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S. Huang, P. Chou

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Song T. Huang and Philip Chou
Lawrence Livermore National Laboratory
P. O. Box 808, Livermore, CA 94551
huang3@llnl.gov

ABSTRACT

This paper presents a way of looking at fissile systems by using parameters that are invariant under the density law transformation. Since the neutron transport equation and the diffusion equation are invariant under the density law transformation, the approach of using density law invariant parameters should offer better physical insights to a criticality safety problem. Two examples were used to illustrate this approach; the use of nonleakage fraction and the use of number of neutron mean free paths to characterize a neutron problem for criticality safety applications. Because of the neutron transport process, the density law gives us certain advantages in simplifying the physical problems to assess criticality issues. A criticality safety engineer may want to utilize these parameters in addition to other parameters such as k_{eff} , neutron spectrum, etc., as the tools in the tool box for criticality safety assessment.

Key Words: Criticality Safety Methods

1 INTRODUCTION

The complexity of neutron transport in terms of space, time, and neutron energy dependence is well recognized. Through the years, there have been many attempts to utilize various parameters to characterize a neutron system to help nuclear engineers to understand various applications. Parameters such as k_{eff}, neutron spectrum, average energy of neutrons causing fissions, and others are used to size up the types of the problems in a particular application. From a neutron physics viewpoint, not all the parameters used are of the same usefulness.

2 DESCRIPTION OF THE WORK

It is interesting to point out that under the density law transformation [1,2], some parameters offer better physics insight than others. For example, parameters such as the number of mean free paths, nonleakage fraction, and surface mass density, are invariant under the density law transformation. Actually, the transport equation and the diffusion equation are invariant under the density law transformation. This means if we use parameters, which are invariant under the density law, we should have a better understanding of the neutron physics for a problem. The density law is an inherent property of the neutron transport process. Given the complexity of the neutron transport process, the density law gives us a certain physics insight that is very helpful for practitioners in the criticality safety field. With this in mind, this paper presents another way of looking at fissile systems by using parameters that are invariant under the density law.

3 RESULTS

3.1 Use of the Nonleakage Fraction (or the Leakage Parameter)

In reactor physics and in criticality applications, various approaches are used to dissect a problem into the geometry and material perspectives. For example, we use B_g^2 for geometric buckling and B_m^2 for material buckling in a way to help us understand what parameters are in the material side and what parameters in the geometry side and how they are related to criticality assessment. As a matter of fact, this approach is very powerful in developing many hand calculation methods such as the J. Thomas's limiting surface density method [3].

In an infinite medium problem, there is no neutron leakage. A parameter such as k_{inf} and its associated four factor formula are used to explain neutron transport for this type of problems. For a finite neutron system, the neutron leakage plays an important role. For example, it is customary to represent the neutron reproduction factor as follows:

$$\begin{aligned} k_{eff} &= k_{inf} * (\text{Nonleakage Fraction}) \\ &= k_{inf} / (1+M^2B^2) \end{aligned}$$

where M^2 may be interpreted as the migration area and B^2 as the geometric buckling under the modified one group model. Although we illustrate the concept with a modified one group model, the overall concept of nonleakage fraction is independent of the model used as the nonleakage fraction may be obtained by various methods including the Monte Carlo method.

Figure 1 shows that the use of M^2B^2 as a parameter will yield a few additional physics insights than just the k_{eff} value. Thus,

$M^2B^2 = k_{inf} - 1 \rightarrow$ Critical

$M^2B^2 < k_{inf} - 1 \rightarrow$ Supercritical

$M^2B^2 > k_{inf} - 1 \rightarrow$ Subcritical

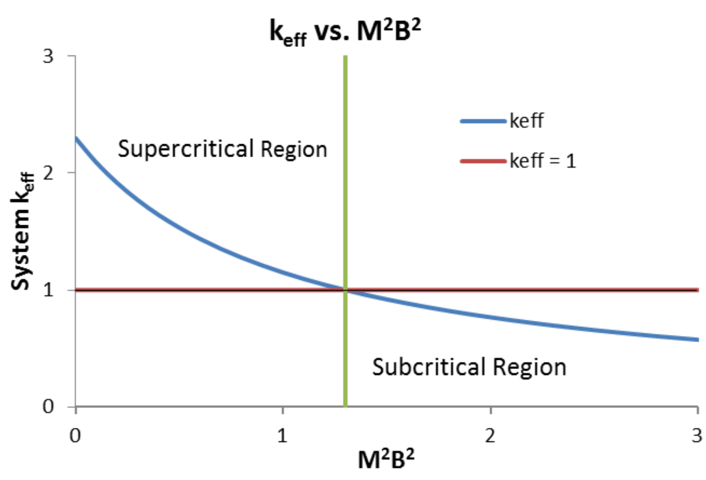

Figure 1. k_{eff} as a function of M^2B^2.

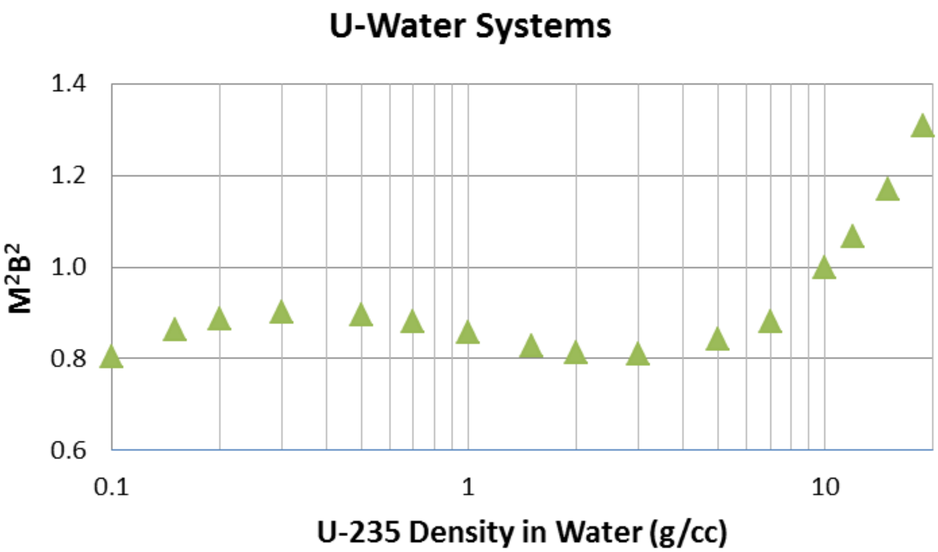

Figure 2. M^2B^2 as a function of the U-235 density in ^{235}U-Water Systems.

Figure 2 shows a M^2B^2 plot against uranium density at critical conditions. For example, the maximum value of k_{inf} of the water moderated uranium systems is about 2.3 at full U-235 density and hence the same systems with M^2B^2 greater than 1.3 are subcritical in general as a first order approximation. This corresponds to a nonleakage fraction less than 0.43 for the subcritical region. Thus, the nonleakage fraction in general or M^2B^2 under the modified one group model provides us with a good parameter to understand the criticality issue. It is noted that there are many water moderated uranium systems with the same M^2B^2 value (smaller M^2 and bigger B^2 or larger M^2 and smaller B^2) which should give the same k_{eff}/k_{inf} ratio. For a given U-235 density in water, the region of M^2B^2 above the M^2B^2-curve in Fig.2 will be subcritical and vice versa. The M^2B^2 parameter gives us additional insights for various systems with the same fissile material regarding neutron leakage.

Figure 3 shows leakage fractions against uranium density at critical conditions. For example, any leakage fraction above the data points indicates subcritical conditions for a given uranium density in water. Similarly, any leakage fraction below the data points indicates supercritical conditions.

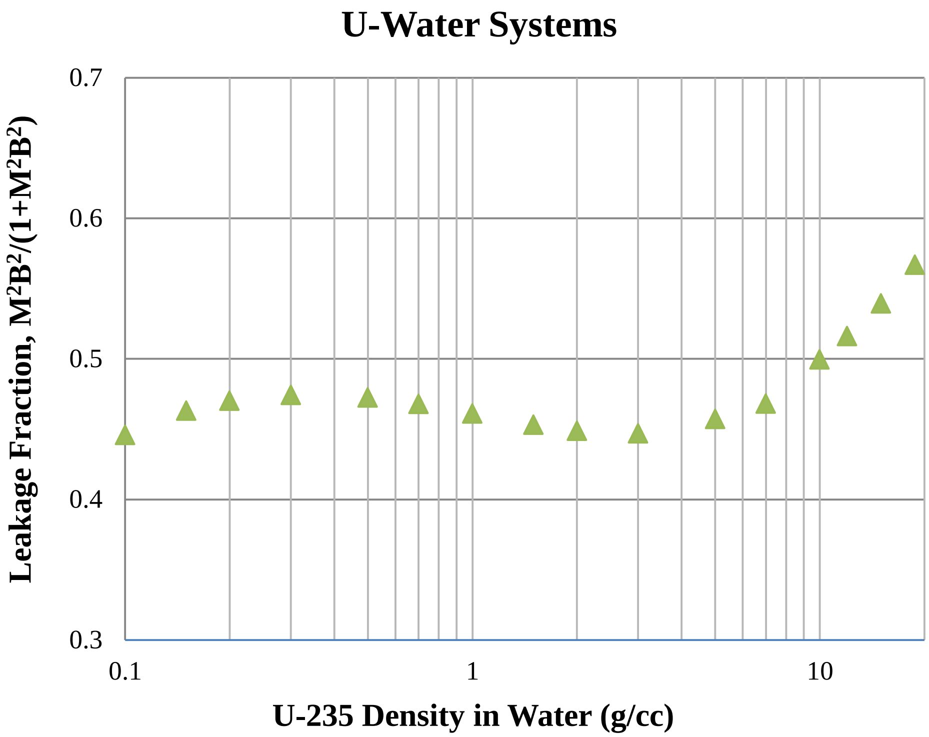

Figure 3. Leakage Fraction as a function of the U-235 density in ^{235}U-Water Systems.

3.2 Use of Number of Neutron Mean Free Paths

In applying the same concept, the use of number of neutron mean free paths to assess criticality safety appears to offer the same benefits. The number of neutron mean free paths for a system is invariant under the density law transformation. For systems with the same material composition, the same number of the mean free path offers very similar neutron physics. Obviously, the length of a neutron mean free path of a system is a material side property but the number of neutron mean free paths of a system depends also upon the geometry side of the system. Of course, the length of mean free path depends upon reaction types and also is neutron energy dependent. Because of the neutron transport process, the density law gives us certain advantages in using the number of the mean free paths to assess criticality issues.

For example, the average escape probability P_0 for the sphere with radius a and the mean free path length *l* is, per the Dirac chord method,

$$P_0 = \left(\frac{3}{8 \cdot \left(a/l \right)^3} \right) \cdot \left(2 \cdot \left(\frac{a}{l} \right)^2 - 1 + \left(1 + \frac{2a}{l} \right) \cdot e^{-\frac{2a}{l}} \right)$$

Since the number of the mean free paths is conserved under the density law, so the average escape probability is also conserved. This means that the number of mean free paths is related to the neutron leakage parameters. Table 1 shows the number of mean free paths versus the average escape probability for our example. It is obvious that a system with a larger number of mean free paths is more reactive than one with a smaller number of mean free paths. It also gives a qualitative sense of the leakage situation as well. Thus, the parameter of the number of mean free paths can give us a first cut understanding about the leakage nature of a problem. Futthermore, it can also be used to distinguish various nuclear systems from a leakage prospective.

Table 1. Number of Mean Free paths versus Average Escape Probability

a/l	1	2	3	4	5
P_0	0.52	0.33	0.23	0.18	0.15

A criticality safety engineer may want to utilize this parameter in addition to other parameters such as k_{eff}, leakage fraction etc., as the tools in the tool box for criticality safety assessment.

4 CONCLUSIONS

The density law offers a few physics insights to the neutron transport process. We have illustrated that the use of parameters which are invariant under the density law offers another interesting way of looking at criticality safety issues. Obviously, because of the complexity of neutron transport process, there is no single silver bullet that resolves all criticality issues. Nevertheless, the use of the number of mean free paths or nonleakage fraction or other parameters from the density law perspective do offer us another venue for helping practitioners deal with never ending varieties of criticality safety applications.

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