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Value of Spatial Information for Determining Geothermal Well Placement

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VALUE OF SPATIAL INFORMATION FOR DETERMINING GEOTHERMAL WELL PLACEMENT[1]

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Abstract

We present a value of information methodology for the exploration of hidden geothermal resources. Specifically, this methodology is applicable for spatial decisions, such as, "Where to drill?" We evaluate how well the magnetotellurics (MT) technique is able to delineate the lateral position of the low resistive materials that are indicative of a hidden resource. The low resistive layer represents where alteration has occurred. However, the existence of a clay cap does not ensure that economic temperatures are still present below; the clay layer represents the historical high temperature of the system.

To represent the lateral uncertainty, we create simplified prior earth models that include a clay cap in one of *N* possible lateral locations. We use these prior models to simulate the data collection, inversion and interpretation of MT data. MT's ability to delineate the correct lateral location can be quantified by comparing the true location in each prior model to what location was interpreted from each respective inverted model. This is called the information reliability. The value of information (VOI) depends on this reliability measure but also is affected by whether or not a resource still exists below the clay cap. Therefore, we include in the methodology the probability of the resource existing under the clay cap. We explicitly model the two different outcomes: the positive value when a resource exists and the loss if one does not.

Table 1: Table of Symbols

Clay cap location	x
Index of Clay Cap Locations	i

Total number of Considered Clay Cap Locations	N
Decision alternative	a
Existence or Non-Existence of Resource	θ
Value: metric to define outcome of decision	v
Vector of earth parameters	$\mathbf{z}$
Index of models with same clay cap location	t
Total number of realizations with clay cap i	T
Decision predictor/function (eg drilling)	$g_a(\cdot)$
Geophysical forward modeling (i.e. MT simulation)	$f(\cdot)$
Electrical resistivity model	ρ
Synthetic data	d
Synthetic data with noise	$\tilde{d}$
Inverted electrical resistivity model	$\tilde{\rho}$
Automatic interpretation function	$h(\cdot)$
Interpreted Location of Clay Cap	$\tilde{x}$
Index of interpreted Clay Cap Locations	j
Prior Value	Vprior
Value with Perfect Information	V_{PI}
Value with Imperfect Information	V_{II}
Value of Imperfect Information	$VOI_{imperfect}$

1. Introduction

The goal of the work presented in this paper is to demonstrate value of information for spatial geothermal decisions, such as “where to drill?” A spatial decision can be defined as any decision whose outcome depends on the spatial distribution of some phenomena (Trainor-Guitton, 2010). This may be especially challenging with hidden (or blind) geothermal resources. We motivate our modeling after the conceptualization of these resources by Cumming (2009). Figure 1 demonstrates a possible blind/hidden geothermal resource where no surface exposure exists to indicate a possible geothermal resource. Figure 1 demonstrates a scenario where faults and fractures allow for the circulation of hot water to accessible depths. As a result, smectite and illite clays are formed just above the shallowest depths where the hot water circulates.

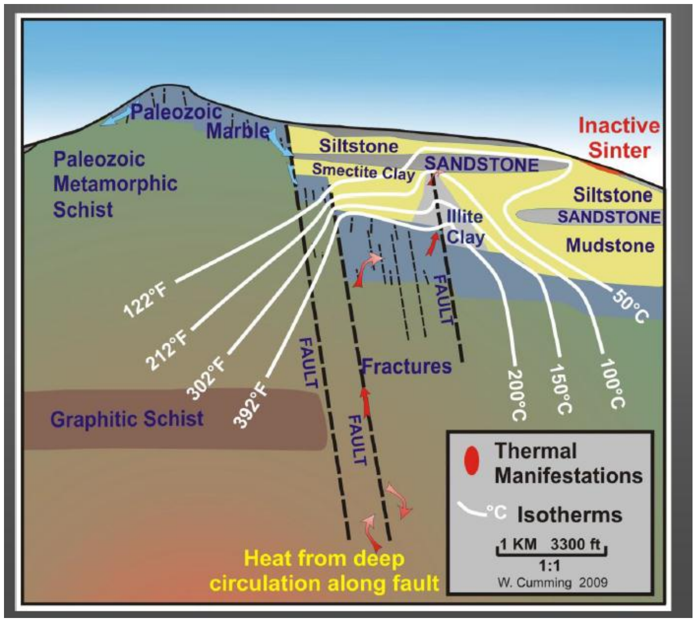

Figure 1: Conceptualization of blind geothermal resource where no surface feature exists to demonstrate existence of a possible resource (from Cummings 2009)

VOI is a decision analysis method that quantifies how relevant and reliable any particular information source is, given a decision with a highly uncertain outcome (Bratvold et al., 2009). VOI can be used to justify the costs of collecting the proposed data. In its simplest form, the VOI equation can be expressed as:

$$VOI = V_{with\ information} - V_{prior} \quad (1)$$

where value *V*, is the metric used to quantify the outcome of a decision; the higher the value, the more "successful" an outcome of a decision is. V_{prior} , which captures the expected outcome of a decision taken without the proposed information, will be addressed in Section 2. Section 2 describes how the prior uncertainty of the subsurface is represented with multiple realizations of simplified geothermal reservoir models representing different possible locations of the resource. In Section 3, we will describe how the value with information ($V_{with\ information}$) can be estimated by simulating the MT response on the prior models. Specifically, we devise a method for estimating MT's reliability to determine the location of the geothermal reservoirs.

Depending on the decision and the models included in the prior, VOI can underscore the strengths and weakness of a particular information source. We demonstrate this by simulating the physics of the MT measurement on many geothermal reservoir models that represent possible exploration scenarios, and by performing inversions of MT data. First we give background of why MT has been used for geothermal exploration.

Information Source considered: Magnetotellurics

MT has strengths and weaknesses when used to explore for geothermal resources. Historically, the MT technique has been used to delineate the low resistive materials that are indicative of alteration caused by the circulation of hot fluids (Gunderson et al, 2000; Newman et al., 2008). Figure 2 is also from Cumming (2009) which provides a conceptual model of electrical resistivity for the geologic representation of Figure 1. The hidden resource is at the apex of the isotherms which coincides with the concave-side of the 10 ohm-m clay cap (yellow). Therefore, for our modeling and VOI demonstration purposes, this clay cap (yellow) is the key **potential** indicator of the resource.

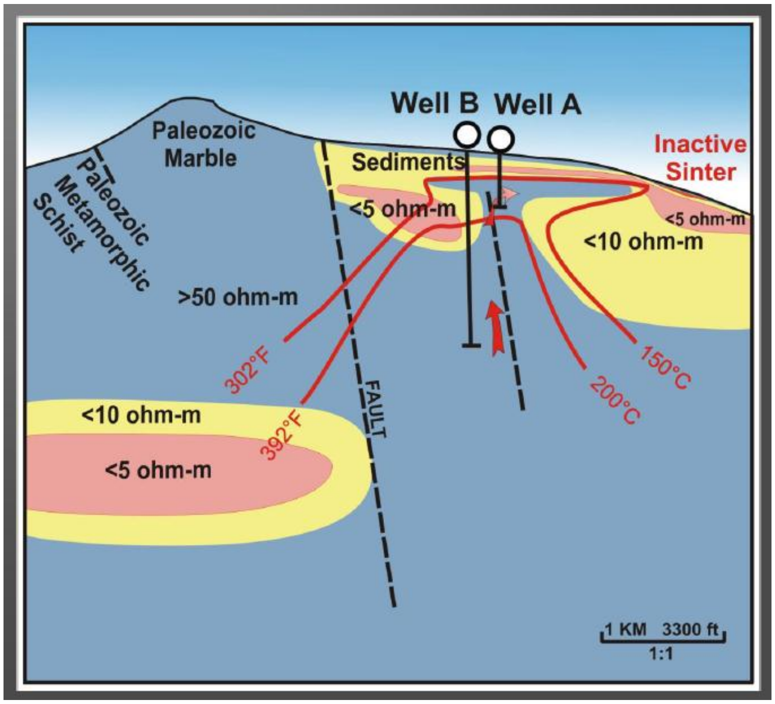

Figure 2: Conceptual model of electrical resistivity for a hidden geothermal resource (from Cumming, 2009)

This alteration reflects the historical high temperature of the system, therefore the existence of a clay cap does not ensure that economic temperatures still exist below it. Karlsdóttir et al. (2012) describe how the resistivity alone cannot confirm a viable geothermal resource:

> *The resistivity reflects the alteration caused by the heating of the rocks and reflects the peak temperature experienced by the system, being it at the present or in the past. … The resistivity structure reflects the temperature, provided there is equilibrium between alteration and present temperature. In case of cooling the alteration may remain and the resistivity will reflect the temperature at which the alteration was formed. Whether the resistivity (and the alteration) indicates the present temperature of the system will only be confirmed by drilling.*

In other words, the MT measurements may help us determine where a clay cap exists, but they can't tell us definitively about the temperature below the cap. Additionally, the cap's lower electrical resistivity tends to shunt electrical currents and greatly reduces sensitivity to the reservoir. VOI will allow us quantify both MT's usefulness (spatial coverage and sensitivity to low resistive clays) and limitations (low resistivity is not uniquely associated with higher temperature i.e. whether a resource exists or not).

2. Uncertainty of Possible Hidden Resource (Clay Cap) Location: Where to drill?

Figure 3 depicts the decision scenario that we have described thus far in a decision tree. The tree represents the decision-to-outcome process chronologically from left to right. First a decision of where to drill is taken (extreme left). The final outcome (extreme right) will depend on where the clay cap is and if a resource exists under the cap. For this work, we only consider how the MT source can help detect the location of the clay cap. In Section 4, we will introduce how we account for the probability of the resource existing (represented by $Pr(\Theta = \theta_k)$).

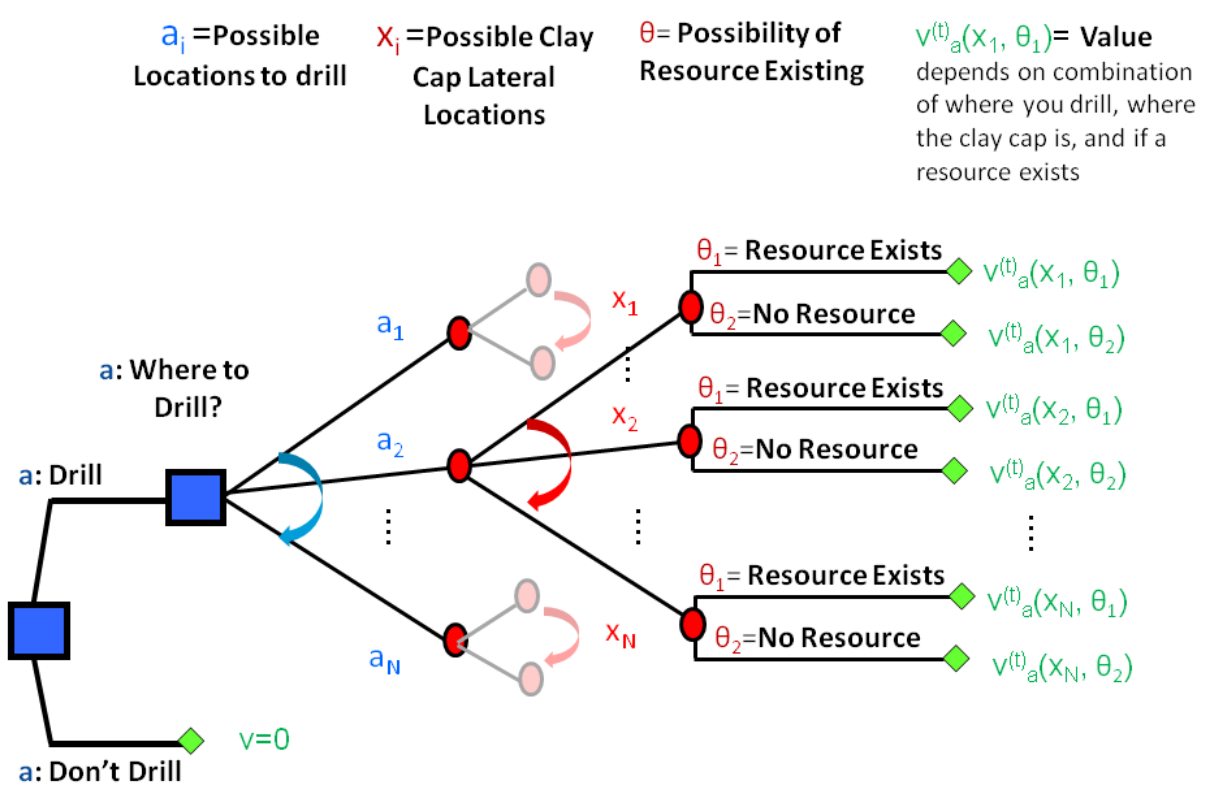

Figure 3: Decision tree where blue squares depict the spatial decision alternatives and the red nodes depict both the uncertainty of the clay cap locations and the resource existence. Lastly, the unique combination of these alternatives and uncertainties result in an outcome measured in value (green diamonds).

To represent our uncertainty in the location of the clay cap, we create prior models with clay caps of varying lateral locations. We assume the hidden resource below the clay cap can only exist in one of *N* discrete locations. Within our prior models, the clay cap is represented at *N*=15 different locations, where the horizontal location (x) of the middle of the clay cap varies between -3500m and +3500m. Let us represent each model by

$$\boldsymbol{z}^{(t)}(X = x_i) \quad t = 1, \ldots, T \;\; i = 1, \ldots, N \tag{2}$$

where vector $\boldsymbol{z}$ contains the electrical resistivity and any other relevant properties (i.e. temperature, porosity, etc.) of the model and *t* indexes all models that have the same clay cap location. Figure 4 demonstrates model $\boldsymbol{z}^{(t)}(X = 0)$; the x location denotes the "throat" of the clay cap. Figure 2 portrays this "throat" as the shallowest location of the highest isotherms. Therefore, the x_i location represents the shallowest access to the potential resource. The clay cap in all models ranges between 0.5 and 1.5 km depth and is 3km wide.

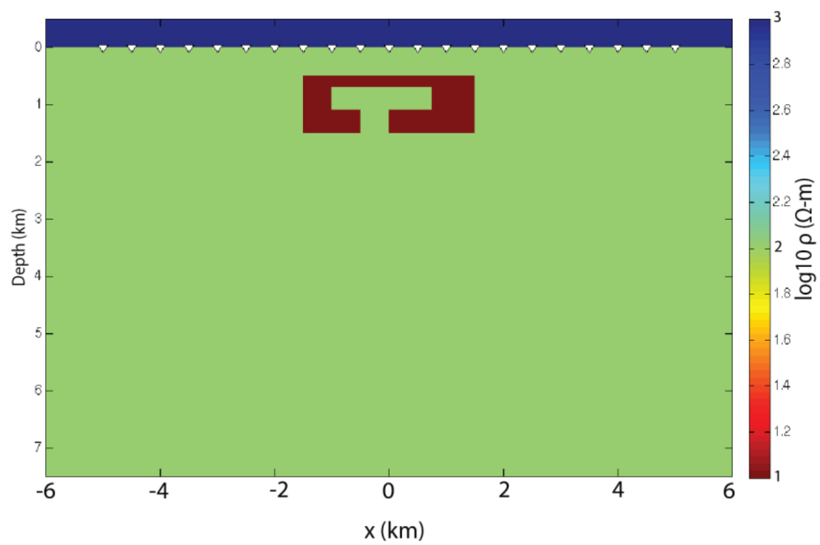

Figure 4: One realization of the electrical resistivity model representing the hidden resource. The red layer represents the yellow 10 Ohm-m layer in Figure 2. The blue layer is the air and the green the background subsurface.

In turn, we assume that we only consider drilling in these *N* locations, if at all. Thus, the spatial alternatives (represented by index a) are to drill in one of the *N* possible clay cap locations or not to drill at all. These are represented as the columns in Table 2, while the different possible clay cap locations (model categories x_i) are represented in the rows of Table 2. The last column of the table represents the option to not drill at all.

Table 2: Table of Decisions alternatives (a) and value outcomes (v) assuming resource exists under clay cap

Alternatives → Clay Cap Locations (in models) ↓	Drill @ x=+3500m (a=1)	Drill @ x=+3000m (a=2)	…	Drill @ x= -3000m (a=14)	Drill @ x= -3500m (a=15)	Don't Drill (a=16)
Clay Cap @ x= +3500m	Highest Value $$	Value $	…	LOSS	LOSS	0
Clay Cap @ x= +3000m	Value $	Highest Value $$	…	LOSS	LOSS	0
⋮				⋮	⋮	⋮
Clay Cap @ x= -3000m	LOSS	LOSS	…	Highest Value $$	Value $	0
Clay Cap @ x= -3500m	LOSS	LOSS	…	Value $	Highest Value $$	0

The outcome of choosing a decision alternative a with a clay location of x_i, is quantified with the "value outcome metric." The value metric allows for comparison between outcomes from different decision alternatives, which can be represented by function g_a.

$$v_a^{(t)}(x_i) = g_a\left(\boldsymbol{z}(\mathrm{X} = x_i)^{(t)}\right) \tag{3}$$
$$a = 1,..,N+1 \quad i = 1,\dots,N \quad t = 1,..T$$

Table 2 depicts that the highest outcomes (most successful decisions) occur along the diagonal: this is when the drilling location aligns with the location of the middle of the clay cap. The value outcomes then drop off as you move away from the diagonal signifying the mismatch between the possible resource location and the drilling location.

V_{prior}: the best decision option given prior uncertainty

Decision analysis concepts are often described in terms of lotteries and prizes (Pratt et al, 1995). By choosing to drill or not, a decision maker is choosing whether or not to participate in a lottery with certain perceived chances of winning a prize (drilling into a profitable reservoir); however, this lottery also involves the chances of losing money (missing the resource or drilling into an uneconomic reservoir). By utilizing V_{prior}, a decision-maker can logically determine when one should participate in this lottery given both the prior uncertainties and possible gains and losses.

V_{prior} is only dependent on the current state of uncertainty ($Pr(X = x_i)$) and the outcomes of the decision ($v_a(x_i)$):

$$V_{prior} = \max_a \left(\sum_{i=1}^{N} Pr(X = x_i) v_a(x_i) \right) \quad a = 1,\dots,N+1 \tag{4}$$

The V_{prior} expression identifies which decision alternative will on average result with the highest value (most successful outcome). The prior distribution is used to calculate a weighted average inside the summation and the $\max_{a}$ captures the highest outcome value among all the different spatial alternatives *a*.

V_{prior} is inherently a very subjective measure, since the definition of the prior is to quantify what you don't know. Therefore, we test three different prior distributions and two different value outcome matrices (Table 2). Figure 5 displays the three prior distributions. The uniform distribution (black) declares that there's an equal likelihood that the clay cap exists at any of the *N* locations between -3,500m and +3,500m. The two Gaussian distributions (red and green curves in Figure 5) reflect a belief that the resource is centered at x=0. The Gaussian with the smaller standard deviation (red curve) reflects less uncertainty of the location than the Gaussian with the larger standard deviation (green curve).

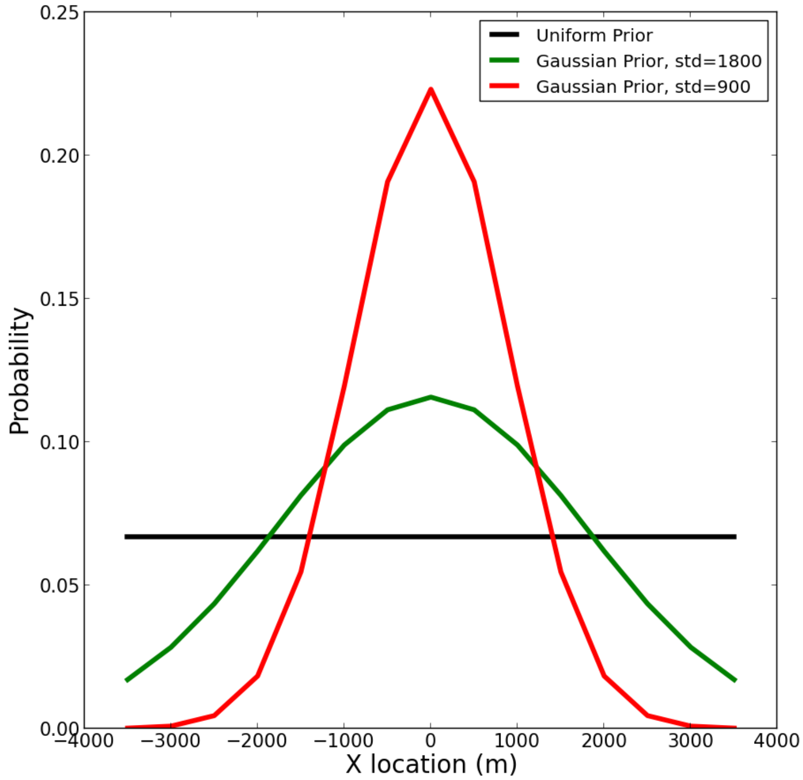

Figure 5: 3 Different Prior Distributions used to test Vprior sensitivity

Two value outcome matrices were assessed. Figure 6 displays a value outcome matrix that penalizes drilling decisions that miss the clay cap by ≥1000m. Whereas Figure 7 is a more "forgiving" value outcome matrix, in that losses are not occurred until the drilling location is quite far (4000m) from the actual location of the clay cap. The individual values in Figure 6 and

Figure 7 are arbitrary and can be replaced by more realistic dollar amounts in order to represent specific locations or particular drilling applications.

	a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11	a12	a13	a14	a15
x = -3500	$500,000	$200,000	$0	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000
x = -3000	$200,000	$500,000	$200,000	$0	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000
x = -2500	$0	$200,000	$500,000	$200,000	$0	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000
x = -2000	-$300,000	$0	$200,000	$500,000	$200,000	$0	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000
x = -1500	-$300,000	-$300,000	$0	$200,000	$500,000	$200,000	$0	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000
x = -1000	-$300,000	-$300,000	-$300,000	$0	$200,000	$500,000	$200,000	$0	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000
x = -500	-$300,000	-$300,000	-$300,000	-$300,000	$0	$200,000	$500,000	$200,000	$0	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000
x = 0	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	$0	$200,000	$500,000	$200,000	$0	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000
x = 500	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	$0	$200,000	$500,000	$200,000	$0	-$300,000	-$300,000	-$300,000	-$300,000
x = 1000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	$0	$200,000	$500,000	$200,000	$0	-$300,000	-$300,000	-$300,000
x = 1500	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	$0	$200,000	$500,000	$200,000	$0	-$300,000	-$300,000
x = 2000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	$0	$200,000	$500,000	$200,000	$0	-$300,000
x = 2500	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	$0	$200,000	$500,000	$200,000	$0
x = 3000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	$0	$200,000	$500,000	$200,000
x = 3500	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	-$300,000	$0	$200,000	$500,000

Figure 6: Value outcomes that drop off quickly (i.e. losses are experienced when drilling 1,500m from actual clay cap). Rows represent the actual clay cap location and columns represent the drilling location (decision alternative). Green equals gain and red equal loss.

	a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11	a12	a13	a14	a15
x = -3500	$500,000	$433,000	$366,000	$299,000	$232,000	$165,000	$98,000	$31,000	-$36,000	-$103,000	-$170,000	-$237,000	-$304,000	-$371,000	-$438,000
x = -3000	$433,000	$500,000	$433,000	$366,000	$299,000	$232,000	$165,000	$98,000	$31,000	-$36,000	-$103,000	-$170,000	-$237,000	-$304,000	-$371,000
x = -2500	$366,000	$433,000	$500,000	$433,000	$366,000	$299,000	$232,000	$165,000	$98,000	$31,000	-$36,000	-$103,000	-$170,000	-$237,000	-$304,000
x = -2000	$299,000	$366,000	$433,000	$500,000	$433,000	$366,000	$299,000	$232,000	$165,000	$98,000	$31,000	-$36,000	-$103,000	-$170,000	-$237,000
x = -1500	$232,000	$299,000	$366,000	$433,000	$500,000	$433,000	$366,000	$299,000	$232,000	$165,000	$98,000	$31,000	-$36,000	-$103,000	-$170,000
x = -1000	$165,000	$232,000	$299,000	$366,000	$433,000	$500,000	$433,000	$366,000	$299,000	$232,000	$165,000	$98,000	$31,000	-$36,000	-$103,000
x = -500	$98,000	$165,000	$232,000	$299,000	$366,000	$433,000	$500,000	$433,000	$366,000	$299,000	$232,000	$165,000	$98,000	$31,000	-$36,000
x = 0	$31,000	$98,000	$165,000	$232,000	$299,000	$366,000	$433,000	$500,000	$433,000	$366,000	$299,000	$232,000	$165,000	$98,000	$31,000
x = 500	-$36,000	$31,000	$98,000	$165,000	$232,000	$299,000	$366,000	$433,000	$500,000	$433,000	$366,000	$299,000	$232,000	$165,000	$98,000
x = 1000	-$103,000	-$36,000	$31,000	$98,000	$165,000	$232,000	$299,000	$366,000	$433,000	$500,000	$433,000	$366,000	$299,000	$232,000	$165,000
x = 1500	-$170,000	-$103,000	-$36,000	$31,000	$98,000	$165,000	$232,000	$299,000	$366,000	$433,000	$500,000	$433,000	$366,000	$299,000	$232,000
x = 2000	-$237,000	-$170,000	-$103,000	-$36,000	$31,000	$98,000	$165,000	$232,000	$299,000	$366,000	$433,000	$500,000	$433,000	$366,000	$299,000
x = 2500	-$304,000	-$237,000	-$170,000	-$103,000	-$36,000	$31,000	$98,000	$165,000	$232,000	$299,000	$366,000	$433,000	$500,000	$433,000	$366,000
x = 3000	-$371,000	-$304,000	-$237,000	-$170,000	-$103,000	-$36,000	$31,000	$98,000	$165,000	$232,000	$299,000	$366,000	$433,000	$500,000	$433,000
x = 3500	-$438,000	-$371,000	-$304,000	-$237,000	-$170,000	-$103,000	-$36,000	$31,000	$98,000	$165,000	$232,000	$299,000	$366,000	$433,000	$500,000

Figure 7: Value outcomes that drop off slowly (i.e. losses are only experienced when one drills >4,000m from the actual clay cap location). Rows represent the actual clay cap location and columns represent the drilling location (decision alternative). Green equals gain and red equal loss.

Table 3 contains the resulting six different V_{prior}'s. The prior uncertainty of the location of the clay cap decreases down the rows of the Table 3 and the overall individual value outcomes increase through the columns. Therefore, V_{prior} is lowest in the top left and highest in the bottom right of the table. This makes sense since a completely uninformed prior is the uniform distribution, and therefore entering the "geothermal lottery" is quite risky. However, if the uncertainty of the clay cap location decreases (represented by the Gaussian distributions) then the V_{prior} increases, and more so with a smaller standard deviation (last row). These Gaussian distributions could represent a situation where prior geological or well information exists that indicates a possible clay cap location. The difference between the two columns is that the individual value outcomes drop-off more slowly for increasing mismatches between the drilling and the clay cap locations (the off-diagonals of Table 2). Therefore, there is less risk of a monetary loss for the furthest right column and all the V_{prior}'s are higher in this column.

Table 3: Vprior for different prior uncertainties (rows) and different individual value outcomes (columns).

Prior Distribution ↓	$v_a(x_i)$: Gains drop quickly (Figure 6)	$v_a(x_i)$: Gains drop off slowly (Figure 7)
Uniform Prior	$0	$249,866.67
Gaussian Prior (μ=0m, σ^2=1800)	$0	$324,818.74
Gaussian Prior (μ=0m, σ^2=900)	$140,859.84	$406,931.52

Value of Perfect Information

The value of perfect information ($VOI_{perfect}$) provides an upper bound on what a new information source could have, given your prior uncertainty and modeled value outcomes. Perfect information for this example assumes that some measurement could reveal without error, the location of the clay cap. With this perfect information, theoretically, one would drill exactly at the neck of the clay cap. The value with this perfect information is expressed as

$$V_{PI} = \sum_{i=1}^{N} Pr(X = x_i) \left(\max_a v_a(x_i) \right) \quad a = 1, \dots, N+1 \tag{5}$$

which only differs from V_{prior} by the placement of the $\max\limits_{\mathbf{a}}$, which is now before the averaging ($\sum_{i=1}^{N} Pr(X = x_i)$). This represents that we will have the information before we choose a location for drilling (*a*), and therefore we can choose the alternative that has the highest value for each clay cap location. For both value outcome matrices (Figure 6 and Figure 7), this is the diagonal: $500,000. Then the average of all best outcomes for each of the clay cap locations is calculated. Since all three of the prior distributions are symmetric, V_{PI} is $500,000 for all 6 combinations of prior uncertainty and value outcomes (Table 4). Following Equation 1, the value *of* perfect information, is the difference between this and V_{prior}.

$$VOI_{perfect} = V_{PI} - V_{prior} \tag{1}$$

As seen in Table 4 information has more value in cases of higher uncertainty (uniform prior) and greater loss when one drills far from the target (Figure 6).

Table 4: $VOI_{perfect}$ for different prior uncertainties (rows) and different individual value outcomes (columns

Prior Distribution ↓	$v_a(x_i)$: Gains drop quickly (Figure 6)	$v_a(x_i)$: Gains drop off slowly (Figure 7)
Uniform Prior	$500,000 - $0 = $500,000	$500,000 - $249,866 = $250,133
Gaussian Prior (μ=0m, σ^2=1800)	$500,000 - $0 = $500,000	$500,000 - $324,818 = $175,181
Gaussian Prior (μ=0m, σ^2=900)	$500,000 - $140,859 = $359,140	$500,000 -$406,931 = $93,068

3. MT: Simulating Data Collection, Inversion and Interpretation of Clay Cap Location

To assess the value of MT information, we must have an estimate of MT's reliability to locate the clay cap. We estimate the reliability by mimicking the data collection, inversion and interpretation processes.

The workflow to estimate the value with imperfect MT information can be described in 7 steps.

1. The MT response is forward modeled (represented with function $(\cdot)$) for each prior model. We utilize the electromagnetic simulation code MARE2DEM (Key & Oval, 2011). Frequencies between 0.1 and 1000 Hz (21 frequencies total, 4 per decade) are observed with 21 receivers. The line of MT receivers covers -5,000m to +5,000m, therefore the entire clay cap is covered in each of the models.

$$d_i^{(t)} = f\left(\boldsymbol{z}\left(x_i^{(t)}\right)\right) \quad i = 1 \dots, N \;\; t = 1, \dots, T \tag{6}$$

2. 4% random Gaussian noise is added to all of the $T*N$ (each of the prior models) MT forward responses.

$$\tilde{d}_i^{(t)} = d_i^{(t)} + 0.04 d_i^{(t)} * N(0,1) \quad i = 1 \dots, N \;\; t = 1, \dots, T \tag{7}$$

3. Geophysical inversion is performed for each noisy data set; one inverted electrical resistivity model ($\tilde{\rho}_i^{(t)}$) is obtained for every prior model. Figure 8 includes 3 prior models (first column) and their respective inversion models (last column).

$$\tilde{\rho}_i^{(t)} = f^{-1}\left(\tilde{d}_i^{(t)}\right) i = 1 \dots, N \;\; t = 1, \dots, T \tag{8}$$

4. For each inversion result, automatic interpretation (denoted by function $h(\cdot)$) is used to locate the clay cap "throat" at fixed depths. The location of the maximum resistivity within the minimum resistivity region is chosen as the interpreted clay cap "throat" location ${\tilde{x}_j}^{(t)}$:

$${\tilde{x}_j}^{(t)} = h\left(\tilde{\rho}_i^{(t)}\right) \quad i, j = 1 \dots, N \;\; t = 1, \dots, T \tag{9}$$

5. The data likelihood/reliability is calculated by comparing the interpreted location of clay cap in the inverted image (${\tilde{x}_j}^{(t)}$) to its prior model's original location (${x_i}^{(t)}$).

$$Pr\left(\tilde{X} = \tilde{x}_j | X = x_i\right) \quad i, j = 1 \dots, N \tag{10}$$

6. The information posterior is calculated: use Bayes rule to estimate the probability of the actual clay cap location given an interpreted clay location $\tilde{x}_j{}^{(t)}$.

$$Pr\left(X = x_i | \tilde{X} = \tilde{x}_j\right) \quad i, j = 1 \ldots, N \tag{11}$$

How the reliability and information posterior are calculated will be further explained later in this section.

7. Lastly, the value with imperfect information (V_{II}) is calculated using the information posterior.

$$V_{II} = \sum_{j=1}^{J} \Pr(\tilde{X} = \tilde{x}_j) \left\{ \max_a \left[\sum_{i=1}^{15} Pr(X = x_i | \tilde{X} = \tilde{x}_j) v_a(x_i) \right] \right\} \quad a = 1, \ldots, N+1 \tag{12}$$

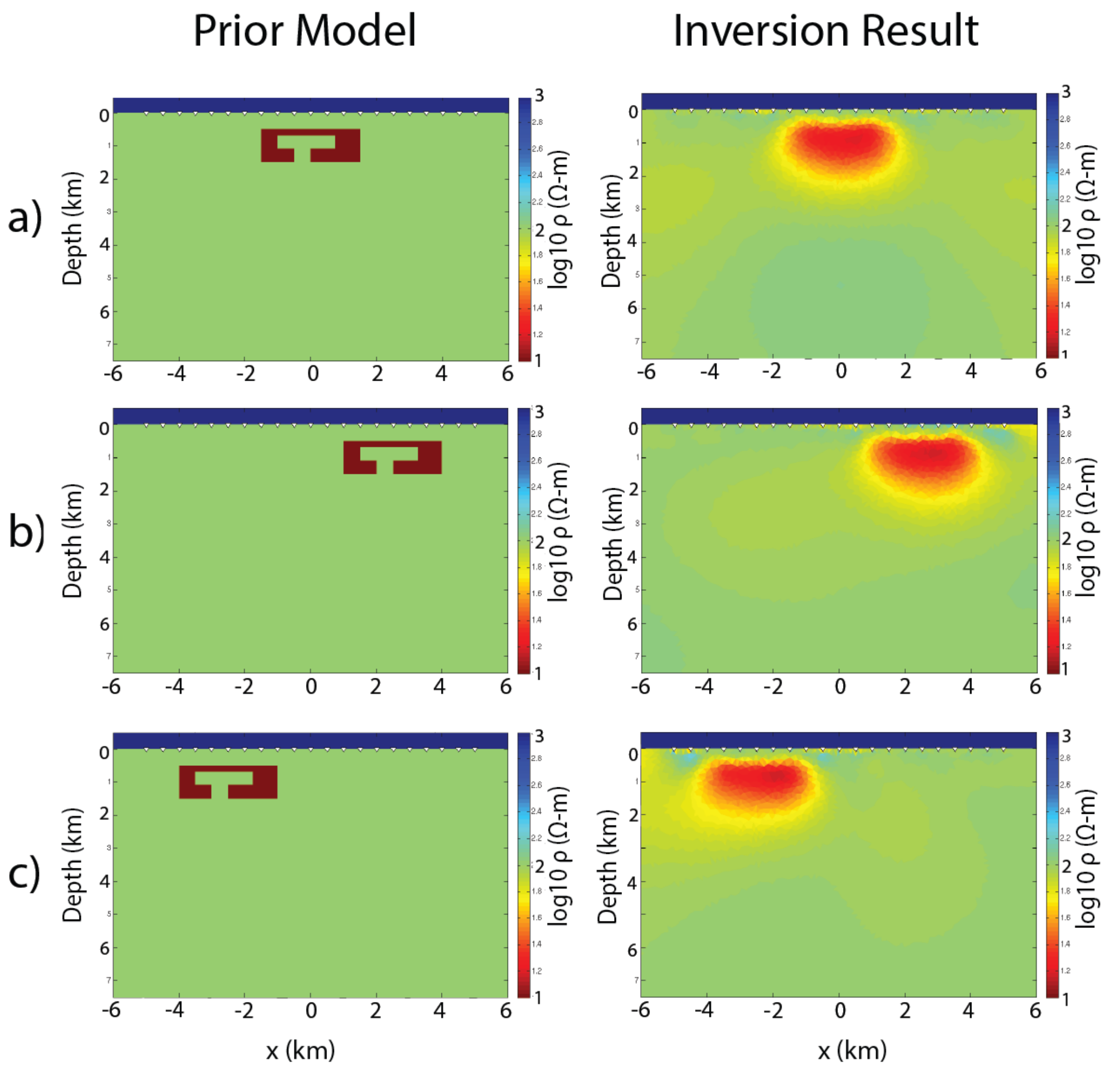

Figure 8: First column contains 3 prior models. The second column represents their respective inversion results. Clay cap located at a) x=0, b) x=+2500m and c) x=-2500m. The locations of MT receivers are shown as triangles on the surface.

Table 5 is one way to visualize the information posterior calculated in Step 6 (Equation 10). The rows represent the actual or true clay cap location and the columns represent the interpreted locations. The frequency (count per each row-column combination) is calculated and used in the value with imperfect information expression (Equation 11).

Table 5: Information reliability of MT to decipher location of clay cap.

Interpreted locations→ True locations ↓	Interpret Clay Cap at x=+3500m (j=1)	Interpret Clay Cap at x=+3000m (j=2)	…	Interpret Clay Cap at x= -3500m (j=N)
Clay Cap @ x= +3500m (i=1)	$Pr(\mathrm{X}=\mathrm{x}_{\mathrm{i}=1} \mid \tilde{X}=\tilde{x}_{j=1})$	$Pr(\mathrm{X}=\mathrm{x}_{\mathrm{i}=1} \mid \tilde{X}=\tilde{x}_{j=2})$	…	$Pr(\mathrm{X}=\mathrm{x}_{\mathrm{i}=1} \mid \tilde{X}=\tilde{x}_{j=N})$
Clay Cap @ x= +3000m (i=2)	$Pr(\mathrm{X}=\mathrm{x}_{\mathrm{i}=2} \mid \tilde{X}=\tilde{x}_{j=1})$	$Pr(\mathrm{X}=\mathrm{x}_{\mathrm{i}=2} \mid \tilde{X}=\tilde{x}_{j=2})$	…	$Pr(\mathrm{X}=\mathrm{x}_{\mathrm{i}=2} \mid \tilde{X}=\tilde{x}_{j=N})$
⋮				
Clay Cap @ x= -3500m (i=N)	$Pr(\mathrm{X}=\mathrm{x}_{\mathrm{i}=\mathrm{N}} \mid \tilde{X}=\tilde{x}_{j=1})$	$Pr(\mathrm{X}=\mathrm{x}_{\mathrm{i}=\mathrm{N}} \mid \tilde{X}=\tilde{x}_{j=2})$	…	$Pr(\mathrm{X}=\mathrm{x}_{\mathrm{i}=\mathrm{N}} \mid \tilde{X}=\tilde{x}_{j=N})$

The value with imperfect information V_{II} (Equation 11) uses the information posterior as a "misinterpretation rate," accounting for how frequently the interpretation of the MT data may correctly or incorrectly locate the clay cap. With this interpretation of the clay location $\tilde{x}_j$ from the information, the alternative with the highest outcome can be selected (represented by the $\max_{a}$). This is calculated for every possible interpretation (index j) and these are weighted by the data marginal, $\Pr(\tilde{X} = \tilde{x}_j)$, which accounts for how often that interpretation may occur. The actual reliability values for this example are shown in Figure 9.

	~x = -3500	~x = -3000	~x = -2500	~x = -2000	~x = -1500	~x = -1000	~x = -500	~x = 0	~x = 500	~x = 1000	~x = 1500	~x = 2000	~x = 2500	~x = 3000	~x = 3500
x = -3500	71.4%	28.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
x = -3000	0.0%	37.5%	62.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
x = -2500	0.0%	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
x = -2000	0.0%	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
x = -1500	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
x = -1000	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
x = -500	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
x = 0	0.0%	0.0%	0.0%	0.0%	0.0%	28.6%	0.0%	42.9%	28.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
x = 500	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
x = 1000	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	33.3%	0.0%	66.7%	0.0%	0.0%	0.0%	0.0%	0.0%
x = 1500	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	16.7%	83.3%	0.0%	0.0%	0.0%	0.0%
x = 2000	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%
x = 2500	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	57.1%	42.9%	0.0%
x = 3000	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%
x = 3500	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	16.7%	0.0%	83.3%

Figure 9: Reliability for example problem. Each row represents actual or true clay cap (prior model) and the columns represent how frequently that inverted clay cap was interpreted at different locations (represented by symbol ~x). Thus, each row sums to 100%.

Two V_{II} measures can be calculated using the two value outcome matrices of Figure 6 and Figure 7. These are shown in Table 6. As expected, both V_{II}'s are lower than V_{PI} of $500,000 (Table 4). Also, V_{II} is lower when the less "forgiving" value outcome matrix (Figure 6) is used. When the interpreted location doesn't match the actual location, this will result in larger losses and consequently a lower V_{II} compared to the case when Figure 7 is used.

Table 6: Two V_{II} results from two different value outcome matrices (Figure 6 and Figure 7)

	$v_a(x_i)$: Gains drop quickly (Figure 6)	$v_a(x_i)$: Gains drop off slowly (Figure 7)
V_{II}	$410,666.67	$478,560.00

4. Value of Imperfect Information Results

Now the value *with* imperfect information, V_{II}, can be put into the VOI equation (equation 1) to calculate the value *of* imperfect information $VOI_{imperfect}$:

$$VOI_{II} = V_{II} - V_{prior} \tag{13}$$

Six different $VOI_{imperfect}$'s are calculated using the previous V_{prior} (Table 3) and the two V_{II}'s (Table 6). These are shown in Table 7. The value of imperfect information is highest ($410K) when the prior uncertainty of the clay cap is highest (uniform prior and Gaussian with σ^2=1800) and the penalties for drilling far from the clay cap are harsher (Figure 6). This is intuitively rational. Data should have more value when our ignorance is highest and the risk for costly outcomes to decisions is greater. Conversely, the value of imperfect information is lowest ($71K) when the prior uncertainty reflects the high confidence in where the clay cap is (Gaussian with σ^2=900) and drilling far from the actual clay cap doesn't result in a severe economic loss.

Table 7: $VOI_{imperfect}$ Results

$VOI_{imperfect}$	$v_a(x_i)$: Gains drop quickly (Figure 6)	$v_a(x_i)$: Gains drop off slowly (Figure 7)
Uniform Prior	\$410,667 - \$0 = \$410,667	\$478,560 - \$249,867 =\$228,693
Gaussian Prior (μ=0m, σ^2=1800)	\$410,667 - \$0 = \$410,667	\$478,560 - \$324,819 =\$153,741
Gaussian Prior (μ=0m, σ^2=900)	\$410,667 - \$140,859 = \$269,806	\$478,560 - \$406,931 = \$71,628

Accounting for No Resource under Clay Cap

Up until now, we've assumed that a resource does exist under the clay cap: $Pr(\Theta = \theta_{k=1}) = 1$. Now we will account for the occurrence of no resource existing under the clay cap which is represented as the second uncertainty in the decision tree of Figure 3. We link each combination of prior model and decision alternative to two possible value outcomes: the value outcome if there is a resource ($\theta_{k=1}$) or not ($\theta_{k=0}$). The average of the two now replaces the quantity of Equation 3:

$$v_a^{(t)}(x_i) = Pr(\Theta = \theta_{k=0})\, v_a^{(t)}(\theta_{k=0}) + Pr(\Theta = \theta_{k=1}) v_a^{(t)}(\theta_{k=1}) \quad (14)$$
$$a = 1, \dots, N+1 \quad i = 1, \dots, N$$

where $Pr(\Theta = \theta_{k=1,2})$ is the probability of an economic resource existing under the clay cap. For now, we assume that the resistivity structure would remain the same whether a resource exists or not under the clay cap since the clay cap is representative of the historical temperature (see Section 1). Table 8 demonstrates how the value of information decreases with decreasing probability of occurrence of an economic reservoir.

Table 8: $VOI_{imperfect}$ for different probabilities of an economic resource occurring under the clay cap.

$VOI_{imperfect}$ $v_a(x_i)$: Gains drop quickly (Figure 6)	$Pr(\Theta = \theta_{k=1})$ = 1.0	$Pr(\Theta = \theta_{k=1})$ = 0.7	$Pr(\Theta = \theta_{k=1})$ = 0.5	$Pr(\Theta = \theta_{k=1})$= 0.3
Uniform Prior	\$410.67 - \$0 = \$410,667	\$244,992.0 - \$0 = \$197,466	\$55,333 - \$0 = \$55,333	\$0 -\$0 = \$0
Gaussian Prior (μ=0m, σ^2=1800)	\$410,667 - \$0 = \$410,667	\$197,466 - \$0= \$197,466	\$55,333 - \$0 = \$55,333	\$0 -\$0 = \$0
Gaussian Prior (μ=0m, σ^2=900)	\$410,667 - \$140,859 = \$269,806	\$197,466 - \$8,601 = \$188,864	\$55,333 - \$0= \$55,333	\$0 -\$0 = \$0

5. Concluding Remarks & Future Work

Our results show how the value of information depends on four factors.

1) The reliability of the geophysical technique considered.

If we compare Table 4 and Table 7, we see the impact of the "imperfect MT message." Because of inaccuracies introduced from the added noise, inversion, interpretation, and MT's limited resolution, we won't always perfectly identify the clay cap's location. We account for this by estimating the reliability and calculating the value of imperfect information.

2) The description of the prior uncertainty (Figure 5).

Table 3 summarizes the different V_{prior}'s calculated. With greater uncertainty, represented by the uniform distribution, a new source of information has more potential to have value since the V_{prior} is lower.

3) The value outcomes (Figure 6 and Figure 7).

The value outcomes represent the estimated gains and losses due to the combination of the location of the clay cap and the choice of drilling location. The value outcomes of Figure 6 penalize drilling decisions that are far from the actual cap. Therefore, information in this situation will have more value since it can help us avoid costly outcomes. This is seen in Table 7.

4) The strength of the relationship between a clay cap and an economic geothermal reservoir.

The last set of results included the possibility that no resource existed under the clay cap. With smaller probabilities of a resource existing (and thus a smaller chance of a high-valued outcome), the value of information decreases.

For this example, VOI increases when the prior uncertainty is higher and the value outcomes decrease quickly for when one drills far from the target clay cap. It should be noted that these results are highly dependent on the framing of the decision problem. Here we focus on hidden resources and assume that a clay cap is indicative of a possible geothermal source. Many more geothermal possibilities could be included, such as a low-enthalpy system, in which there would be no clay cap. Future work will include more complex prior models by adding more heterogeneity to the prior models to mimic the structure seen in Figure 2.

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