEEE International Conference on Evolutionary Computation.

[5] Glass, R.J., Beyeler, W.E., and Stamber, K.L., 2004, Advanced Simulation for Analysis of Critical Infrastructure: Abstract Cascades, the Electric power grid, and Fedwire, Sandia National Laboratories, SAND 2004-4239.

[6] Holland, J.H., 1996, Hidden Order: How Adaptation Builds Complexity, Basic Books.

[7] Testafatsion, L, 2006, Agent-Based Computational Economics: A Constructive Approach to Economic Theory in L. Tesfatsion and K. L. Judd (editors), Handbook of Computational Economics, Volume 2: Agent-Based Computational Economics, Handbooks in Economics Series, North-Holland.