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Qualitative Evaluation of the Accuracy of Maps for Release of Hazardous Materials

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Qualitative Evaluation of the Accuracy of Maps for Release of Hazardous Materials

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Abstract

The LinguisticBelief© software tool developed by Sandia National Laboratories was applied to provide a qualitative evaluation of the accuracy of various maps that provide information on releases of hazardous material, especially radionuclides. The methodology, "Uncertainty for Qualitative Assessments," includes uncertainty in the evaluation.

The software tool uses the mathematics of fuzzy sets, approximate reasoning, and the belief/plausibility measure of uncertainty. SNL worked cooperatively with the Remote Sensing Laboratory (RSL) and the National Atmospheric Release Advisory Center (NARAC) at Lawrence Livermore National Laboratory (LLNL) to develop models for three types of maps for use in this study. SNL and RSL developed the maps for "Accuracy Plot for Area" and "Aerial Monitoring System (AMS) Product Confidence". SNL and LLNL developed the "LLNL Model". For each of the three maps, experts from RSL and LLNL created a model in the LinguisticBelief software.

This report documents the three models and provides evaluations of maps associated with the models, using example data. Future applications will involve applying the models to actual graphs to provide a qualitative evaluation of the accuracy of the maps, including uncertainty, for use by decision makers. A "Quality Thermometer" technique was developed to rank-order the quality of a set of maps of a given type. A technique for pooling expert option from different experts was provided using the PoolEvidence© software.

Acknowledgments

This effort was a joint project between the National Nuclear Security Agency's RSL and Sandia National Laboratories. Craig Marianno of RSL was the RSL lead for the effort and assembled the experts from RSL and LLNL to create the models. Brenda Pobanz of LLNL NARAC was the primary contact for the LLNL NARAC model. The assistance of these individuals is greatly appreciated, as is the assistance of the numerous staff members at RSL and LLNL NARAC who participated in the development of the models. Bruce Berry, John Fulton, and Carla Ulibarri of Sandia National Laboratories provided peer review of this report. Diane Ross edited and prepared the document for publication.

Contents

Figures

Executive Summary

This study applies a methodology to qualitatively evaluate the accuracy of various maps that provide information on releases of hazardous material, especially radionuclides. The methodology includes uncertainty in the evaluation. This work was sponsored by the Department of Energy (DOE) and was performed during fiscal year 2008.

The methodology, "Uncertainty for Qualitative Assessments," uses the mathematics of fuzzy sets, approximate reasoning, and the belief/plausibility measure of uncertainty, as implemented in the LinguisticBelief© software tool developed by Sandia National Laboratories.

SNL worked cooperatively with the Remote Sensing Laboratory (RSL) and the National Atmospheric Release Advisory Center (NARAC) at Lawrence Livermore National Laboratory (LLNL) to develop models for three types of maps for use in this study. SNL and RSL developed the maps for "Accuracy Plot for Area" and "Aerial Monitoring System (AMS) Product Confidence". SNL and LLNL developed the "LLNL Model". For each of the three maps, experts from RSL and LLNL attended a three-hour training session on the methodology and then SNL, RSL, and LLNL created the model in the LinguisticBelief software. This report documents the three models.

This report uses example data to show how the accuracy of a given map is evaluated using the model. Future applications will involve application of the models to actual graphs, which will provide a qualitative evaluation of the accuracy of the maps, including uncertainty, for use by decision makers.

A "Quality Thermometer" technique, documented in this report, was developed to rank-order the accuracy of a set of maps of a specific type.

This effort successfully generated qualitative uncertainty models for the three types of graphs, and the models are available for application to actual maps. This report documents how information from more than one expert can be pooled using the PoolEvidence© software developed by Sandia National Laboratories.

Acronyms

AMS	Aerial Monitoring System
DOE	Department of Energy
LLNL	Lawrence Livermore National Laboratory
NARAC	National Atmospheric Release Advisory Center
PAG	Protective Action Guideline
RSL	Remote Sensing Laboratory

1 Introduction

1.1 Background

The Remote Sensing Laboratory (RSL) and the National Atmospheric Release Advisory Center (NARAC) at Lawrence Livermore National Laboratory (LLNL) generate graphical maps that provide information on releases of hazardous material, especially radionuclides.

Sandia National Laboratories has developed a methodology that provides qualitative information, including uncertainty, that can aid decision making. The methodology uses the mathematics of fuzzy sets, approximate reasoning, and the belief/plausibility measure of uncertainty, as implemented in the LinguisticBelief© software tool developed by Sandia National Laboratories. [Methodology] [LinguisticBelief] Qualitative information is addressed using purely linguistic fuzzy sets for variables. Approximate reasoning is used to combine different variables. Belief/Plausibility is a superset of probability that addresses epistemic (state of knowledge) uncertainty; probability is a special case of belief/plausibility.

1.2 Project Overview

Sponsored by the Department of Energy (DOE) for fiscal year 2008, SNL uncertainty analysts applied the methodology, Uncertainty for Qualitative Assessments, to the uncertainty of maps (produced by RSL and LLNL) that show hazardous material releases. There is a need to provide qualitative information on the quality of the maps for decision makers, and this report documents the application of the methodology for providing that information.

Using example data, this report shows how the accuracy of a given map is evaluated using the model; examples for all three models are provided. Future applications will involve application of the models to actual graphs. The application to actual graphs will provide a qualitative evaluation of the accuracy of the maps, including uncertainty, for use by decision makers.

The methodology was applied to three types of maps, all developed by SNL in cooperation with RSL and the LLNL NARAC. One of the RSL maps, “Accuracy Plot for Area”, is associated with information gathered on the ground. The second RSL map, “AMS (Aerial Monitoring System) Product Confidence”, is associated with information gathered from the air. The LLNL NARAC map, called the “LLNL Model”, generates a plume concentration.

To prepare for the map development, experts from RSL and LLNL attended a three-hour training session on the methodology and spent a day creating the model in the LinguisticBelief software. The training session material is documented in SAND2007-6684P. [Training Material] The model for the “Accuracy Plot for Area” map was created at RSL in Las Vegas, NV on January 29 and 30, 2008. The model for the “LLNL Model” was developed at LLNL in Livermore, CA on March 6 and 7, 2008. The model for “AMS Product Confidence” was generated at RSL in Las Vegas, NV on July 1 and 2, 2008.

1.3 Summary of Methodology

This section summarizes the methodology used in this project; the references provide more details. The three mathematical techniques used are:

1. fuzzy sets,
2. approximate reasoning, and
3. the belief/plausibility measure of uncertainty.

Fuzzy sets are appropriate for describing vague concepts, such as a person's height as "Tall" or a day as "Sunny". Vagueness means that the membership of a given element may not be totally in (or not in) a given fuzzy set. For example, a man who is 6 feet tall may have partial membership in the "Tall" fuzzy set, and partial membership in the "Medium" fuzzy set. Fuzzy sets are sometimes fuzzy numbers in that they describe a numerical variable; however, in general, a fuzzy set is just a linguistic (word) and does not have to be associated with a numerical variable. For example, "Happiness" may be described by the fuzzy sets "Very Happy", "Accepting", and Depressed". It is misleading to force a numerical scale on "Happiness. Is the scale [0, 10], or $[0, 10^6]$, or $[-13, 42^{1/2}]$? The purely linguistic fuzzy sets describe the variable more appropriately than any arbitrary numerical scale.

Variables described by linguistic fuzzy sets can be combined using approximate reasoning. Approximate reasoning is a rule base that specifies the fuzzy sets for a variable in terms of the fuzzy sets of its constituent variables. For example, we may reason on "Happiness" as a combination of "Quality of Life" and "Outlook on Life". "Quality of Life" may be described by the fuzzy sets "Not so Good" and "Good"; "Outlook on Life" may be described by the fuzzy sets "Optimist" and "Pessimist". An example rule base for approximate reasoning for "Happiness" is given in Figure 1-1; this figure was generated using the LinguisticBelief code.

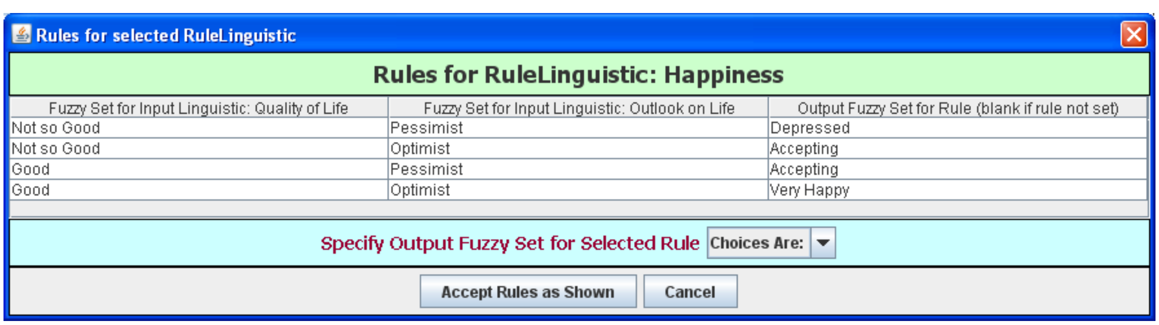
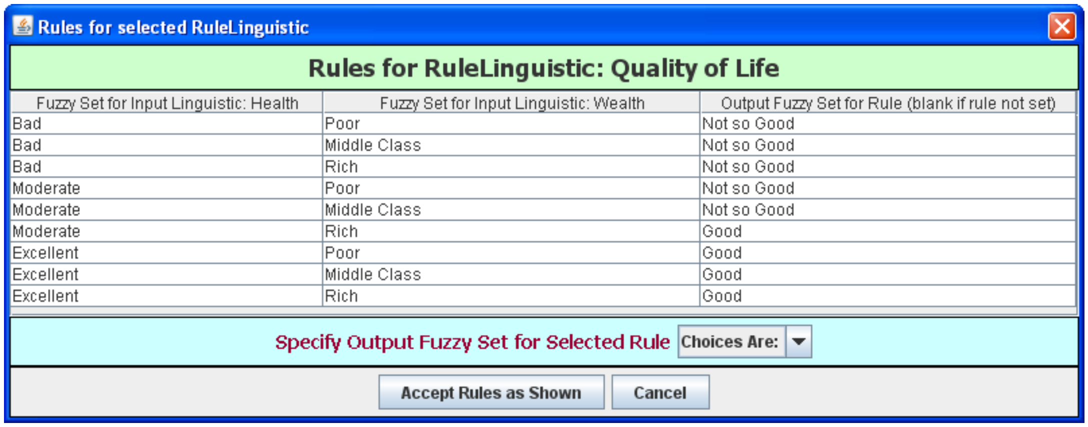


Figure 1-1. Approximate Reasoning Rule Base for "Happiness"

Similarly, “Quality of Life” can be modeled as a combination of “Health” and “Wealth” as indicated in Figure 1-2.



Fuzzy Set for Input Linguistic: Health	Fuzzy Set for Input Linguistic: Wealth	Output Fuzzy Set for Rule (blank if rule not set)
Bad	Poor	Not so Good
Bad	Middle Class	Not so Good
Bad	Rich	Not so Good
Moderate	Poor	Not so Good
Moderate	Middle Class	Not so Good
Moderate	Rich	Good
Excellent	Poor	Good
Excellent	Middle Class	Good
Excellent	Rich	Good

Figure 1-2. Approximate Reasoning Rule Base for “Quality of Life”

Belief/plausibility is a measure of uncertainty that is a superset of the probability measure of uncertainty. Belief/plausibility allows consideration of epistemic (state of knowledge) uncertainty, while probability focuses on aleatory (random) uncertainty. For, example a fair coin has aleatory uncertainty in that the likelihood of heads is not known with certainty; it has a probability of ½, as does tails. However, if we cannot examine the coin it may be biased heads or may be two-headed. We have state-of-knowledge uncertainty about the coin that is not random at all; the coin is either fair, biased, or two-headed (or two-tailed); we just do not know. Belief/plausibility considers epistemic uncertainty by providing lower and upper bounds on probability, called belief and plausibility, respectively. The unknown coin is an example of total ignorance; the belief that the coin will be heads (or tails) is 0.0 and the plausibility that the coin will be heads (or tails) is 1.0. With no epistemic uncertainty, belief and plausibility both reduce to the single measure “probability”.

Belief/plausibility can be calculated based on the evidence assigned over the sample space for a variable. The sample space can be linguistic fuzzy sets. For example, for our earlier example, we may assign evidence for “Health” as indicated in Figure 1-3.

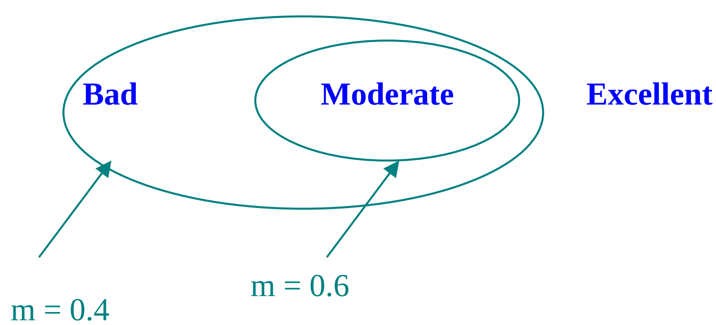


Figure 1-3. Evidence for “Health”

Subsets of the sample space with evidence are called focal elements. Evidence is denoted by "m". In Figure 1-3 we have two focal elements: {Bad, Moderate} with evidence 0.4 and {Moderate} with evidence 0.6. For any subset A, the belief and plausibility of A can be evaluated from the focal elements as follows:

$$Bel(A) = \sum_{B|B\subseteq A} m(B)$$
$$Pl(A) = \sum_{B|A\cap B\neq 0} m(B) \quad (Eqn.1.1)$$

where B is a focal element.

For example, the subset {Moderate} has belief 0.6 and plausibility 1.0.

If all the focal elements are singletons–that is, each subset B with evidence has only one element–both belief and plausibility are the same, the probability. If all the evidence is assigned to one element of the sample space, there is no uncertainty; the probability of that element is 1.0.

Figure 1-4 is an example of the assignment of evidence in Figure 1-3 in the LinguisticBelief code.

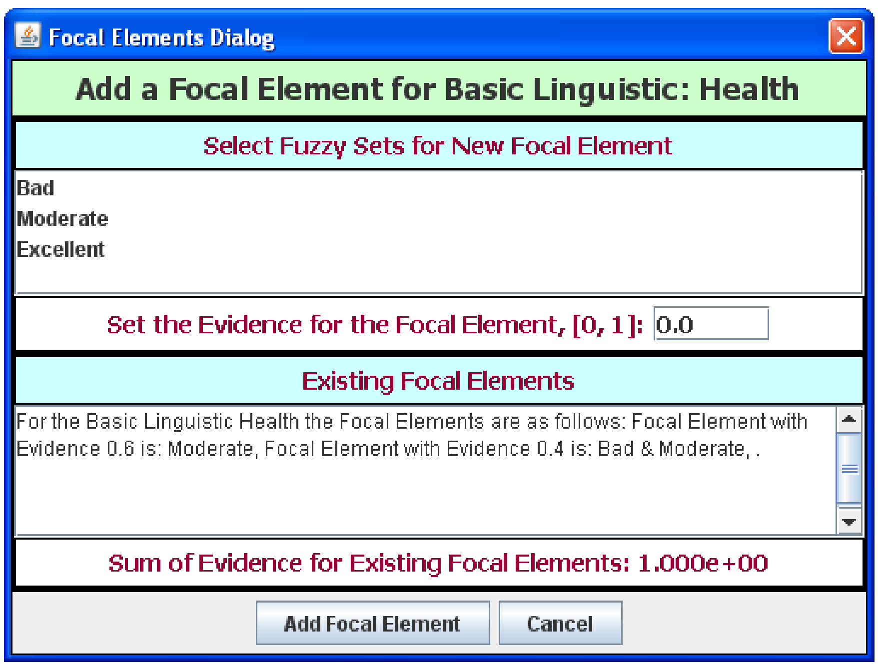


Figure 1-4. Evidence for "Health" in LinguisticBelief

Figure 1-5 is the result for “Health” calculated by LinguisticBelief.

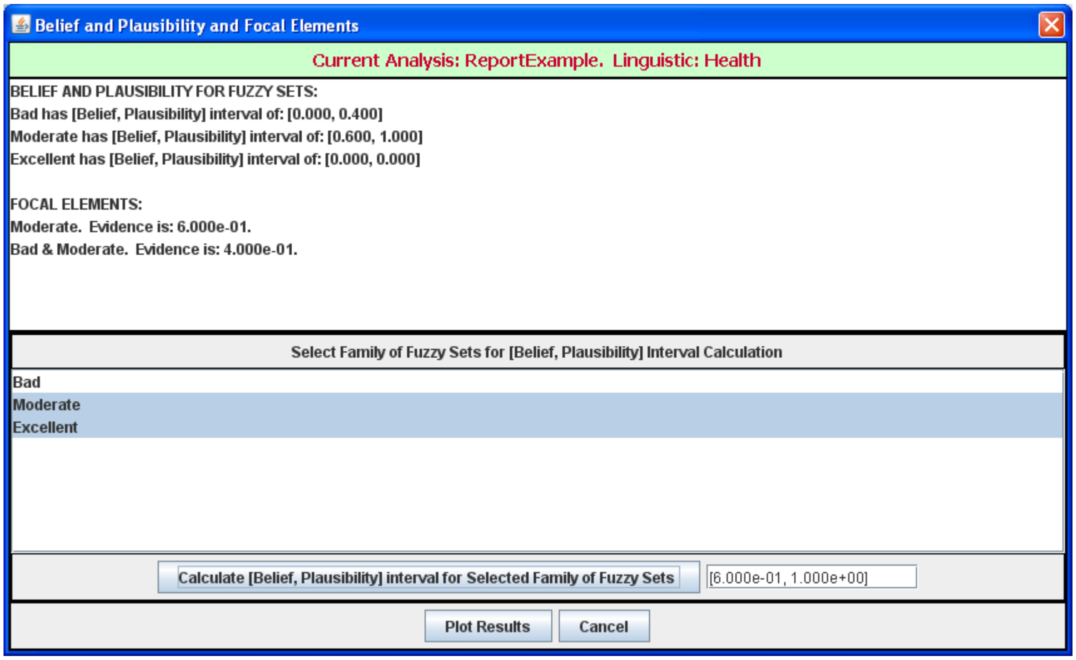


Figure 1-5. Belief/Plausibility for “Health” in LinguisticBelief

Figure 1-6 summarizes the results for “Health” in graphical form.

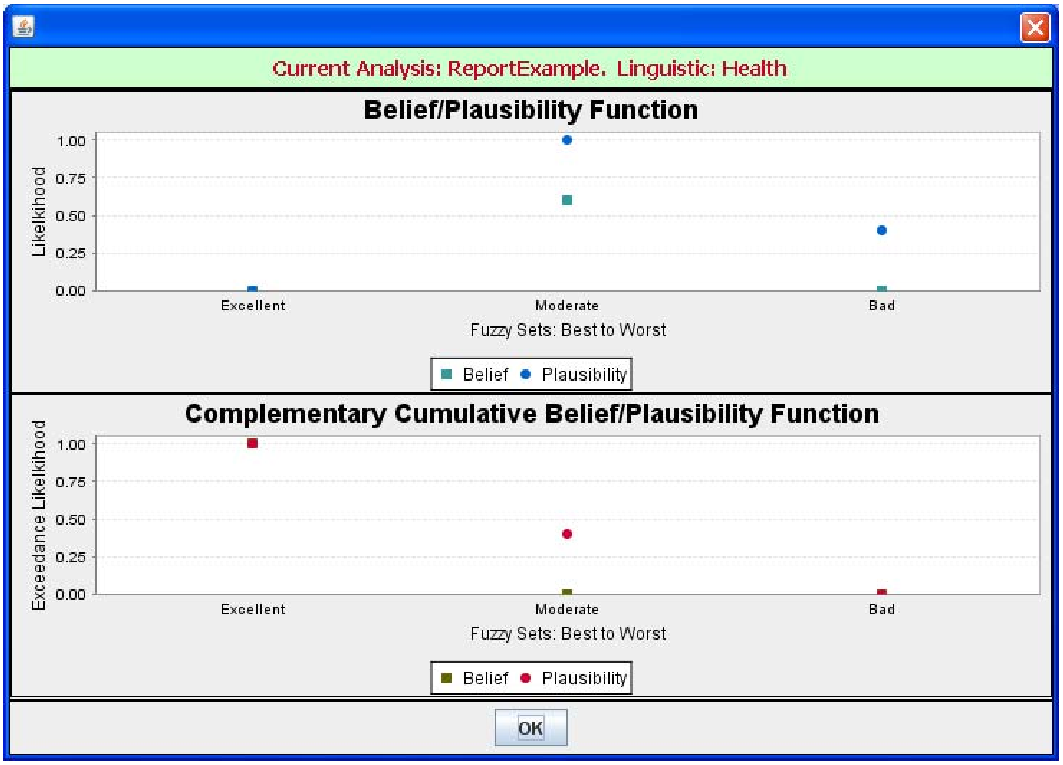


Figure 1-6. Graphical Result for “Health” in LinguisticBelief

Note that the graphical result contains two plots. The top plot is the likelihood of each fuzzy set for the variable of concern; this is the belief/plausibility function. It is an extension of the probability density function for a probability measure extended to belief/plausibility over fuzzy sets. The bottom plot is the likelihood of *exceedance* for each fuzzy set for the variable of concern; this is the complementary cumulative belief/plausibility function. It is an extension of the complementary cumulative probability distribution function for a probability measure extended to belief/plausibility over fuzzy sets. For example, the likelihood that "Health" exceeds "Excellent" (and is either "Moderate" or "Bad") in Figure 1-6 is the belief/plausibility interval 0/0.4. The likelihood that "Health" exceeds "Bad" is always 0.0.[1]

LinguisticBelief models a set of variables as an approximate reasoning rule base, using user-defined rules. Each variable is defined by linguistic fuzzy sets. For a given set of evidence, belief/plausibility is propagated up the rule base. The result is the belief/plausibility for the "top" variable (or any other variable). For example, the belief/plausibility for "Happiness" calculated using LinguisticBelief using example evidence is given in Figure 1-7.

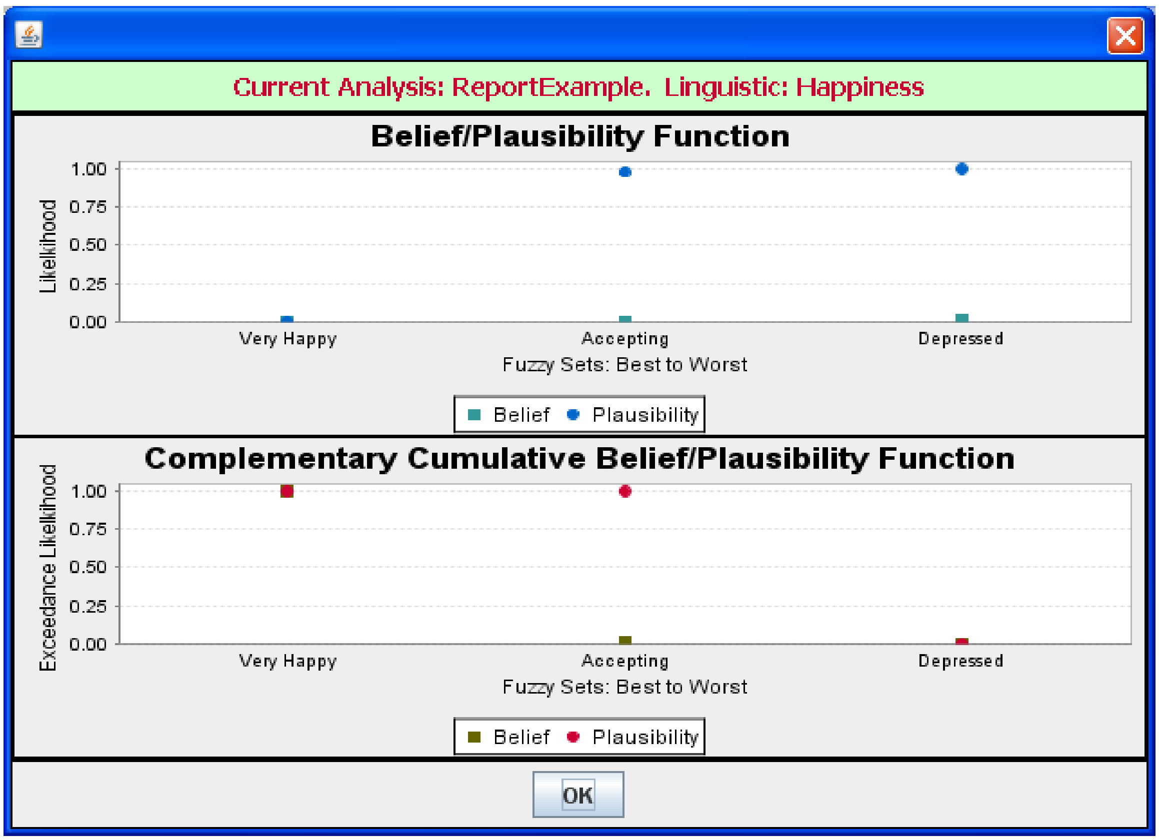


Figure 1-7. Graphical Result for "Happiness" in LinguisticBelief

[1] Following traditional convention, the cumulative distribution is the likelihood of being less than or equal to a value; the complementary cumulative distribution is the likelihood of exceeding a value.

These techniques as implemented in the LinguisticBelief code were applied to develop models for the qualitative uncertainty in the types of maps of interest as described in the following sections of this report.

In application, different experts may assign different evidence over the fuzzy sets for a given variable. The PoolEvidence© software tool can be used to pool the evidence for use in LinguisticBelief. This capability is also addressed in the following sections.

2 RSL Models

2.1 Accuracy Plot for Area

The top-level variable "Accuracy Plot for Area" is composed of the following three input variables:

- "***Quality Measurements***", which addresses the accuracy of the measurements over the area where the measurements were taken. This variable is composed of the variables: "Who Measured" and "Radionuclide ID".
- "***Measurement Conditions***", which also addresses the accuracy of the measurements over the area where the measurements were taken. This variable is composed of the variables: "Type Radiation", "Weather", and "Surface Type for Measurement".
- "***Extension to Desired Area***", which addresses the accuracy to which the measurements can be extended to the area of interest for the appropriate map. "Extension to Desired Area" is a basic variable.

The model for "Accuracy Plot for Area" in the LinguisticBelief software is shown in Figure 2-1. The figure indicates the structure of the variables, rule-based and basic, in the model. Each of the rule-based variables is composed of other variables as subsequently discussed. (The right side panel in the figure provides information about the state of the selected node in the code. Here, the selected node is the name of the current analysis. If other nodes are selected for variables, the information panel summarizes all the information about that variable: fuzzy sets, focal elements if a basic variable, inputs if a rule variable, and belief/plausibility.)

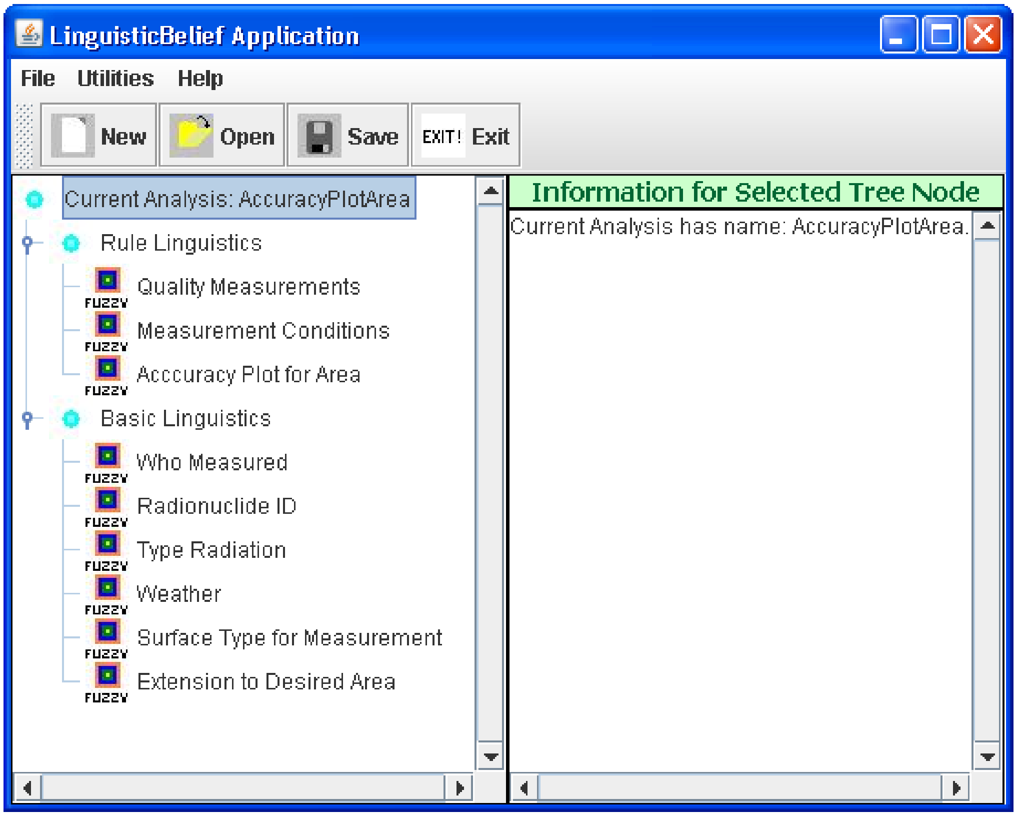
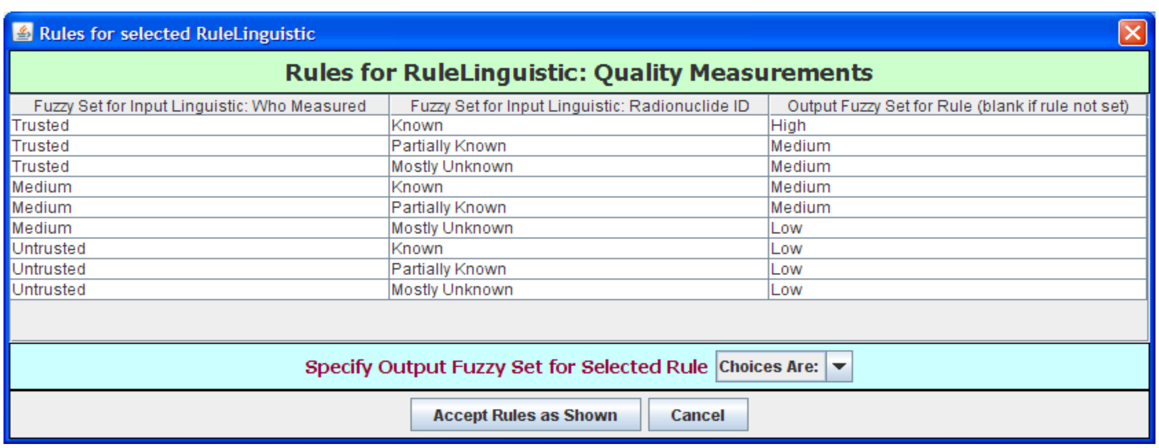


Figure 2-1. Model for "AccuracyPlotArea" in LinguisticBelief

The approximate reasoning rule base for each rule-based variable is given in Figures 2-2 through 2-4. The fuzzy sets for each variable (rule-based and basic) are as indicated in these figures. The fuzzy sets were selected by the RSL team.


Rules for selected RuleLinguistic

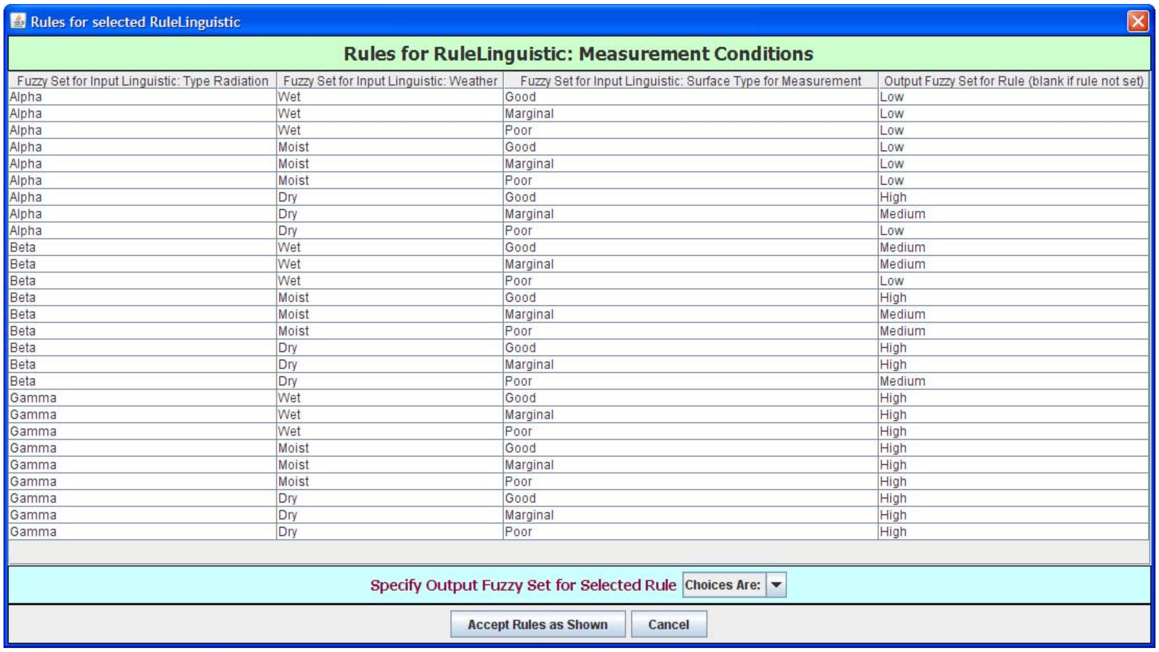
Rules for RuleLinguistic: Quality Measurements

Fuzzy Set for Input Linguistic: Who Measured	Fuzzy Set for Input Linguistic: Radionuclide ID	Output Fuzzy Set for Rule (blank if rule not set)
Trusted	Known	High
Trusted	Partially Known	Medium
Trusted	Mostly Unknown	Medium
Medium	Known	Medium
Medium	Partially Known	Medium
Medium	Mostly Unknown	Low
Untrusted	Known	Low
Untrusted	Partially Known	Low
Untrusted	Mostly Unknown	Low

Specify Output Fuzzy Set for Selected Rule Choices Are:

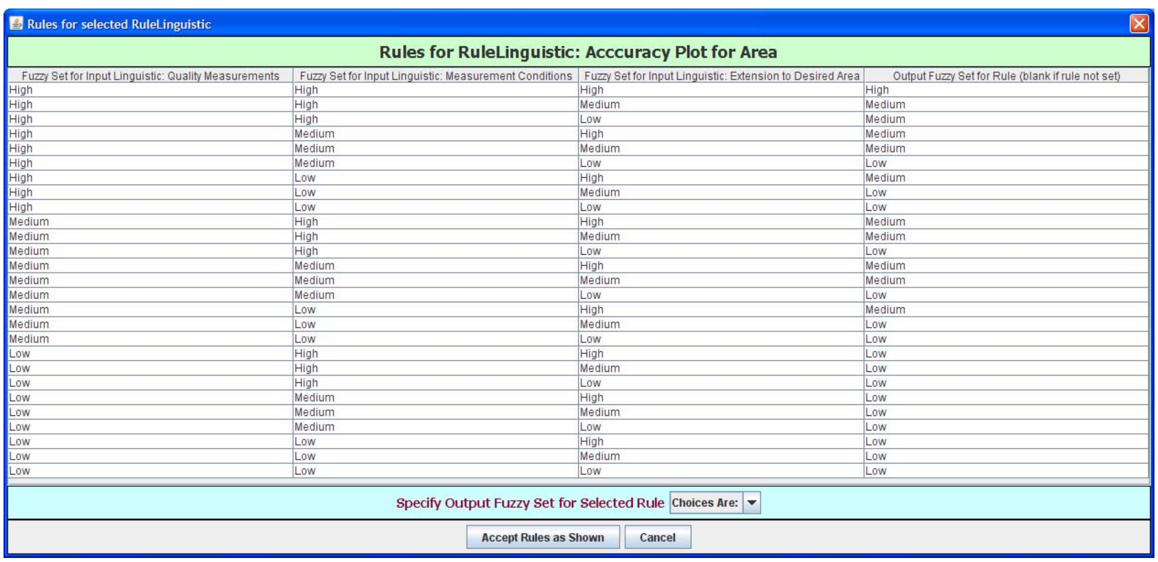
Accept Rules as Shown Cancel

Figure 2-2. Rule Base for "Quality Measurements"



Fuzzy Set for Input Linguistic: Type Radiation	Fuzzy Set for Input Linguistic: Weather	Fuzzy Set for Input Linguistic: Surface Type for Measurement	Output Fuzzy Set for Rule (blank if rule not set)
Alpha	Wet	Good	Low
Alpha	Wet	Marginal	Low
Alpha	Wet	Poor	Low
Alpha	Moist	Good	Low
Alpha	Moist	Marginal	Low
Alpha	Moist	Poor	Low
Alpha	Dry	Good	High
Alpha	Dry	Marginal	Medium
Alpha	Dry	Poor	Low
Beta	Wet	Good	Medium
Beta	Wet	Marginal	Medium
Beta	Wet	Poor	Low
Beta	Moist	Good	High
Beta	Moist	Marginal	Medium
Beta	Moist	Poor	Medium
Beta	Dry	Good	High
Beta	Dry	Marginal	High
Beta	Dry	Poor	Medium
Gamma	Wet	Good	High
Gamma	Wet	Marginal	High
Gamma	Wet	Poor	High
Gamma	Moist	Good	High
Gamma	Moist	Marginal	High
Gamma	Moist	Poor	High
Gamma	Dry	Good	High
Gamma	Dry	Marginal	High
Gamma	Dry	Poor	High

Figure 2-3. Rule Base for "Measurement Conditions"



Fuzzy Set for Input Linguistic: Quality Measurements	Fuzzy Set for Input Linguistic: Measurement Conditions	Fuzzy Set for Input Linguistic: Extension to Desired Area	Output Fuzzy Set for Rule (blank if rule not set)
High	High	High	High
High	High	Medium	Medium
High	High	Low	Medium
High	Medium	High	Medium
High	Medium	Medium	Medium
High	Medium	Low	Low
High	Low	High	Medium
High	Low	Medium	Low
High	Low	Low	Low
Medium	High	High	Medium
Medium	High	Medium	Medium
Medium	High	Low	Low
Medium	Medium	High	Medium
Medium	Medium	Medium	Medium
Medium	Medium	Low	Low
Medium	Low	High	Medium
Medium	Low	Medium	Low
Medium	Low	Low	Low
Low	High	High	Low
Low	High	Medium	Low
Low	High	Low	Low
Low	Medium	High	Low
Low	Medium	Medium	Low
Low	Medium	Low	Low
Low	Low	High	Low
Low	Low	Medium	Low
Low	Low	Low	Low

Figure 2-4. Rule Base for "Accuracy Plot for Area"

Using the model, the qualitative uncertainty for a *specific* map can be evaluated by assigning evidence to the focal elements for each of the six basic variables in the model.

Figure 2-5 is an example of assignment of evidence to the basic variable "Extension to Desired Area". This example uses dummy evidence to illustrate the technique.

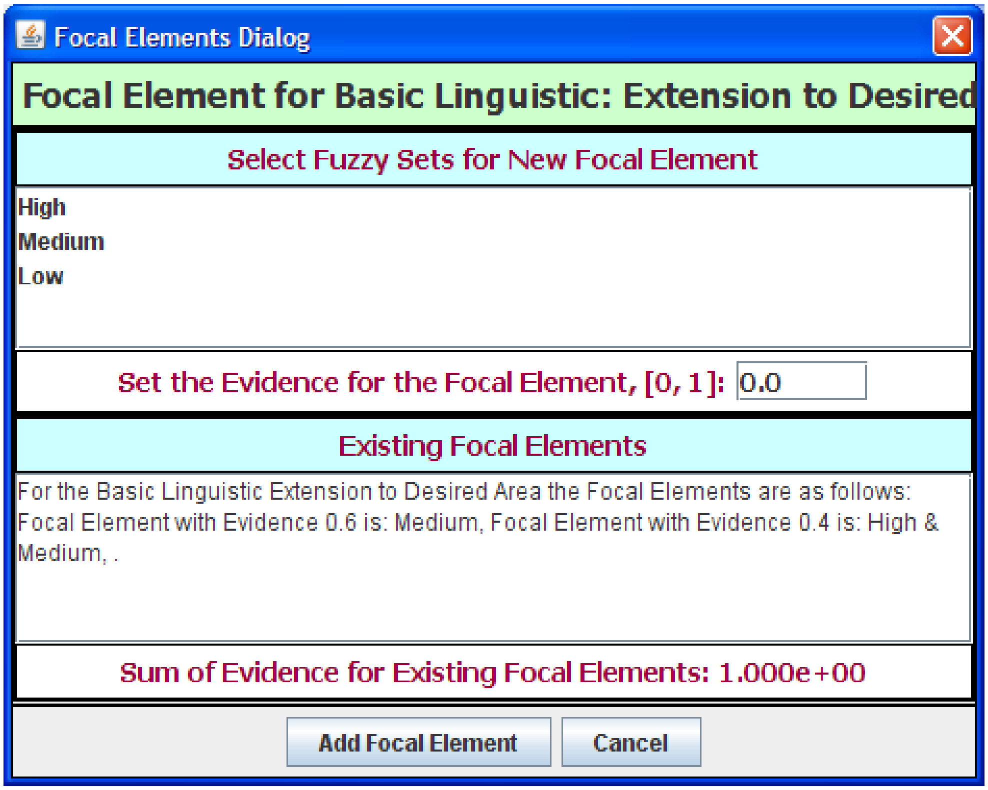


Figure 2-5. Example Body of Evidence for “Extension to Desired Area”

Once evidence has been assigned to each basic variable, the uncertainty for any variable (basic or rule-based) can be evaluated. Figure 2-6 shows the uncertainty for the basic variable “Extension to Desired Area”.

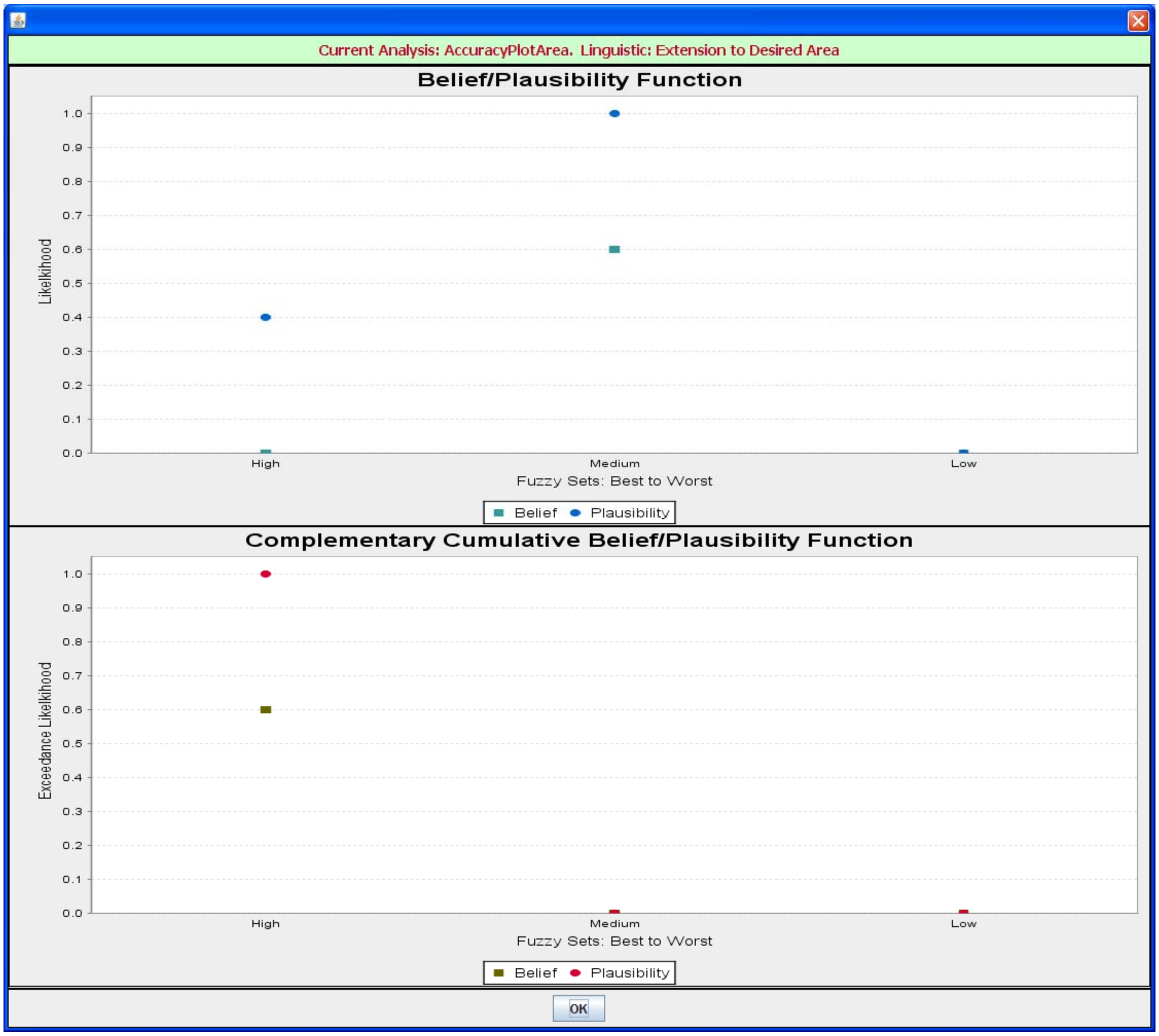


Figure 2-6. Uncertainty for “Extension to Desired Area”

Using example data for all the basic variables, Figure 2-7 shows the uncertainty for the top level, rule-based variable “Accuracy Plot for Area”.

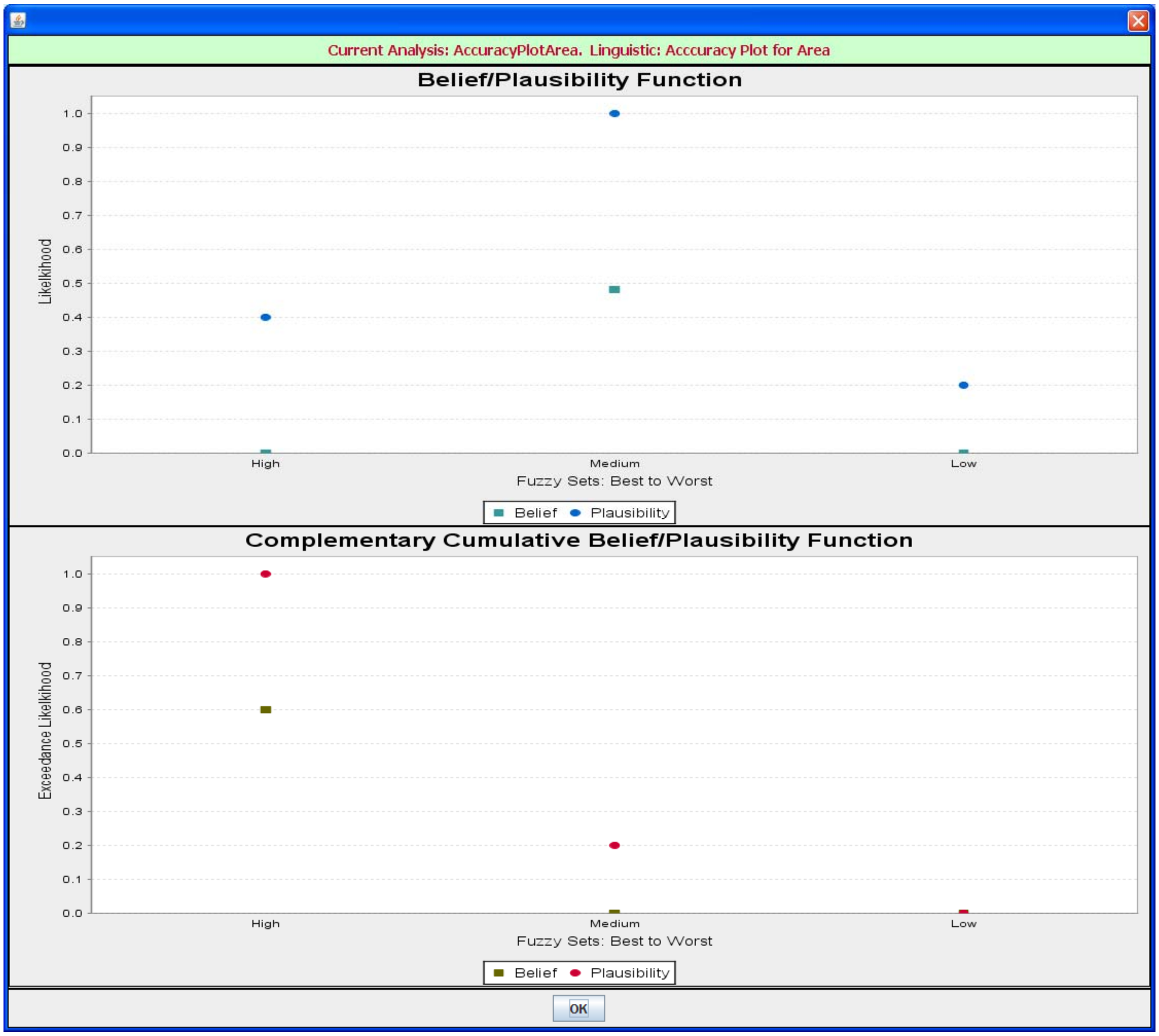


Figure 2-7. Uncertainty for "Accuracy Plot for Area"

The uncertainty graphs for a given variable provide the result for that variable. For example, assuming the evidence used in this example Figure 2-7 summarizes the accuracy of a given map as the uncertainty in the variable "Accuracy Plot for Area". For this map, based on the upper graph the likelihood that the accuracy is "High" is between 0 and 0.4, the likelihood that the accuracy is "Medium" is between about 0.5 and 1.0, and the likelihood that the accuracy is "Low" is between 0 and 0.2. Based on the lower graph, the likelihood that the accuracy of the map is worse than "High" is between 0.6 and 1.0. Therefore, this graph is not likely to be of "High" accuracy.

Section 4 discusses a technique for summarizing results for a variable and comparing different maps using a "Quality Thermometer".

Appendix A provides all the input data and results for an example evaluation for "Accuracy Plot for Area".

2.2 AMS Product Confidence

The top-level variable "Accuracy of PAG Map" (PAG is "Protective Action Guideline") is composed of the following two input variables:

- "***Accuracy of Count Rate at Platform***", which addresses the accuracy of the measurements obtained by the airborne platform (fixed wing or helicopter mounted). This variable is composed of the basic variables: "Who Took Measurement" and "Accuracy Count Rate above Background for Radionuclide ID".
- "***Extrapolation to Ground Measurement***", which addresses the accuracy to which the measurements can be extended to evaluate the radionuclides on the ground. This variable is composed of the basic variables: "Fineness of Measurement Grid" and "Topography".

Figure 2-8 shows the model for "AMS Product Confidence" in the LinguisticBelief software. The figure indicates the structure of the variables, rule-based and basic, in the model. Each of the rule-based variables is composed of other variables as subsequently discussed.

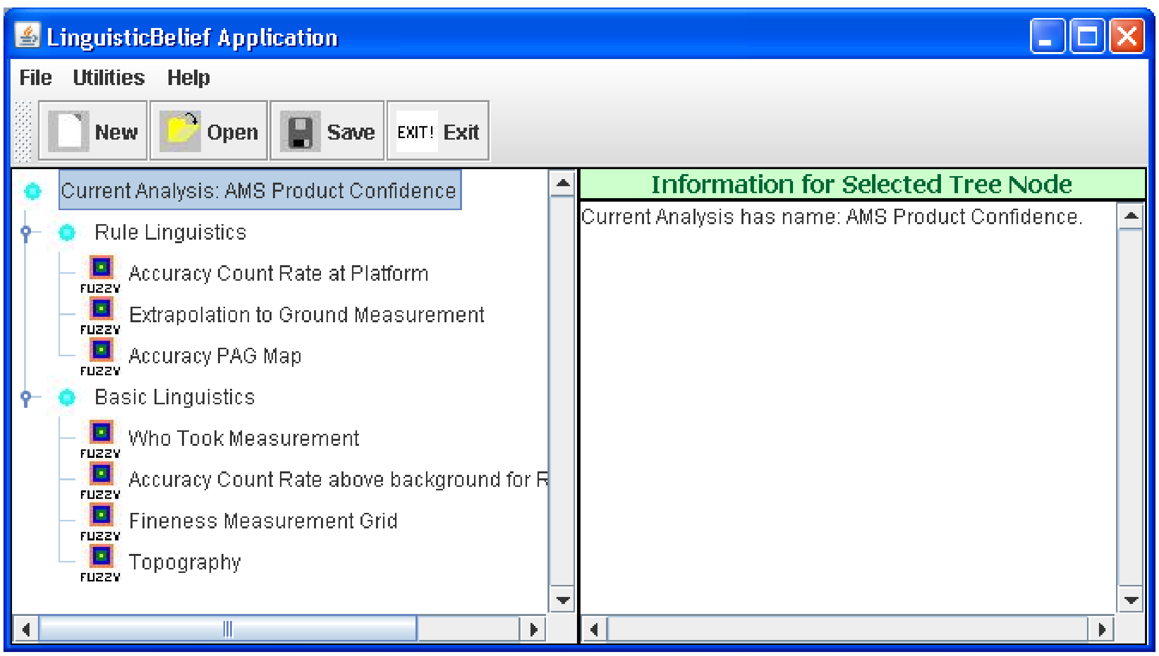
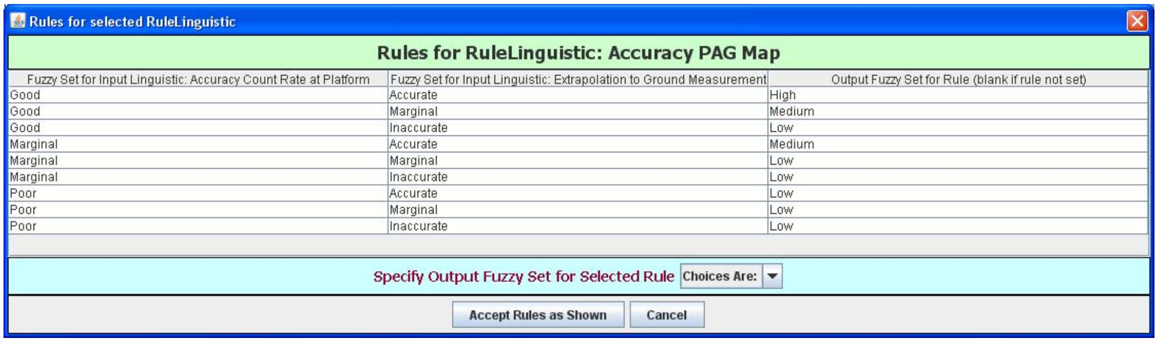


Figure 2-8. Model for "AMS Product Confidence" in LinguisticBelief

The approximate reasoning rule base for each rule-based variable is given in Figures 2-9 through 2-11. The fuzzy sets for each variable (rule-based and basic) are as indicated in these figures. The fuzzy sets were selected by the RSL team.



Rules for selected RuleLinguistic

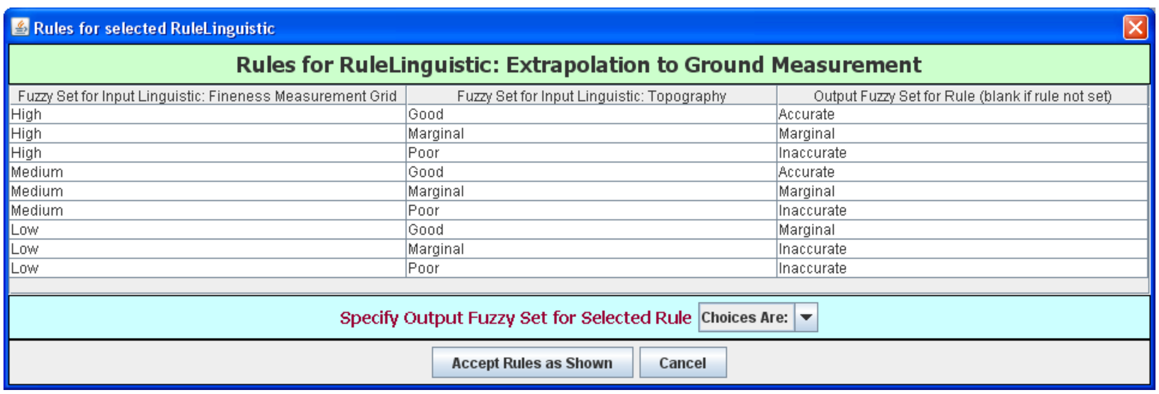
Rules for RuleLinguistic: Accuracy PAG Map

Fuzzy Set for Input Linguistic: Accuracy Count Rate at Platform	Fuzzy Set for Input Linguistic: Extrapolation to Ground Measurement	Output Fuzzy Set for Rule (blank if rule not set)
Good	Accurate	High
Good	Marginal	Medium
Good	Inaccurate	Low
Marginal	Accurate	Medium
Marginal	Marginal	Low
Marginal	Inaccurate	Low
Poor	Accurate	Low
Poor	Marginal	Low
Poor	Inaccurate	Low

Specify Output Fuzzy Set for Selected Rule Choices Are:

Accept Rules as Shown Cancel

Figure 2-9. Rule Base for "Accuracy PAG Map"



Rules for selected RuleLinguistic

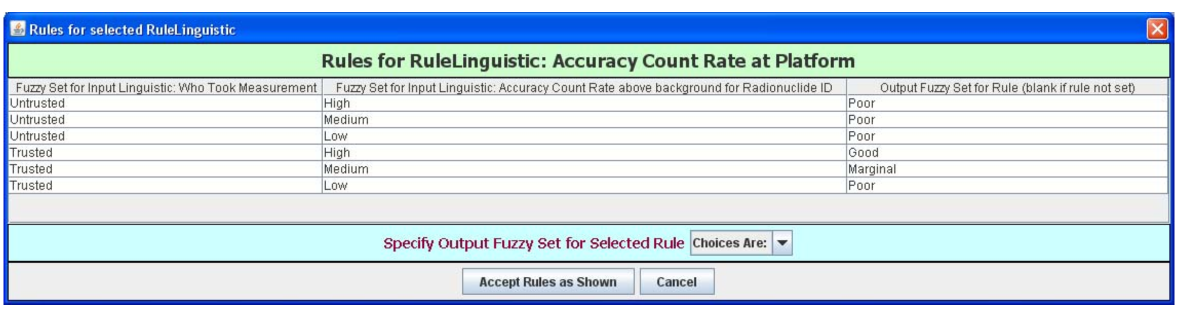
Rules for RuleLinguistic: Extrapolation to Ground Measurement

Fuzzy Set for Input Linguistic: Fineness Measurement Grid	Fuzzy Set for Input Linguistic: Topography	Output Fuzzy Set for Rule (blank if rule not set)
High	Good	Accurate
High	Marginal	Marginal
High	Poor	Inaccurate
Medium	Good	Accurate
Medium	Marginal	Marginal
Medium	Poor	Inaccurate
Low	Good	Marginal
Low	Marginal	Inaccurate
Low	Poor	Inaccurate

Specify Output Fuzzy Set for Selected Rule Choices Are:

Accept Rules as Shown Cancel

Figure 2-10. Rule Base for "Extrapolation to Ground Measurement"



Rules for selected RuleLinguistic

Rules for RuleLinguistic: Accuracy Count Rate at Platform

Fuzzy Set for Input Linguistic: Who Took Measurement	Fuzzy Set for Input Linguistic: Accuracy Count Rate above background for Radionuclide ID	Output Fuzzy Set for Rule (blank if rule not set)
Untrusted	High	Poor
Untrusted	Medium	Poor
Untrusted	Low	Poor
Trusted	High	Good
Trusted	Medium	Marginal
Trusted	Low	Poor

Specify Output Fuzzy Set for Selected Rule Choices Are:

Accept Rules as Shown Cancel

Figure 2-11. Rule Base for "Accuracy Count Rate at Platform"

Using the model, the qualitative uncertainty for a specific map can be evaluated by assigning evidence to the focal elements for each of the six basic variables in the model.

Figure 2-12 is an example of assignment of evidence to the basic variable "Topography".

This example uses dummy evidence to illustrate the technique.

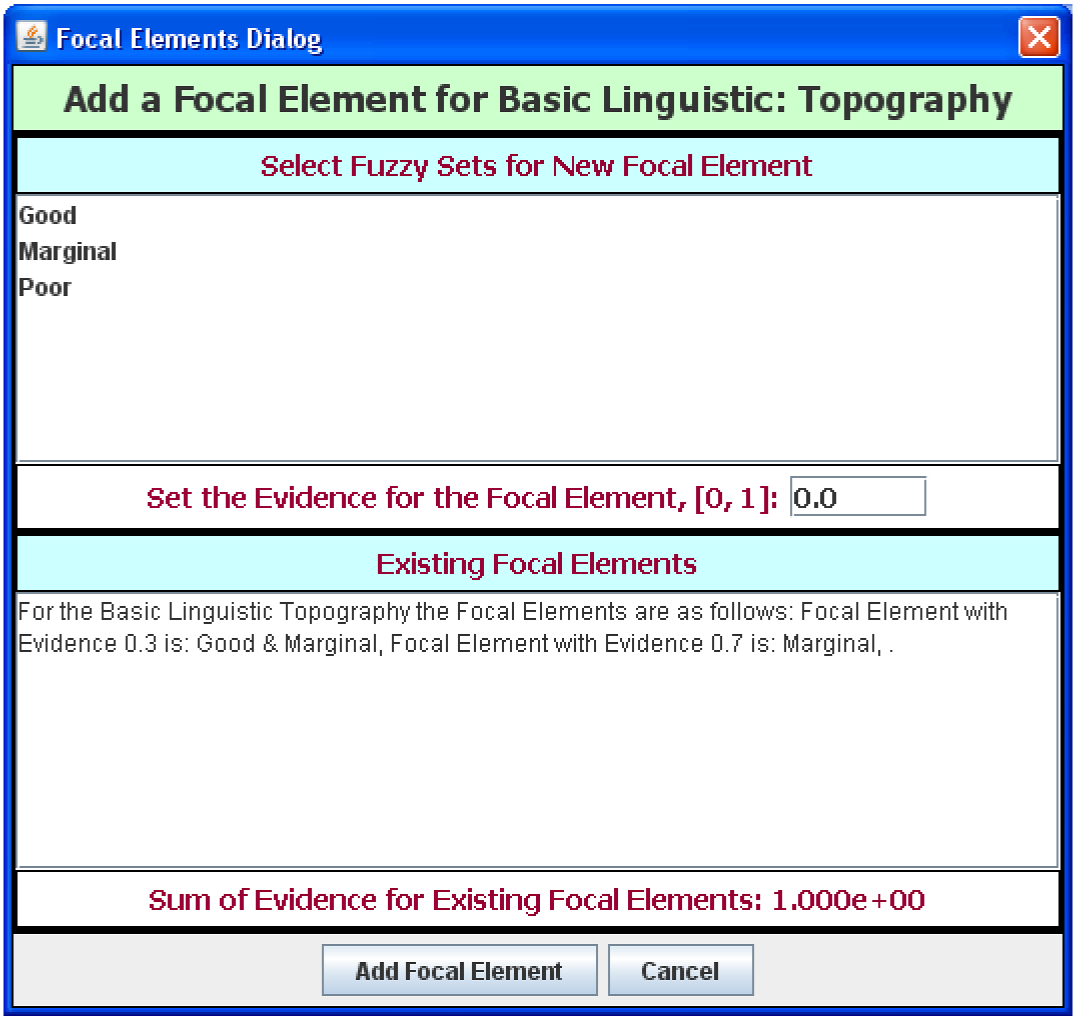


Figure 2-12. Example Body of Evidence for "Topography"

Once evidence has been assigned to each basic variable, the uncertainty for any variable (basic or rule-based) can be evaluated. Figure 2-13 shows the uncertainty for the basic variable "Topography".

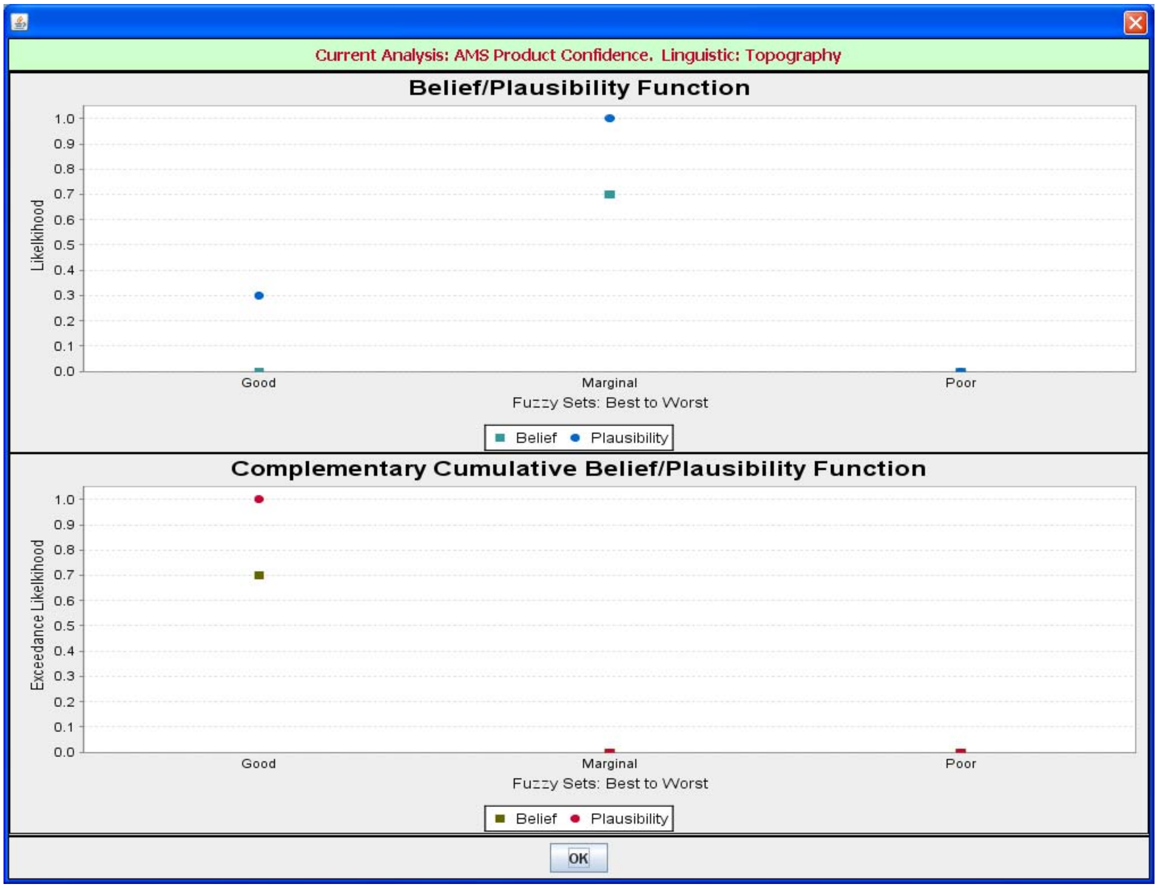


Figure 2-13. Uncertainty for "Topography"

Using example data for all the basic variables, Figure 2-14 shows the uncertainty for the top level rule-based variable “Accuracy of PAG Map”.

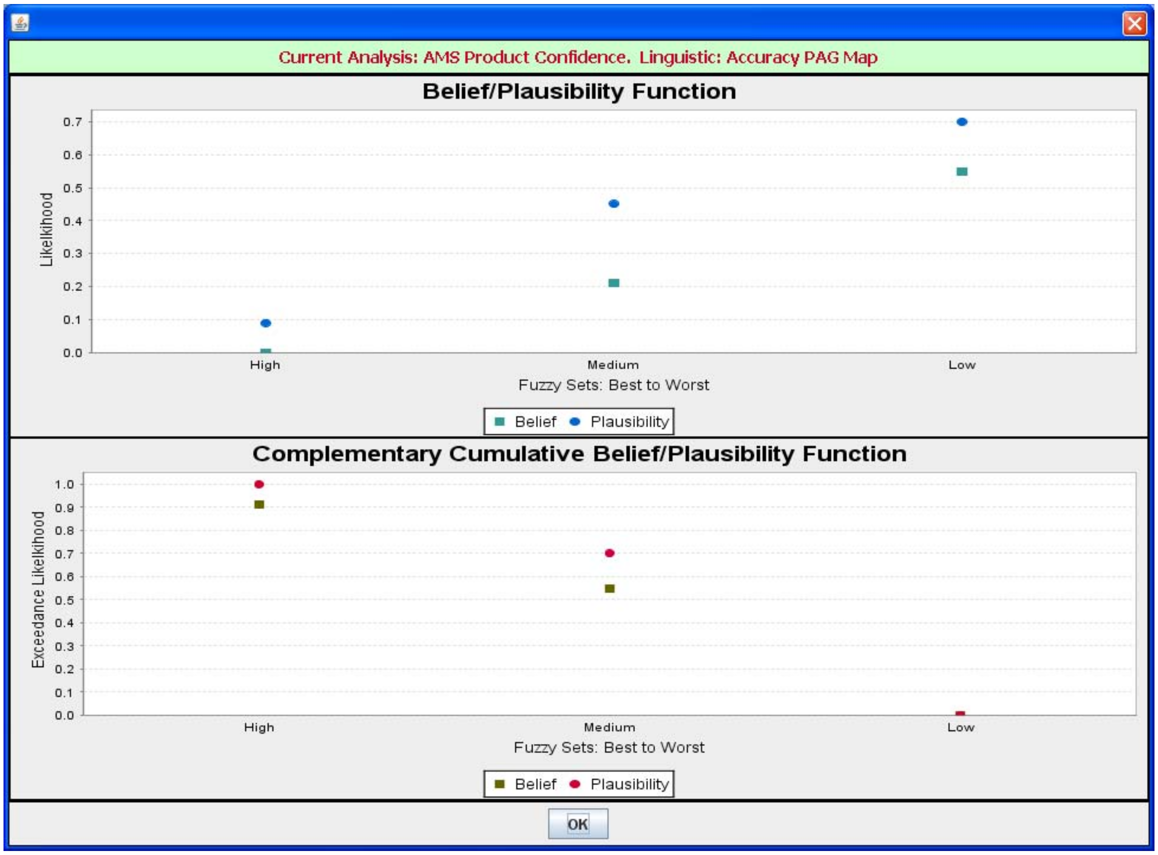


Figure 2-14. Uncertainty for “Accuracy of PAG Map”

The uncertainty graphs for a given variable provide the result for that variable. For example, assuming the evidence used in this example Figure 2-14 summarizes the accuracy of a given map as the uncertainty in the variable “Accuracy of PAG Map”. For this map, based on the upper graph, the likelihood that the accuracy is “High” is between 0 and 0.1, the likelihood that the accuracy is “Medium” is between about 0.2 and 0.45, and the likelihood that the accuracy is “Low” is between 0.55 and 0.7. Based on the lower graph, the likelihood that the accuracy of the map is worse than “High” is between 0.9 and 1.0. Therefore, it is very unlikely that this map is of “High” accuracy.

Section 4 discusses a technique for summarizing results and comparing different maps.

3 LLNL Model

For the LLNL Model, the top-level variable "Accuracy Plume Map" is composed of the following three input variables:

- ***"Accuracy of Location"***, which addresses the accuracy to which the location of the plume is known. This variable is composed of three basic variables: "Weather Confidence", "Source Location and Time", and "Geometry (cloud height, etc.)".
- ***"Accuracy of Magnitude (Source Term)"***, which addresses the accuracy to which the type and quantity of contaminant in the plume (radionuclides or chemicals) is known. This variable is composed of three basic variables: "Radionuclides (types)", "Amount (rate)", and "Geometry (cloud height, etc.)".
- ***"Accuracy of Timing"***, which addresses the accuracy to which the projected timing for the plume is known. "Accuracy of Timing" is a basic variable.

The LLNL Model in the LinguisticBelief software is shown in Figure 3-1. The figure indicates the structure of the variables, rule-based and basic, in the model. Each of the rule-based variables is composed of other variables as subsequently discussed.

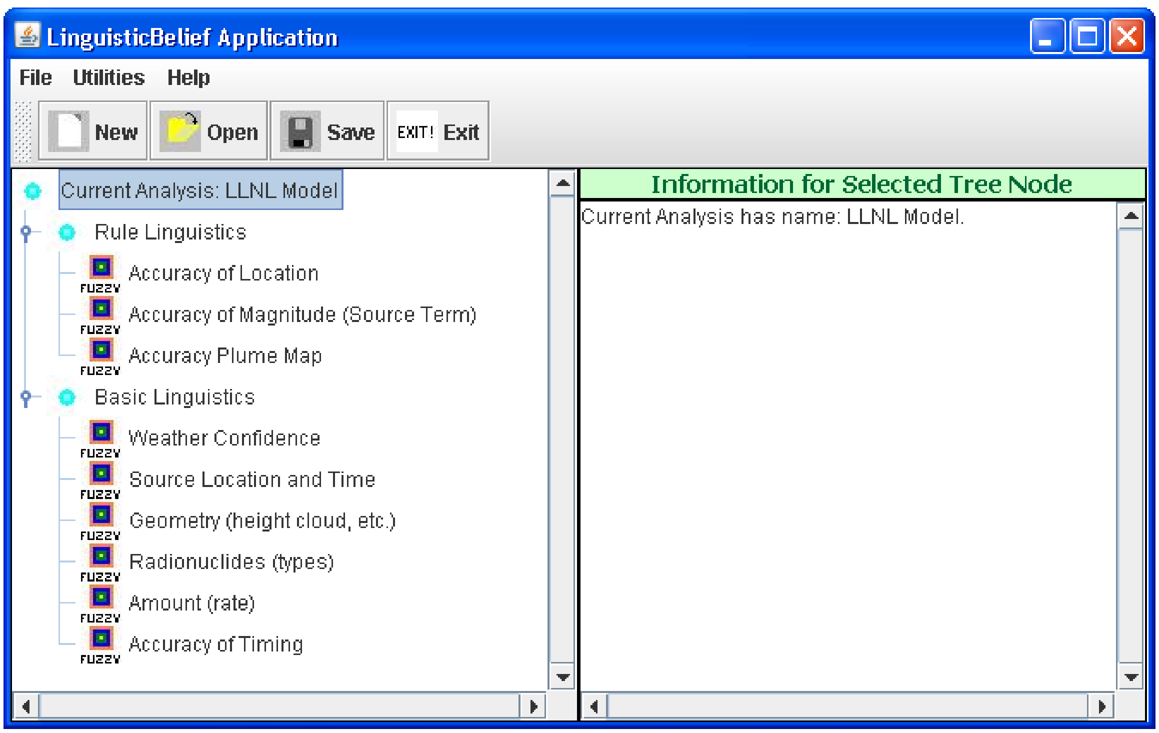


Figure 3-1. Model for "LLNL Model" in LinguisticBelief

The approximate reasoning rule base for each rule-based variable is given in Figures 3-2 through 3-4. The fuzzy sets for each variable (rule-based and basic) are as indicated in these figures. The fuzzy sets were selected by the LLNL team.

Rules for selected RuleLinguistic

Rules for RuleLinguistic: Accuracy of Location

Fuzzy Set for Input Linguistic: Weather Confidence	Fuzzy Set for Input Linguistic: Source Location and Time	Fuzzy Set for Input Linguistic: Geometry (height cloud, etc.)	Output Fuzzy Set for Rule (blank if rule not set)
Low	Unknown	Unknown	Low 270 - 360
Low	Unknown	Partially known	Low 270 - 360
Low	Unknown	Known	Low 270 - 360
Low	Partially known	Unknown	Low 270 - 360
Low	Partially known	Partially known	Low 270 - 360
Low	Partially known	Known	Low 270 - 360
Low	Known	Unknown	Low 270 - 360
Low	Known	Partially known	Low 270 - 360
Low	Known	Known	Low 270 - 360
Medium	Unknown	Unknown	Low 270 - 360
Medium	Unknown	Partially known	Low 270 - 360
Medium	Unknown	Known	Low 270 - 360
Medium	Partially known	Unknown	Medium 90 -270
Medium	Partially known	Partially known	Medium 90 -270
Medium	Partially known	Known	Medium 90 -270
Medium	Known	Unknown	Medium 90 -270
Medium	Known	Partially known	Medium 90 -270
Medium	Known	Known	Medium 90 -270
High	Unknown	Unknown	Low 270 - 360
High	Unknown	Partially known	Low 270 - 360
High	Unknown	Known	Low 270 - 360
High	Partially known	Unknown	Medium 90 -270
High	Partially known	Partially known	Medium 90 -270
High	Partially known	Known	Medium 90 -270
High	Known	Unknown	Medium 90 -270
High	Known	Partially known	High 45 - 90
High	Known	Known	High 45 - 90

Specify Output Fuzzy Set for Selected Rule Choices Are:

Accept Rules as Shown Cancel

Figure 3-2. Rule Base for "Accuracy of Location"

Rules for selected RuleLinguistic

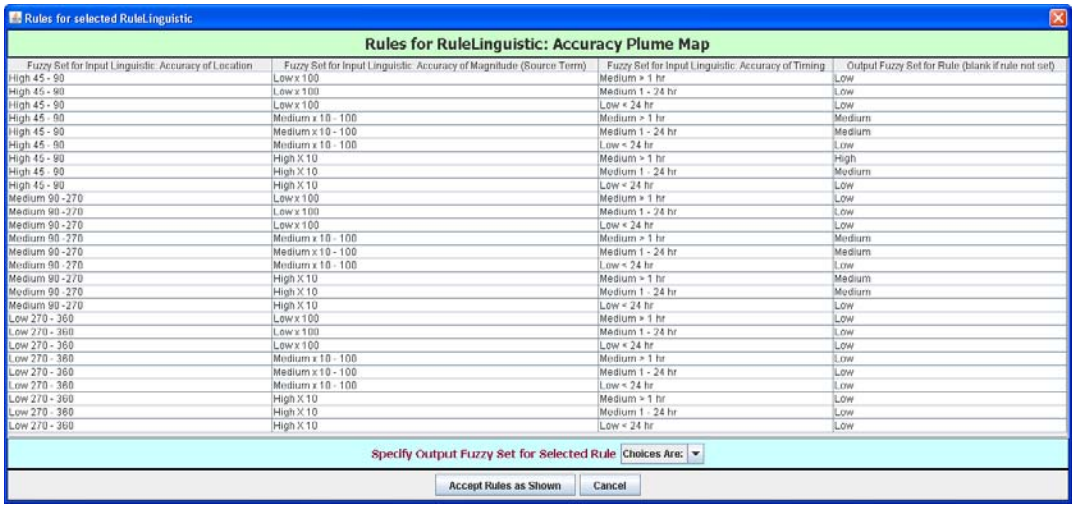
Rules for RuleLinguistic: Accuracy of Magnitude (Source Term)

Fuzzy Set for Input Linguistic: Radionuclides (types)	Fuzzy Set for Input Linguistic: Amount (rate)	Fuzzy Set for Input Linguistic: Geometry (height cloud, etc.)	Output Fuzzy Set for Rule (blank if rule not set)
Known	Low X 100	Unknown	Low x 100
Known	Low X 100	Partially known	Low x 100
Known	Low X 100	Known	Low x 100
Known	Medium X 10 - 100	Unknown	Medium x 10 - 100
Known	Medium X 10 - 100	Partially known	Medium x 10 - 100
Known	Medium X 10 - 100	Known	Medium x 10 - 100
Known	High X 2 - 10	Unknown	Medium x 10 - 100
Known	High X 2 - 10	Partially known	High X 10
Known	High X 2 - 10	Known	High X 10
Partially known	Low X 100	Unknown	Low x 100
Partially known	Low X 100	Partially known	Low x 100
Partially known	Low X 100	Known	Low x 100
Partially known	Medium X 10 - 100	Unknown	Medium x 10 - 100
Partially known	Medium X 10 - 100	Partially known	Medium x 10 - 100
Partially known	Medium X 10 - 100	Known	Medium x 10 - 100
Partially known	High X 2 - 10	Unknown	Medium x 10 - 100
Partially known	High X 2 - 10	Partially known	Medium x 10 - 100
Partially known	High X 2 - 10	Known	Medium x 10 - 100
Unknown	Low X 100	Unknown	Low x 100
Unknown	Low X 100	Partially known	Low x 100
Unknown	Low X 100	Known	Low x 100
Unknown	Medium X 10 - 100	Unknown	Low x 100
Unknown	Medium X 10 - 100	Partially known	Low x 100
Unknown	Medium X 10 - 100	Known	Low x 100
Unknown	High X 2 - 10	Unknown	Low x 100
Unknown	High X 2 - 10	Partially known	Low x 100
Unknown	High X 2 - 10	Known	Low x 100

Specify Output Fuzzy Set for Selected Rule Choices Are:

Accept Rules as Shown Cancel

Figure 3-3. Rule Base for "Accuracy of Magnitude (Source Term)"



Fuzzy Set for Input Linguistic: Accuracy of Location	Fuzzy Set for Input Linguistic: Accuracy of Magnitude (Source Term)	Fuzzy Set for Input Linguistic: Accuracy of Timing	Output Fuzzy Set for Rule (blank if rule not set)
High 45 - 90	Low x 100	Medium > 1 hr	Low
High 45 - 90	Low x 100	Medium 1 - 24 hr	Low
High 45 - 90	Low x 100	Low < 24 hr	Low
High 45 - 90	Medium x 10 - 100	Medium > 1 hr	Medium
High 45 - 90	Medium x 10 - 100	Medium 1 - 24 hr	Medium
High 45 - 90	Medium x 10 - 100	Low < 24 hr	Low
High 45 - 90	High X 10	Medium > 1 hr	High
High 45 - 90	High X 10	Medium 1 - 24 hr	Medium
High 45 - 90	High X 10	Low < 24 hr	Low
Medium 90 -270	Low x 100	Medium > 1 hr	Low
Medium 90 -270	Low x 100	Medium 1 - 24 hr	Low
Medium 90 -270	Low x 100	Low < 24 hr	Low
Medium 90 -270	Medium x 10 - 100	Medium > 1 hr	Medium
Medium 90 -270	Medium x 10 - 100	Medium 1 - 24 hr	Medium
Medium 90 -270	Medium x 10 - 100	Low < 24 hr	Low
Medium 90 -270	High X 10	Medium > 1 hr	Medium
Medium 90 -270	High X 10	Medium 1 - 24 hr	Medium
Medium 90 -270	High X 10	Low < 24 hr	Low
Low 270 - 360	Low x 100	Medium > 1 hr	Low
Low 270 - 360	Low x 100	Medium 1 - 24 hr	Low
Low 270 - 360	Low x 100	Low < 24 hr	Low
Low 270 - 360	Medium x 10 - 100	Medium > 1 hr	Low
Low 270 - 360	Medium x 10 - 100	Medium 1 - 24 hr	Low
Low 270 - 360	Medium x 10 - 100	Low < 24 hr	Low
Low 270 - 360	High X 10	Medium > 1 hr	Low
Low 270 - 360	High X 10	Medium 1 - 24 hr	Low
Low 270 - 360	High X 10	Low < 24 hr	Low

Figure 3-4. Rule Base for "Accuracy Plume Map"

Using the model, the qualitative uncertainty for a *specific* map can be evaluated by assigning evidence to the focal elements for each of the six basic variables in the model.

Figure 3-5 is an example of assignment of evidence to the basic variable "Weather Confidence". This example uses dummy evidence to illustrate the technique.

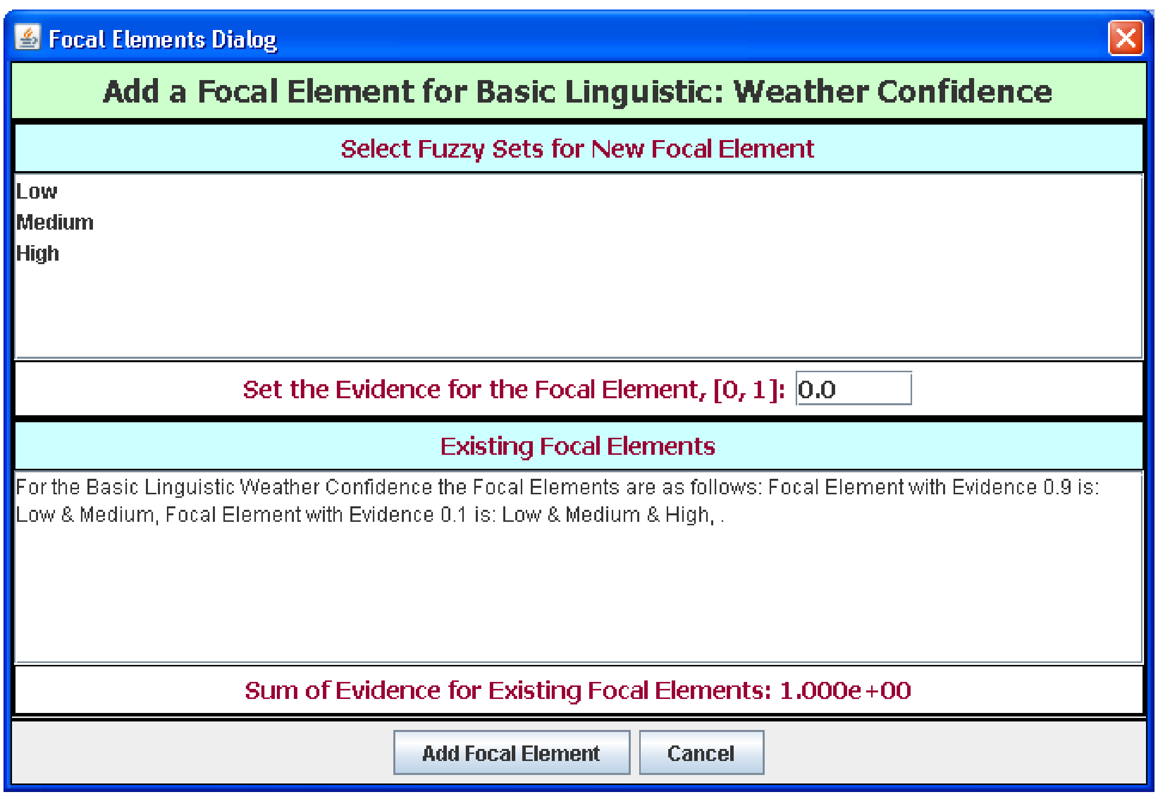


Figure 3-5. Example Body of Evidence for "Weather Confidence"

Once evidence has been assigned to each basic variable, the uncertainty for any variable (basic or rule-based) can be evaluated. Figure 3-6 shows the uncertainty for the basic variable “Weather Confidence”.

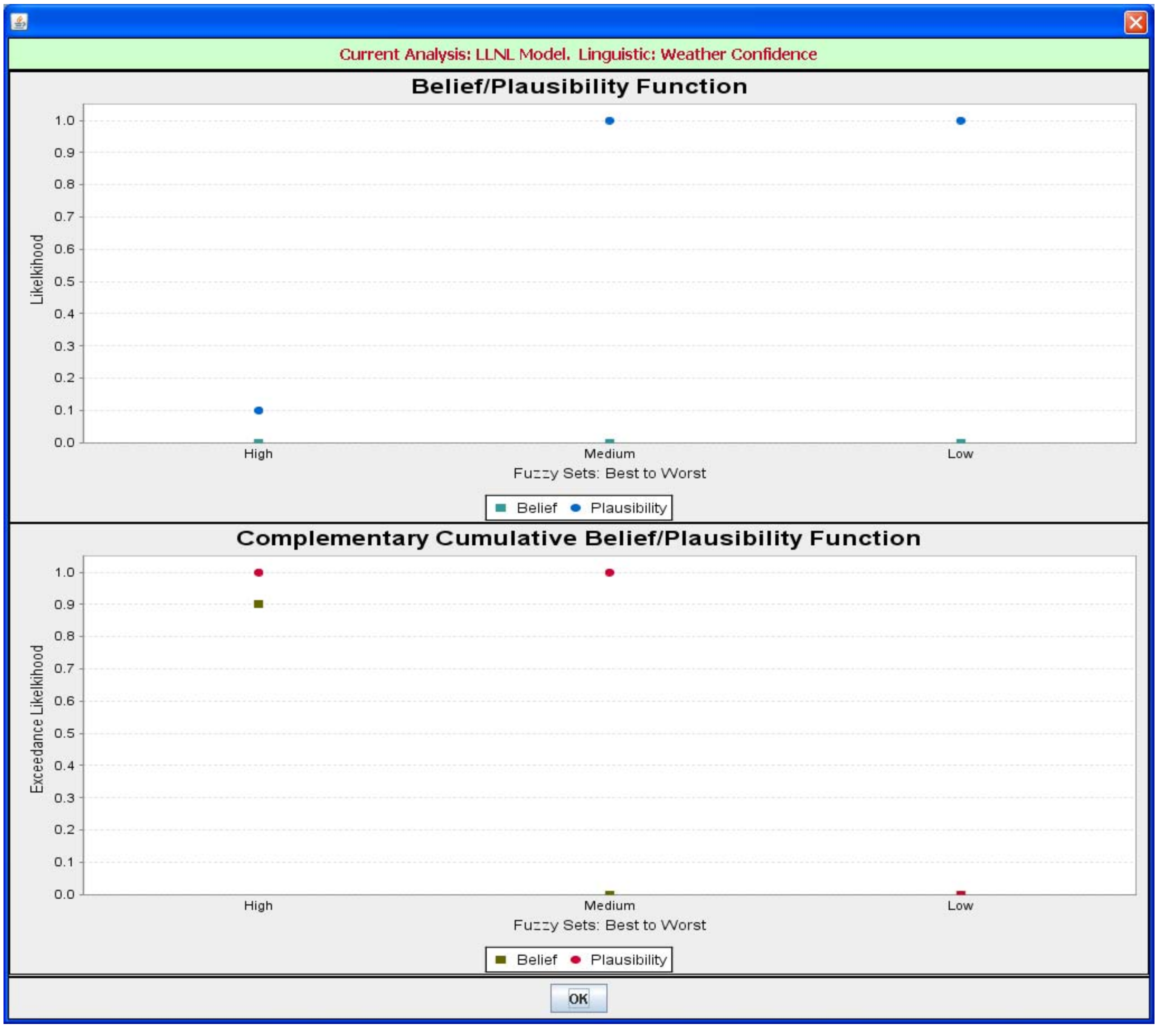


Figure 3-6. Uncertainty for “Weather Confidence”

Using example data for all the basic variables, Figure 3-7 shows the uncertainty for the top-level, rule-based variable "Accuracy Plume Map".

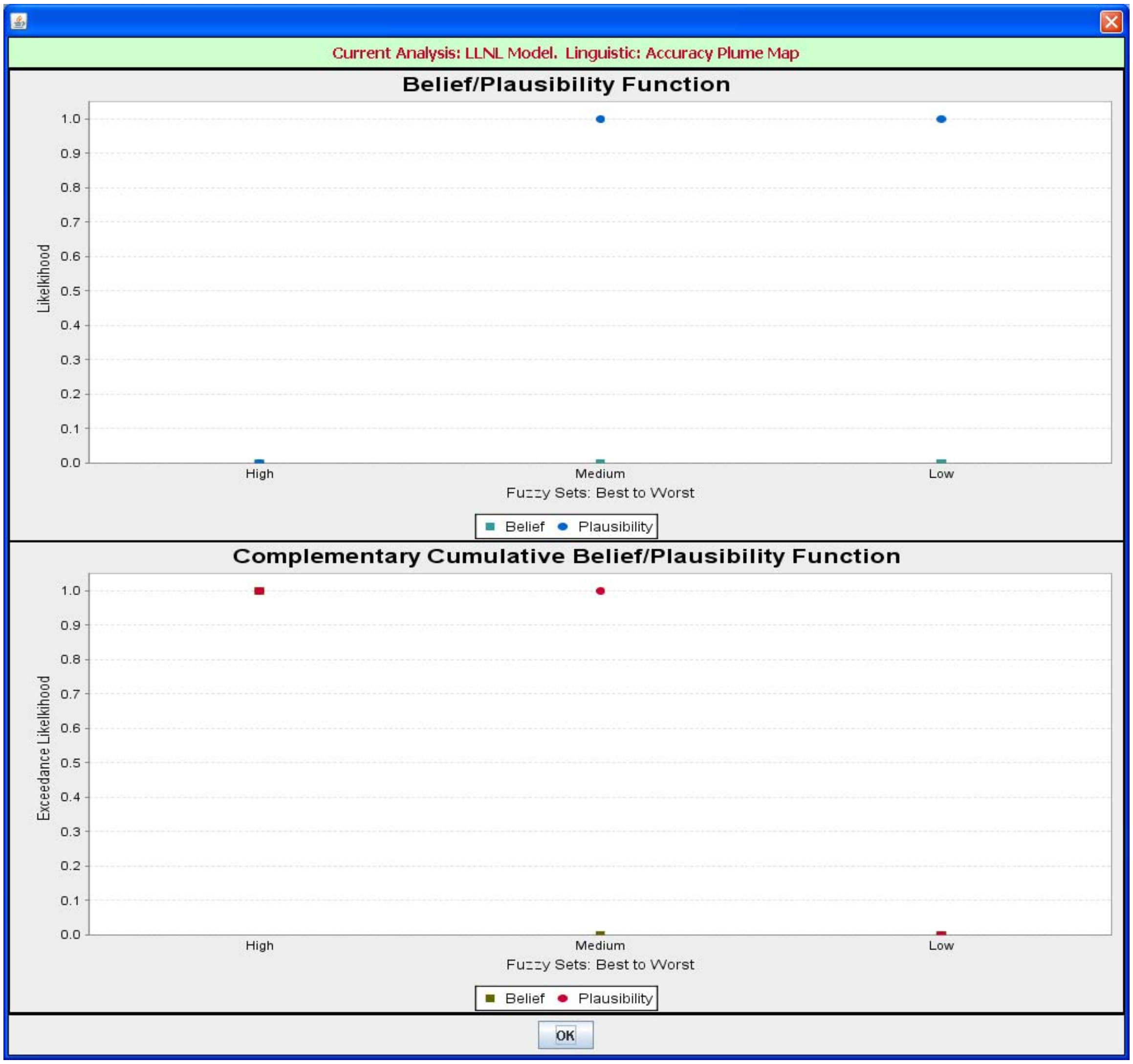


Figure 3-7. Uncertainty for "Accuracy Plume Map"

The uncertainty graphs for a given variable provide the result for that variable. Assuming the evidence used in this example, Figure 3-7 summarizes the accuracy of a given map as the uncertainty in the variable "Accuracy Plume Map". For this map, based on the upper graph, the likelihood that the accuracy is "High" is 0 with certainty, the likelihood that the accuracy is "Medium" is between 0 and 1, and the likelihood that the accuracy is "Low" is between 0 and 1. Based on the lower graph, the likelihood that the accuracy of the map is worse than "High" is 1.0; that is, we are certain that the accuracy is not "High". The likelihood that the accuracy is worse that "Medium" is between 0 and 1. Therefore, this map may be of "Medium" accuracy but that is highly uncertain.

Section 4 discusses a technique for summarizing results and comparing different maps.

4 Summarizing and Rank-Ordering Results

Sections 2 and 3 summarized the models and provided example results. This section develops a simple measure for the accuracy of a map using the results of the models.

The result of an evaluation can be summarized with a “Quality Thermometer” that considers the uncertainty in the result. The RSL model for “Accuracy Plot for Area” will be used to illustrate this approach, where the result is “Accuracy Plot for Area” with the fuzzy sets “High”, “Medium”, and “Low”. For the “Accuracy Plot for Area” variable, the LinguisticBelief code will produce an uncertainty distribution over the fuzzy sets for the variable as indicated in Figure 2-7 presented earlier. The fuzzy sets in Figure 2-7 are ordered from “best” to “worst”, or “High” to “Low” in this case.

In general, as in Figure 2-7, the result is a belief/plausibility interval distribution over each fuzzy set. This general situation is summarized conceptually in Figure 4-1.

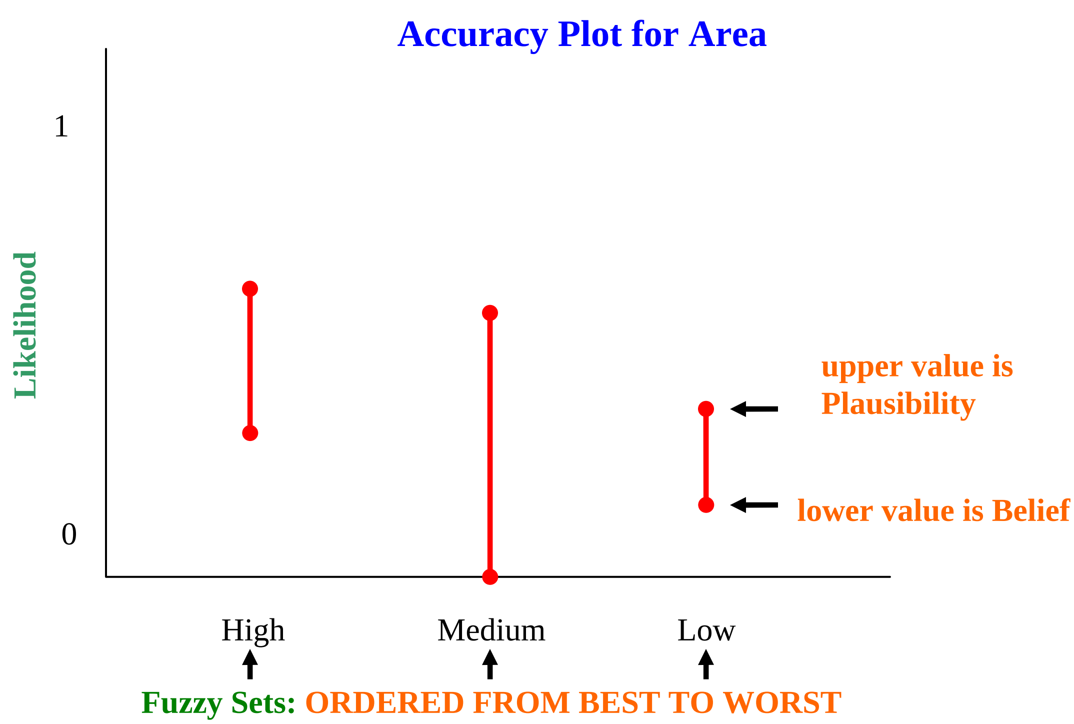


Figure 4-1. Fuzzy Sets are Ordered and Uncertainty Is an Interval

If all the evidence for the variable is probabilistic, than the belief/plausibility interval reduces to a point value, the probability. This degenerate case is shown in Figure 4-2.[2]

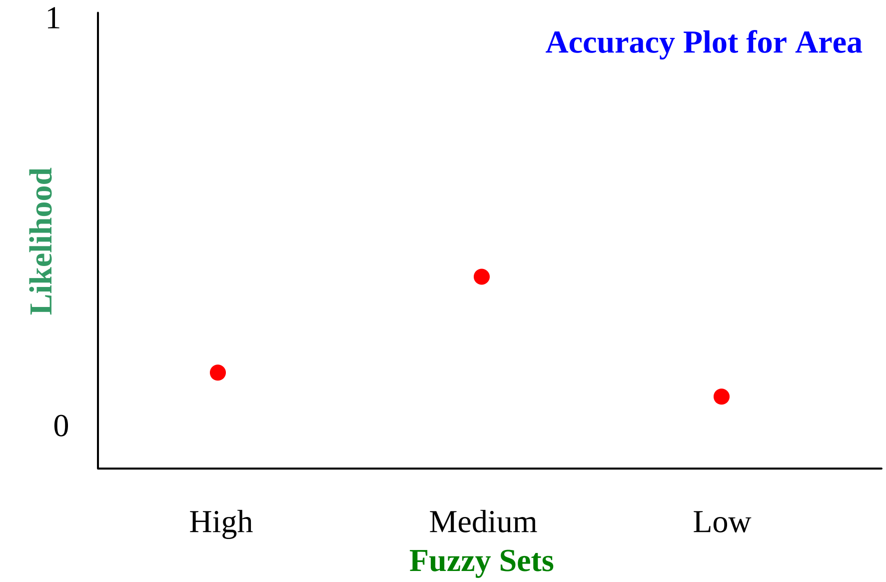


Figure 4-2. Degenerate Case: Probability Distribution

If there is no uncertainty in the evidence, there is no uncertainty for the variable. This degenerate case is shown in Figure 4-3. Only one fuzzy set has a non-zero likelihood, and the likelihood for that fuzzy set is 1.0.

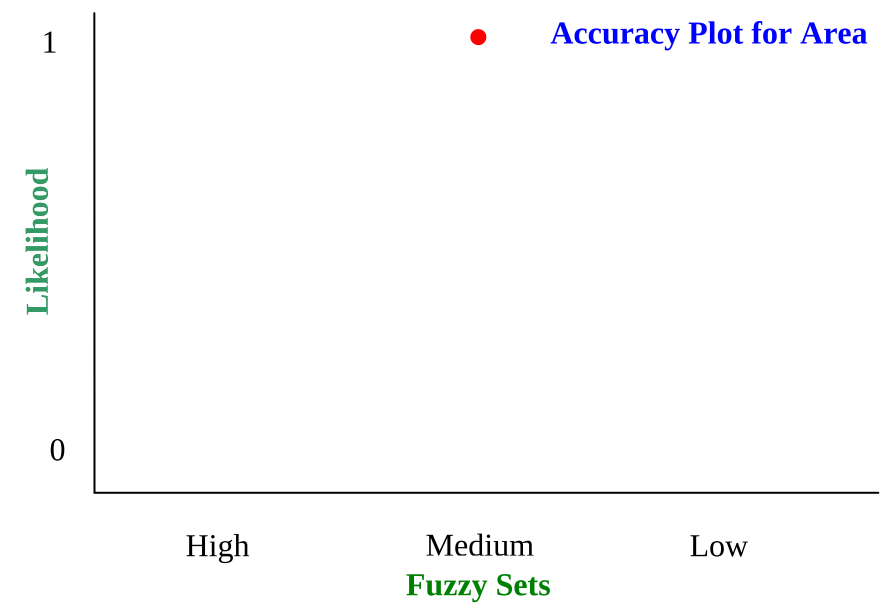


Figure 4-3. Degenerate Case: Certainty

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[2] "Degenerate case" means "special case." Here, both belief and plausibility "degenerate" to the single value probability.

Certainty is a special case of probability, and probability is a special case of belief/plausibility. That is, Figure 4-3 is a special case of Figure 4-2, and Figure 4-2 is a special case of Figure 4-1.

To summarize the "goodness" of the variable, a simple measure is desired to reflect "How good is the graph?" An appropriate measure is "the non-zero likelihood of the best fuzzy set". More precisely, "the belief/plausibility interval of the best fuzzy set for which the plausibility is greater than 0". For the degenerate case of probability, the belief/plausibility interval is a point value–the probability–and the measure becomes "the probability of the best fuzzy set for which the probability is greater than 0". For the degenerate case of no uncertainty, only one fuzzy set has a probability greater than 0–that fuzzy set has a probability of 1.0–and the measure becomes "the fuzzy set that has probability 1.0". This approach can be summarized using a "Quality Thermometer" graph.

For Figure 4-1, the "best" fuzzy set with non-zero likelihood is "High", and the simple measure is the belief/plausibility interval for "High" produced by the code. The summary result for Figure 4-1 is given in Figure 4-4, a "Quality Thermometer" for the result.

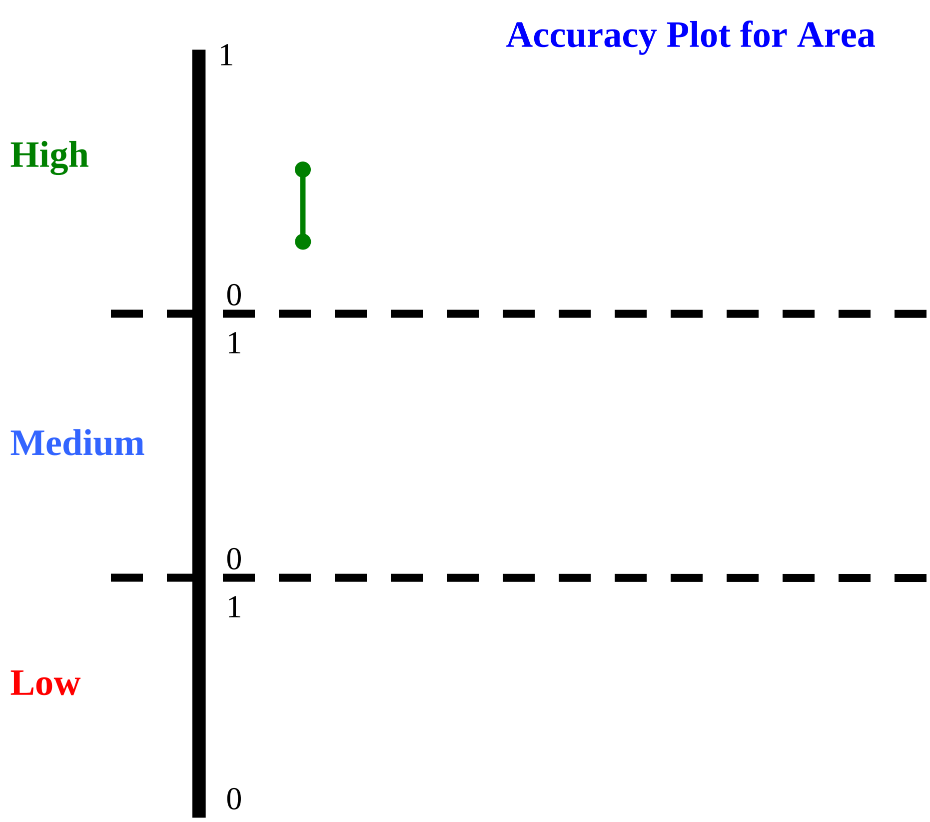

Figure 4-4. "Quality Thermometer" for Accuracy Plot for Area for Figure 4-1

Similarly, the simple results for the degenerate cases of Figures 4-2 and 4-3 are shown in Figures 4-5 and 4-6, respectively.

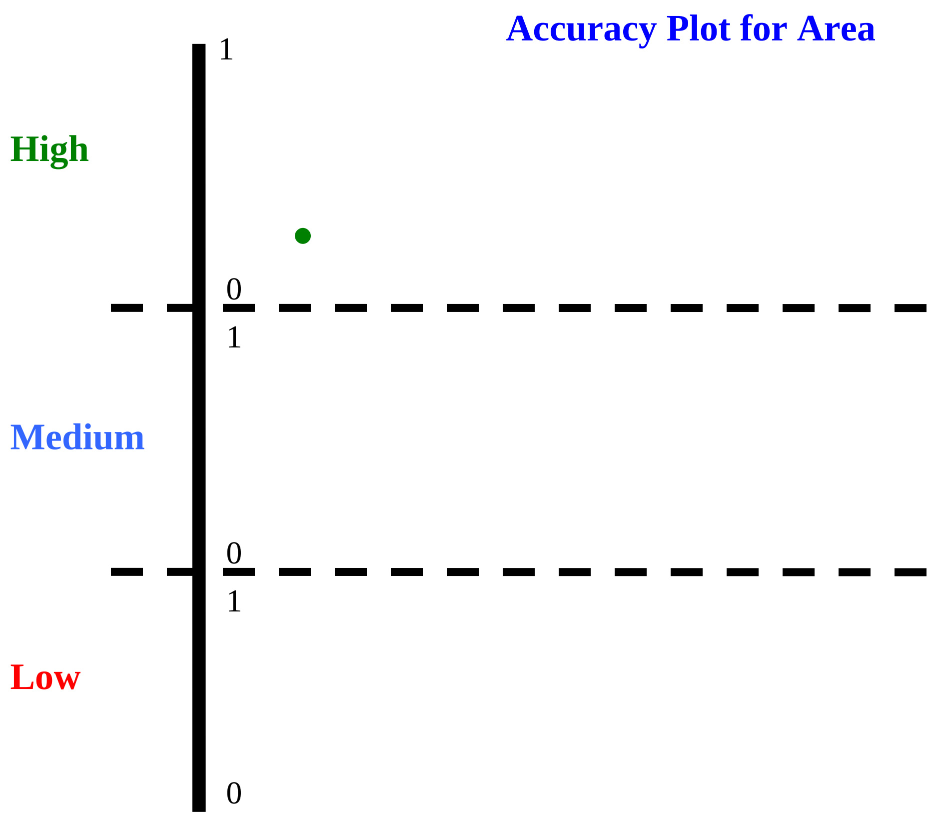

Figure 4-5. "Quality Thermometer" for Accuracy Plot for Area for Figure 4-2

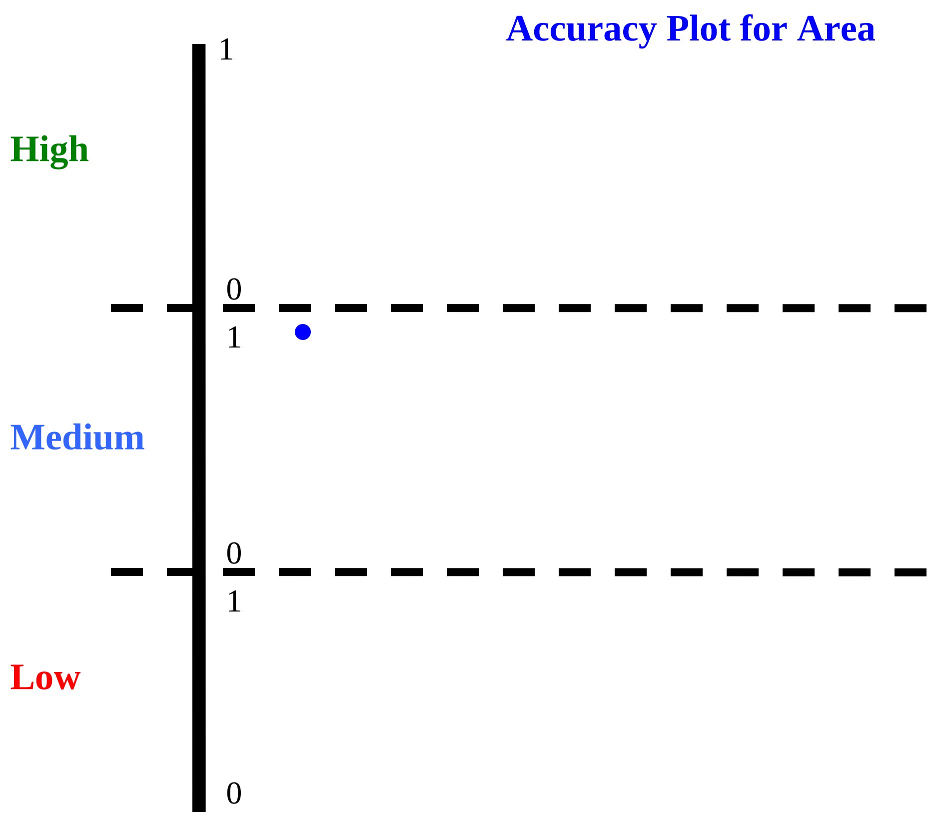

Figure 4-6. "Quality Thermometer" for Accuracy Plot for Area for Figure 4-3

The quality of one graph can be compared to another using the "Quality Thermometer".

The ranking–by decreasing quality–is by highest belief, sub-ranked by highest plausibility, for the best fuzzy set with non-zero plausibility.[3]

[3] Here we reason on how *good* the graph is (the best fuzzy set), so we rank by belief, then sub-rank by plausibility for the *best* fuzzy set. If we wish to reason on how *bad* a variable is (e.g., terrorist risk), we rank by plausibility, then sub-rank by belief for the *worst* fuzzy set.

For example, assume the result in Figure 4-7 for four different graphs: A, B, C, and D.

Graphs A and B have the same Belief for "Medium", but the Plausibility for "Medium" for A is higher than the Plausibility of "Medium" for B. The Belief and Plausibility for Graph C is equal for "Medium", which is the Probability. Therefore, the Belief for C is higher than the Belief for A and B for "Medium".

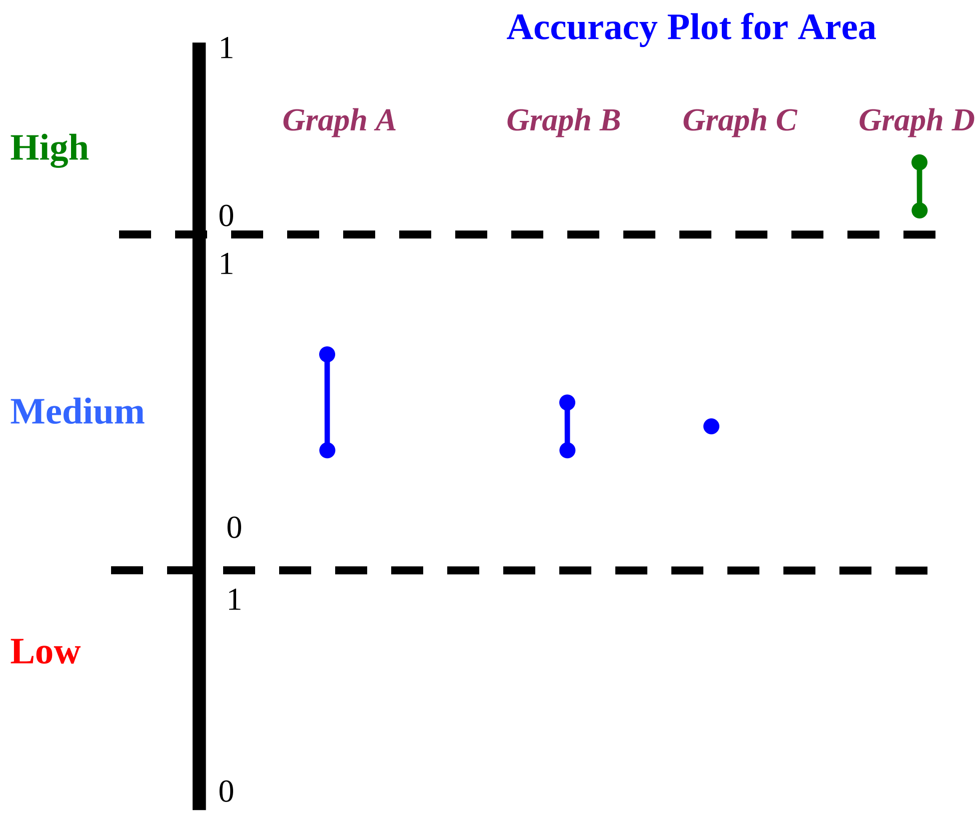

Figure 4-7. Quality Thermometer for Accuracy for Plot Area for Four Maps: A, B, C, and D

The ranking of the graphs–by decreasing quality–is: D, C, A, and B.

5 Combining Expert Judgment

The assignment of evidence to evaluate a given map may be performed by more than one expert and these experts may generate different evidence. The PoolEvidence© software tool developed by Sandia National Laboratories can be used to pool the evidence from different experts into pooled evidence for input into LinguisticBelief.

Figure 5-1 is the model for all the basic variables for "Accuracy of PAG Map" in PoolEvidence© assuming four experts (A, B, C, and D) provide evidence. Figure 5-1 highlights the evidence (focal elements) for the variable for "Accuracy of Count Rate greater than Background for Radionuclide ID" provided by Expert B.

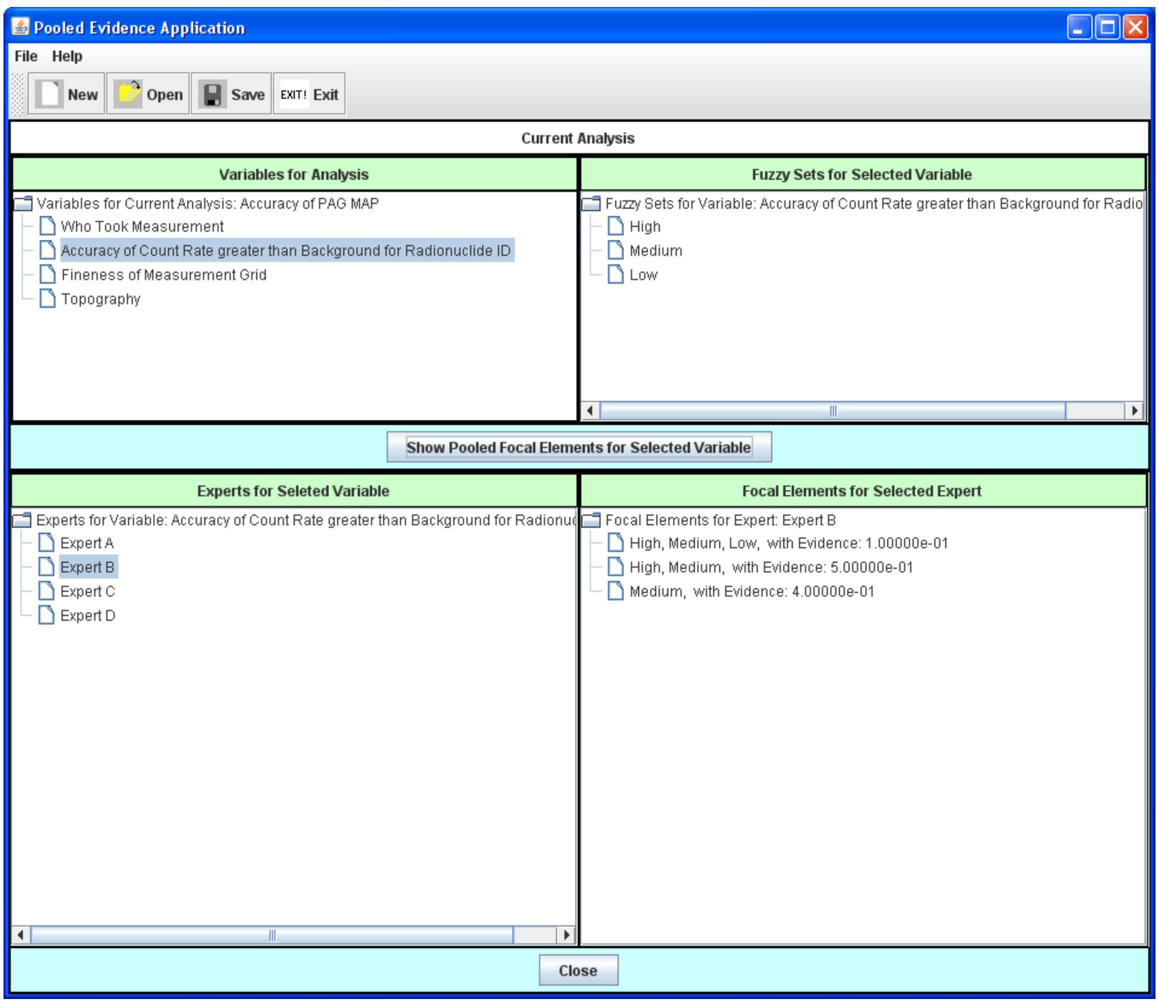

Figure 5-1. "Accuracy of PAG Map" in PoolEvidence

Using example data from each expert, Figure 5-2 shows the pooled evidence for the basic variable “Accuracy of Count Rate greater than Background for Radionuclide ID”.

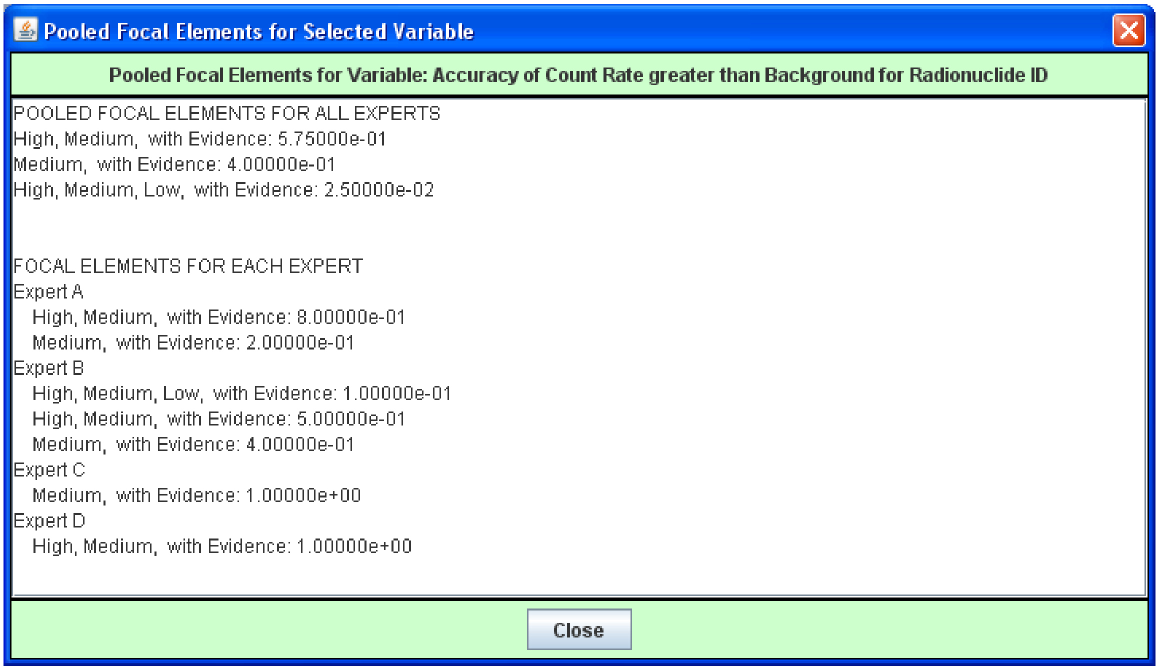

Figure 5-2. Example of Pooled Evidence for “Accuracy of Count Rate greater than Background for Radionuclide ID”

The pooled evidence summarized in Figure 5-2 is calculated from the evidence (focal elements) provided by each individual expert as also summarized in Figure 5-2. For Expert B, the evidence summarized in Figure 5-2 is the evidence shown in Figure 5-1.

This technique can be used to combine evidence from different experts into pooled evidence for each variable to be used to evaluate the qualitative accuracy of maps. Then, this pooled evidence is input into LinguisticBelief to evaluate the accuracy of the map.

6 Summary and Next Steps

A methodology has been applied to provide qualitative uncertainty for maps associated with the release of hazardous materials. The methodology was developed and implemented by Sandia National Laboratories in the LinguisticBelief software, and applied to three types of maps generated by RSL and LLNL NARAC.

For each of the three maps, subject matter experts at RSL and LLNL developed a model in the LinguisticBelief software tool. This report documents these models, and provides example results using dummy data.

Evaluations of maps for all three models are provided, using example data. Future applications involve application of the models to actual graphs. The application to actual graphs will provide a qualitative evaluation of the accuracy of the maps, including uncertainty, for use by decision makers.

A "Quality Thermometer" technique was developed to rank order the quality of a set of maps of a given type.

A technique for pooling expert option from different experts was provided using the PoolEvidence© software.

References

[Methodology] *Evaluation of Risk from Acts of Terrorism: The Adversary/Defender Model using Belief and Fuzzy Sets*, John Darby, Sandia National Laboratories, SAND2006-5777, September 2006.

[LinguisticBelief] *LinguisticBelief: A Java Application for Linguistic Evaluation using Belief, Fuzzy Sets, and Approximate Reasoning*, John Darby, Sandia National Laboratories, SAND2007-1299, March 2007.

[Training Material] *Uncertainty for Qualitative Variables*, John Darby, Sandia National Laboratories, SAND2007-6684P, November 12, 2007.

Appendix A: A Complete Example

This appendix provides an example of all the data required to evaluate "AccuracyPlotArea", the RSL model discussed in Section 2.1. Example evidence for each basic variable is provided. Note that many of the basic variables have no uncertainty for this example. The output for each rule-based variable is provided.

A.1 Map with Uncertainty

This section provides an example for the accuracy of a map where some of the variables have significant uncertainty, resulting in considerable uncertainty in the accuracy of that map.

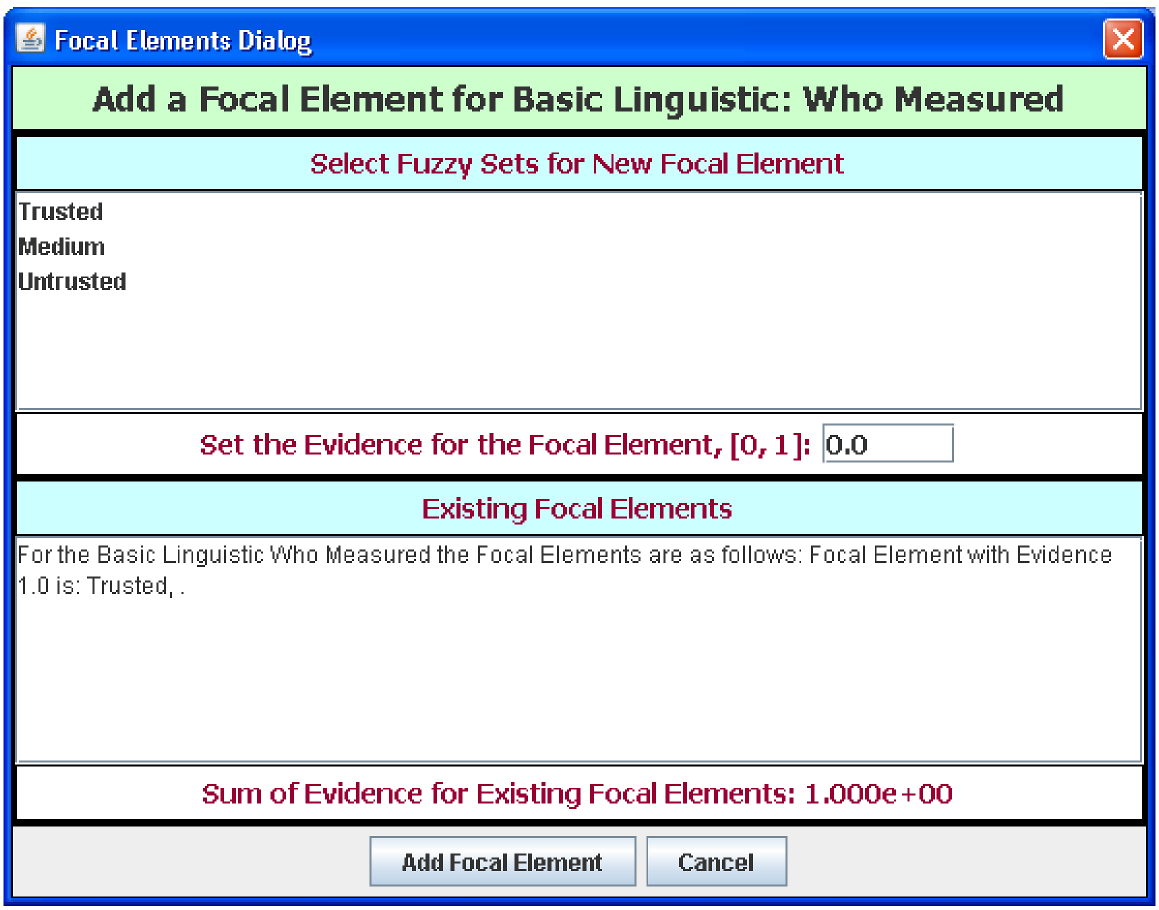

Figure A-1. Focal Elements for "Who Measured"

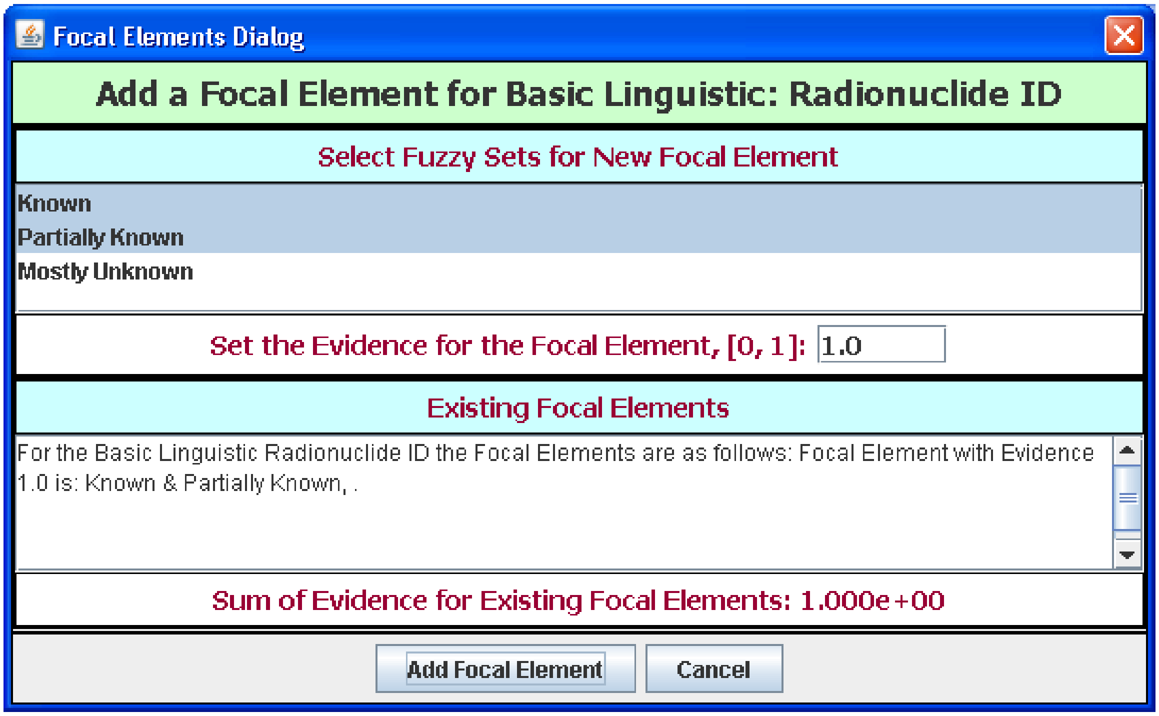

Figure A-2. Focal Elements for "Radionuclide ID"

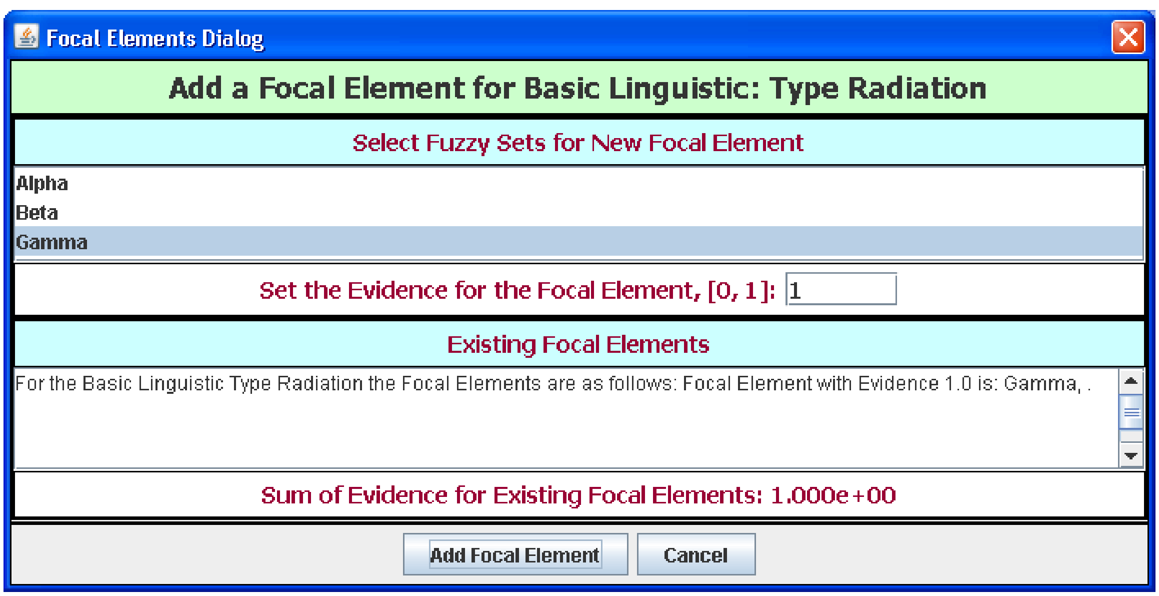

Figure A-3. Focal Elements for "Type Radiation"

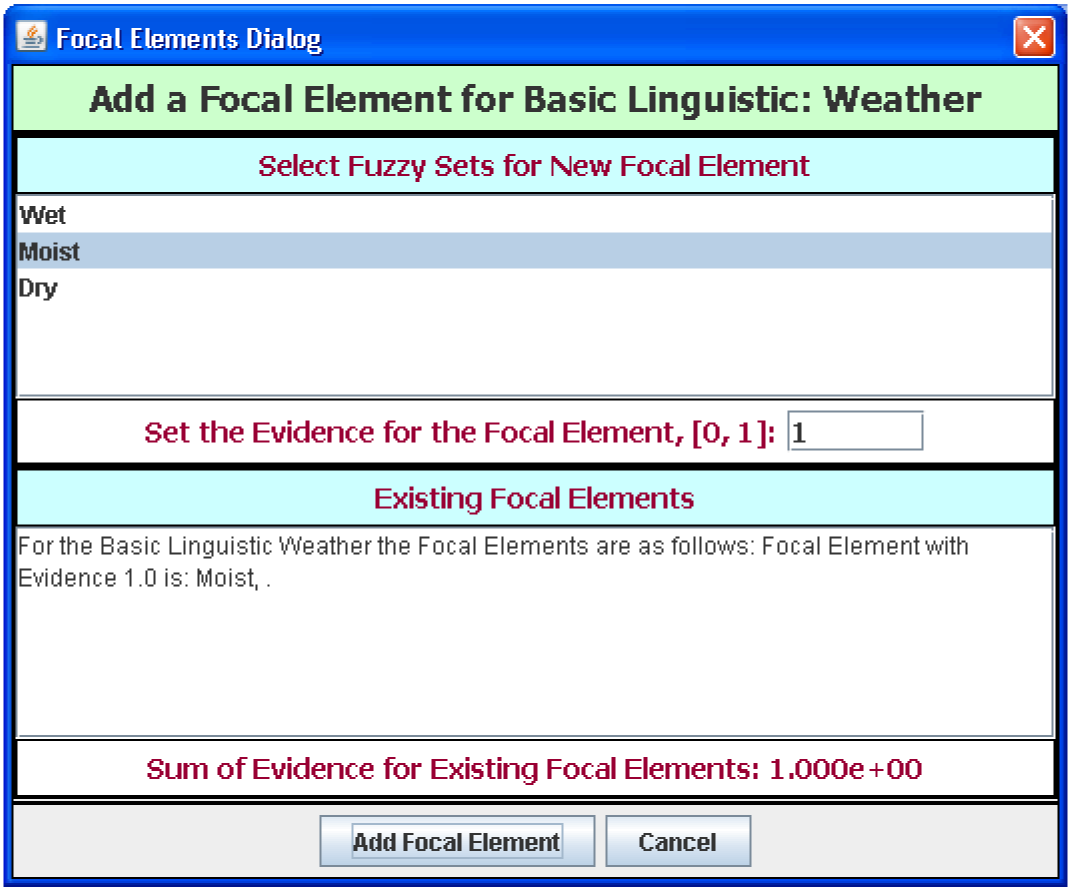

Figure A-4. Focal Elements for "Weather"

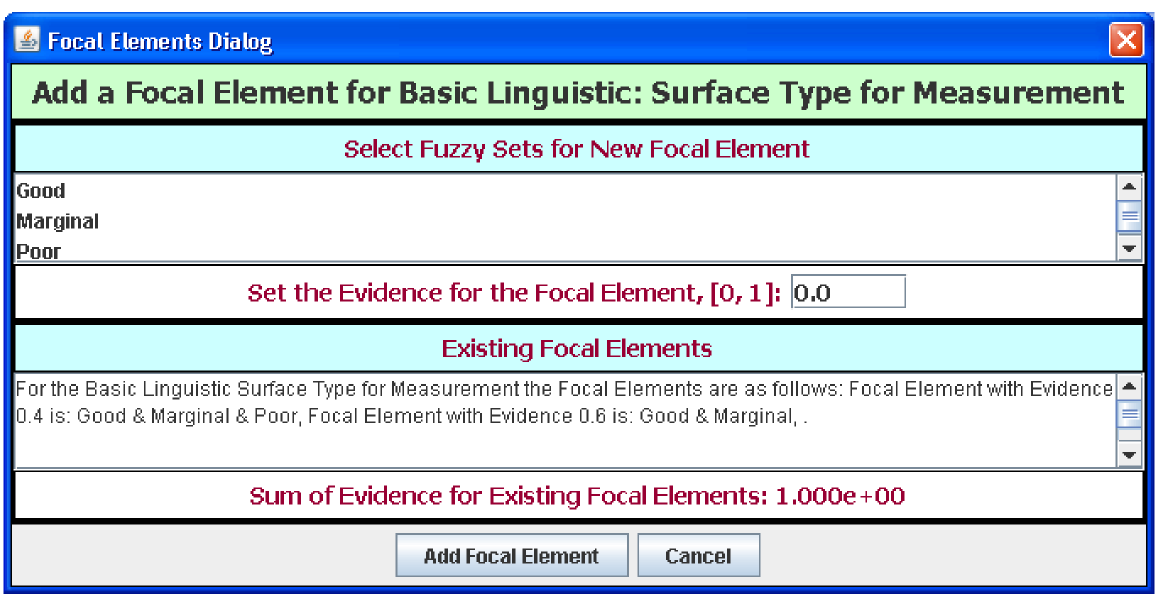

Figure A-5. Focal Elements for "Surface Type for Measurement"

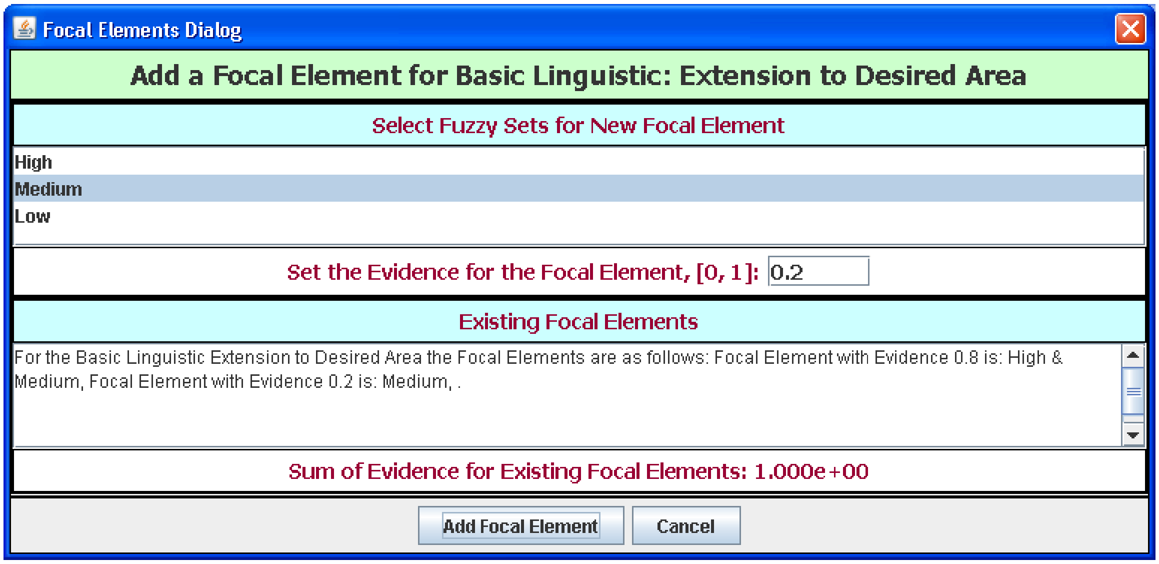

Figure A-6. Focal Elements for "Extension to Desired Area"

Using these inputs, the result for each rule-based variable is as follows.

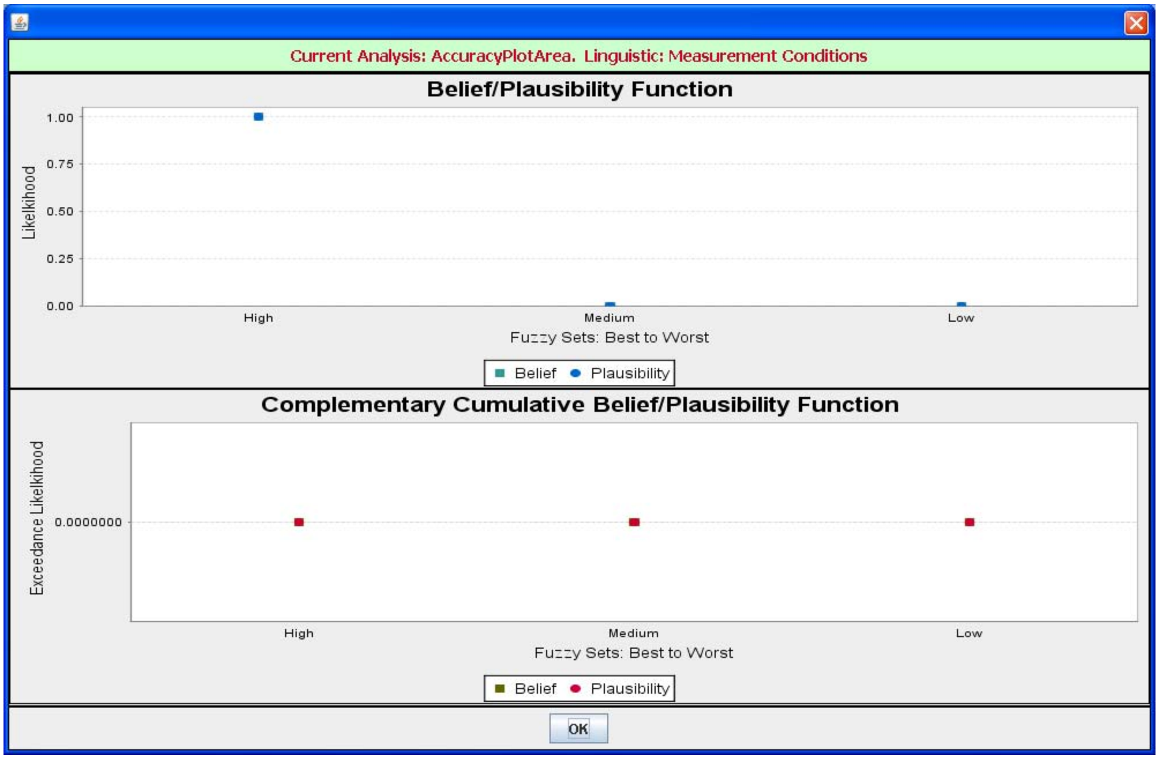

Figure A-7. Results for "Measurement Conditions"

Note that there is no uncertainty in "Measurement Conditions", it is "High". This is due to no uncertainty in two of the input variables for "Measurement Conditions", specifically: "Type Radiation" is "Gamma" with certainty and "Weather" is "Moist" with certainty as indicated in Figures A-3 and A-4. Although there is uncertainty in the third input variable "Surface Type For Measurement" as indicated in Figure A-5, the rule base for "Measurement Conditions" in Figure 2-3 indicates that for "Type Radiation" "Gamma" and "Weather" "Moist", "Measurement Conditions" is "High" regardless of the value of "Surface Type for Measurement".

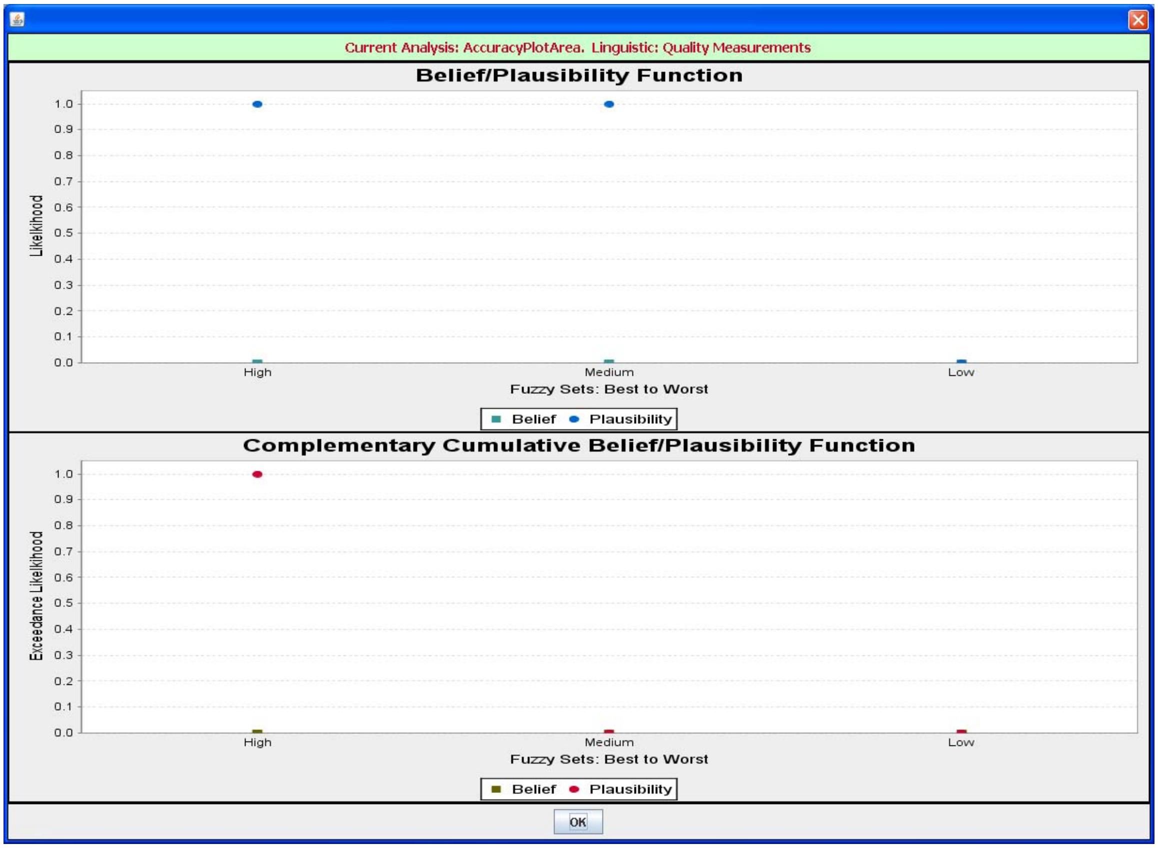

Figure A-8. Results for "Quality Measurements"

Note that there is considerable uncertainty in "Quality Measurements".

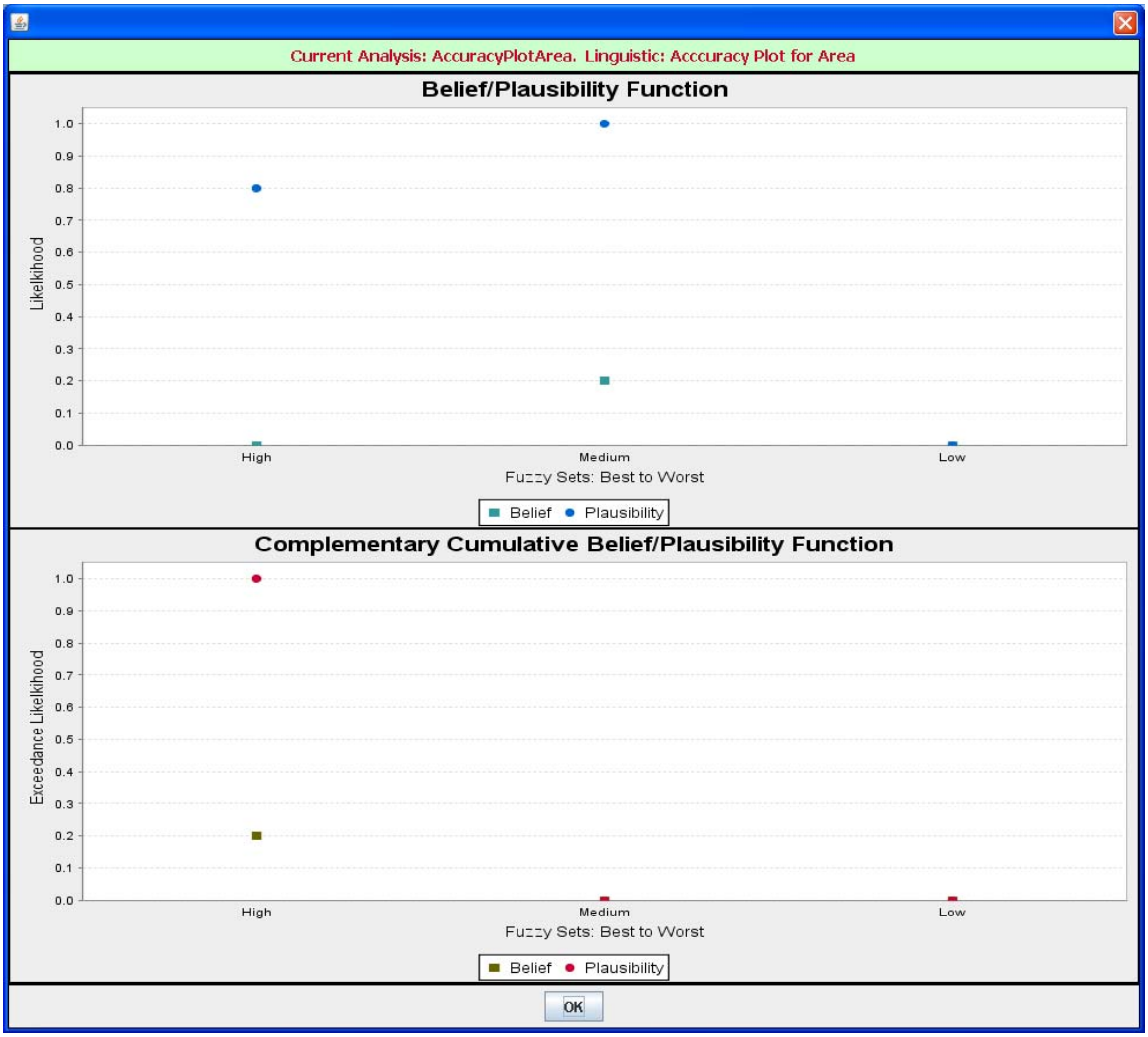

Figure A-9. Results for "Accuracy Plot for Area"

"Accuracy Plot for Area" is a combination of: "Quality Measurements", "Measurement Conditions", and "Extension to Desired Area", as indicated by the rules in Figure 2-4. As shown in Figure A-7, "Measurement Conditions" has no uncertainty; however, "Extension to Desired Area" has uncertainty as indicated in Figure A-6 and "Quality Measurements" has significant uncertainty as indicated in Figure A-8.

From Figure A-9, the accuracy of the specific map has a likelihood of between 0 and 0.8 of "High" and a likelihood of between 0.2 and 1.0 of "Medium". The likelihood of the accuracy being worse than "High" is between 0.2 and 1.0.

For the "Quality Thermometer", this map has belief/plausibility of 0/0.8 for "High".

A.2 Map with No Uncertainty

This section provides an example for the accuracy of a map where most of the variables have no uncertainty, resulting in little uncertainty in the accuracy of that map.

If better information is known about the map, the result will be more accurate. For example, assume that the evidence for “Surface Type for Measurement” is as indicated in Figure A-10.

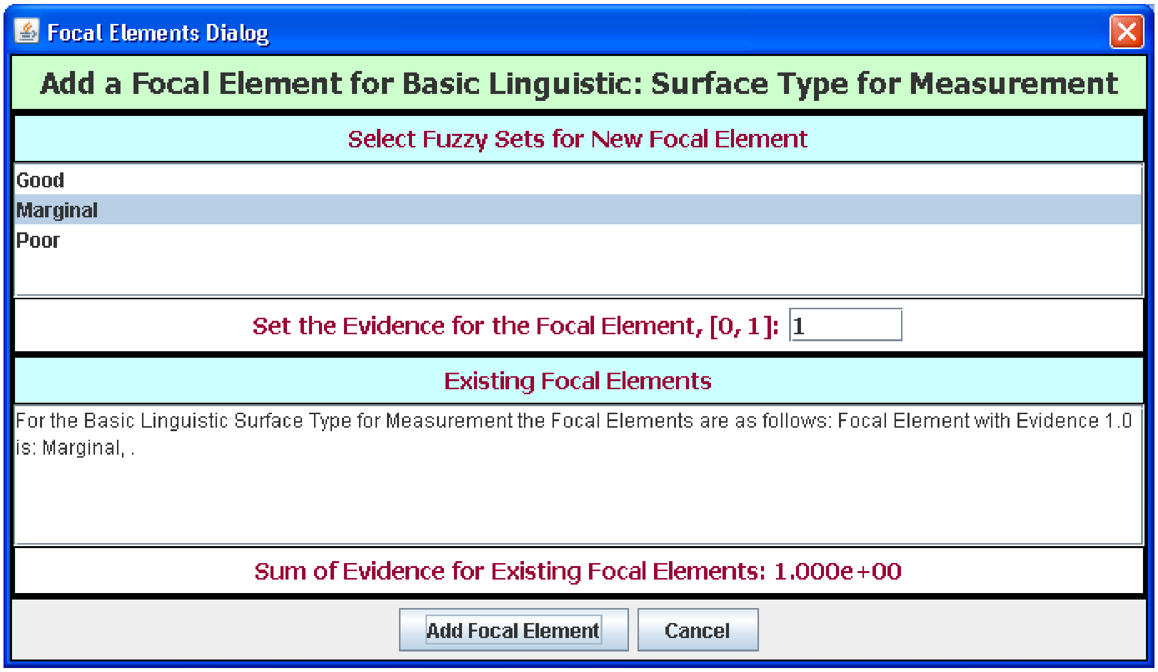

Figure A-10. No Uncertainty for “Surface Type for Measurement”

Assume the evidence for “Extension to Desired Area” is as indicated in Figure A-11.

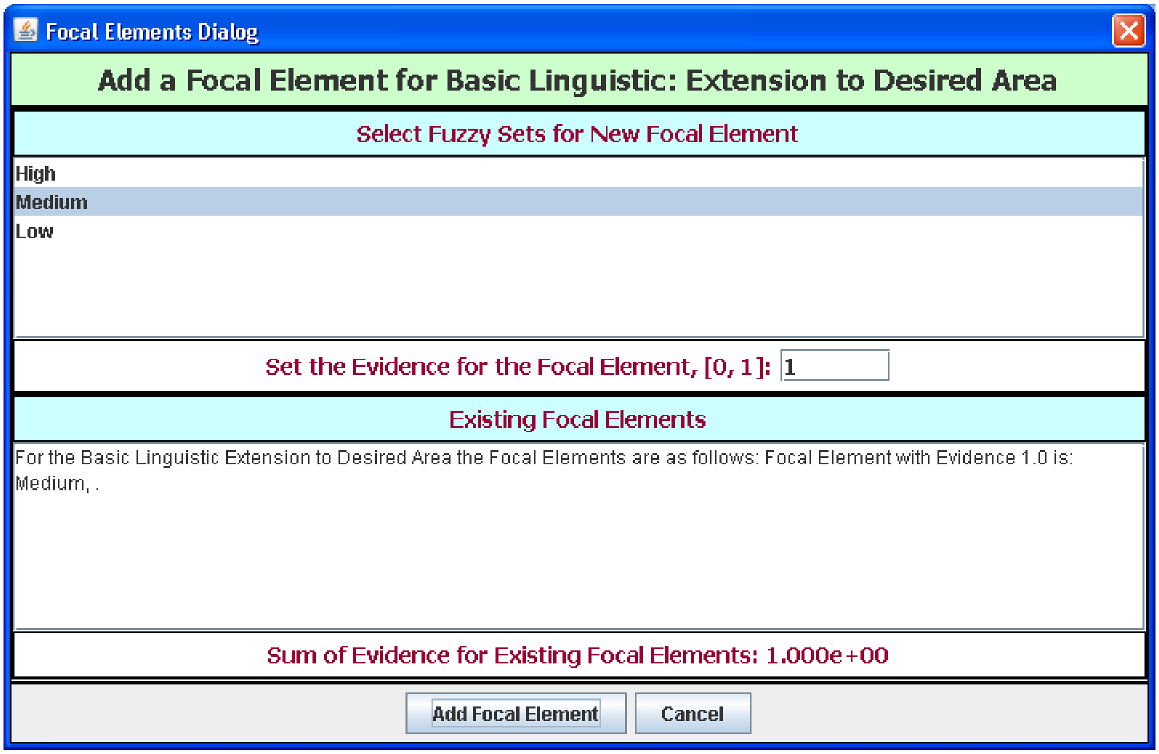

Figure A-11. No Uncertainty for “Extension to Desired Area”

Assume the evidence for “Radionuclide ID” is as indicated in Figure A-12.

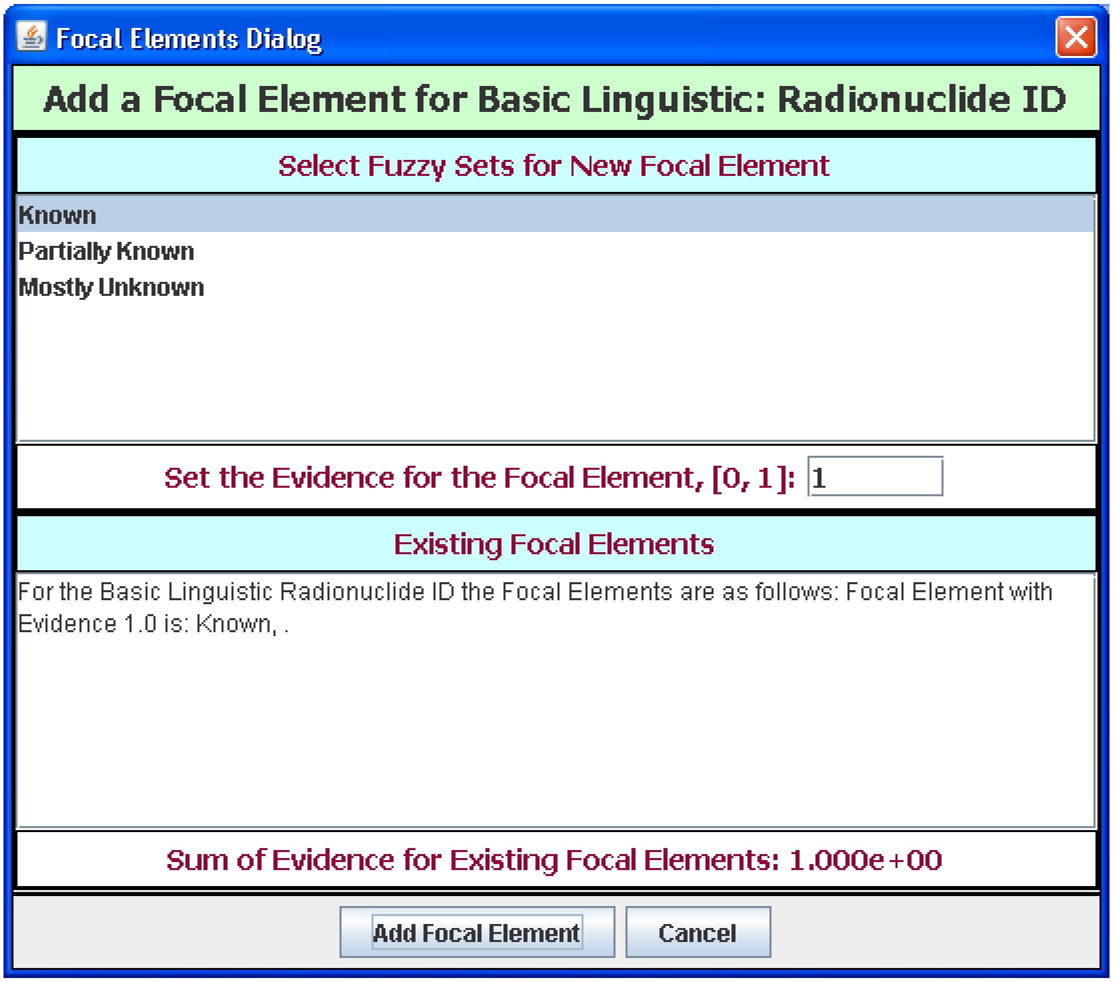

Figure A-12. No Uncertainty for “Radionuclide ID”

Assume the rest of the evidence is as before for Section A.1. The result for "Accuracy Plot for Area" is given in Figure A-13.

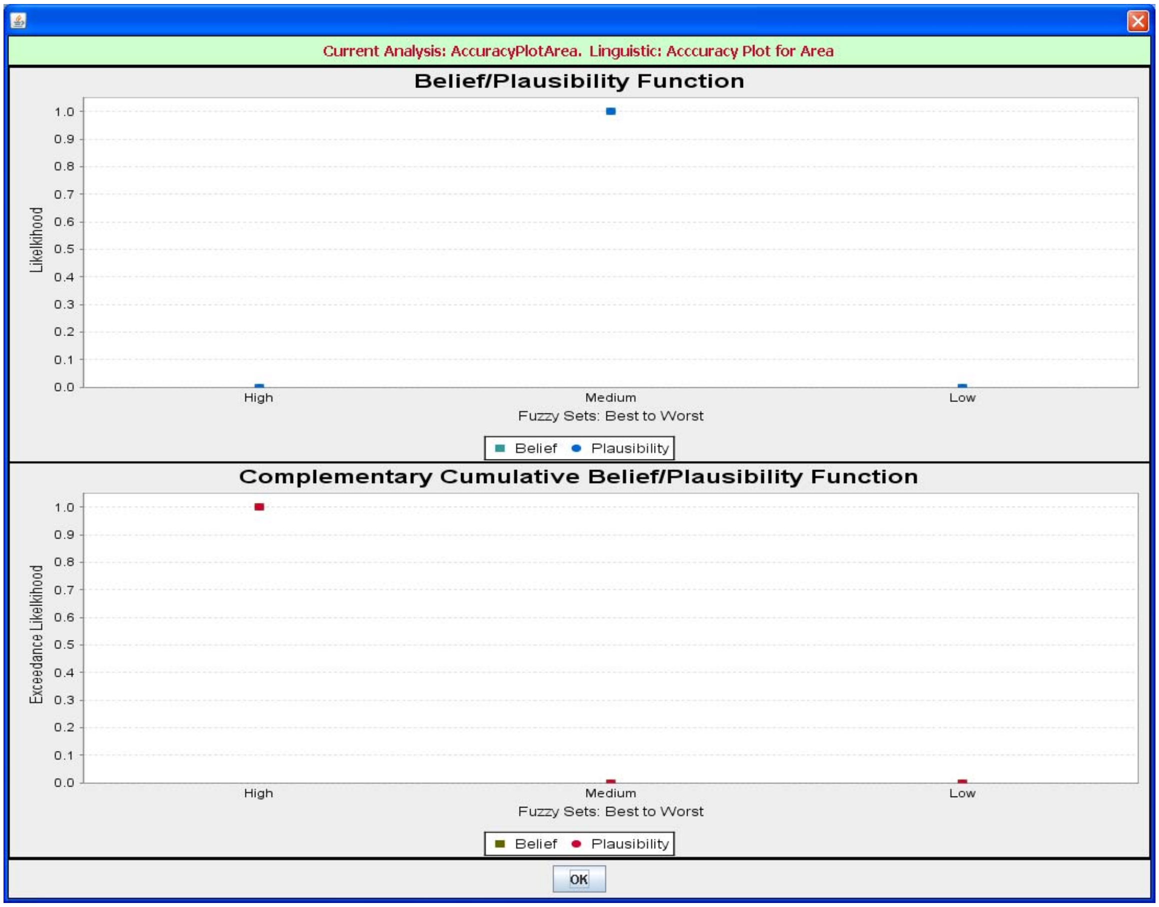

Figure A-13. Results for "Accuracy Plot for Area"

From Figure A-13, the accuracy of the specific map has a likelihood of 1 of "Medium". The likelihood of the accuracy being worse than "High" is 1.0 and the likelihood of the accuracy being worse than "Medium" is 0. This map has no uncertainty in its accuracy: it is "Medium".

For the "Quality Thermometer", this map has belief/plausibility of 1.0/1.0 for "Medium".

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