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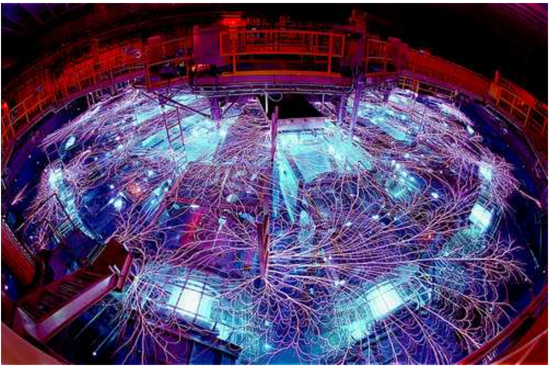

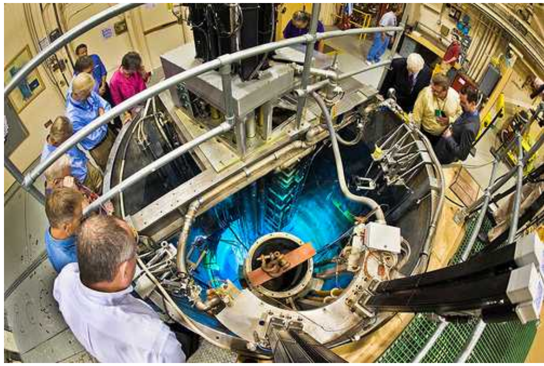

Construction and Use of Tolerance Bounds Based on Binary Data to Assess Margin and Uncertainty

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Considerations for Binary Data QMU

- QMU objectives are similar for binary and variables data:
  - Measure vulnerability to change due to low margin to requirements
  - Detect a trend with age
  - Estimate proportion of units that would fail to achieve required output

- Statistical considerations are special for binary data:
  - Unit sensitivity is observed only as censored "go/no go" observations
  - Less information per test so more units must be tested
    - Adaptive sampling schemes (e.g., Neyer D-Optimal) conserve assets
  - Maximum likelihood (ML) methods accommodate censored data, *but*
    - Tolerance bounds rely on asymptotic results
      - Small sample properties should be checked by simulation
    - ML results are sensitive to distributional assumptions
      - Distributional tests have low power
      - Sensitivity analysis highlights resulting uncertainty

Considerations for Binary Data QMU

- This talk considers example of go/no go performance in detonation of explosive devices where both reliability and safety are at issue. Tolerance bounds are calculated for unit sensitivity to stimulus inputs:
  - ***All-fire level***: Stimulus at which at least 99.9% of units can be expected to function at 95% confidence.
  - ***No-fire level***: Stimulus at which no more than 0.1% of units can be expected to function at 95% confidence.

- Many of the statistical issues apply to other applications, e.g.:
  - Medical trials (e.g., LD50 testing)
  - Electronics (e.g., transistors)
  - Mechanical design (e.g., airplane snack packaging)

Focus on Statistician's Contributions

- Explain rationale and assumptions of standard techniques:
  - Adaptive test schemes (Neyer D-Optimal)
  - Maximum likelihood estimation
  - Approximate confidence regions
  - Tolerance ratio
- Design/Assess test adequacy:
  - Precision of parameter estimates
  - Quality of approximate confidence regions
- Improve on standard methods:
  - Develop experimental and analysis strategies to incorporate the time to fire (as well as whether the unit fired)

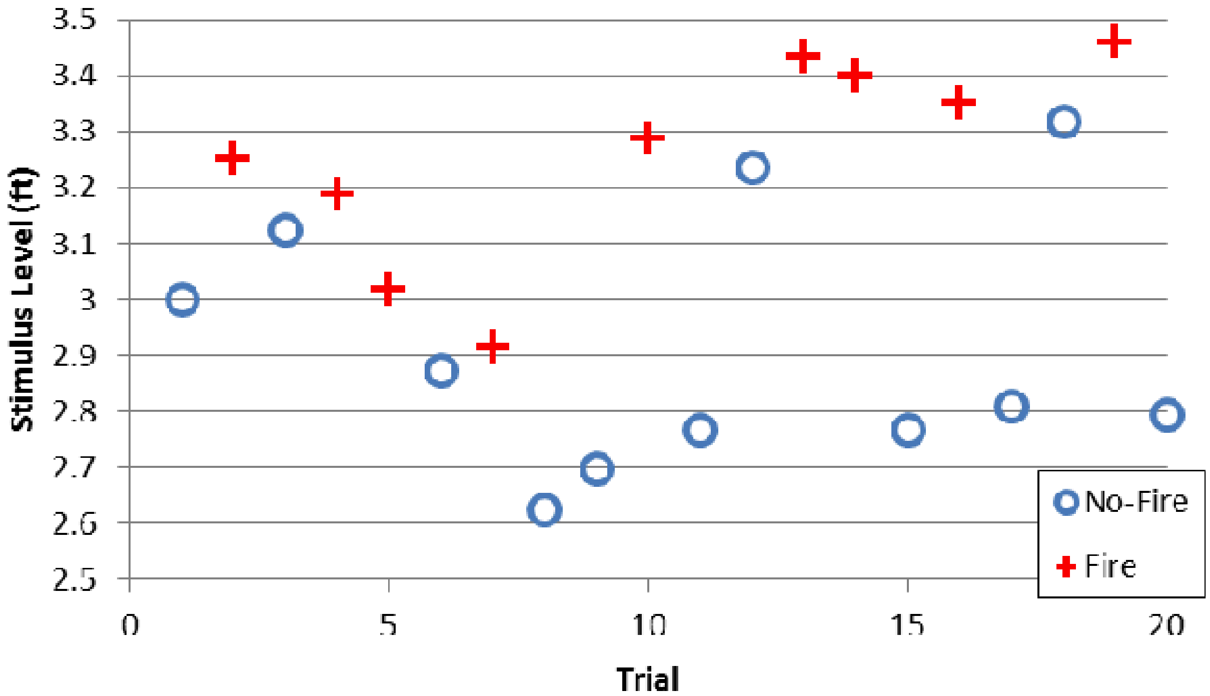
Binary Testing of Explosives

- Each unit tested at controlled stimulus level

Familiar example?

Drop table

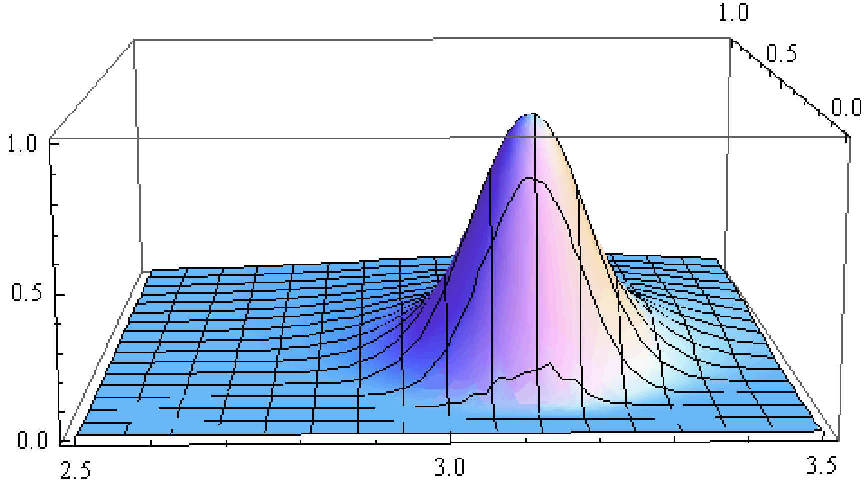
Binary Testing of Explosives: Neyer D-Optimal



- Simulated example:
  - 20 units randomly selected from Normal μ = 3, σ = 0.3 inches
  - Neyer D-Optimal with μ guess = 2.5 to 3.5, σ guess = 0.1
  - Maximize information by testing at μ +/- 1.138 σ

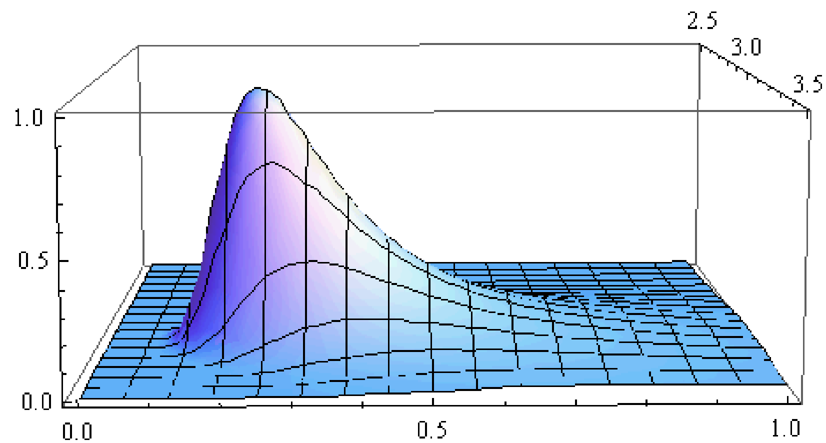
Maximum Likelihood Estimation

Relative likelihood (relative to MLEs) for example simulation from Normal μ = 3, σ = 0.3 inches



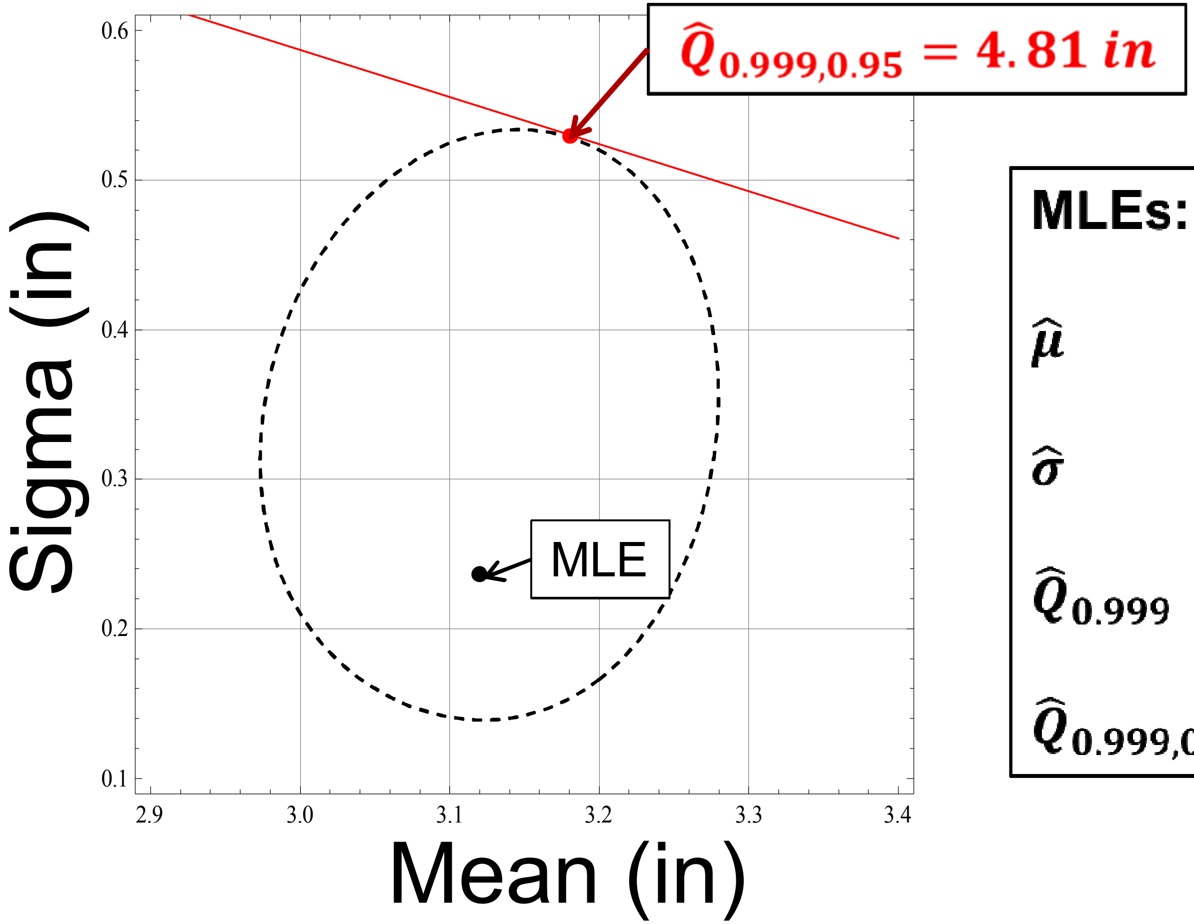
Mean

Sigma



Tolerance Bound Estimation

Asymptotic 74.2% joint confidence region for example simulation from Normal μ = 3, σ = 0.3 inches



MLEs:

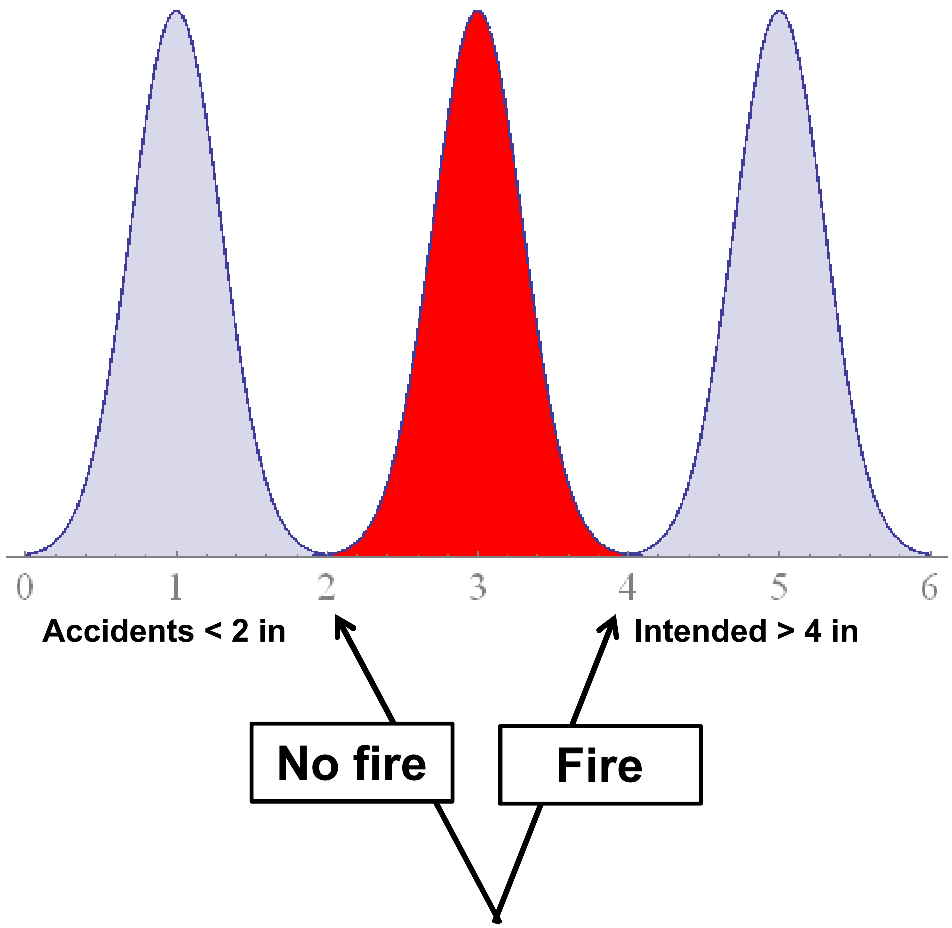
$\hat{\mu}$ = 3.12 in

$\hat{\sigma}$ = 0.24 in

$\hat{Q}_{0.999}$ = 3.85 in

$\hat{Q}_{0.999,0.95}$ = 4.81 in

Tolerance Ratio



Performance Requirements (PRs)

$$TR = \frac{PR - \hat{Q}_{0.999}}{\hat{Q}_{0.999,0.95} - \hat{Q}_{0.999}}$$

$$TR = \frac{4 - 3.85}{4.81 - 3.85}$$

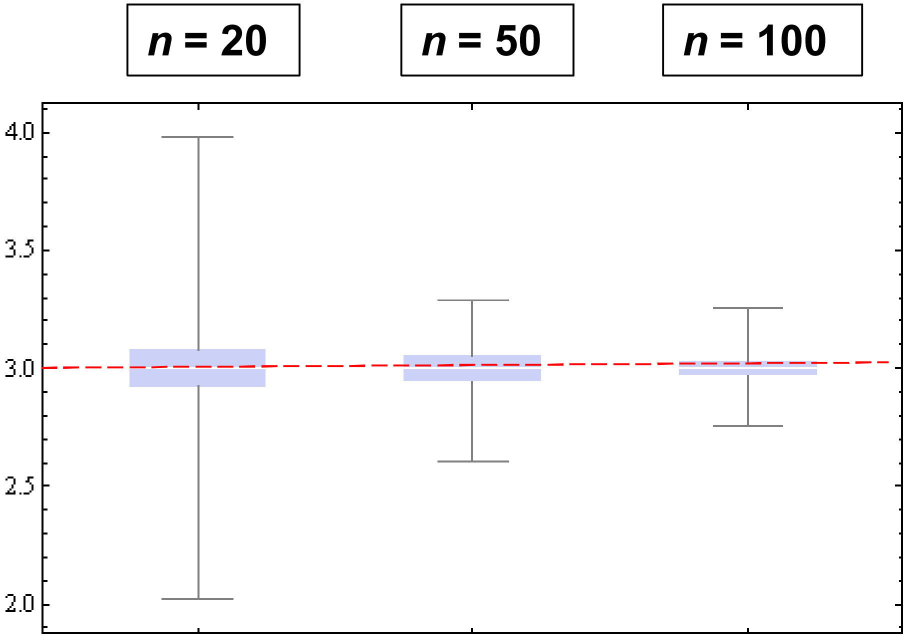
$$TR < 1$$

$$\Rightarrow$$

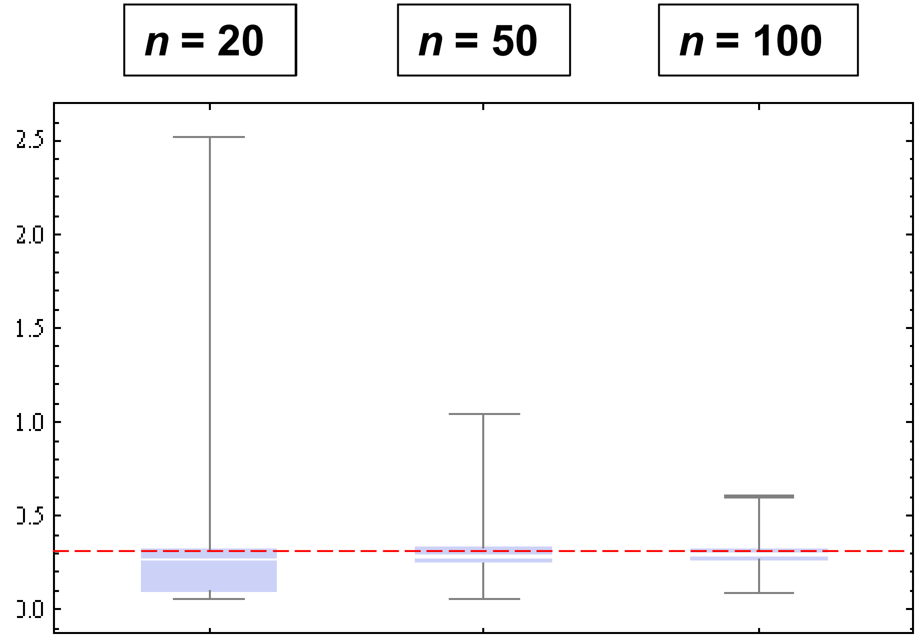
Need to modify product or inputs or test additional units

Effect of Sample Size on Estimation

10,000 simulated experiments at μ +/- 1.138 σ



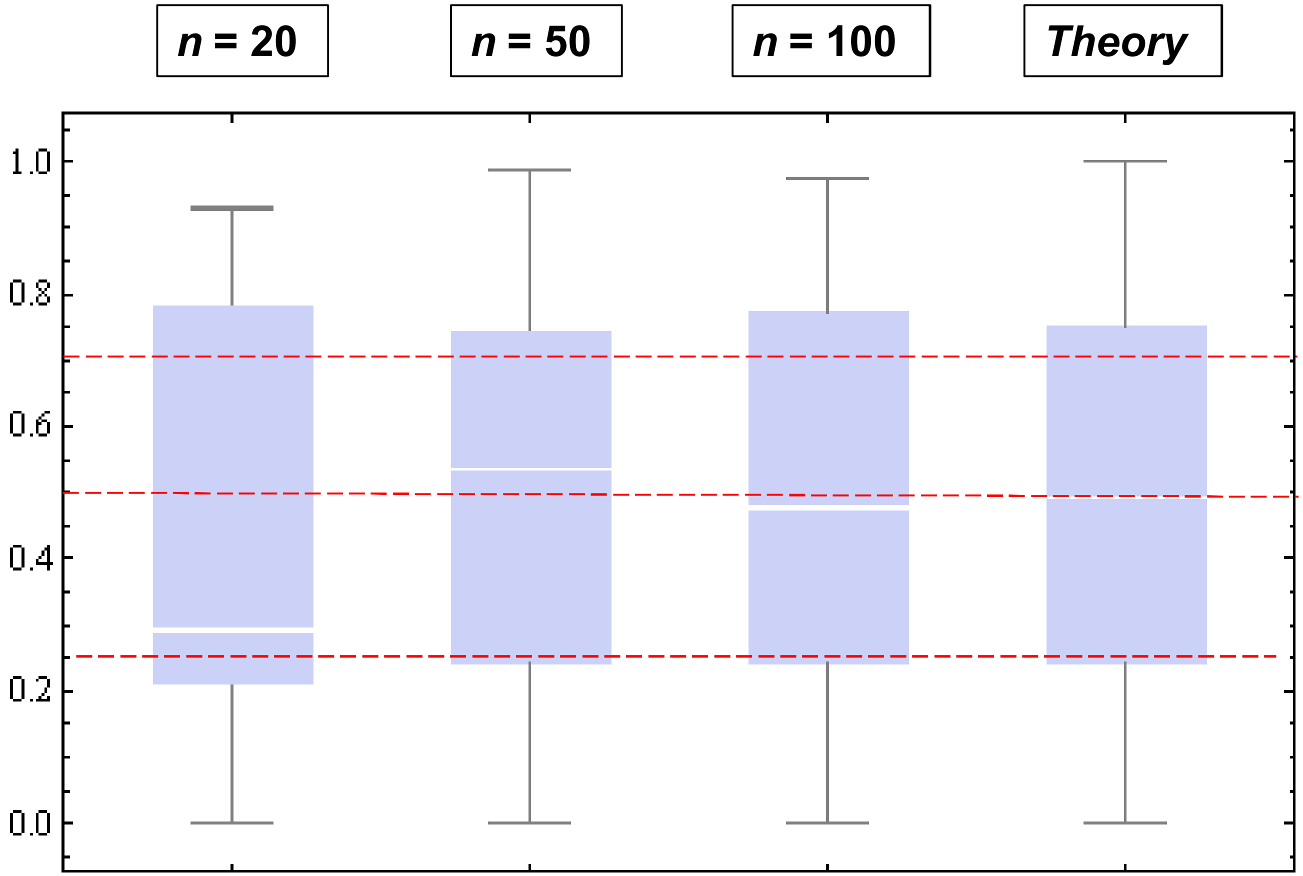
Estimated Mean



Estimated Sigma

Effect of Sample Size on Asymptotics

10,000 simulated experiments at μ +/- 1.138 σ



Likelihood Ratio (L)

$$L = \frac{L(\mu, \sigma)}{L(\hat{\mu}, \hat{\sigma})}$$

$$-2 lnL \sim {\chi_2}^2$$

(for large samples)

$$\Rightarrow$$

$$P[L < \alpha] = \alpha$$

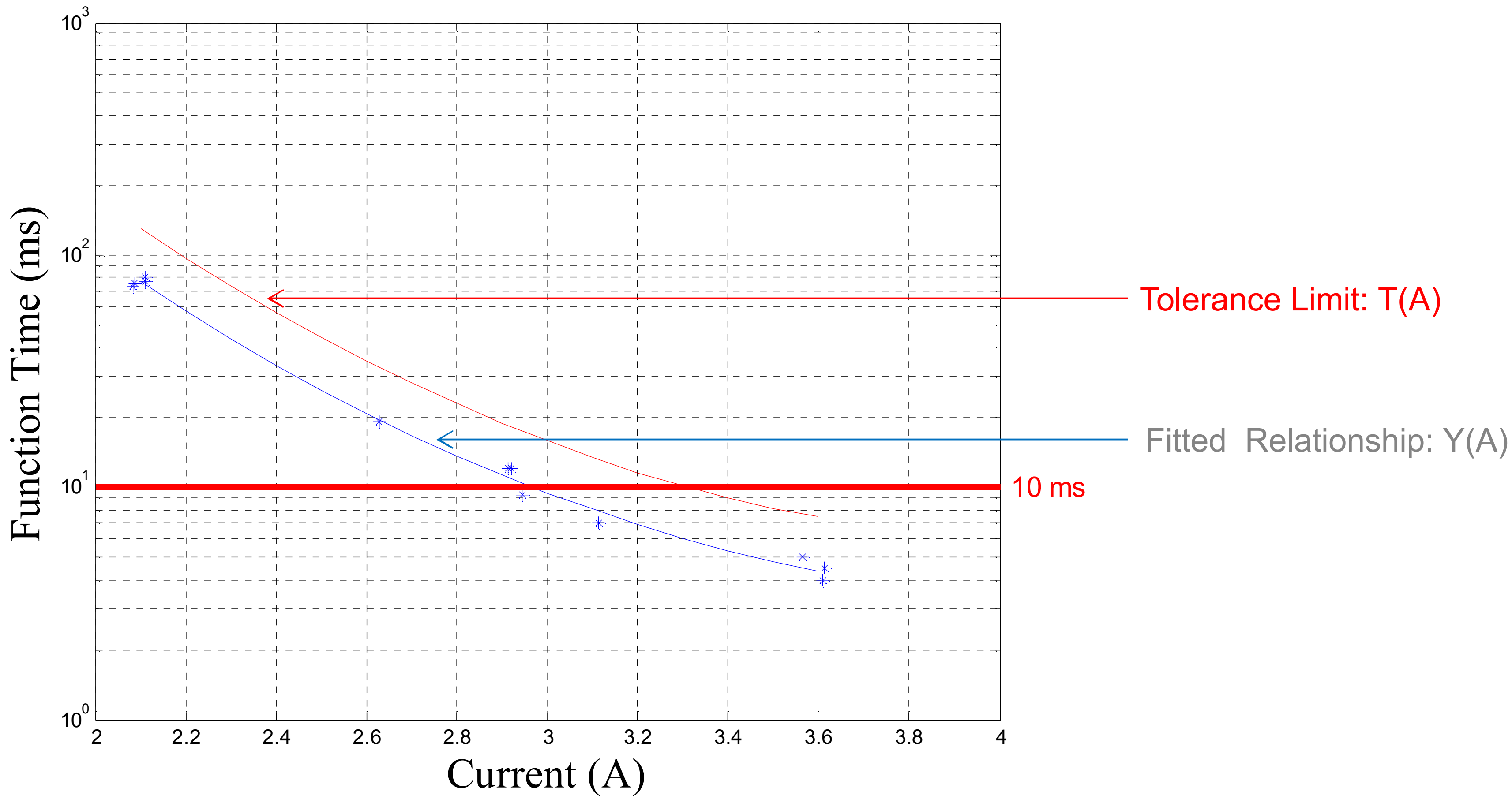
Potential Areas for Improvement

- Experimental Design
  - Better incorporation of information from testing of previous lots
  - Allow experimenter to adapt number of trials to obtain target level of uncertainty
  - Modify Neyer method to emphasize reduction of uncertainty in σ

- Data Analysis and Modeling
  - Provide measure of efficiency of parameter estimates
  - Develop methods (graphical/inferential) to assess goodness-of-fit
  - Modify analysis for situation where stimulus levels are not precisely controlled/measured

- Develop experimental and analysis strategies to incorporate the time to fire (as well as whether the unit fired)
  - Considers both energy and power
  - Note: Time-to-fire could be complete or right-censored

More Efficient/Informative Approach



With modeling assumptions (quadratic model is accurate + normality):
Given a particular stimulus (A), we are 95% confident that no more than .1% of units will produce a function time exceeding T(A)
Example: Suppose a 10 ms requirement: ➔ All-fire is ~ 3.3A

Useful References

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